

Space-filling branes & gaugings

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ABSTRACT: We consider in any dimension the supersymmetric \mathbb{Z}_2 truncations of the maximal supergravity theories. In each dimension and for each truncation we determine all the sets of 1/2-BPS space-filling branes, i.e. branes whose world-volume invades the whole of space-time, that preserve the supersymmetry of the truncated theory and the representations of the symmetry of such theory to which they belong. We show that in any dimension below eight these sets always contain exotic branes, that are objects that do not have a ten-dimensional origin. We repeat the same analysis for half-maximal theories and for the quarter-maximal theories in four and three dimensions. We then discuss all the possible gaugings of these theories as described in terms of the embedding tensor. In general, the truncation acts on the quadratic constraints of the embedding tensor in such a way that some representations survive the truncation although they are not required by the supersymmetry of the truncated theory. We show that for any theory, among these representations, the highest-dimensional ones are precisely those of the 1/2-BPS space-filling branes that preserve the same supersymmetry of the truncated theory, and we interpret this result as the fact that these quadratic constraints after the truncation become tadpole conditions for such branes.

KEYWORDS: Global Symmetries, p-branes, Supergravity Models, String Duality

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1 Introduction

It is well known that the $SO(32)$ type-I string theory in ten dimensions is obtained from the type-IIB theory by performing the orientifold projection [1, 2]. In the closed sector, the projection is due to the O9-plane, while the open sector arises due to the presence of D9-branes [3], and RR and NSNS tadpole cancellations correspond to the fact that the charge and tension of the O9-plane are cancelled by those of the D9-branes. In the low-energy theory, the projection in the closed sector acts as a \mathbb{Z}_2 truncation to $\mathcal{N} = 1$ supergravity, in which the spinors are halved and, among the gauge potentials, the NSNS 2-form B_2 and the RR 4-form C_4 are projected out, while the RR 2-form C_2 survives.

From the point of view of supergravity, there is another consistent supersymmetric \mathbb{Z}_2 truncation, in which all RR fields are projected out, leading to the gravity sector of the heterotic theory. The two truncations are related by S-duality. Denoting with ψ_μ the gravitino of the IIB theory, which is a doublet of Majorana-Weyl spinors of the same

chirality, using the conventions of [4] the gravitino-dependent part of the supersymmetry transformations of B_2 , C_2 and C_4 can be schematically written in the string frame as

$$\begin{aligned}\delta C_{\mu\nu} &= ie^{-\phi}\bar{\epsilon}\gamma_{[\mu}\sigma_1\psi_{\nu]} + \dots \\ \delta B_{\mu\nu} &= i\bar{\epsilon}\gamma_{[\mu}\sigma_3\psi_{\nu]} + \dots \\ \delta C_{\mu\nu\rho\sigma} &= e^{-\phi}\bar{\epsilon}\gamma_{[\mu\nu\rho}\sigma_2\psi_{\sigma]} + \dots \quad ,\end{aligned}\tag{1.1}$$

where the Pauli matrices act on the doublets of spinors. The O9 truncation is then realised in the spinor sector as the projection

$$\text{O9} : \Psi = \pm\sigma_1\Psi\tag{1.2}$$

while the S-dual truncation, which we label SO9, acts as

$$\text{SO9} : \Psi = \pm\sigma_3\Psi \quad ,\tag{1.3}$$

where with Ψ we denote any spinor in the theory [4]. From eq. (1.1) one can see that the truncation $\Psi = \pm\sigma_2\Psi$ projects out both B_2 and C_2 while keeping C_4 , and hence does not lead to a supersymmetric theory.

In the low-energy theory, the occurrence of D9-branes is signalled by the fact that one can consistently introduce a RR 10-form in the supersymmetry algebra, whose transformation contains the Pauli matrix σ_1 consistently with the fact that the field survives the O9 truncation [4]. Analogously, one can consider the S-dual of the RR 10-form potential, and write an effective action for the 1/2-BPS brane that is charged under it [5]. The tension of such brane scales like g_s^{-4} [6, 7], and the presence of the Pauli matrix σ_3 in the supersymmetry variation of the potential signals that it survives the SO9 truncation.¹ The presence of two space-filling 1/2-BPS branes, each of the two surviving each of the two truncations, is also signalled by the presence of the doublet of central charges Z_μ^a , $a = 1, 2$, in the supersymmetry algebra. Indeed, if μ is along the time direction, this can be dualised to $Z_{i_1\dots i_9}^a$, where the i 's are space indices, which is a doublet of 9-brane central charges [6]. For each \mathbb{Z}_2 truncation, the supersymmetry preserved by the 1/2-BPS 9-brane that survives the projection is exactly the supersymmetry of the truncated theory.

The 10-forms that couple to the 9-branes in IIB belong to a quadruplet (i.e. a spin-3/2 representation) of the global symmetry $\text{SL}(2, \mathbb{R})$ of IIB supergravity [8], and are more precisely the spin 3/2 and $-3/2$ components (i.e. the longest weights) of that representation [5, 9]. The same applies to maximal theories in lower dimensions: in any dimension D one can determine the representation of the global symmetry group G to which the RR D -form potentials belong [10, 11], and the space-filling 1/2-BPS branes turn out to correspond to the long weights of that representation [12]. The analysis of [10, 11] was performed by suitably decomposing the very-extended Kac-Moody algebra E_{11} [13], and

¹In [7] it was conjectured that the $\text{SO}(32)$ heterotic theory can be obtained from type-IIB by performing the S-dual of the orientifold projection, and the charge and tension of the S-dual of the O9-plane are cancelled by these branes, that are the S-duals of the D9-branes and are defined as end-points of D-strings. We will not discuss this issue in this paper.

D	G	repr.	branes	R-symmetry	Z_μ	deg.
IIB	$SL(2, \mathbb{R})$	4	2	U(1)	2	1
9	$\mathbb{R}^+ \times SL(2, \mathbb{R})$	4	2	U(1)	2	1
8	$SL(3, \mathbb{R}) \times SL(2, \mathbb{R})$	(15, 1)	6	U(2)	3	2
7	$SL(5, \mathbb{R})$	70	20	USp(4)	5	4
6	$SO(5, 5)$	320 $\overline{126}$	80 16	USp(4) \times USp(4)	(5, 1) + (1, 5) (1, 1)	8 16
5	$E_{6(6)}$	$\overline{1728}$	432	USp(8)	27	16
4	$E_{7(7)}$	8645	2016	SU(8)	63	32
3	$E_{8(8)}$	147250	17280	SO(16)	135	128

Table 1. The 1/2-BPS space-filling branes of the maximal theories in any dimension and their degeneracy [12]. The number of branes is given in the fourth column, while the third column contains the representation of the corresponding D -form potential. The sixth column contains the representation of the central charge and in the last column we list the degeneracy, which is simply the ratio of the number of branes to the dimension of the representation of the central charge. In six dimensions the first line corresponds to branes supporting a vector multiplet, and the second line to branes supporting a tensor multiplet.

we will especially make use of the results of [10], where the representations of the potentials in the lower-dimensional theories were shown to arise from the dimensional reduction of both standard potentials and mixed-symmetry potentials in ten dimensions, that follow from the decomposition of the E_{11} algebra [14].

A crucial result that applies to all the maximal theories in dimension less than ten is the fact that the 1/2-BPS condition for space-filling branes is degenerate, which means that different branes can preserve the same supersymmetry [12].² This degeneracy was determined in [12] by simply observing that the number of 1/2-BPS space-filling branes, that are the long weights of the representation of the D -forms, is always a multiple of the dimension of the R-symmetry group of the vector central charge Z_μ . As in the IIB theory, we can associate to each space-filling brane a \mathbb{Z}_2 truncation to the half-supersymmetric theory, and given that the degenerate branes all preserve the supersymmetry of the same truncation, we arrive at the obvious conclusion that the number of different supersymmetric \mathbb{Z}_2 truncations is precisely the dimension of the representation of the central charge Z_μ . The first result of this paper will be to identify these truncations, and for each truncation to identify the branes that are not projected out, i.e. the branes that preserve the same supersymmetry of the truncated theory. We give in table 1 the number of branes and the corresponding degeneracy in any dimension, as well as the dimension of the vector central charge, which gives the number of different supersymmetric \mathbb{Z}_2 truncations.

We will start considering explicitly the eight-dimensional case.³ We will determine the supersymmetry transformations of all the fields in a manifestly $SL(3, \mathbb{R}) \times SL(2, \mathbb{R})$ -covariant

²The same applies to 1/2-BPS defect branes [15] and domain walls [16] of the maximal theories.

³In $D = 9$ the dimension of the central charge and the degeneracy of the space-filling branes are identical to the IIB case.

notation,⁴ and we will use this to show that there are three different \mathbb{Z}_2 truncations to minimal supergravity coupled to two vector multiplets. The R-symmetry of the theory is $\text{SO}(3) \times \text{SO}(2)$, and we will therefore introduce two sets of Pauli matrices: the matrices σ_i , $i = 1, 2, 3$, generate the $\text{SO}(3)$ Clifford algebra, while τ_a , $a = 1, 2$, which numerically are equal to the first two Pauli matrices, generate the $\text{SO}(2)$ Clifford algebra, and τ_3 , which is the third Pauli matrix, is the $\text{SO}(2)$ chirality matrix. There are three 2-forms in the theory, coming from B_2 , C_2 and the compactified C_4 in IIB, and we will find that the gravitino-dependent part of their supersymmetry transformations can be schematically written in the string frame as

$$\begin{aligned} \delta C_{\mu\nu} &= ie^{-\phi} \bar{\epsilon} \gamma_{[\mu} \sigma_1 \tau_3 \psi_{\nu]} + \dots \\ \delta B_{\mu\nu} &= i \bar{\epsilon} \gamma_{[\mu} \sigma_3 \tau_3 \psi_{\nu]} + \dots \\ \delta C_{\mu\nu x^1 x^2} &= ie^{-\phi} \bar{\epsilon} \gamma_{[\mu} \sigma_2 \tau_3 \psi_{\nu]} + \dots \quad , \end{aligned} \tag{1.4}$$

where x^i ($i = 1, 2$) are the two compact directions. It is easy to identify the three \mathbb{Z}_2 truncations as

$$\begin{aligned} \text{O9} : \quad \Psi &= \pm \sigma_1 \tau_3 \Psi \\ \text{SO9} : \quad \Psi &= \pm \sigma_3 \tau_3 \Psi \\ \text{O7} : \quad \Psi &= \pm \sigma_2 \tau_3 \Psi \quad , \end{aligned} \tag{1.5}$$

and only one 2-form survives each truncation. The 8-forms that couple to the space-filling 7-branes belong to the $(\mathbf{15}, \mathbf{1})$. We will write down the variation of these potentials and show that for each truncation in eq. (1.5) there are two space-filling branes that survive.

The fact that there are two space-filling branes preserved by each truncation is not surprising if one considers in particular the O7 truncation. Indeed, we know that the D7-brane and its S-dual preserve the same supersymmetry [17, 18]. Performing T-dualities in x^1 and x^2 , the D7-brane is mapped to the D9-brane, while the S-dual of the D7-brane is mapped to an *exotic* space-filling brane, i.e. a brane charged with respect to an 8-form potential whose IIB origin is a mixed-symmetry potential. These branes survive the O9 truncation in eight dimensions. Similarly, by S-duality one obtains the branes that survive the SO9 truncation. All these arguments can then be repeated in all lower dimensions, and by multiple T and S-duality transformations one obtains all the different truncations and all the branes that preserve the same supersymmetry of each truncation. Most of these branes are exotic, and we identify them with the corresponding components of the mixed-symmetry potential using the universal T-duality rules derived in [19].

The \mathbb{Z}_2 truncation of the maximal theory in D dimensions gives half-maximal supergravity coupled to $d = 10 - D$ vector multiplets. This theory has global symmetry $\mathbb{R}^+ \times \text{SO}(d, d)$ in dimension higher than four, $\text{SL}(2, \mathbb{R}) \times \text{SO}(6, 6)$ in four dimensions and $\text{SO}(8, 8)$ in three dimensions, and we identify in all dimensions the irreducible representations of these groups that contain the branes preserving the same supersymmetry of the truncated theory. The particular truncation such that \mathbb{R}^+ is identified with the string dilaton scaling (and therefore $\text{SO}(d, d)$ is T-duality) is always the SO9 truncation.

⁴As far as we know, this result was not available in the literature.

D	G	repr.	branes	R-symmetry	Z_μ	deg.
6A	$\mathbb{R}^+ \times \text{SO}(4, 4)$	35_V	8	$\text{USp}(2) \times \text{USp}(2)$	(1, 1)	8
		35_S	8			8
		35_C	8			8
6B	$\text{SO}(5, 5)$	320	80	$\text{USp}(4)$	5	16
5	$\mathbb{R}^+ \times \text{SO}(5, 5)$	320	80	$\text{USp}(4)$	5	16
		210	80			16
4	$\text{SL}(2, \mathbb{R}) \times \text{SO}(6, 6)$	(3, 495)	480	$\text{U}(4)$	15	32
		(1, 2079)	480			32
3	$\text{SO}(8, 8)$	60060	8960	$\text{SO}(8)$	35	256

Table 2. The number and the degeneracy of the 1/2-BPS space-filling branes of the half-maximal theories which arise as \mathbb{Z}_2 truncations of the maximal ones [20]. In the 6A theory, 8 of the branes support tensor multiplets and the remaining 16 support hypermultiplets. In the 6B theory the branes support vector multiplets. In five and four dimensions half of the branes support vector multiplets and the other half support hypermultiplets [20].

Starting from six dimensions, apart from the branes that preserve the same supersymmetry of the truncation, there are additional space-filling branes surviving the truncation which are 1/2-BPS states of the truncated theory. As in the maximal case, one can determine the vector central charge Z_μ as a representation of the R-symmetry of the half-maximal theory, and relate it to the number of space-filling branes to determine their degeneracy [20]. We list in table 2 the number of 1/2-BPS space-filling branes, the central charge and the degeneracy for the truncated theories. In the table we denote with 6A the $\mathcal{N} = (1, 1)$ theory and with 6B the $\mathcal{N} = (2, 0)$ theory, and the latter case corresponds to IIB compactified on T^4/\mathbb{Z}_2 , so that the truncation is geometric. Exactly as in the maximal theory, the number of vector central charges gives the number of \mathbb{Z}_2 truncations to quarter-maximal theories, and the degeneracy gives the number of space-filling branes that preserve the same supersymmetry of the truncation. We will be able to show that in all cases the branes that preserve the same supersymmetry of a given truncation of the half-maximal theory are the union of two different sets of degenerate branes of the maximal theory.

The analysis can be further extended to consider the \mathbb{Z}_2 truncation of the quarter-maximal theories. Indeed, starting from four dimensions, apart from the branes of the half-maximal theories that preserve the same supersymmetry of the truncation, there are additional space-filling branes surviving the truncation which are 1/2-BPS states of the truncated quarter-maximal theory. In [21] the number of space-filling branes of the quarter-maximal theories in four and three dimensions was determined and then compared to the number of vector central charges to obtain the degeneracy. We list the results in table 3. Again, the number of vector central charges gives the number of \mathbb{Z}_2 truncations to theories with four supercharges, i.e. $\mathcal{N} = 1$ in four dimensions, and the degeneracy gives the number of space-filling branes that preserve the same supersymmetry of the truncation. We will show that the branes that preserve the same supersymmetry of a given truncation of the quarter-maximal theory are the union of four different sets of degenerate branes of the maximal theory.

D	G	repr.	branes	R-symmetry	Z_μ	deg.
4	$\text{SL}(2, \mathbb{R})^3 \times \text{SO}(4, 4)$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{350})$	96	U(2)	$\mathbf{3}$	32
		$(\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{28})$	96			32
		$(\mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{28})$	96			32
		$(\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{28})$	96			32
3	$\text{SO}(4, 4) \times \text{SO}(4, 4)$	$(\mathbf{28}, \mathbf{350})$	2304	$\text{SU}(2) \times \text{SU}(2)$	$(\mathbf{3}, \mathbf{3})$	256
		$(\mathbf{350}, \mathbf{28})$	2304			256

Table 3. The number and the degeneracy of 1/2-BPS space-filling branes of the quarter-maximal theories resulting from \mathbb{Z}_2 truncations [21] (see also tables 7 and 8 of [22]).

The truncation of the maximal theory to the half-maximal one can also be performed in the presence of gaugings. In particular, the truncation of $\mathcal{N} = 8$ gauged supergravity to $\mathcal{N} = 4$ gauged supergravity coupled to six vector multiplets was studied in [23] using the embedding tensor formalism [24–27]. Decomposing the embedding tensor of the maximal theory [28] under $\text{SL}(2, \mathbb{R}) \times \text{SO}(6, 6)$ and projecting out the representations that are odd under \mathbb{Z}_2 , one is left with the embedding tensor of the half-maximal theory [29]. On the other hand, by projecting out the representations of the quadratic constraints that are odd under \mathbb{Z}_2 , one is left with more than the quadratic constraints of the half-maximal theory. Among the representations of the quadratic constraints that survive the \mathbb{Z}_2 truncation but are not required by supersymmetry, the highest-dimensional one contains space-filling branes that preserve the same supersymmetry of the \mathbb{Z}_2 truncation. The fact that this quadratic constraint is not required in $\mathcal{N} = 4$ although it is not projected out has therefore the natural interpretation that it becomes a tadpole condition for the corresponding brane [23].

Using the results of the first part of this paper, we will generalise this to any maximal theory. All the space-filling branes that preserve the same supersymmetry of the \mathbb{Z}_2 truncation belong to the representation of the symmetry of the half-maximal theory which is the highest-dimensional representation of the quadratic constraint which survives the truncation but is not required by the supersymmetry of the truncated theory. We will also show that exactly the same applies for the truncation from the half-maximal to the quarter-maximal theories, using the quadratic constraints of the embedding tensor of $\mathcal{N} = 2$ theories discussed in [27]. The truncation of the four-dimensional $\mathcal{N} = 2$ theory whose symmetry appears in table 3 gives the $\mathcal{N} = 1$ theory with $\text{SL}(2, \mathbb{R})^7$ global symmetry. Minimal supersymmetry does not require any quadratic constraint for the embedding tensor, and consistently we find that all the highest-dimensional representations of the quadratic constraints of the $\mathcal{N} = 2$ theory that survive the \mathbb{Z}_2 truncation coincide with the representations of the space-filling branes which preserve the same supersymmetry of the truncation. To obtain this result, we will use the analysis of [30], where the space-filling branes of the $\text{SL}(2, \mathbb{R})^7$ $\mathcal{N} = 1$ model that arises from the IIB O3/O7 $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold were derived.

Finally, we will discuss the truncations of the gauged theories with lower supersymmetry from the point of view of the maximal theories. Considering again the IIB O3/O7

$T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, the embedding tensor of the four-dimensional theory arises from geometric and non-geometric IIB fluxes. These fluxes satisfy Bianchi identities, and we will show that these Bianchi identities are in the same representations as the space-filling branes that preserve the same supersymmetry of the orbifold. Again, the $SL(2, \mathbb{R})^7$ analysis performed in [30] will be crucial to get this result. The result also applies to $T^4/\mathbb{Z}_2 \times T^n$ orientifolds.

The plan of the paper is as follows. In section 2 we derive the supersymmetry transformations of the fields of maximal supergravity in a manifestly $SL(3, \mathbb{R}) \times SL(2, \mathbb{R})$ -covariant notation, and we use this to derive the three independent \mathbb{Z}_2 truncations to the half-maximal theory coupled to two vector multiplets. We determine the space-filling branes that for each truncation preserve the same supersymmetry of the truncated theory. In section 3 we generalise this result to any dimension and any supersymmetry. In section 4 we discuss gauged supergravities, and we show that in general the highest-dimensional representations of the quadratic constraint that survive the \mathbb{Z}_2 truncation but are not required by the supersymmetry of the truncated theory precisely coincide with the representations containing the space-filling branes that preserve the same supersymmetry of the truncation. This is also done for the truncation of theories with lower supersymmetry. In section 5 we discuss the particular case of the IIB O3/O7 $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, and we show that the Bianchi identities are in the representations of the space-filling branes that preserve the same supersymmetry of the orbifold truncation. Finally, section 6 contains our conclusions. The paper also contains an appendix, in which the details of the $D = 8$ notations and conventions used in section 2 are explained.

2 $D = 8$ supergravity and its truncations

The $SU(2)$ gauged maximal $D = 8$ supergravity was originally constructed in [31] by dimensional reduction from eleven dimensions on an $SU(2)$ group manifold. This was later generalised in [32] to include more general gaugings. The supersymmetry transformations in the ungauged case can be recovered from these papers, but they are not suitable for our purposes, because we need them in a formulation which is manifestly covariant under $SL(3, \mathbb{R}) \times SL(2, \mathbb{R})$. In the first subsection we will derive these transformations imposing the closure of the supersymmetry algebra, and in particular we will write down the gravitino-dependent part of the supersymmetry transformation of the 8-form potentials. In the second subsection we will show that the theory admits three different \mathbb{Z}_2 truncations to the half-maximal theory coupled to two vector multiplets, and by considering the action of these projections on the 8-forms we will determine the space-filling branes that are not projected out in each truncation.

2.1 Supersymmetry algebra

We first introduce the notation. We work with a mostly-minus space-time signature, and we denote the curved space-time indices with Greek letters μ, ν, \dots , while the tangent-space indices are α, β, \dots . We denote with upstairs indices $M = 1, 2, 3$ and $A = 1, 2$ the fundamentals of $SL(3, \mathbb{R})$ and $SL(2, \mathbb{R})$, and with $m = 1, 2, 3$ and $a = 1, 2$ the vector

indices of their maximal compact subgroups $SO(3)$ and $SO(2)$.⁵ The seven scalars in the theory parametrise the coset-space $SL(3, \mathbb{R})/SO(3) \otimes SL(2, \mathbb{R})/SO(2)$. We describe them introducing the matrices L_M^m and V_A^a , together with the inverse matrices \tilde{L}_m^M and \tilde{V}_a^A , satisfying the identities

$$\begin{aligned} L_M^m \tilde{L}^{Nn} \delta_{mn} &= \delta_M^N & L_M^m \tilde{L}^{Mn} &= \delta^{mn} & L_M^m L_N^n L_P^p \epsilon_{mnp} &= \epsilon_{MNP} \\ V_A^a \tilde{V}^{Bb} \delta_{ab} &= \delta_A^B & V_A^a \tilde{V}^{Ab} &= \delta^{ab} & V_A^a V_B^b \epsilon_{ab} &= \epsilon_{AB} \end{aligned} \quad (2.1)$$

We define the Maurer-Cartan forms as

$$\begin{aligned} \tilde{L}_m^M \partial_\mu L_{Mn} &= Q_{\mu mn} + P_{\mu mn} \\ \tilde{V}_a^A \partial_\mu V_{Ab} &= Q_{\mu ab} + P_{\mu ab} \end{aligned} \quad (2.2)$$

where the $SO(3)$ and $SO(2)$ connections $Q_{\mu mn}$ and $Q_{\mu ab}$ are antisymmetric while $P_{\mu mn}$ and $P_{\mu ab}$ are symmetric and traceless. The other bosonic fields are the vielbein e_μ^α , the 1-form $A_{\mu MA}$ in the $(\mathbf{3}, \mathbf{2})$, the 2-form $A_{\mu\nu}^M$ in the $(\mathbf{3}, \mathbf{1})$ and the 3-forms $A_{\mu\nu\rho}^A$ in the $(\mathbf{1}, \mathbf{2})$. The field-strengths of the 3-forms satisfy a self-duality condition.

We now move to discuss the fermionic sector. The eight-dimensional chirality matrix γ_9 is defined in terms of the gamma matrices γ_μ as

$$\gamma_9 = -\frac{i}{8!} \epsilon_{\mu_1 \dots \mu_8} \gamma^{\mu_1 \dots \mu_8} \quad (2.3)$$

We also introduce the Pauli matrices σ_m which act on $SO(3)$ spinor indices. Similarly, we introduce the matrices τ_a acting on the spinor indices of $SO(2)$. Numerically, τ_1 and τ_2 coincide with σ_1 and σ_2 . We will also need the $SO(2)$ chirality matrix τ_3 , which coincides numerically with σ_3 . The eight-dimensional fermions are the gravitino ψ_μ and the spinors χ_m and χ_a , while we denote with ϵ the supersymmetry parameter. They all have also spinor indices of $SO(3) \times SO(2)$, and satisfy a chirality condition with respect to $\gamma_9 \tau_3$. In particular

$$\gamma_9 \tau_3 \psi_\mu = \psi_\mu \quad \gamma_9 \tau_3 \chi_m = -\chi_m \quad \gamma_9 \tau_3 \chi_a = \chi_a \quad \gamma_9 \tau_3 \epsilon = \epsilon \quad (2.4)$$

and thus χ_m has opposite ‘chirality’ with respect to all the other fermions. All the fermions also satisfy the ‘symplectic’ Majorana condition

$$\Psi = C \bar{\Psi}^T \quad (2.5)$$

where C is defined as

$$C = C_8 \sigma_2 \tau_1 \quad (2.6)$$

and the eight-dimensional Majorana matrix C_8 is symmetric and satisfies

$$C_8^\dagger \gamma_\mu C_8 = -\gamma_\mu^T \quad (2.7)$$

⁵Repeated m and a indices, regardless of whether they are up or down, are meant to be contracted by δ^{mn} and δ^{ab} .

It can be shown that the symplectic Majorana condition of eq. (2.5) is compatible with the chirality conditions defined in eq. (2.4). Finally, the spinors χ_m and χ_a also satisfy the irreducibility conditions

$$\sigma_m \chi_m = \tau_a \chi_a = 0 \quad . \quad (2.8)$$

The number of on-shell degrees of freedom that these fermions propagate match those of the bosons. We discuss in more detail the fermionic sector in appendix A, where we also derive the properties of the various bilinears under Majorana flip.

The way we proceed to derive the supersymmetry transformations of the fields is by imposing that the supersymmetry algebra closes. We first write down the final outcome of our analysis, and then we discuss in more detail how the algebra closes on the various fields. The supersymmetry transformations of the fermionic fields are

$$\begin{aligned} \delta\psi_\mu &= D_\mu \epsilon - \frac{1}{48} F_{MA}^{\nu\rho} \tilde{L}_m^M \tilde{V}_a^A \gamma_{\mu\nu\rho} \sigma_m \tau_a \epsilon + \frac{5}{24} F_{\mu\nu MA} \tilde{L}_m^M \tilde{V}_a^A \gamma^\nu \sigma_m \tau_a \epsilon \\ &\quad - \frac{1}{36} F^{\nu\rho\sigma M} L_{Mm} \gamma_{\mu\nu\rho\sigma} \sigma_m \tau_3 \epsilon + \frac{1}{6} F_{\mu\nu\rho}^M L_{Mm} \gamma^{\nu\rho} \sigma_m \tau_3 \epsilon \\ &\quad - \frac{i}{16} F_{\mu\nu\rho\sigma}^A V_{Aa} \gamma^{\nu\rho\sigma} \tau_a \epsilon \\ \delta\chi_m &= -\frac{i}{2} P_{\mu mn} \gamma^\mu \sigma_n \tau_3 \epsilon + \frac{i}{12} F_{\mu\nu MA} \tilde{L}_m^M \tilde{V}_a^A \gamma^{\mu\nu} \tau_a \tau_3 \epsilon + \frac{1}{24} F_{\mu\nu MA} \tilde{L}_n^M \tilde{V}_a^A \epsilon_{mnp} \gamma^{\mu\nu} \sigma_p \tau_a \tau_3 \epsilon \\ &\quad + \frac{i}{18} F_{\mu\nu\rho}^M L_{Mm} \gamma^{\mu\nu\rho} \sigma_m \epsilon + \frac{1}{36} F_{\mu\nu\rho}^M L_{Mn} \epsilon_{mnp} \gamma^{\mu\nu\rho} \sigma_p \epsilon \\ \delta\chi_a &= -\frac{i}{2} P_{\mu ab} \gamma^\mu \tau_b \epsilon - \frac{i}{16} F_{\mu\nu MA} \tilde{L}_m^M \tilde{V}_a^A \gamma^{\mu\nu} \sigma_m \epsilon - \frac{1}{16} F_{\mu\nu MA} \tilde{L}_m^M \tilde{V}_b^A \epsilon_{ab} \gamma^{\mu\nu} \sigma_m \tau_3 \epsilon \\ &\quad + \frac{1}{64} F_{\mu\nu\rho\sigma}^A V_{Aa} \gamma^{\mu\nu\rho\sigma} \epsilon \quad . \end{aligned} \quad (2.9)$$

In the transformation of the gravitino, the derivative D_μ is covariant with respect to local Lorentz, local SO(3) and local SO(2), that is

$$D_\mu \epsilon = \partial_\mu \epsilon + \frac{1}{4} \omega_{\mu\alpha\beta} \gamma^{\alpha\beta} \epsilon + \frac{i}{4} Q_{\mu mn} \epsilon_{mnp} \sigma_p \epsilon + \frac{i}{4} Q_{\mu ab} \epsilon_{ab} \tau_3 \epsilon \quad . \quad (2.10)$$

The field-strengths $F_{\mu\nu MA}$, $F_{\mu\nu\rho}^M$ and $F_{\mu\nu\rho\sigma}^A$ are defined as

$$\begin{aligned} F_{\mu\nu MA} &= 2\partial_{[\mu} A_{\nu] MA} \\ F_{\mu\nu\rho}^M &= 3\partial_{[\mu} A_{\nu\rho]}^M + \frac{3}{8} \epsilon^{MNP} \epsilon^{AB} A_{[\mu NA} F_{\nu\rho] PB} \\ F_{\mu\nu\rho\sigma}^A &= 4\partial_{[\mu} A_{\nu\rho\sigma]}^A - \frac{8}{9} \epsilon^{AB} A_{[\mu MB} F_{\nu\rho\sigma]}^M - \frac{8}{9} \epsilon^{AB} A_{[\mu\nu}^M F_{\rho\sigma] MB} \end{aligned} \quad (2.11)$$

and they are invariant with respect to the gauge transformations

$$\begin{aligned} \delta A_{\mu MA} &= \partial_\mu \Lambda_{MA} \\ \delta A_{\mu\nu}^M &= 2\partial_{[\mu} \Sigma_{\nu]}^M - \frac{1}{8} \epsilon^{MNP} \epsilon^{AB} \Lambda_{NA} F_{\mu\nu PB} \\ \delta A_{\mu\nu\rho}^A &= 3\partial_{[\mu} \Xi_{\nu\rho]}^A + \frac{2}{9} \epsilon^{AB} \Lambda_{MB} F_{\mu\nu\rho}^M + \frac{4}{3} \epsilon^{AB} \Sigma_{[\mu}^M F_{\nu\rho] MB} \quad . \end{aligned} \quad (2.12)$$

The supersymmetry transformations of the bosons are

$$\begin{aligned}
\delta e_\mu^\alpha &= -i\bar{\epsilon}\gamma^\alpha\psi_\mu \\
\delta L_{Mm} &= L_{Mn}\bar{\epsilon}\sigma_n\tau_3\chi_m \\
\delta V_{Aa} &= V_{Ab}\bar{\epsilon}\tau_b\chi_a \\
\delta A_{\mu MA} &= L_{Mm}V_{Aa}(i\bar{\epsilon}\sigma_m\tau_a\psi_\mu - \bar{\epsilon}\gamma_\mu\sigma_m\chi_a - \bar{\epsilon}\gamma_\mu\tau_a\tau_3\chi_m) \\
\delta A_{\mu\nu}^M &= \tilde{L}_m^M\left(i\bar{\epsilon}\gamma_{[\mu}\sigma_m\tau_3\psi_{\nu]} + \frac{1}{2}\bar{\epsilon}\gamma_{\mu\nu}\chi_m\right) + \frac{1}{4}\epsilon^{MNP}\epsilon^{AB}A_{[\mu NA}\delta A_{\nu]PB} \\
\delta A_{\mu\nu\rho}^A &= \tilde{V}_a^A\left(\bar{\epsilon}\gamma_{[\mu\nu}\tau_a\psi_{\rho]} - \frac{i}{3}\bar{\epsilon}\gamma_{\mu\nu\rho}\chi_a\right) - \frac{2}{3}\epsilon^{AB}A_{[\mu MB}\delta A_{\nu\rho]}^M \\
&\quad + \frac{4}{3}\epsilon^{AB}A_{[\mu\nu}^M\delta A_{\rho]}MB + \frac{1}{6}\epsilon^{AB}\epsilon^{CD}\epsilon^{MNP}A_{[\mu MB}A_{\nu NC}\delta A_{\rho]PD} \quad . \quad (2.13)
\end{aligned}$$

We now discuss in some detail how the analysis of the closure of the supersymmetry algebra was performed. We have computed the commutator of two supersymmetry transformations of parameters ϵ_2 and ϵ_1 on the bosonic fields, and we have imposed that this closes on all the local symmetries of the theory. In particular, on the vielbein one obtains

$$[\delta_1, \delta_2]e_\mu^\alpha = \partial_\mu\xi^\nu e_\nu^\alpha + \xi^\nu\partial_\nu e_\mu^\alpha + \Lambda^{\alpha\beta}e_{\mu\beta} \quad , \quad (2.14)$$

where the general coordinate transformation parameter is

$$\xi_\mu = -i\bar{\epsilon}_2\gamma_\mu\epsilon_1 \quad (2.15)$$

and the local Lorentz parameter is

$$\begin{aligned}
\Lambda_{\alpha\beta} &= \xi^\nu\omega_{\nu\alpha\beta} + \tilde{L}_m^M\tilde{V}_a^A\left(\frac{i}{24}F_{MA}^{\mu\nu}\bar{\epsilon}_2\gamma_{\alpha\beta\mu\nu}\sigma_m\tau_a\epsilon_1 + \frac{5i}{12}F_{\alpha\beta MA}\bar{\epsilon}_2\sigma_m\tau_a\epsilon_1\right) \\
&\quad + L_{Mm}\left(\frac{i}{18}F^{\mu\nu\rho M}\bar{\epsilon}_2\gamma_{\alpha\beta\mu\nu\rho}\sigma_m\tau_3\epsilon_1 + \frac{2i}{3}F_{\alpha\beta\mu}\bar{\epsilon}_2\gamma^\mu\sigma_m\tau_3\epsilon_1\right) \\
&\quad + \frac{3}{8}F_{\alpha\beta\mu\nu}^AV_{Aa}\bar{\epsilon}_2\gamma^{\mu\nu}\tau_a\epsilon_1 \quad . \quad (2.16)
\end{aligned}$$

All the other fields also transform correctly under general coordinate transformations. One can show that on top of this, on the scalars one produces local $SO(3)$ and $SO(2)$ transformations. To prove that the supersymmetry algebra closes on the vector $A_{\mu MA}$ one needs the identities

$$\begin{aligned}
D_\mu L_{Mm} &= \partial_\mu L_{Mm} + Q_{\mu mn}L_{Mn} = P_{\mu mn}L_{Mn} & D_\mu \tilde{L}_m^M &= -P_{\mu mn}\tilde{L}_n^M \\
D_\mu V_{Aa} &= \partial_\mu V_{Aa} + Q_{\mu ab}V_{Ab} = P_{\mu ab}V_{Ab} & D_\mu \tilde{V}_a^A &= -P_{\mu ab}\tilde{V}_b^A
\end{aligned} \quad (2.17)$$

which follow from the definitions given in eq. (2.2). The final result is that the algebra produces a gauge transformation of parameter

$$\Lambda_{MA} = \Lambda_{MA}^{\text{susy}} - \xi^\mu A_{\mu MA} \quad , \quad (2.18)$$

where

$$\Lambda_{MA}^{\text{susy}} = i\tilde{L}_{Mm}V_{Aa}\bar{\epsilon}_2\sigma_m\tau_a\epsilon_1 \quad . \quad (2.19)$$

The gauge transformation parameter of the 2-forms is

$$\Sigma_\mu^M = \Sigma_\mu^{\text{susy}M} - \xi^\nu A_{\nu\mu}^M - \frac{1}{8}\epsilon^{MNP}\epsilon^{AB}A_{\mu NA}\Lambda_{PB}^{\text{susy}} \quad , \quad (2.20)$$

where

$$\Sigma_\mu^{\text{susy}M} = -\frac{i}{2}\tilde{L}_m^M\bar{\epsilon}_2\gamma_\mu\sigma_m\tau_3\epsilon_1 \quad . \quad (2.21)$$

Finally, the gauge parameter of the 3-forms is

$$\Xi_{\mu\nu}^A = \Xi_{\mu\nu}^{\text{susy}A} - \xi^\rho A_{\rho\mu\nu}^A + \frac{4}{9}\epsilon^{AB}A_{[\mu MB}\Sigma_{\nu]}^{\text{susy}M} + \frac{4}{9}\epsilon^{AB}A_{\mu\nu}^M\Lambda_{MB}^{\text{susy}} \quad , \quad (2.22)$$

where

$$\Xi_{\mu\nu}^{\text{susy}A} = \frac{1}{3}\tilde{V}_a^A\bar{\epsilon}_2\gamma_{\mu\nu}\tau_a\epsilon_1 \quad . \quad (2.23)$$

A crucial ingredient to prove the closure of the supersymmetry algebra on the 3-form doublet is the self-duality relation

$$F_{\mu_1\dots\mu_4}^A V_{Aa} = -\frac{1}{4!}\epsilon_{\mu_1\dots\mu_4\nu_1\dots\nu_4}\epsilon_{ab}F^{\nu_1\dots\nu_4 A}V_{Ab} \quad . \quad (2.24)$$

We refer to appendix A for more details on the self-duality properties in eight dimensions.

Following [8], one can proceed and derive the supersymmetry transformations of the higher-rank forms by imposing the closure of the supersymmetry algebra, provided that the first-order duality conditions are imposed. In particular, the algebra closes on the 4-forms A_{4M} in the $(\bar{\mathbf{3}}, \mathbf{1})$ that are dual to the 2-forms, on the 5-forms A_5^{MA} in the $(\mathbf{3}, \mathbf{2})$ that are dual to the 1-forms, and on the 6-forms A_{6M}^N in the $(\mathbf{8}, \mathbf{1})$ and A_{6AB} in the $(\mathbf{1}, \mathbf{3})$, that are dual to the scalars. Moreover, the algebra closes on the non-propagating 7-forms in the $(\mathbf{6}, \mathbf{2}) \oplus (\bar{\mathbf{3}}, \mathbf{2})$ and 8-forms in the $(\mathbf{15}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{1})$ [10, 11]. In particular, we are interested in the highest-dimensional representation of the 8-forms, which is the $(\mathbf{15}, \mathbf{1})$. Indeed, in general the p -branes of the maximal theory are associated to the long weights of the highest-dimensional representation of the $(p+1)$ -forms [12, 33]. The $\mathbf{15}$ is the irreducible representation with two symmetric indices up and one down. To determine how the field $A_8^{MN}{}_P$ behaves with respect to the different \mathbb{Z}_2 truncations, we only need the gravitino-dependent part of its supersymmetry transformation, which is

$$\delta A_{\mu_1\dots\mu_8}{}^{MN}{}_P = \tilde{L}_m^M\tilde{L}_m^N L_{Pn}\bar{\epsilon}\gamma_{[\mu_1\dots\mu_7}\sigma_n\psi_{\mu_8]} + \dots \quad . \quad (2.25)$$

The 1/2-BPS space-filling branes correspond to the components of the 8-form potential that satisfy the highest-weight constraint. These in general are all the long weights of the representation, which in the case of the $\mathbf{15}$ of $\text{SL}(3, \mathbb{R})$ are the six components of $A_8^{MN}{}_P$ such that $M = N$ and $M \neq P$ [12]. In the next subsection we will determine the three different \mathbb{Z}_2 truncations to the half-maximal theory coupled to two vector multiplets, and we will show that for each truncation there are two space-filling branes that survive the projection. These are the branes that preserve the supersymmetry of the truncated theory.

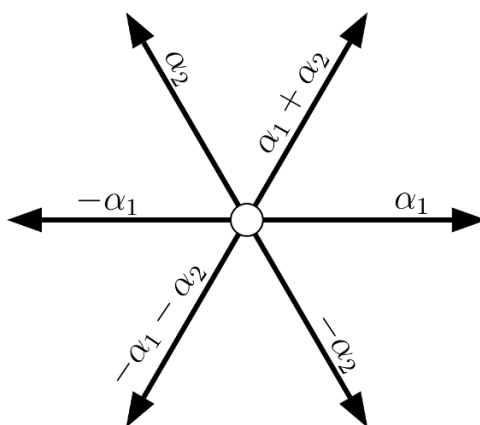


Figure 1. The roots of $SL(3, \mathbb{R})$.

2.2 \mathbb{Z}_2 truncations to half-supersymmetry and space-filling branes

The maximal theory in $D = 8$ can be truncated to half-maximal supergravity coupled to two vector multiplets. The resulting theory has global continuous symmetry $\mathbb{R}^+ \times SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$, and therefore there are three independent truncations because there are three different ways of embedding $\mathbb{R}^+ \times SL(2, \mathbb{R})$ inside $SL(3, \mathbb{R})$. The three different embeddings can be easily visualised by looking at the root diagram of $SL(3, \mathbb{R})$, which we draw in figure 1. Each of the three $SL(2, \mathbb{R})$'s are generated by one positive root, the corresponding negative root and the corresponding Cartan generator.

We first discuss the scalar sector. The scalars V_A^a are obviously not projected out because the truncation does not act on the $SL(2, \mathbb{R})$ factor of the maximal theory. The index M in the fundamental of $SL(3, \mathbb{R})$ splits as $M = (\sharp, \dot{A})$, where $\dot{A} = 1, 2$ is the index of the fundamental of the $SL(2, \mathbb{R})$ inside $SL(3, \mathbb{R})$, and similarly the $SO(3)$ index m splits as $m = (\sharp, \dot{a})$, where \dot{a} is the vector index of the maximal compact subgroup $SO(2)$ of $SL(2, \mathbb{R})$. The scalars L_{Mm} are truncated to

$$L_{Mm} \rightarrow (e^\Phi, e^{-\Phi/2} V_{\dot{A}\dot{a}}^a) \quad , \quad (2.26)$$

where the dilaton Φ parametrises \mathbb{R}^+ and the matrix $V_{\dot{A}\dot{a}}^a$ satisfies the same identities as V_{Aa} given in eq. (2.1).

We then derive how the truncation acts on the fermions. The gravity multiplet of the truncated theory contains a Majorana gravitino and a Majorana spinor, while each vector multiplet contains a single Majorana spinor. We find that the truncation (up to an overall sign) is

$$\begin{aligned} \psi_\mu &= \sigma_\sharp \tau_3 \psi_\mu \\ \chi_\sharp &= \sigma_\sharp \tau_3 \chi_\sharp \\ \chi_{\dot{a}} &= -\sigma_\sharp \tau_3 \chi_{\dot{a}} \\ \chi_a &= -\sigma_\sharp \tau_3 \chi_a \quad . \end{aligned} \quad (2.27)$$

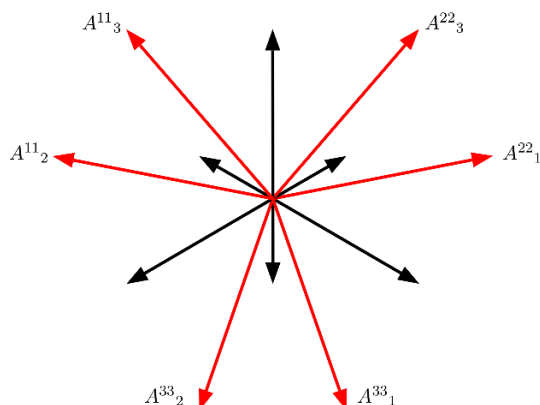


Figure 2. The $\mathbf{15}$ of $SL(3, \mathbb{R})$ to which the 8-forms belong. The longest weights are painted in red, and for each longest weight we have written the corresponding component of the potential. For simplicity the space-time indices are omitted. The shortest weights have multiplicity two.

The supersymmetry parameter ϵ is truncated like the gravitino. It can be checked that the constraints of eq. (2.8) and the Majorana condition of eq. (2.5) are consistent with the truncation and this implies that one ends up with the correct number of fermions.⁶ The chirality condition of eq. (2.4) on the truncated fermions gives two spinors of opposite chirality that can be recast in a single Dirac spinor Ψ satisfying the standard $D = 8$ Majorana condition $\Psi = \tilde{C}_8 \bar{\Psi}^T$, where $\tilde{C}_8 = C_8 \gamma_9$ satisfies the condition

$$\tilde{C}_8^\dagger \gamma_\mu \tilde{C}_8 = \gamma_\mu^T \quad , \quad (2.28)$$

which has opposite sign with respect to eq. (2.7).

We can now figure out how the truncation acts on the supersymmetry algebra. First of all, it is straightforward to check that the truncation on the scalars and the one on the fermions are consistent. On the 1-forms, the fermionic truncation is consistent with keeping only the components $A_{\mu \dot{A}A}$, because the supersymmetry variation of $A_{\mu \sharp A}$ is identically zero. Similarly, for the 2-form only the singlet component survives because the variation of $A_{\mu\nu}^{\dot{A}}$ vanishes identically. Finally, the 3-form is fully projected out. The variation of the 1-forms and 2-form that survive the projection is

$$\begin{aligned} \delta A_{\mu \dot{A}A} &= e^{-\Phi/2} V_{\dot{A}\dot{a}} V_{Aa} (i\bar{\epsilon}\sigma_{\dot{a}}\tau_a\psi_\mu - \bar{\epsilon}\gamma_\mu\sigma_{\dot{a}}\chi_a - \bar{\epsilon}\gamma_\mu\tau_a\tau_3\chi_{\dot{a}}) \\ \delta A_{\mu\nu}^\sharp &= e^{-\Phi} \left(i\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]} + \frac{1}{2}\bar{\epsilon}\gamma_{\mu\nu}\chi^\sharp \right) + \frac{1}{4}\epsilon^{A\dot{B}}\epsilon^{AB} A_{[\mu \dot{A}A} \delta A_{\nu] \dot{B}B} \quad . \end{aligned} \quad (2.29)$$

To summarise, the gauge fields that survive are four vectors and one 2-form, which is precisely the content of the half-maximal theory.

By projecting the fermions according to eq. (2.27) in the supersymmetry transformation of the 8-forms whose gravitino term is given in eq. (2.25), one obtains that only the

⁶In particular the constraint (2.8) implies that $\chi_\sharp = \sigma_{\dot{a}}\chi_{\dot{a}}$.

components $A_{\mu_1 \dots \mu_8}{}^{MN}{}_{\sharp}$ survive. Out of these, only the 8-forms $A_{\mu_1 \dots \mu_8}{}^{\dot{A}\dot{B}}{}_{\sharp}$ in the $\mathbf{3}$ of $\text{SL}(2, \mathbb{R})$ couple to 7-branes, and their supersymmetry transformations have the form

$$\delta A_{\mu_1 \dots \mu_8}{}^{\dot{A}\dot{B}}{}_{\sharp} = e^{2\Phi} \tilde{V}_a^{\dot{A}} \tilde{V}_a^{\dot{B}} \tilde{\epsilon}^{\gamma}{}_{[\mu_1 \dots \mu_7} \sigma_{\sharp} \psi_{\mu_8]} + \dots \quad (2.30)$$

In particular, we are interested in the brane components, which are the long weights of the $\mathbf{3}$, i.e. the two components $A_{\mu_1 \dots \mu_8}{}^{\dot{A}\dot{A}}{}_{\sharp}$. We can understand better how the truncation acts by looking at the weight diagram of the $\mathbf{15}$ in figure 2. We fix our conventions so that $\sharp = 3$ corresponds to taking the $\text{SL}(2, \mathbb{R})$ subgroup as the one generated by the root α_1 in figure 1. This $\text{SL}(2, \mathbb{R})$ acts on the indices 1 and 2, and the 8-form components that survive the projection are $A_{\mu_1 \dots \mu_8}{}^{11}{}_{\mathbf{3}}$ and $A_{\mu_1 \dots \mu_8}{}^{22}{}_{\mathbf{3}}$. If $\sharp = 2$, the $\text{SL}(2, \mathbb{R})$ subgroup is generated by the root α_2 and acts on the indices 1 and 3. In this case the 8-form components that survive are $A_{\mu_1 \dots \mu_8}{}^{11}{}_{\mathbf{2}}$ and $A_{\mu_1 \dots \mu_8}{}^{33}{}_{\mathbf{2}}$. Finally, if $\sharp = 1$, the $\text{SL}(2, \mathbb{R})$ subgroup is generated by $\alpha_1 + \alpha_2$ and acts on the indices 2 and 3, and the 8-form components that survive are $A_{\mu_1 \dots \mu_8}{}^{22}{}_{\mathbf{1}}$ and $A_{\mu_1 \dots \mu_8}{}^{33}{}_{\mathbf{1}}$. To summarise, we find that for each truncation there are two space-filling branes that preserve the same supersymmetry of the truncation, precisely as expected from the analysis of the central charges [12].

We now want to understand this result from the perspective of the IIB theory. From IIB, one expects only four space-filling branes to arise by reducing to eight dimensions, which are the D9, the D7 and their S-duals. The remaining two 7-branes are exotic and couple to 8-forms that arise from mixed-symmetry potentials in IIB. These potentials are derived from a suitable decomposition of the E_{11} algebra [13], and can be found for instance in section 3.1 of ref. [20]. One can classify all the mixed-symmetry potentials that give rise to branes in lower dimensions in terms of the non-positive integer number α denoting how the tension of the corresponding brane scales with respect to the string coupling g_s , and obviously T-duality relates different potentials with the same value of α . Following [34], we denote the potentials with $\alpha = -1, -2, -3 \dots$ with the letters C, D, E and so on. The 8-forms in eight dimensions then arise from the RR potentials C_8 and C_{10} (with $\alpha = -1$), their S-duals E_8 and F_{10} (with $\alpha = -3$ and -4 respectively) and the mixed-symmetry potentials $E_{10,2,2}$ and $F_{10,2,2}$ (again with $\alpha = -3$ and -4 respectively).⁷ Denoting with $x^i, i = 1, 2$ the internal directions in the reduction from ten to eight dimensions, these two mixed-symmetry potentials give rise to the 8-form potentials $E_{\mu_1 \dots \mu_8} x^1 x^2, x^1 x^2, x^1 x^2$ and $F_{\mu_1 \dots \mu_8} x^1 x^2, x^1 x^2, x^1 x^2$.

To derive which is the pair of 7-branes that is not projected out in each truncation, we move to the string frame, which corresponds to performing the redefinitions

$$e_{\mu}{}^{\alpha} = e^{-\frac{1}{3}\phi} e_{\mu}{}^s{}^{\alpha} \quad \psi_{\mu} = e^{-\frac{1}{6}\phi} \psi_{\mu}^s \quad \epsilon = e^{-\frac{1}{6}\phi} \epsilon^s \quad (2.31)$$

As a convention, we associate to the case $\sharp = 3$ the reduction to $D = 8$ of the SO9 truncation. In this case the global symmetry of the truncated theory is perturbative and the dilaton Φ is proportional to the eight-dimensional string dilaton ϕ . To get the right

⁷In general we denote with $A_{p,q,r,\dots}$ a mixed-symmetry potential in a representation such that p, q, r, \dots (with $p \geq q \geq r \dots$) denote the length of each column of its Young tableau.

scaling in the string frame we impose

$$\text{SO9} : \Phi = -\frac{2}{3}\phi \quad . \quad (2.32)$$

As a result, the supersymmetry transformations of the four vectors and the 1-form have no dilaton dependence in front of the gravitino term, as expected from the reduction of the SO9-truncated ten-dimensional theory. In particular, the 2-form is the NS-NS 2-form $B_{\mu\nu}$ and transforms exactly as the second equation in (1.4), while the vectors are $B_{\mu x^i}$ and $g_{\mu x^i}$. Performing the same rescaling on eq. (2.30), we find that the transformation of both $A_{\mu_1 \dots \mu_8}{}^{11}{}_3$ and $A_{\mu_1 \dots \mu_8}{}^{22}{}_3$ has a factor $e^{-4\phi}$ in front of the gravitino term, which implies that the corresponding branes have both $\alpha = -4$. These are the branes coming from the IIB potentials F_{10} and $F_{10,2,2}$.

We take the truncation identified by $\sharp = 2$ to be the O7 truncation. In this case the $\text{SL}(2, \mathbb{R})$ symmetry is non-perturbative, and the components of the matrix $V_{\dot{A}\dot{a}}$ scale differently with respect to the string dilaton. In particular, we take the component with $\dot{A} = \dot{a} = 1$ to scale like $e^{\phi/2}$, and the one with $\dot{A} = \dot{a} = 3$ to scale like $e^{-\phi/2}$. On top of this, the scalar Φ contains a term proportional to the string dilaton. The precise dependence on the string dilaton of Φ , V_{11} and V_{33} is

$$\text{O7} : \begin{cases} \Phi = \frac{1}{3}\phi + \dots \\ V_{11} = e^{\phi/2} \dots \\ V_{33} = e^{-\phi/2} \dots \end{cases} \quad , \quad (2.33)$$

where we have ignored the contribution of the additional scalars. One obtains that the transformation of the 2-form has an $e^{-\phi}$ factor, as expected because this is the RR 2-form $C_{\mu\nu x^1 x^2}$ and transforms as the third equation in (1.4). Out of the four vectors, two have no dilaton factor (corresponding to $B_{\mu x^i}$) and two have a factor $e^{-\phi}$ (corresponding to $C_{\mu x^i}$). By performing the rescaling on eq. (2.30), we find that the transformation of $A_{\mu_1 \dots \mu_8}{}^{11}{}_2$ has a factor $e^{-3\phi}$, while the one of $A_{\mu_1 \dots \mu_8}{}^{33}{}_2$ has a factor $e^{-\phi}$. We thus identify the former with the potential E_8 and the latter with the potential C_8 .

Finally, the truncation identified by $\sharp = 1$ is the reduction of the O9 truncation. Also in this case the $\text{SL}(2, \mathbb{R})$ symmetry is non-perturbative, and we take the component of $V_{\dot{A}\dot{a}}$ with $\dot{A} = \dot{a} = 2$ to scale like $e^{\phi/2}$, and the one with $\dot{A} = \dot{a} = 3$ to scale like $e^{-\phi/2}$. On top of this, the scalar Φ contains a term proportional to the string dilaton precisely as in the O7 case. We thus get

$$\text{O9} : \begin{cases} \Phi = \frac{1}{3}\phi + \dots \\ V_{22} = e^{\phi/2} \dots \\ V_{33} = e^{-\phi/2} \dots \end{cases} \quad . \quad (2.34)$$

One obtains that the transformation of the 2-form has an $e^{-\phi}$ factor, as expected because this is the RR 2-form $C_{\mu\nu}$ and transforms as the first equation in (1.4). Out of the four vectors, two have no dilaton factor (corresponding to $g_{\mu x^i}$) and two have a factor $e^{-\phi}$ (corresponding to $C_{\mu x^i}$). From eq. (2.30) we read that the potential $A_{\mu_1 \dots \mu_8}{}^{22}{}_1$ has a factor

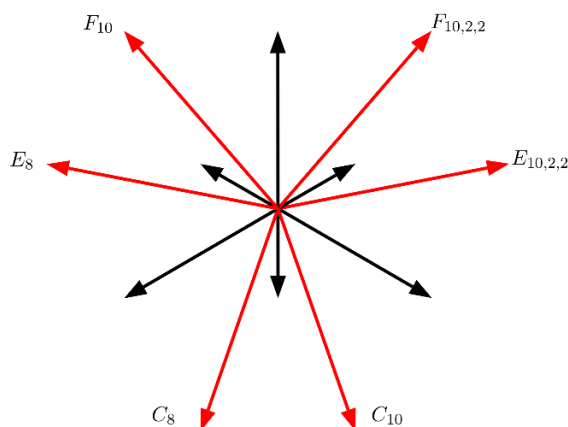


Figure 3. The identification of the long weights of the **15** of $SL(3, \mathbb{R})$ with the IIB potentials. The table should be compared with table 2.

$e^{-3\phi}$ and thus corresponds to $E_{10,2,2}$, while the one of $A_{\mu_1 \dots \mu_8}{}^{33}{}_{11}$ has a factor $e^{-\phi}$ and corresponds to C_{10} .

We draw again in figure 3 the weight diagram of the **15**, where now the long weights are identified with the potentials of the IIB theory. We see from the diagram that the branes on the same horizontal line share the same value of α . This is obvious from the fact that the $SL(2, \mathbb{R})$ associated to the root α_1 is part of the T-duality symmetry. In particular, the branes with $\alpha = -4$ belong to the **3**, the branes with $\alpha = -3$ belong to the **4** and the branes with $\alpha = -1$ belong to the **2** of this $SL(2, \mathbb{R})$. The table also shows that the $SL(2, \mathbb{R})$ of the IIB theory is the one generated by α_2 . As we know, the 8-forms belong to the **3**, the 10-forms to the **4** and the mixed-symmetry potentials to the **2** of this other $SL(2, \mathbb{R})$. Finally, the third $SL(2, \mathbb{R})$, generated by $\alpha_1 + \alpha_2$, mixes E_8 and F_{10} , C_8 and $F_{10,2,2}$ and C_{10} and $E_{10,2,2}$. For each truncation, it is the branes in the **3** that survive, as we have already shown. It is known that in the case of the O7 truncation the potentials C_8 and E_8 both survive because the corresponding branes preserve the same supersymmetry, but what this analysis shows is that in the SO9 truncation one gets that both F_{10} and $F_{10,2,2}$ are not projected out, while in the case of the O9 truncation both C_{10} and $E_{10,2,2}$ survive the projection. For clarity, we summarise this result in table 4. In the next section we will show how this result can be generalised to identify in any dimension all truncations and the various space-filling branes that preserve the same supersymmetry of each truncation.

3 \mathbb{Z}_2 truncations in any dimension

In the previous section we have determined the three different \mathbb{Z}_2 truncations of $D = 8$ maximal supergravity to the half-maximal theory, and for each truncation we have identified the two space-filling branes that preserve the same supersymmetry of the truncation. We have shown that in the case of the O9 and SO9 truncations, one of the two 7-branes is an exotic brane, which corresponds to the IIB mixed symmetry potentials $E_{10,2,2}$ in the

$D = 8$ truncations	potentials	brane components
O9	C_{10}	$C_{8x^1x^2}$
	$E_{10,2,2}$	$E_{8x^1x^2,x^1x^2,x^1x^2}$
SO9	F_{10}	$F_{8x^1x^2}$
	$F_{10,2,2}$	$F_{8x^1x^2,x^1x^2,x^1x^2}$
O7	C_8	C_8
	E_8	E_8

Table 4. The \mathbb{Z}_2 truncations of the maximal theory in $D = 8$ from the IIB perspective. The indices x^i , $i = 1, 2$, label the internal directions.

O9 case and $F_{10,2,2}$ in the SO9 case. In general, exotic branes are associated to specific components of the ten-dimensional mixed-symmetry potentials $A_{p,q,r,\dots}$ (with $p \geq q \geq r \dots$) determined as follows: first of all, only the p set can contain space-time indices, while all the other sets of indices must be internal, because the space-time indices must be antisymmetric. On top of this, the p indices must contain all the internal indices q , which must contain all the internal indices r and so on [15, 34–36]. In [19] a universal rule was derived that relates different brane components of mixed-symmetry potentials by a T-duality transformation in a given direction. Specifically, given an $\alpha = -n$ brane associated to a mixed-symmetry potential such that the internal x index occurs N times (in N different sets of antisymmetric indices), this is mapped by T-duality along x to the brane associated to the potential in which the x index occurs $n - N$ times. Schematically, this can be written as

$$\alpha = -n : \quad \underbrace{x, x, \dots, x}_N \xleftrightarrow{T_x} \underbrace{x, x, \dots, x}_{n-N} . \quad (3.1)$$

Using this T-duality rule, if one performs two T-dualities in the directions x^1 and x^2 not only C_8 is mapped to $C_{8x^1x^2}$ as one naturally expects, but also E_8 is mapped to $E_{8x^1x^2,x^1x^2,x^1x^2}$ and $F_{8x^1x^2}$ is mapped to $F_{8x^1x^2,x^1x^2,x^1x^2}$. We stress that performing two T-dualities maps states in IIB to other states in the same theory, and as far as representations of the perturbative $SL(2, \mathbb{R})$ inside $SL(3, \mathbb{R})$ are concerned, it maps one long weight to the other. In other words, using the universal T-duality rule in eq. (3.1) we could have immediately declared that the O7 truncation, in which C_8 and E_8 are not projected out, is mapped by two T-dualities to the O9 truncation, in which $C_{8x^1x^2}$ and $E_{8x^1x^2,x^1x^2,x^1x^2}$ are not projected out, and the latter truncation is mapped to the SO9 truncation, in which the 8-form potentials that survive are $F_{8x^1x^2}$ and $F_{8x^1x^2,x^1x^2,x^1x^2}$.

The aim of this section is to show that using eq. (3.1) and S-duality, one can characterise all truncations in any dimension, and for each truncation determine all the space-filling branes that are not projected out. We will first discuss the maximal case in any dimension from seven to three, and we will then move to the \mathbb{Z}_2 truncations of the half-maximal theories listed in table 2 and finally the \mathbb{Z}_2 truncations of the quarter-maximal theories listed in table 3.

3.1 From maximal to half-maximal supergravity

D = 7: we want to consider the truncation of maximal supergravity in $D = 7$ to the half-maximal theory coupled to three vector multiplets, with symmetry $\mathbb{R}^+ \times \text{SO}(3, 3)$, which is isomorphic to $\text{GL}(4, \mathbb{R})$. In the truncation, the vectors in the $\overline{\mathbf{10}}$ are truncated to the $\mathbf{6}$ and the 2-forms in the $\mathbf{5}$ are truncated to a singlet. There are 5 different ways of performing this truncation, corresponding to the five different ways in which $\text{SL}(4, \mathbb{R})$ can be embedded in $\text{SL}(5, \mathbb{R})$, and this agrees with the dimension of the vector central charge, which indeed belongs to the vector representation of the R-symmetry $\text{SO}(5)$. We denote with $M = 1, \dots, 5$ the index of the fundamental of $\text{SL}(5, \mathbb{R})$ and with $m = 1, \dots, 5$ the vector index of $\text{SO}(5)$. As in the previous section, the scalars are encoded in the matrix L_{Mm} satisfying identities analogous to those in eq. (2.1), with the Maurer-Cartan form defined as in eq. (2.2).⁸

The 7-forms $A_{\mu_1 \dots \mu_7}{}^{MN}{}_P$ belong to the $\mathbf{70}$ of $\text{SL}(5, \mathbb{R})$, which as in the eight-dimensional case is the irreducible representation with two symmetric upper indices and one lower index. The gravitino-dependent part of its supersymmetry transformation is

$$\delta A_{\mu_1 \dots \mu_7}{}^{MN}{}_P = i \tilde{L}_m^M \tilde{L}_m^N L_{Pn} \bar{\epsilon} \gamma_{[\mu_1 \dots \mu_6} \Gamma_n \psi_{\mu_7]} + \dots \quad , \quad (3.2)$$

where we denote with Γ_m the $\text{SO}(5)$ gamma-matrices. The 1/2-BPS space-filling branes are the 20 components such that $M = N$ and $M \neq P$ [12].

We truncate the theory by splitting the M index as $M = (\sharp, A)$, where $A = 1, \dots, 4$ is the index of the fundamental of $\text{SL}(4, \mathbb{R})$. Similarly, m splits as $m = (\sharp, a)$. The scalars are truncated to

$$L_{Mm} \rightarrow (e^\Phi, e^{-\Phi/4} V_{Aa}) \quad , \quad (3.3)$$

where the dilaton Φ parametrises \mathbb{R}^+ and the matrix V_{Aa} contains the scalars parametrising the coset $\text{SL}(4, \mathbb{R})/\text{SO}(4)$. On ψ_μ and ϵ the truncation acts as

$$\psi_\mu = \Gamma_\sharp \psi_\mu \quad \epsilon = \Gamma_\sharp \epsilon \quad . \quad (3.4)$$

As a result, after the truncation only the 7-forms $A_{\mu_1 \dots \mu_7}{}^{MN}{}_\sharp$ survive, and in particular only the components $A_{\mu_1 \dots \mu_7}{}^{AB}{}_\sharp$ in the $\mathbf{10}$ of $\text{SL}(4, \mathbb{R})$ couple to 6-branes. Their supersymmetry transformations have the form

$$\delta A_{\mu_1 \dots \mu_7}{}^{AB}{}_\sharp = i e^{\frac{3}{2}\Phi} \tilde{V}_a^A \tilde{V}_a^B \bar{\epsilon} \gamma_{[\mu_1 \dots \mu_6} \psi_{\mu_7]} + \dots \quad . \quad (3.5)$$

In particular, there are four 6-branes in the $\mathbf{10}$, that all preserve the same supersymmetry which is the supersymmetry preserved by the truncation.

From the ten-dimensional IIB perspective, the five truncations are the SO_9 , preserving the 2-form $B_{\mu\nu}$, the O_9 , preserving the 2-form $C_{\mu\nu}$ and the three different $\text{O}7_{x^i}$ truncations, preserving the 2-form $C_{\mu\nu x^j x^k}$ (with i, j, k all different). As in eight dimensions, we go to the string frame to get the tension of the 6-branes that are preserved in each truncation. In seven dimensions, this corresponds to performing the redefinitions

$$e_\mu{}^\alpha = e^{-\frac{2}{5}\phi} e_\mu^{s\alpha} \quad \psi_\mu = e^{-\frac{1}{5}\phi} \psi_\mu^s \quad \epsilon = e^{-\frac{1}{5}\phi} \epsilon^s \quad . \quad (3.6)$$

⁸We use here exactly the same index notation as for the $\text{SL}(3, \mathbb{R})/\text{SO}(3)$ coset of the previous section. We do not expect this to cause any confusion to the reader.

$D = 7$ truncations	#	potentials	brane components	# branes
O9	1	C_{10}	$C_{7x^1x^2x^3}$	1
		$E_{10,2,2}$	$E_{7x^1x^2x^3,x^px^q,x^px^q}$	3
SO9	1	F_{10}	$F_{7x^1x^2x^3}$	1
		$F_{10,2,2}$	$F_{7x^1x^2x^3,x^px^q,x^px^q}$	3
O7_{xⁱ}	3	C_8	C_{7x^i}	1
		E_8	E_{7x^i}	1
		$E_{9,2,1}$	$E_{7x^ix^p,x^ix^p,x^i}$	2

Table 5. The \mathbb{Z}_2 truncations of the maximal theory in $D = 7$. In the case of the O7 truncation, the $i = 1, 2, 3$ index labels the truncation while the p and q indices are different from i .

In the case of the SO9 truncation, the scalar Φ is proportional to the string dilaton, and $SL(4, \mathbb{R})$ is the perturbative T-duality symmetry. More precisely, one gets

$$\text{SO9} : \Phi = -\frac{4}{5}\phi \quad , \quad (3.7)$$

and from eq. (3.5) one can then check that the four preserved 6-branes have $\alpha = -4$. For the other four truncations, the dilaton identification can be taken to be

$$\text{O9, O7}_{x^i} : \begin{cases} \Phi = \frac{1}{5}\phi + \dots \\ \text{diag}(V_{Aa}) = (e^{-\frac{3}{4}\phi} \dots, e^{\frac{1}{4}\phi} \dots, e^{\frac{1}{4}\phi} \dots, e^{\frac{1}{4}\phi} \dots) \end{cases} \quad , \quad (3.8)$$

where as in eight dimensions the dots stand for contributions of the other scalars. One can check that in this case from eq. (3.5) one gets one $\alpha = -1$ 6-brane and three $\alpha = -3$ 6-branes.

These results could have been easily deduced by compactification from eight dimensions. Indeed, we know from the analysis of the previous section that F_{10} and $F_{10,2,2}$ give rise to 7-branes that are both preserved under the SO9 truncation. This implies that in seven dimensions the 6-branes coupled to $F_{\mu_1 \dots \mu_7 x^1 x^2 x^3, x^1 x^2, x^1 x^2}$, $F_{\mu_1 \dots \mu_7 x^1 x^2 x^3, x^1 x^3, x^1 x^3}$ and $F_{\mu_1 \dots \mu_7 x^1 x^2 x^3, x^2 x^3, x^2 x^3}$ must all preserve the same supersymmetry of the one coupled to $F_{\mu_1 \dots \mu_7 x^1 x^2 x^3}$. The same applies to the O9 truncation, where C_{10} and $E_{10,2,2}$ give rise to four 6-branes. We can then use the T-duality rules in eq. (3.1) to determine the 6-branes that are not projected out in each of the O7 truncations. The truncation O7_{x^i} is obtained by performing two T-dualities in the directions x^p and x^q different from x^i . One obtains, together with C_{7x^i} and E_{7x^i} coming from C_8 and E_8 , also the two additional branes $E_{7x^ix^p,x^ix^p,x^i}$ coming from the mixed-symmetry potential $E_{9,2,1}$. The full result of the different truncations in seven dimensions is summarised in table 5.

In any dimension $D = 10 - d$, the potentials $F_{10,2n,2n}$ are known to belong to a specific representation of $SO(d, d)$, which is the self-dual representation with d antisymmetric indices [20]. As we have seen, the corresponding branes are those preserved by the SO9 truncation. In the following we will show that one can continue the analysis starting from the SO9 truncation and mapping this to all the other possible truncations in any dimensions using S-duality and the T-duality transformations in eq. (3.1).

$D = 6A$ truncations	#	potentials	brane components	# branes
O9	1	C_{10}	$C_{6x^1\dots x^4}$	1
		$E_{10,2,2}$	$E_{6x^1\dots x^4, x^p x^q, x^p x^q}$	6
		$G_{10,4,4}$	$G_{6x^1\dots x^4, x^1\dots x^4, x^1\dots x^4}$	1
SO9	1	F_{10}	$F_{6x^1\dots x^4}$	1
		$F_{10,2,2}$	$F_{6x^1\dots x^4, x^p x^q, x^p x^q}$	6
		$F_{10,4,4}$	$F_{6x^1\dots x^4, x^1\dots x^4, x^1\dots x^4}$	1
O7 $_{x^i x^j}$	6	C_8	$C_{6x^i x^j}$	1
		E_8	$E_{6x^i x^j}$	1
		$E_{9,2,1}$	$E_{6x^i x^j x^p, x^i x^p, x^i}$	4
		$E_{10,4,2}$	$E_{6x^1\dots x^4, x^1\dots x^4, x^p x^q}$	1
		$G_{10,4,2}$	$G_{6x^1\dots x^4, x^1\dots x^4, x^p x^q}$	1
O5	1	C_6	C_6	1
		$E_{8,2}$	$E_{6x^p x^q, x^p x^q}$	6
		$G_{10,4}$	$G_{6x^1\dots x^4, x^1\dots x^4}$	1
SO5	1	D_6	D_6	1
		$D_{8,2}$	$D_{6x^p x^q, x^p x^q}$	6
		$D_{10,4}$	$D_{6x^1\dots x^4, x^1\dots x^4}$	1

Table 6. The \mathbb{Z}_2 truncations of the maximal theory in $D = 6$ leading to the $\mathcal{N} = (1, 1)$ theory. The x 's denote the four internal directions. As everywhere else in this section, for each truncation the i, j, \dots indices label the truncation while the p, q, \dots indices are the remaining ones.

D = 6: the maximal theory in six dimensions has global symmetry $SO(5, 5)$, and the 6-form potentials that couple to 5-branes can either support vector or tensor multiplets. The former belong to the **320**, which has 80 long weights, while the latter belong to the **126**, which has 16 long weights [12, 37]. The theory can be truncated to either $\mathcal{N} = (1, 1)$ supergravity coupled to four vector multiples, with global symmetry $\mathbb{R}^+ \times SO(4, 4)$, or $\mathcal{N} = (2, 0)$ supergravity coupled to five vector multiples, with global symmetry $SO(5, 5)$. As table 1 shows, in the first case there are ten different truncations, which can be easily understood by observing that there are ten ways in which one can embed $SO(4, 4)$ in $SO(5, 5)$, while in the latter there is only one truncation because the truncated theory has the same symmetry of the maximal one.

We start considering the truncations to the $\mathcal{N} = (1, 1)$ theory, which we also denote as 6A. The SO9 truncation is the one that leaves the perturbative T-duality symmetry $SO(4, 4)$ intact. The eight 5-branes charged under the potentials $F_{6x^1\dots x^4}$, $F_{6x^1\dots x^4, x^p x^q, x^p x^q}$ and $F_{6x^1\dots x^4, x^1\dots x^4, x^1\dots x^4}$ are all preserved by the truncation, and correspond to the eight long weights of the **35_V** of $SO(4, 4)$. This is mapped by S-duality to the O9 truncation, preserving the branes coupled to C_{10} and $E_{10,2,2}$ together with the $\alpha = -5$ brane coupled to $G_{10,4,4}$. By T-dualising in the directions x^p and x^q one then gets the branes of the **O7** $_{x^i x^j}$ truncations, with i, j different from p, q . There are six pairs of coordinates that one can choose, corresponding to six different O7 truncations. One can also perform four T-dualities on O9, which leads to the O5 truncation. Finally, the tenth truncation is obtained by performing an S-duality transformation of O5, and we dub this latter truncation SO5. We give in table 6 the full result, which can be reproduced using eq. (3.1) and the S-duality transformation rules of the various mixed-symmetry potentials.

$D = 6\text{B}$ truncation	potentials	brane components	# branes
tensor	$D_{7,1}$	D_{6x^p,x^p}	4
	$D_{9,3}$	$D_{6x^px^qx^r,x^px^qx^r}$	4
	$F_{9,3}$	$F_{6x^px^qx^r,x^px^qx^r}$	4
	$F_{10,4,1,1}$	$F_{6x^1\dots x^4,x^1\dots x^4,x^p,x^p}$	4

Table 7. The \mathbb{Z}_2 truncation of the maximal theory in $D = 6$ leading to the $\mathcal{N} = (2, 0)$ theory. The branes that survive the projection are 16 and in the maximal theory support tensor multiplets in their world-volume.

The truncation to the $\mathcal{N} = (2, 0)$ or 6B theory is unique. All the 5-branes that support tensor multiplets on their world-volume preserve the same supersymmetry and all survive the projection, that we simply label “tensor” truncation. These branes are charged under the mixed-symmetry potentials $D_{7,1}$, $D_{9,3}$ with $\alpha = -2$ and $F_{9,3}$ and $F_{10,4,1,1}$ with $\alpha = -4$, and they correspond to the 16 long weights of the $\overline{\mathbf{126}}$ of $\text{SO}(5, 5)$. We report the result in table 7.

Apart from the space-filling branes that preserve the same supersymmetry of the truncation, starting from six dimensions there are additional space-filling branes that survive the truncation and are 1/2-BPS states of the truncated theory. In the case of the 6A truncation, these branes are in the $\mathbf{35}_V \oplus \mathbf{35}_S \oplus \mathbf{35}_C$, as shown in table 2. This result can be understood in the case of the SO9 truncation, in which case the $\text{SO}(4, 4)$ symmetry is perturbative and the branes that are left are those of the heterotic theory [20]. In particular, the eight branes in the $\mathbf{35}_V$ are the SO5 branes in table 6 and the 16 branes in the $\mathbf{35}_S \oplus \mathbf{35}_C$ are all those in table 7 (more precisely the first representation contains the $\alpha = -2$ branes and the second the $\alpha = -4$ branes [37]). In [20, 38] the world-volume multiplets for each of these branes in the truncated theory were determined, and in particular it was shown that the SO5 branes and the $\alpha = -2$ branes in table 7 support a hypermultiplet, while the $\alpha = -4$ branes in table 7 support a tensor multiplet.

For any other truncation to the 6A theory, the space-filling branes that remain as 1/2-BPS states of the truncated theory can be determined using the properties of the various truncations under S and T dualities. In particular, the branes in table 7 are present in every truncation. On top of this, in the O9 case one gets the O5 branes and vice-versa, in the SO5 case one gets the SO9 branes and in the $\text{O7}_{x^i x^j}$ case one gets the $\text{O7}_{x^k x^l}$ branes, where $i, j \neq k, l$.

The 6A truncation has also a natural interpretation as the untwisted sector of IIA reduced on the orbifold T^4/\mathbb{Z}_2 . The space-filling branes of this theory were discussed in [20, 38, 39], and performing a single T-duality transformation to translate the results of those papers in the IIB language, one finds that the \mathbb{Z}_2 truncation in this case is the SO5 one, while the remaining 1/2-BPS space-filling branes in the truncated theory are the SO9 branes and the branes in table 7. Indeed, one can for instance show that the mixed-symmetry potentials that are coupled to 5-branes in table 9 of [38] are mapped to the SO9 potentials in table 6 and to the potentials in table 7 by performing a single T-duality using eq. (3.1).

In the case of the unique 6B truncation, as table 2 shows, all the 80 5-branes in the $\mathbf{320}$, i.e. all the branes in table 6, are 1/2-BPS states of the truncated theory. The truncation

$D = 5$ truncations	#	potentials	brane components	# branes
O9	1	C_{10}	$C_5 x^1 \dots x^5$	1
		$E_{10,2,2}$	$E_5 x^1 \dots x^5, x^p x^q, x^p x^q$	10
		$G_{10,4,4}$	$G_5 x^1 \dots x^5, x^p x^q x^r x^s, x^p x^q x^r x^s$	5
SO9	1	F_{10}	$F_5 x^1 \dots x^5$	1
		$F_{10,2,2}$	$F_5 x^1 \dots x^5, x^p x^q, x^p x^q$	10
		$F_{10,4,4}$	$F_5 x^1 \dots x^5, x^p x^q x^r x^s, x^p x^q x^r x^s$	5
O7 _{$x^i x^j x^k$}	10	C_8	$C_5 x^i x^j x^k$	1
		E_8	$E_5 x^i x^j x^k$	1
		$E_{9,2,1}$	$E_5 x^i x^j x^k x^p, x^i x^p, x^i$	6
		$E_{10,4,2}$	$E_5 x^1 \dots x^5, x^i x^j x^p x^q, x^i x^j$	3
		$G_{10,4,2}$	$G_5 x^1 \dots x^5, x^i x^j x^p x^q, x^i x^j$	3
		$G_{10,5,4,1}$	$G_5 x^1 \dots x^5, x^1 \dots x^5, x^i x^j x^k x^p, x^p$	2
O5 _{x^i}	5	C_6	$C_5 x^i$	1
		$E_{8,2}$	$E_5 x^i x^p x^q, x^p x^q$	6
		$E_{9,4,1}$	$E_5 x^i x^p x^q x^r, x^i x^p x^q x^r, x^i$	4
		$G_{10,4}$	$G_5 x^1 \dots x^5, x^p x^q x^r x^s$	1
		$G_{10,5,2,1}$	$G_5 x^1 \dots x^5, x^1 \dots x^5, x^i x^p, x^p$	4
SO5 _{x^i}	5	D_6	$D_5 x^i$	1
		$D_{8,2}$	$D_5 x^i x^p x^q, x^p x^q$	6
		$D_{10,4}$	$D_5 x^1 \dots x^5, x^p x^q x^r x^s$	1
		$F_{9,4,1}$	$F_5 x^i x^p x^q x^r, x^i x^p x^q x^r, x^i$	4
		$F_{10,5,2,1}$	$F_5 x^1 \dots x^5, x^1 \dots x^5, x^i x^p, x^p$	4
T²SO5 _{x^i}	5	$D_{7,1}$	$D_5 x^i x^p, x^p$	4
		$D_{9,3}$	$D_5 x^i x^p x^q x^r, x^p x^q x^r$	4
		$F_{9,3}$	$F_5 x^i x^p x^q x^r, x^p x^q x^r$	4
		$F_{10,4,1,1}$	$F_5 x^1 \dots x^5, x^p x^q x^r x^s, x^p, x^p$	4

Table 8. The \mathbb{Z}_2 truncations of the maximal theory in $D = 5$.

has a natural geometric interpretation as the untwisted sector of IIB reduced on T^4/\mathbb{Z}_2 , and indeed it is easy to show that the 5-branes listed in table 10 of [38], where the orbifold analysis was performed in detail, are exactly the branes in table 6.

D = 5: as table 1 shows, there are 432 1/2-BPS 4-branes in five dimensions, which are the long weights of the $\overline{\mathbf{1728}}$ of $E_{6(6)}$. There are 27 different \mathbb{Z}_2 truncations to the half-maximal theory coupled to five vector multiplets, with symmetry $\mathbb{R}^+ \times \text{SO}(5, 5)$, and in each of these we expect 16 space-filling branes to preserve the same supersymmetry of the truncation [12].

As in the 6A case, we can start from the SO9 truncation and then obtain all the others using dualities. The 16 4-branes that preserve the same supersymmetry of the SO9 truncation couple to the potentials $F_5 x^1 \dots x^5$, $F_5 x^1 \dots x^5, x^p x^q, x^p x^q$ and $F_5 x^1 \dots x^5, x^p x^q x^r x^s, x^p x^q x^r x^s$, that correspond to the long weights of the $\overline{\mathbf{126}}$ of the $\text{SO}(5, 5)$ symmetry of the truncated theory. By S-duality, these are mapped to the O9 truncation, and then by T-dualities the latter is mapped to 10 $\text{O7}_{x^i x^j x^k}$ truncations and 5 O5_{x^i} truncations. The O5 truncations can then be mapped by S-duality to 5 SO5_{x^i} truncations. Finally, we want to derive what happens if one performs two T-dualities on the SO5_{x^i} branes. Using eq. (3.1), one can show

that if both T-dualities are along directions different from x^i , then the SO_{5,x^i} branes are mapped into themselves. On the other hand, if one T-duality is along x^i , then the set of branes that one ends up with is always the same regardless of which direction one chooses for the other T-duality transformation. We call the corresponding truncation T^2SO_{5,x^i} . The full result of this analysis is summarised in table 8.

As in six dimensions, for each truncation there are additional space-filling branes that survive the projection and are 1/2-BPS states of the truncated theory. As table 2 shows, these branes are in the $\mathbf{320} \oplus \mathbf{210}$ of $SO(5,5)$, and they are 80 for each representation. To determine what these branes are, we can again consider the SO_9 truncation, in which case the $SO(5,5)$ symmetry is perturbative. From table 5 of ref. [34] we find that for this truncation the branes in the $\mathbf{210}$ have $\alpha = -2$ and those in the $\mathbf{320}$ have $\alpha = -4$. Therefore these branes are all the SO_{5,x^i} and T^2SO_{5,x^i} branes. Similarly, for different truncations one can determine the branes that are 1/2-BPS states of the truncated theory using dualities. As a particular case, the T^2SO_{5,x^i} truncation is the geometric truncation corresponding to IIB on $T^4/\mathbb{Z}_2 \times S^1$, where x^i is the circle direction.

D = 4: from table 1 we read that the maximal theory in $D = 4$ possesses 63 different \mathbb{Z}_2 truncations to the half-maximal theory, whose symmetry is $SL(2, \mathbb{R}) \times SO(6,6)$. There are 2016 space-filling branes belonging to the $\mathbf{8645}$ of $E_{7(7)}$, and the degeneracy for each truncation is 32. The mixed-symmetry potentials that couple to the 3-branes that preserve the same supersymmetry of the SO_9 truncation are F_{10} , $F_{10,2,2}$, $F_{10,4,4}$ and $F_{10,6,6}$, and the 32 brane components are the long weights of the $(\mathbf{1}, \mathbf{462})$ representation of the symmetry of the truncated theory. By S and T dualities, one can determine all the branes that preserve the same supersymmetry of the truncation for each of the 63 truncations. The result is summarised in table 9.

We can also determine the 3-branes that in each truncation are not projected out and become 1/2-BPS states of the truncated theory. These branes are in the $(\mathbf{3}, \mathbf{495}) \oplus (\mathbf{1}, \mathbf{2079})$ of the $SL(2, \mathbb{R}) \times SO(6,6)$ symmetry of the truncated theory. In particular, in the case of the SO_9 truncation one finds (see e.g. table 6 of ref. [34]) that there are 240 $\alpha = -2$ and 240 $\alpha = -6$ branes in the $\mathbf{495}$ and 480 $\alpha = -4$ branes in the $\mathbf{2079}$ of $SO(6,6)$. These are all the $SO_{5,x^i x^j}$ and $T^2SO_{5,x^i x^j}$ branes. Using S and T dualities one can determine the 960 3-branes that are 1/2-BPS states of the truncated theory for all the other truncations. The truncations $T^2SO_{5,x^i x^j}$ are identified with the geometric compactifications of IIB on $T^4/\mathbb{Z}_2 \times T^2$, where $x^i x^j$ are the two torus directions.

D = 3: finally we consider the three-dimensional case. In the maximal theory there are 17280 space-filling 1/2-BPS 2-branes belonging to the $\mathbf{147250}$ of $E_{8(8)}$. The \mathbb{Z}_2 truncation leads to the half-maximal theory with symmetry $SO(8,8)$. There are 135 different truncations, and for each truncation there are 128 1/2-BPS 2-branes preserving the same supersymmetry of the truncation. To determine these branes in each truncation, one first has to further decompose $SO(8,8)$ as $\mathbb{R}^+ \times SO(7,7)$. In the case of the SO_9 truncation, as usual the $SO(7,7)$ symmetry is perturbative. The $\alpha = -4$ potentials F_{10} , $F_{10,2,2}$, $F_{10,4,4}$ and $F_{10,6,6}$ give rise to 64 2-branes that are the long weights of the $\overline{\mathbf{1716}}$ of $SO(7,7)$. This representation embeds in the $\mathbf{6435}$ of $SO(8,8)$, which also contains the $\mathbf{1716}$ of $SO(7,7)$,

$D = 4$ truncations	#	potentials	brane components	# branes
O9	1	C_{10}	$C_{4x^1\dots x^6}$	1
		$E_{10,2,2}$	$E_{4x^1\dots x^6, x^p x^q, x^p x^q}$	15
		$G_{10,4,4}$	$G_{4x^1\dots x^6, x^p x^q x^r x^s, x^p x^q x^r x^s}$	15
		$I_{10,6,6}$	$I_{4x^1\dots x^6, x^1\dots x^6, x^1\dots x^6}$	1
SO9	1	F_{10}	$F_{4x^1\dots x^6}$	1
		$F_{10,2,2}$	$F_{4x^1\dots x^6, x^p x^q, x^p x^q}$	15
		$F_{10,4,4}$	$F_{4x^1\dots x^6, x^p x^q x^r x^s, x^p x^q x^r x^s}$	15
		$F_{10,6,6}$	$F_{4x^1\dots x^6, x^1\dots x^6, x^1\dots x^6}$	1
O7 $_{x^i x^j x^k x^l}$	15	C_8	$C_{4x^i x^j x^k x^l}$	1
		E_8	$E_{4x^i x^j x^k x^l}$	1
		$E_{9,2,1}$	$E_{4x^i x^j x^k x^l x^p, x^i x^p, x^i}$	8
		$E_{10,4,2}$	$E_{4x^1\dots x^6, x^i x^j x^p x^q, x^i x^j}$	6
		$G_{10,4,2}$	$G_{4x^1\dots x^6, x^i x^j x^p x^q, x^i x^j}$	6
		$G_{10,5,4,1}$	$G_{4x^1\dots x^6, x^i x^j x^k x^p x^q, x^i x^j x^k x^p, x^p}$	8
		$G_{10,6,6,2}$	$G_{4x^1\dots x^6, x^1\dots x^6, x^1\dots x^6, x^p x^q}$	1
		$I_{10,6,6,2}$	$I_{4x^1\dots x^6, x^1\dots x^6, x^1\dots x^6, x^p x^q}$	1
O5 $_{x^i x^j}$	15	C_6	$C_{4x^i x^j}$	1
		$E_{8,2}$	$E_{4x^i x^j x^p x^q, x^p x^q}$	6
		$E_{9,4,1}$	$E_{4x^i x^j x^p x^q x^r, x^i x^p x^q x^r, x^i}$	8
		$E_{10,6,2}$	$E_{4x^1\dots x^6, x^1\dots x^6, x^i x^j}$	1
		$G_{10,4}$	$G_{4x^1\dots x^6, x^p x^q x^r x^s}$	1
		$G_{10,5,2,1}$	$G_{4x^1\dots x^6, x^i x^p x^q x^r x^s, x^i x^p, x^p}$	8
		$G_{10,6,4,2}$	$G_{4x^1\dots x^6, x^1\dots x^6, x^i x^j x^p x^q, x^p x^q}$	6
		$I_{10,6,6,4}$	$I_{4x^1\dots x^6, x^1\dots x^6, x^1\dots x^6, x^p x^q x^r x^s}$	1
O3	1	C_4	C_4	1
		$E_{8,4}$	$E_{4x^p x^q x^r x^s, x^p x^q x^r x^s}$	15
		$G_{10,6,2,2}$	$G_{4x^1\dots x^6, x^1\dots x^6, x^p x^q, x^p x^q}$	15
		$I_{10,6,6,6}$	$I_{4x^1\dots x^6, x^1\dots x^6, x^1\dots x^6, x^1\dots x^6}$	1
SO5 $_{x^i x^j}$	15	D_6	$D_{4x^i x^j}$	1
		$D_{8,2}$	$D_{4x^i x^j x^p x^q, x^p x^q}$	6
		$D_{10,4}$	$D_{4x^1\dots x^6, x^p x^q x^r x^s}$	1
		$F_{9,4,1}$	$F_{4x^i x^j x^p x^q x^r, x^i x^p x^q x^r, x^i}$	8
		$F_{10,5,2,1}$	$F_{4x^1\dots x^6, x^i x^p x^q x^r x^s, x^i x^p, x^p}$	8
		$H_{10,6,2}$	$H_{4x^1\dots x^6, x^1\dots x^6, x^i x^j}$	1
		$H_{10,6,4,2}$	$H_{4x^1\dots x^6, x^1\dots x^6, x^i x^j x^p x^q, x^p x^q}$	6
		$H_{10,6,6,4}$	$H_{4x^1\dots x^6, x^1\dots x^6, x^1\dots x^6, x^p x^q x^r x^s}$	1
T²SO5 $_{x^i x^j}$	15	$D_{7,1}$	$D_{4x^i x^j x^p, x^p}$	4
		$D_{9,3}$	$D_{4x^i x^j x^p x^q x^r, x^p x^q x^r}$	4
		$F_{9,3}$	$F_{4x^i x^j x^p x^q x^r, x^p x^q x^r}$	4
		$F_{9,5,2}$	$F_{4x^i x^j x^p x^q x^r, x^i x^j x^p x^q x^r, x^i x^j}$	4
		$F_{10,4,1,1}$	$F_{4x^1\dots x^6, x^p x^q x^r x^s, x^p, x^p}$	4
		$F_{10,6,3,1}$	$F_{4x^1\dots x^6, x^1\dots x^6, x^i x^j x^p, x^p}$	4
		$H_{10,6,3,1}$	$H_{4x^1\dots x^6, x^1\dots x^6, x^i x^j x^p, x^p}$	4
		$H_{10,6,5,3}$	$H_{4x^1\dots x^6, x^1\dots x^6, x^i x^j x^p x^q x^r, x^p x^q x^r}$	4

Table 9. The \mathbb{Z}_2 truncations of the maximal theory in $D = 4$.

whose 64 long weights correspond to the $\alpha = -8$ potentials $J_{10,7,7,1,1}$, $J_{10,7,7,3,3}$, $J_{10,7,7,5,5}$ and $J_{10,7,7,7,7}$. These $\alpha = -4$ and $\alpha = -8$ branes together give the whole set of 128 branes that preserve the supersymmetry of the SO9 truncations. By performing all possible S and T duality transformations, one then determines all the other truncations. The result is summarised in table 10.

As in any other dimension below seven, there are additional space-filling branes that are preserved by the truncation and become 1/2-BPS states of the truncated theory. These branes are 8960 and correspond to the long weights of the **60060** of SO(8, 8). In the case of the SO9 truncation, this representation contains 560 $\alpha = -2$ and 560 $\alpha = -10$ branes in the **1001** of SO(7, 7), 2240 $\alpha = -4$ and 2240 $\alpha = -8$ in the **11648** of SO(7, 7), and finally 3360 $\alpha = -6$ branes in the **24024** of SO(7, 7) (see table 7 of ref. [34]). These are all the $\text{SO5}_{x^i x^j x^k}$ and $\text{T}^2\text{SO5}_{x^i x^j x^k}$ branes in table 10. We end the analysis of the maximal theories by observing that the truncations $\text{T}^2\text{SO5}_{x^i x^j x^k}$ are identified with the geometric compactifications of IIB on $T^4/\mathbb{Z}_2 \times T^3$, where $x^i x^j x^k$ are the three torus directions.

3.2 From half-maximal to quarter-maximal supergravity

D = 6: both the $\mathcal{N} = (1, 1)$ (or 6A) and the $\mathcal{N} = (2, 0)$ (or 6B) theories admit \mathbb{Z}_2 truncations to the $\mathcal{N} = (1, 0)$ supergravity theory coupled to one tensor multiplet and four hypermultiplets, with the hyper-scalars parametrising the coset manifold $\text{SO}(4, 4)/(\text{SO}(4) \times \text{SO}(4))$. The truncation is unique in the case of the $\mathcal{N} = (1, 1)$ theory because the global symmetry stays the same, while in the case of the $\mathcal{N} = (2, 0)$ there are five different truncations. We now want to determine for any of these truncations what are the branes that preserve the same supersymmetry of the truncation.

We start considering the 6A theory. As table 2 shows, the 1/2-BPS space-filling branes of the 6A theory belong to the $\mathbf{35}_V \oplus \mathbf{35}_S \oplus \mathbf{35}_C$, and all preserve the same supersymmetry of the truncated theory. As already discussed in the previous subsection, in the case of the SO9 truncation to the half-maximal theory, the branes in the $\mathbf{35}_V$ are the SO5 branes while the branes in the $\mathbf{35}_S \oplus \mathbf{35}_C$ are the “tensor” branes in table 7. As we mentioned already, the SO5 truncation of the maximal theory is the T-dual of the untwisted sector of IIA on T^4/\mathbb{Z}_2 , and in this case the 1/2-BPS space-filling branes of the 6A theory are the SO9 and the tensor branes. In general, from the point of view of the maximal theory, the $\mathbf{35}_V$ comes from the **320**, while the other two representations come from the **126**.

In the case of the 6B theory, there are five different truncations. As mentioned already, the truncated theory can be viewed as the untwisted sector of IIB on T^4/\mathbb{Z}_2 , and we know that on this theory the O9 and O5 orientifolds preserve the same supersymmetry.⁹ We therefore label the corresponding truncation as O9/O5. By dualities one gets the other four truncations, and the final result is

$$\text{O9/O5}, \text{ SO9/SO5}, 3 \times \text{O7}_{x^i x^j} / \text{O7}_{x^k x^l} \quad . \quad (3.9)$$

In each case, these branes are the long weights of the $\mathbf{35}_V \oplus \mathbf{35}_V$ representation of the SO(4, 4) symmetry of the truncated theory.

⁹This model was originally constructed in [40]. For the systematics of orientifold model building, see [41, 42].

$D = 3$ truncations	#	potentials	brane components	# branes
O9	1	C_{10}	$C_{3x^1\dots x^7}$	1
		$E_{10,2,2}$	$E_{3x^1\dots x^7, x^p x^q, x^p x^q}$	21
		$G_{10,4,4}$	$G_{3x^1\dots x^7, x^p x^q x^r x^s, x^p x^q x^r x^s}$	35
		$G_{10,7,7,1,1}$	$G_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^p, x^p}$	7
		$I_{10,6,6}$	$I_{3x^1\dots x^7, x^p x^q x^r x^s x^t x^u, x^p x^q x^r x^s x^t x^u}$	7
		$I_{10,7,7,3,3}$	$I_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^p x^q x^r, x^p x^q x^r}$	35
		$K_{10,7,7,5,5}$	$K_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^p x^q x^r x^s x^t, x^p x^q x^r x^s x^t}$	21
		$M_{10,7,7,7,7}$	$M_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^1\dots x^7}$	1
SO9	1	F_{10}	$F_{3x^1\dots x^7}$	1
		$F_{10,2,2}$	$F_{3x^1\dots x^7, x^p x^q, x^p x^q}$	21
		$F_{10,4,4}$	$F_{3x^1\dots x^7, x^p x^q x^r x^s, x^p x^q x^r x^s}$	35
		$F_{10,6,6}$	$F_{3x^1\dots x^7, x^p x^q x^r x^s x^t x^u, x^p x^q x^r x^s x^t x^u}$	7
		$J_{10,7,7,1,1}$	$J_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^p, x^p}$	7
		$J_{10,7,7,3,3}$	$J_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^p x^q x^r, x^p x^q x^r}$	35
		$J_{10,7,7,5,5}$	$J_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^p x^q x^r x^s x^t, x^p x^q x^r x^s x^t}$	21
		$J_{10,7,7,7,7}$	$J_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^1\dots x^7}$	1
O7 $x^i x^j x^k x^l x^m$	21	C_8	$C_{3x^i x^j x^k x^l x^m}$	1
		E_8	$E_{3x^i x^j x^k x^l x^m}$	1
		$E_{9,2,1}$	$E_{3x^i x^j x^k x^l x^m x^p, x^i x^p, x^i}$	10
		$E_{10,4,2}$	$E_{3x^1\dots x^7, x^i x^j x^p x^q, x^i x^j}$	10
		$G_{9,6,5}$	$G_{3x^i x^j x^k x^l x^m 5x^p, x^i x^j x^k x^l x^m x^p, x^i x^j x^k x^l x^m}$	2
		$G_{10,4,2}$	$G_{3x^1\dots x^7, x^i x^j x^p x^q, x^i x^j}$	10
		$G_{10,5,4,1}$	$G_{3x^1\dots x^7, x^i x^j x^k x^p x^q, x^i x^j x^k x^p, x^p}$	20
		$G_{10,6,6,2}$	$G_{3x^1\dots x^7, x^i x^j x^k x^l x^p x^q, x^i x^j x^k x^l x^p x^q, x^p x^q}$	5
		$G_{10,7,5,1,1}$	$G_{3x^1\dots x^7, 7, x^i x^j x^k x^l x^m, x^i, x^i}$	5
		$I_{10,6,6,2}$	$I_{3x^1\dots x^7, x^i x^j x^k x^l x^p x^q, x^i x^j x^k x^l x^p x^q, x^p x^q}$	5
		$I_{10,7,7,2,1,1}$	$I_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^p x^q, x^p, x^p}$	2
		$I_{10,7,7,5,3}$	$I_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^i x^j x^k x^p x^q, x^i x^j x^k}$	10
		$I_{10,7,5,1,1}$	$I_{10,7, x^1 x^2 x^3 x^4 x^5, x^i, x^i}$	5
		$I_{10,7,6,3,2}$	$I_{3x^1\dots x^7, x^1\dots x^7, x^i x^j x^k x^l x^m x^p, x^i x^j x^p, x^i x^j}$	20
		$K_{10,7,7,6,5,1}$	$K_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^i x^j x^k x^l x^p x^q, x^i x^j x^k x^l x^p, x^p}$	10
		$K_{10,7,7,5,3}$	$K_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^i x^j x^k x^p x^q, x^i x^j x^k}$	10
		$K_{10,7,7,7,2}$	$K_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^p x^q}$	1
		$M_{10,7,7,7,7,2}$	$I_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^p x^q}$	1
O5 $x^i x^j x^k$	35	C_6	$C_{3x^i x^j x^k}$	1
		$E_{8,2}$	$E_{3x^i x^j x^k x^p x^q, x^p x^q}$	6
		$E_{9,4,1}$	$E_{3x^i x^j x^k x^p x^q x^r, x^i x^p x^q x^r, x^i}$	12
		$E_{10,6,2}$	$E_{3x^1\dots x^7, x^i x^j x^p x^q x^r x^s, x^i x^j}$	3
		$G_{9,6,3}$	$G_{3x^1\dots x^7, x^i x^j x^k x^p x^q x^r, x^i x^j x^k}$	4
		$G_{10,4}$	$G_{3x^1\dots x^7, x^p x^q x^r x^s}$	1
		$G_{10,5,2,1}$	$G_{3x^1\dots x^7, x^i x^p x^q x^r x^s, x^i x^p, x^p}$	12
		$G_{10,6,4,2}$	$G_{3x^1\dots x^7, x^i x^j x^p x^q x^r x^s, x^i x^j x^p x^q, x^p x^q}$	18
		$G_{10,7,6,3}$	$G_{3x^1\dots x^7, x^1\dots x^7, x^i x^j x^k x^p x^q x^r, x^p x^q x^r}$	4
		$G_{10,7,3,1,1}$	$G_{3x^1\dots x^7, x^1\dots x^7, x^i x^j x^k, x^i, x^i}$	3
		$I_{10,6,6,4}$	$I_{3x^1\dots x^7, x^i x^j x^p x^q x^r x^s, x^i x^j x^p x^q x^r x^s, x^p x^q x^r x^s}$	3
		$I_{10,7,7,4,1,1}$	$I_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^p x^q x^r x^s, x^p, x^p}$	4
		$I_{10,7,7,7,3}$	$I_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^i x^j x^k}$	1
		$I_{10,7,6,5,2}$	$I_{3x^1\dots x^7, x^1\dots x^7, x^i x^j x^k x^p x^q x^r, x^i x^j x^p x^q x^r, x^i x^j}$	12
		$I_{10,7,5,3,1}$	$I_{3x^1\dots x^7, x^1\dots x^7, x^i x^j x^k x^p x^q, x^i x^p x^q, x^i}$	18
		$I_{10,7,4,1}$	$I_{3x^1\dots x^7, x^1\dots x^7, x^i x^j x^k x^p, x^p}$	4
		$K_{10,7,7,7,5,2}$	$K_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^i x^j x^k x^p x^q, x^p x^q}$	6
		$K_{10,7,7,5,1}$	$K_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^i x^p x^q x^r x^s, x^i}$	3
$K_{10,7,7,6,3,1}$	$K_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^i x^j x^p x^q x^r x^s, x^i x^j x^p, x^p}$	12		
$M_{10,7,7,7,7,4}$	$M_{3x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^1\dots x^7, x^p x^q x^r x^s}$	1		

Table 10. (Part 1 of 2) The \mathbb{Z}_2 truncations of the maximal theory in $D = 3$.

$D = 3$ truncations	#	potentials	brane components	# branes
O3_{xⁱ}	7	C_4	C_{3x^i}	1
		$E_{8,4}$	$E_{3x^i x^p x^q x^r x^s, x^p x^q x^r x^s}$	15
		$E_{9,6,1}$	$E_{3x^i x^p x^q x^r x^s x^t, x^i x^p x^q x^r x^s x^t, x^i}$	6
		$G_{10,6,2,2}$	$G_{3x^1 \dots x^7, x^p x^q x^r x^s x^t x^u, x^p x^q, x^p x^q}$	15
		$G_{10,7,4,3}$	$G_{3x^1 \dots x^7, x^1 \dots x^7, x^i x^p x^q x^r, x^p x^q x^r}$	20
		$G_{9,6,1}$	$G_{3x^i x^p x^q x^r x^s x^t, x^i x^p x^q x^r x^s x^t, x^i}$	6
		$G_{10,7,1,1,1}$	$G_{3x^1 \dots x^7, x^1 \dots x^7, x^i, x^i, x^i}$	1
		$I_{10,6,6,6}$	$I_{3x^1 \dots x^7, x^p x^q x^r x^s x^t x^u, x^p x^q x^r x^s x^t x^u, x^p x^q x^r x^s x^t x^u}$	1
		$I_{10,7,7,6,1,1}$	$I_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^p x^q x^r x^s x^t x^u, x^p, x^p}$	6
		$I_{10,7,5,5,1}$	$I_{3x^1 \dots x^7, x^1 \dots x^7, x^i x^p x^q x^r x^s, x^i x^p x^q x^r x^s, x^i}$	15
		$I_{10,7,4,3}$	$I_{3x^1 \dots x^7, x^1 \dots x^7, x^i x^p x^q x^r, x^p x^q x^r}$	20
		$K_{10,7,7,6,1,1}$	$K_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^p x^q x^r x^s x^t x^u, x^p, x^p}$	6
		$K_{10,7,7,7,3,2}$	$K_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^i x^p x^q, x^p x^q}$	15
		$M_{10,7,7,7,7,6}$	$M_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^p x^q x^r x^s x^t x^u}$	1
SO5_{xⁱx^jx^k}	35	D_6	$D_{3x^i x^j x^k}$	1
		$D_{8,2}$	$D_{3x^i x^j x^k x^p x^q, x^p x^q}$	6
		$D_{10,4}$	$D_{3x^1 \dots x^7, x^p x^q x^r x^s}$	1
		$F_{9,4,1}$	$F_{3x^i x^j x^k x^p x^q x^r, x^i x^p x^q x^r, x^i}$	12
		$F_{9,6,3}$	$F_{3x^i x^j x^k x^p x^q x^r, x^i x^j x^k x^p x^q x^r, x^i x^j x^k}$	4
		$F_{10,5,2,1}$	$F_{3x^1 \dots x^7, x^i x^p x^q x^r x^s, x^i x^p, x^p}$	12
		$F_{10,7,4,1}$	$F_{3x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k x^p, x^p}$	4
		$H_{10,6,2}$	$H_{3x^1 \dots x^7, x^i x^j x^p x^q x^r x^s, x^i x^j}$	3
		$H_{10,6,4,2}$	$H_{3x^1 \dots x^7, x^i x^j x^p x^q x^r x^s, x^i x^j x^p x^q, x^p x^q}$	18
		$H_{10,6,6,4}$	$H_{3x^1 \dots x^7, x^i x^j x^p x^q x^r x^s, x^i x^j x^p x^q x^r x^s, x^p x^q x^r x^s}$	3
		$H_{10,7,3,1,1}$	$H_{3x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k, x^i, x^i}$	3
		$H_{10,7,5,3,1}$	$H_{3x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k x^p x^q, x^i x^p x^q, x^i}$	18
		$H_{10,7,7,5,1}$	$H_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^i x^p x^q x^r x^s, x^i}$	3
		$J_{10,7,6,3}$	$J_{3x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k x^p x^q x^r, x^p x^q x^r}$	4
		$J_{10,7,7,4,1,1}$	$J_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^p x^q x^r x^s, x^p, x^p}$	4
		$J_{10,7,6,5,2}$	$J_{3x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k x^p x^q x^r, x^i x^j x^p x^q x^r, x^i x^j}$	12
		$J_{10,7,7,6,3,1}$	$J_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^p x^q x^r x^s, x^i x^j x^p, x^p}$	12
		$L_{10,7,7,7,3}$	$L_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k}$	1
$L_{10,7,7,7,5,2}$	$L_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k x^p x^q, x^p x^q}$	6		
$L_{10,7,7,7,7,4}$	$L_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^p x^q x^r x^s}$	1		
T²SO5_{xⁱx^jx^k}	35	$D_{7,1}$	$D_{3x^i x^j x^k x^p, x^p}$	4
		$D_{9,3}$	$D_{3x^i x^j x^k x^p x^q x^r, x^p x^q x^r}$	4
		$F_{9,3}$	$F_{3x^i x^j x^k x^p x^q x^r, x^p x^q x^r}$	4
		$F_{9,5,2}$	$F_{3x^i x^j x^k x^p x^q x^r, x^i x^j x^p x^q x^r, x^i x^j}$	12
		$F_{10,4,1,1}$	$F_{3x^1 \dots x^7, x^p x^q x^r x^s, x^p, x^p}$	4
		$F_{10,6,3,1}$	$F_{3x^1 \dots x^7, x^i x^j x^p x^q x^r x^s, x^i x^j x^p, x^p}$	12
		$H_{10,6,3,1}$	$H_{3x^1 \dots x^7, x^i x^j x^p x^q x^r x^s, x^i x^j x^p, x^p}$	12
		$H_{10,7,4,2,1}$	$H_{3x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k x^p, x^i x^p, x^i}$	12
		$H_{10,6,5,3}$	$H_{3x^1 \dots x^7, x^i x^j x^p x^q x^r x^s, x^i x^j x^p x^q x^r, x^p x^q x^r}$	12
		$H_{10,7,6,4,1}$	$H_{3x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k x^p x^q x^r, x^i x^p x^q x^r, x^i}$	12
		$J_{10,7,7,5,2,1}$	$J_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^i x^p x^q x^r x^s, x^i x^p, x^p}$	12
		$J_{10,7,7,7,4,1}$	$J_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k x^p, x^p}$	4
		$J_{10,7,6,4,1}$	$J_{3x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k x^p x^q x^r, x^i x^p x^q x^r, x^i}$	12
		$J_{10,7,6,6,3}$	$J_{3x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k x^p x^q x^r, x^i x^j x^k x^p x^q x^r, x^i x^j x^k}$	4
		$L_{10,7,7,7,4,1}$	$L_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k x^p, x^p}$	4
		$L_{10,7,7,7,6,3}$	$L_{3x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^1 \dots x^7, x^i x^j x^k x^p x^q x^r, x^p x^q x^r}$	4

 Table 10. (Part 2 of 2) The \mathbb{Z}_2 truncations of the maximal theory in $D = 3$.

D = 5: the five-dimensional half-maximal theory admits five different \mathbb{Z}_2 truncations to the quarter-maximal theory with symmetry $(\mathbb{R}^+)^2 \times \text{SO}(4, 4)$, containing gravity coupled to two vector multiplets and four hypermultiplets. As table 2 shows, the space-filling branes belong to the $\mathbf{320} \oplus \mathbf{210}$ of $\text{SO}(5, 5)$. A particular way of realising the half-maximal theory is by compactifying IIB on SO9, which corresponds to the heterotic truncation, so that the $\text{SO}(5, 5)$ symmetry is perturbative. In particular, the branes in the $\mathbf{210}$ have $\alpha = -2$ while the branes in the $\mathbf{320}$ have $\alpha = -4$ [20]. These branes are the SO5_{x^i} and $\text{T}^2\text{SO5}_{x^i}$ ones in table 8, where $i = 1, \dots, 5$ labels each of the five truncations. In other words, the $\text{SO}(4, 4)$ of the truncated theory is the one that fixes the particular x^i that identifies the truncation. By looking at table 8, one can show that out of the $\mathbf{320}$ one selects the $\mathbf{35}_V \oplus \mathbf{35}_V$, while out of the $\mathbf{210}$ one selects the $\mathbf{35}_S \oplus \mathbf{35}_C$.

One can also obtain the half-maximal theory by compactifying IIB on $(T^4/\mathbb{Z}_2) \times S^1$, corresponding to the truncation $\text{T}^2\text{SO5}_y$, where y is the S^1 coordinate. It is instructive to show how using duality symmetries one can find the five truncations of the $\text{T}^2\text{SO5}_y$ theory starting from the truncations of the SO9 theory. Denoting the five internal coordinates as x^i, y , with $i = 1, \dots, 4$, we first perform an S-duality transformation to go to O9, then four T-dualities along the x^i directions to go to O5_y , then again S-duality to go to SO5_y , and finally two T-dualities along y and any other x direction to go to $\text{T}^2\text{SO5}_y$. If one performs this chain of transformations on the truncations of the SO9 theory,

$$\text{SO9} : \quad 4 \times \text{SO5}_{x^i} / \text{T}^2\text{SO5}_{x^i}, \quad \text{SO5}_y / \text{T}^2\text{SO5}_y \quad , \quad (3.10)$$

one finds the truncations of the $\text{T}^2\text{SO5}_y$ theory, which are

$$\text{T}^2\text{SO5}_y : \quad \text{O9/O5}_y, \quad 3 \times \text{O7}_{x^i x^j y} / \text{O7}_{x^k x^l y}, \quad \text{SO9/SO5}_y \quad . \quad (3.11)$$

D = 4: the $\text{SL}(2, \mathbb{R}) \times \text{SO}(6, 6)$ theory in four dimensions can be truncated to the $\mathcal{N} = 2$ theory with symmetry $\text{SL}(2, \mathbb{R})^3 \times \text{SO}(4, 4)$, describing gravity coupled to three vector multiplets and four hypermultiplets. There are 15 different truncations, corresponding to the different ways in which one can embed $\text{SO}(4, 4)$ into $\text{SO}(6, 6)$, and correspondingly the vector central charge belongs to the $\mathbf{15}$ of the R-symmetry group $\text{U}(4)$. As reported in table 2, the 960 space-filling branes are the long weights of the $(\mathbf{3}, \mathbf{495}) \oplus (\mathbf{1}, \mathbf{2079})$ [20], and for each truncation one expects 64 branes to preserve the same supersymmetry of the truncation. To figure out what these branes are in each truncation, we focus on the particular half-maximal theory that results from the SO9 truncation of the maximal one. In this case the $\text{SO}(6, 6)$ symmetry is perturbative, and we expect only the branes with even α to survive as 1/2-BPS states. Therefore we get the 15 truncations

$$\text{SO9} : \quad 15 \times \text{SO5}_{x^i x^j} / \text{T}^2\text{SO5}_{x^i x^j} \quad , \quad (3.12)$$

where for any $i, j = 1, \dots, 6$ the $\text{SO5}_{x^i x^j}$ and $\text{T}^2\text{SO5}_{x^i x^j}$ branes preserve the same supersymmetry. Fixing i and j to identify a specific truncation to the $\mathcal{N} = 2$ theory, the symmetry $\text{SO}(4, 4)$ acts on the remaining coordinates, and by looking at table 9 one finds that the 64 branes that preserve the same supersymmetry of the truncated theory are

$$\begin{aligned} (\mathbf{3}, \mathbf{495}) &\rightarrow (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{35}_S) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{35}_C) \\ (\mathbf{1}, \mathbf{2079}) &\rightarrow (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{35}_V) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{35}_V) \quad . \end{aligned} \quad (3.13)$$

Apart from the branes in eq. (3.13), that preserve the same supersymmetry of the truncated theory, in four dimensions there are additional space-filling branes that survive the projection and are 1/2-BPS states of the $\mathcal{N} = 2$ theory. These branes are the 384 long weights of the following representations [21]:

$$(\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{28}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{28}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{28}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{350}) \quad . \quad (3.14)$$

In the particular case of the $\text{SO5}_{x^i x^j} / \text{T}^2 \text{SO5}_{x^i x^j}$ truncation of the SO9-truncated theory, these all the $\text{SO5}_{x^k x^l}$ and $\text{T}^2 \text{SO5}_{x^k x^l}$ branes with $k, l \neq i, j$.

Similarly to the five-dimensional case, one can obtain the half-maximal theory by compactifying IIB on $(T^4/\mathbb{Z}_2) \times T^2$, corresponding to the truncation $\text{T}^2 \text{SO5}_{y^1 y^2}$, where y^1 and y^2 are the T^2 coordinates. As in $D = 5$, one can show how using duality symmetries one can find the 15 truncations of the $\text{T}^2 \text{SO5}_{y^1 y^2}$ half-maximal theory starting from the truncations of the SO9 half-maximal theory given in eq (3.12). Denoting the six internal coordinates as x^i, y^m , with $i = 1, \dots, 4$ and $m = 1, 2$, we first perform an S-duality transformation to go to O9, then four T-dualities along the x^i directions to go to $\text{O5}_{y^1 y^2}$, then again S-duality to go to $\text{SO5}_{y^1 y^2}$, and finally two T-dualities along one y and any other x direction to go to $\text{T}^2 \text{SO5}_{y^1 y^2}$. One finds

$$\begin{aligned} \text{T}^2 \text{SO5}_{y^1 y^2} : \quad & \text{O9/O5}_{y^1 y^2}, \quad \text{O7}_{x^1 \dots x^4} / \text{O3}, \quad 3 \times \text{O7}_{x^i x^j y^1 y^2} / \text{O7}_{x^k x^l y^1 y^2}, \\ & 3 \times \text{O5}_{x^i x^j} / \text{O5}_{x^k x^l}, \quad \text{SO9/SO5}_{y^1 y^2}, \quad 3 \times \text{SO5}_{x^i x^j} / \text{SO5}_{x^k x^l}, \\ & 3 \times \text{T}^2 \text{SO5}_{x^i x^j} / \text{T}^2 \text{SO5}_{x^k x^l} \quad . \end{aligned} \quad (3.15)$$

D = 3: in three dimensions the $\text{SO}(8, 8)$ theory is truncated to the $\text{SO}(4, 4) \times \text{SO}(4, 4)$ theory, describing supergravity coupled to eight hyper-multiplets. The vector central charge belongs to the **35** of the R-symmetry $\text{SO}(8)$, and correspondingly there are 35 different truncations, which can be seen as the $\frac{1}{2} \cdot \binom{8}{4}$ ways in which one can embed $\text{SO}(4, 4) \times \text{SO}(4, 4)$ into $\text{SO}(8, 8)$. As table 2 shows, there are 8960 1/2-BPS 2-branes which are the long weights of the **600060**, and for each truncation one expects 256 branes preserving the same supersymmetry of the truncation [20]. As in the higher-dimensional cases, to identify each truncation and the corresponding branes we consider the half-maximal theory resulting from the SO9 truncation of the maximal one. We get

$$\text{SO9} : \quad 35 \times \text{SO5}_{x^i x^j x^k} / \text{T}^2 \text{SO5}_{x^i x^j x^k} \quad , \quad (3.16)$$

where the indices i, j, k run from 1 to 7. Fixing i, j, k identifies a specific truncation to the quarter-maximal theory, and by carefully looking at table 10 and identifying one $\text{SO}(4, 4)$ as acting on the four unfixed indices and the other as grouping different potentials together, one can show that the $\text{SO5}_{x^i x^j x^k} / \text{T}^2 \text{SO5}_{x^i x^j x^k}$ branes belong to the representation

$$(\mathbf{35}_V, \mathbf{35}_S) \oplus (\mathbf{35}_V, \mathbf{35}_C) \oplus (\mathbf{35}_S, \mathbf{35}_V) \oplus (\mathbf{35}_S, \mathbf{35}_C) \quad . \quad (3.17)$$

Apart from the branes in eq. (3.17), there are additional space-filling branes that survive the truncation and are 1/2-BPS of the truncated theory. The representation of such branes is [21]

$$(\mathbf{350}, \mathbf{28}) \oplus (\mathbf{28}, \mathbf{350}) \quad (3.18)$$

corresponding to 4608 branes. In the particular case of the $\text{SO}5_{x^i x^j x^k} / \text{T}^2 \text{SO}5_{x^i x^j x^k}$ truncation of the SO9-truncated theory, these all the $\text{SO}5_{x^i x^l x^m}$ and $\text{T}^2 \text{SO}5_{x^i x^l x^m}$ branes with $l, m \neq j, k$. This indeed gives in total $3 \cdot \binom{4}{2} \cdot 2 \cdot 128 = 4608$ branes.

Analogously to the five and four-dimensions cases, one can also obtain the half-maximal theory as a geometric orbifold $(T^4/\mathbb{Z}_2) \times T^3$ truncation of IIB. Denoting with $x^i, i = 1, \dots, 4$ the orbifold coordinates and with $y^m, m = 1, 2, 3$, the torus coordinates, this corresponds to the truncation $\text{T}^2 \text{SO}5_{y^1 y^2 y^3}$. One can obtain the 35 truncations of the $\text{T}^2 \text{SO}5_{y^1 y^2 y^3}$ half-maximal theory starting from the truncations of the SO9 half-maximal theory given in eq (3.16). We first perform an S-duality transformation to go to O9, then four T-dualities along the x^i directions to go to $\text{O}5_{y^1 y^2 y^3}$, then again S-duality to go to $\text{SO}5_{y^1 y^2 y^3}$, and finally two T-dualities along one y and one x direction to go to $\text{T}^2 \text{SO}5_{y^1 y^2 y^3}$. The result is

$$\begin{aligned} \text{T}^2 \text{SO}5_{y^1 y^2 y^3} : \quad & \text{O}9 / \text{O}5_{y^1 y^2 y^3}, \quad 3 \times \text{O}7_{x^1 \dots x^4 y^m} / \text{O}3_{y^m}, \quad 3 \times \text{O}7_{x^i x^j y^1 y^2 y^3} / \text{O}7_{x^k x^l y^1 y^2 y^3}, \\ & 9 \times \text{O}5_{x^i x^j y^m} / \text{O}5_{x^k x^l y^m}, \quad \text{SO}9 / \text{SO}5_{y^1 y^2 y^3}, \quad 9 \times \text{SO}5_{x^i x^j y^m} / \text{SO}5_{x^k x^l y^m}, \\ & 9 \times \text{T}^2 \text{SO}5_{x^i x^j y^m} / \text{T}^2 \text{SO}5_{x^k x^l y^m} \quad . \end{aligned} \tag{3.19}$$

3.3 From quarter-maximal to 1/8-maximal supergravity

D = 4: the four-dimensional $\mathcal{N} = 2$ theory with symmetry $(\text{SL}(2, \mathbb{R}))^3 \times \text{SO}(4, 4)$ can be finally truncated to minimal supergravity coupled to seven chiral multiplets and global symmetry $(\text{SL}(2, \mathbb{R}))^7$. The vector central charge belongs to the **3** of the R-symmetry $\text{U}(2)$, and correspondingly there are three different \mathbb{Z}_2 truncations, as expected from the fact that there are three different ways of embedding $\text{SO}(2, 2) \times \text{SO}(2, 2)$ inside $\text{SO}(4, 4)$, with $\text{SO}(2, 2)$ being isomorphic to $(\text{SL}(2, \mathbb{R}))^4$. The 384 1/2-BPS space-filling branes are the long weights of the representations given in eq. (3.14), and one expects that for each of the three truncations 128 branes are not projected out and preserve the same supersymmetry of the truncation. We can for instance identify these truncations as follows: first truncate from the maximal theory to the half-maximal one via SO9, and then from the half-maximal to the $\mathcal{N} = 2$ theory via $\text{SO}5_{x^5 x^6} / \text{T}^2 \text{SO}5_{x^5 x^6}$. One is then left with the $\text{SO}5_{x^i x^j}$ and $\text{T}^2 \text{SO}5_{x^i x^j}$ space-filling branes in table 9, with $i, j = 1, \dots, 4$, and in each of the three truncations one identifies the branes as follows:

$$\text{SO}9 \rightarrow \text{SO}5_{x^5 x^6} / \text{T}^2 \text{SO}5_{x^5 x^6} : \quad 3 \times \text{SO}5_{x^i x^j} / \text{SO}5_{x^k x^l} / \text{T}^2 \text{SO}5_{x^i x^j} / \text{T}^2 \text{SO}5_{x^k x^l} \quad , \tag{3.20}$$

where the indices i, j, k, l are all different. By analysing table 9 one can determine the representations of the $\text{SO}5_{x^i x^j} / \text{SO}5_{x^k x^l} / \text{T}^2 \text{SO}5_{x^i x^j} / \text{T}^2 \text{SO}5_{x^k x^l}$ branes for fixed i, j, k, l that preserve the supersymmetry of a given truncation. Starting from the representations of the 1/2-BPS branes in the $\mathcal{N} = 2$ theory given in eq. (3.14) one gets

$$\begin{aligned} (\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{28}) & \rightarrow (\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1}) \\ & \oplus (\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}) \\ (\mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{28}) & \rightarrow (\mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1}) \\ & \oplus (\mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}) \\ (\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{28}) & \rightarrow (\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1}) \\ & \oplus (\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}) \end{aligned}$$

$$\begin{aligned}
 (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{350}) \rightarrow & (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{3}) \\
 & \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{3}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{3}) \quad . \quad (3.21)
 \end{aligned}$$

The $\mathcal{N} = 2$ theory has also a natural geometric origin as IIB compactified on the orbifold $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$. In this case the torus T^6 is factorised as $T^6 = \otimes_{i=1}^3 T_i^2$, where T_i^2 indicates the two-dimensional torus with coordinates x^i and y^i , and the two \mathbb{Z}_2 's act as $(+, -, -)$ and $(-, +, -)$ on the three pairs of coordinates. We can think of the orbifold action as the sequence of two truncations, which are the $T^2\text{SO}5_{x^1y^1}$ from the maximal to the half-maximal theory, and then $T^2\text{SO}5_{x^2y^2}/T^2\text{SO}5_{x^3y^3}$ from the half-maximal to the $\mathcal{N} = 2$ theory. The three different truncations of the $\mathcal{N} = 2$ theory can then be derived by a suitable series of dualities starting from eq. (3.20), and the result is

$$\begin{aligned}
 \text{O9/O5}_{x^1y^1}/\text{O5}_{x^2y^2}/\text{O5}_{x^3y^3} \quad , \quad \text{O3/O7}_{x^1y^1x^2y^2}/\text{O7}_{x^1y^1x^3y^3}/\text{O7}_{x^2y^2x^3y^3} \quad , \\
 \text{SO9/SO5}_{x^1y^1}/\text{SO5}_{x^2y^2}/\text{SO5}_{x^3y^3} \quad . \quad (3.22)
 \end{aligned}$$

One can recognise the first two as the two T-dual orientifold projections of the IIB theory on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ [43, 44], and the last as the projection on the heterotic sector. As we will show in detail in section 5, the 128 branes in each of these truncations are precisely those that are responsible for the cancellation of the tadpole conditions when all possible geometric and non-geometric fluxes are turned on, and for the $\text{O3/O7}_{x^1y^1x^2y^2}/\text{O7}_{x^1y^1x^3y^3}/\text{O7}_{x^2y^2x^3y^3}$ orientifold case, they have been determined in [19, 30].

D = 3: the three-dimensional theory with symmetry $\text{SO}(4, 4) \times \text{SO}(4, 4)$ can be truncated to the supergravity theory with four supersymmetry coupled to eight scalar multiplets and global symmetry $(\text{SL}(2, \mathbb{R}))^8$. As table 3 shows, there are nine different truncations, corresponding to the fact that each of the two $\text{SO}(4, 4)$ can be decomposed in $(\text{SL}(2, \mathbb{R}))^4$ in three different ways. We expect that for each of the nine truncations 512 out of the 4608 2-branes in the representations of eq. (3.18) survive the are not projected out. These are the branes that preserve the same supersymmetry of the truncation. To identify these branes and the corresponding truncations, we proceed as in four dimensions, considering the particular case of the SO9 truncation of the maximal theory, and the further $\text{SO5}_{x^5x^6x^7}/T^2\text{SO5}_{x^5x^6x^7}$ truncation of the half-maximal theory to the theory with eight supersymmetries. One gets

$$\text{SO9} \rightarrow \text{SO5}_{x^5x^6x^7}/T^2\text{SO5}_{x^5x^6x^7} : 9 \times \text{SO5}_{x^ix^jx^k}/\text{SO5}_{x^ix^lx^m}/T^2\text{SO5}_{x^ix^jx^k}/T^2\text{SO5}_{x^ix^lx^m}, \quad (3.23)$$

where $i = 5, 6, 7$ and $j, k, l, m = 1, \dots, 4$ and all different. Fixing i, j, k, l, m to identify a particular truncation, one reads off the corresponding branes from table 10. These branes belong to representations made of four triplets and four singlets of $(\text{SL}(2, \mathbb{R}))^8$ which arise from decomposing all the **350** and **28** representations in eq. (3.18) as

$$\begin{aligned}
 \mathbf{350} \rightarrow & (\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{3}, \mathbf{3}, \mathbf{1}) \\
 \mathbf{28} \rightarrow & (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}) \quad . \quad (3.24)
 \end{aligned}$$

As in four dimensions, this theory can also be considered as IIB compactified on the orbifold $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2) \times S^1$. Denoting the orbifold coordinates $x^i y^i$, $i = 1, 2, 3$ as in four

dimensions, and the circle coordinate z , we can think of the orbifold action as the sequence of two truncations, which are the $T^2SO5_{x^1y^1z}$ from the maximal to the half-maximal theory, and then $T^2SO5_{x^2y^2z}/T^2SO5_{x^3y^3z}$ from the half-maximal to the quarter-maximal theory. The nine different truncations of the $\mathcal{N} = 2$ theory can then be derived by a suitable series of dualities starting from eq. (3.23), and the result is

$$\begin{aligned}
 & O9/O5_{x^1y^1z}/O5_{x^2y^2z}/O5_{x^3y^3z} \\
 & SO9/SO5_{x^1y^1z}/SO5_{x^2y^2z}/SO5_{x^3y^3z} \\
 & O5_{y^1y^2y^3}/O5_{y^1x^2x^3}/O5_{x^1y^2x^3}/O5_{x^1x^2y^3} \\
 & O5_{x^1x^2x^3}/O5_{x^1y^2y^3}/O5_{y^1x^2y^3}/O5_{y^1y^2x^3} \\
 & SO5_{y^1y^2y^3}/SO5_{y^1x^2x^3}/SO5_{x^1y^2x^3}/SO5_{x^1x^2y^3} \\
 & SO5_{x^1x^2x^3}/SO5_{x^1y^2y^3}/SO5_{y^1x^2y^3}/SO5_{y^1y^2x^3} \\
 & O3_z/O7_{x^1y^1x^2y^2z}/O7_{x^1y^1x^3y^3z}/O7_{x^2y^2x^3y^3z} \\
 & T^2SO5_{x^1x^2x^3}/T^2SO5_{x^1y^2y^3}/T^2SO5_{y^1x^2y^3}/T^2SO5_{y^1y^2x^3} \\
 & T^2SO5_{y^1y^2y^3}/T^2SO5_{y^1x^2x^3}/T^2SO5_{x^1y^2x^3}/T^2SO5_{x^1x^2y^3} \quad , \quad (3.25)
 \end{aligned}$$

where the corresponding branes can be read in table 10.

To conclude this section, we give in table 11 the summary of all the results, in which in any dimension and for any supersymmetry we denote in red the representations of the branes that preserve the same supersymmetry of the truncation, and in black the ones that are 1/2-BPS states. The three-dimensional theory with four supersymmetries admits a further truncation to the minimal $D = 3$ theory. This means that in the three-dimensional theory with four supersymmetries there are also space-filling branes that are 1/2-BPS states, and this should correspond to representations denoted in black in the table, that we have not determined. These branes should be the ones that preserve the same supersymmetry as the truncation to the minimal theory. As we will discuss at the end of the next section, we leave a careful analysis of this truncation and its relation to minimal models with fluxes as a future project.

4 Embedding tensor, quadratic constraints and space-filling branes

The previous analysis yields a concrete prescription for finding the irreducible representations of the quadratic constraint (QC) containing all space-filling branes preserving the same supersymmetry as the given truncation. The argument we used is inspired by the properties of mixed symmetry potentials and how they are used to source exotic branes through Wess-Zumino coupling. The aim of this section is to show that the very same conclusion can be drawn independently by purely studying \mathbb{Z}_2 truncations within gauged supergravities in various dimensions to obtain theories with less supersymmetries. The concrete prescription is to compare all the QC's required by the theory with more supersymmetry when restricted to its \mathbb{Z}_2 -even sector with the ones imposed by the consistency of the theory with halved amount of supersymmetry. The mismatch between the two exactly identifies the irreducible representations containing space-filling branes allowed by the

$D \backslash \# \text{ susy}$	32	16	8	4
8	$\text{SL}(3, \mathbb{R}) \times \text{SL}(2, \mathbb{R})$	$\mathbb{R}^+ \times (\text{SL}(2, \mathbb{R}))^2$		
	(15, 1)	(3, 1)		
7	$\text{SL}(5, \mathbb{R})$	$\mathbb{R}^+ \times \text{SL}(4, \mathbb{R})$		
	70	10		
6	$\text{SO}(5, 5)$	$\mathbb{R}^+ \times \text{SO}(4, 4)$	$\mathbb{R}^+ \times \text{SO}(4, 4)$	
		$\text{SO}(5, 5)$		
	320	35_V 35_V	35_V	
		320	35_V \oplus 35_V	
	$\overline{126}$	35_S \oplus 35_C	35_S \oplus 35_C	
$\overline{126}$				
5	$\text{E}_{6(6)}$	$\mathbb{R}^+ \times \text{SO}(5, 5)$	$(\mathbb{R}^+)^2 \times \text{SO}(4, 4)$	
	$\overline{1728}$	126 320	35_V \oplus 35_V	
		210	35_S \oplus 35_C	
4	$\text{E}_{7(7)}$	$\text{SL}(2, \mathbb{R}) \times \text{SO}(6, 6)$	$(\text{SL}(2, \mathbb{R}))^3 \times \text{SO}(4, 4)$	$(\text{SL}(2, \mathbb{R}))^7$
	8645	(1, $\overline{462}$) (3, 495) (1, 2079)	(3, 35) (3², 28) (1, 350)	3³
3	$\text{E}_{8(8)}$	$\text{SO}(8, 8)$	$\text{SO}(4, 4) \times \text{SO}(4, 4)$	$(\text{SL}(2, \mathbb{R}))^8$
	147250	6435 60060	(35, 35) (350, 28) (28, 350)	3⁴

Table 11. The representations of the space-filling branes that preserve the same supersymmetry of the truncation (in red) and those that are 1/2-BPS states (in black). In six dimensions the case with 16 supersymmetry is divided in two rows, with the upper row corresponding to the 6A and the lower row to the 6B truncation. In four dimensions, the representations of the cases with eight and four supersymmetry are written in a short-hand notation which stands for eqs. (3.13) and (3.14) (eight supersymmetries) and (3.21). Similarly, in three dimensions the red representations of the theory with eight supersymmetries are given in eqs. (3.17), while those of the theory with four supersymmetries can be derived from (3.24).

less supersymmetric theory. From the technical viewpoint of gauged supergravity, such a mismatch represents the set of all closure conditions specified within the [odd, odd] sector.

We will now carry out the aforementioned analysis by treating each number of dimensions separately. This will involve the study of the allowed irreducible representations of the embedding tensor and the corresponding set of QC's it is subject to throughout a chain of \mathbb{Z}_2 truncations.

4.1 Gauged supergravities in $D = 9$

In nine spacetime dimensions 32 and 16 are the only amounts of real supercharges which are compatible with Lorentz symmetry. We shall refer to these cases as the maximal and half-maximal theory, respectively. Hence there is only one \mathbb{Z}_2 truncation to be considered here, i.e. the one relating them (see [45] for details). The most general gauged maximal theory was studied in [46], it enjoys global symmetry $\mathbb{R}^+ \times \text{SL}(2, \mathbb{R})$ and it has embedding

tensor

$$\Theta \in \underbrace{\mathbf{2}_{(+3)}}_{\theta^i} \oplus \underbrace{\mathbf{3}_{(-4)}}_{\kappa^{(ij)}} . \quad (4.1)$$

The discrete truncation yielding the half-maximal theory reads

$$\mathbb{R}^+ \times \mathrm{SL}(2, \mathbb{R}) \longrightarrow (\mathbb{R}^+)^2 , \quad (4.2)$$

however this case turns out to be rather trivial since the embedding tensor of the half-maximal theory belongs to irreducible representations without long weights. This means that even the only space-filling brane that one expects cannot be sourced by flux tadpoles.

4.2 Gauged supergravities in $D = 8$

Moving to the $D = 8$ case, the only two possibilities available are still just the maximal and half-maximal theories, which are then related again by a \mathbb{Z}_2 truncation. The most general gauged maximal theory was studied in [47–49], and it has $\mathrm{SL}(3, \mathbb{R}) \times \mathrm{SL}(2, \mathbb{R})$ global symmetry. The truncation relating the two theories is given by [45]¹⁰

$$\begin{aligned} \mathrm{SL}(3, \mathbb{R}) \times \mathrm{SL}(2, \mathbb{R}) &\longrightarrow \mathbb{R}^+ \times (\mathrm{SL}(2, \mathbb{R}))^2 , \\ \Theta \in \underbrace{(\mathbf{3}, \mathbf{2})}_{\xi_{\alpha m}} \oplus \underbrace{(\overline{\mathbf{6}}, \mathbf{2})}_{f_{\alpha}{}^{(mn)}} &\quad \underbrace{(\mathbf{2}, \mathbf{2})_{(-1)}}_{a_{\alpha i}} \oplus \underbrace{(\mathbf{2}, \mathbf{2})_{(-1)}}_{b_{\alpha i}} . \end{aligned} \quad (4.3)$$

The QC of the maximal theory

$$\begin{aligned} \epsilon^{\alpha\beta} \xi_{\alpha m} \xi_{\beta n} &= 0 , & (\overline{\mathbf{3}}, \mathbf{1}) \\ f_{(\alpha}{}^{np} \xi_{\beta)p} &= 0 , & (\overline{\mathbf{3}}, \mathbf{3}) \\ \epsilon^{\alpha\beta} (\epsilon_{mqr} f_{\alpha}{}^{qn} f_{\beta}{}^{rp} + f_{\alpha}{}^{np} \xi_{\beta m}) &= 0 , & (\overline{\mathbf{15}}, \mathbf{1}) \oplus (\overline{\mathbf{3}}, \mathbf{1}) \end{aligned} \quad (4.4)$$

reduce upon truncating to the following set of QC

$$\begin{aligned} \epsilon^{\alpha\beta} \epsilon^{ij} a_{\alpha i} a_{\beta j} &= 0 , & (\mathbf{1}, \mathbf{1})_{(-2)} \\ \epsilon^{\alpha\beta} \epsilon^{ij} b_{\alpha i} b_{\beta j} &= 0 , & (\mathbf{1}, \mathbf{1})_{(-2)} \\ \epsilon^{\alpha\beta} \epsilon^{ij} a_{\alpha i} b_{\beta j} &= 0 , & (\mathbf{1}, \mathbf{1})_{(-2)} \\ \epsilon^{ij} a_{(\alpha i} b_{\beta)j} &= 0 , & (\mathbf{3}, \mathbf{1})_{(-2)} \\ \epsilon^{\alpha\beta} a_{\alpha(i} b_{\beta j)} &= 0 , & (\mathbf{1}, \mathbf{3})_{(-2)} \end{aligned} \quad (4.5)$$

while the consistency of the half-maximal theory only requires the last two constraints (that are the constraints on the triplets) and two combinations of the three singlets. However, that does not affect our counting of space-filling objects. As far as these are concerned, the peculiarity here is that the decomposition of the QC in (4.4) would in principle yield an extra $(\mathbf{3}, \mathbf{1})$ containing one space-filling brane which cannot be sourced by any fluxes. This is due to the fact that the three-form representation of $\mathrm{SO}(2,2) \sim (\mathrm{SL}(2, \mathbb{R}))^2$ does not have any long weights.

¹⁰We are using here the notation commonly used in the gauged supergravity literature, which differs from the one adopted in section 2. This should not cause any confusion to the reader.

4.3 Gauged supergravities in $D = 7$

As far as theories in seven dimensions are concerned, the situation remains unchanged, i.e. only maximal and half-maximal supersymmetry are allowed and they are related by a \mathbb{Z}_2 truncation. The embedding tensor formulation of gauged maximal $D = 7$ supergravities was developed in [50]. The global symmetry is given by $SL(5, \mathbb{R})$, which can be subsequently broken as follows to yield a half-maximal theory

$$\begin{aligned}
 SL(5, \mathbb{R}) & \longrightarrow \mathbb{R}^+ \times SL(4, \mathbb{R}) \quad , \\
 \Theta \in & \underbrace{\overline{\mathbf{15}}}_{Y_{(MN)}} \oplus \underbrace{\mathbf{40}}_{Z^{[MN],P}} \quad \underbrace{\mathbf{1}_{(-8)}}_{\theta} \oplus \underbrace{\mathbf{6}_{(+2)}}_{\xi_{[mn]}} \oplus \underbrace{\overline{\mathbf{10}}_{(+2)}}_{Q_{(mn)}} \oplus \underbrace{\mathbf{10}_{(+2)}}_{\tilde{Q}^{(mn)}} . \quad (4.6)
 \end{aligned}$$

The QC constraints of the maximal theory

$$Y_{MQ} Z^{QN,P} + 2\epsilon_{MRSTU} Z^{RS,N} Z^{TU,P} = 0 \quad , \quad (\mathbf{5} \oplus \mathbf{45} \oplus \mathbf{70}) \quad (4.7)$$

reduce to

$$\begin{aligned}
 \theta \xi_{mn} & = 0 \quad , \quad (\mathbf{6}_{(-6)}) \\
 (\tilde{Q}^{mp} + \xi^{mp}) Q_{pn} & = 0 \quad , \quad (\mathbf{1}_{(+4)} \oplus \mathbf{15}_{(+4)}) \\
 Q_{mp} \xi^{pn} + \xi_{mp} (\tilde{Q}^{pn} + \xi^{pn}) & = 0 \quad , \quad (\mathbf{1}_{(+4)} \oplus \mathbf{15}_{(+4)}) \\
 \theta \tilde{Q}^{mn} & = 0 \quad , \quad (\mathbf{10}_{(-6)})
 \end{aligned} \quad (4.8)$$

where all of the above constraints are demanded for consistency of the half-maximal theory, except for the singlet part of the second constraint and the last constraint transforming in the $\mathbf{10}$. As we are interested in the space-filling branes, we immediately see that the four objects that we are looking for exactly coincide with the long weights in the $\mathbf{10}$. Note that this is perfect agreement with what presented in table 11.

4.4 Gauged supergravities in $D = 6$

Moving down to six dimensions, one encounters for the first time the possibility of constructing quarter-maximal theories, i.e. preserving only eight real supercharges arranged within a symplectic Majorana-Weyl (SMW) doublet. Starting from the maximal theory [51] enjoying $SO(5, 5)$ global symmetry, one can perform the following \mathbb{Z}_2 truncation

$$\begin{aligned}
 SO(5, 5) & \longrightarrow \mathbb{R}^+ \times SO(4, 4) \quad , \\
 \Theta \in & \underbrace{\mathbf{144}_C}_{\theta^{\alpha A}} \quad \underbrace{\mathbf{8}_C^{(+3)}}_{\zeta_M} \oplus \underbrace{\mathbf{8}_C^{(-1)}}_{\xi_M} \oplus \underbrace{\mathbf{56}_C^{(-1)}}_{f_{[MNP]}} \quad , \quad (4.9)
 \end{aligned}$$

to obtain a theory with sixteen supercharges realising $\mathcal{N} = (1, 1)$ supersymmetry and where the gravity multiplet is coupled to four vector multiplets.

The QC of the $\mathcal{N} = (2, 2)$ theory read

$$\begin{aligned}
 \theta^{\alpha A} \theta^{\beta B} \eta_{AB} & = 0 \quad , \quad (\mathbf{10} \oplus \mathbf{126}_C) \\
 \theta^{\alpha A} \theta^{\beta[B} (\gamma^C]_{\alpha\beta} & = 0 \quad , \quad (\mathbf{320})
 \end{aligned} \quad (4.10)$$

where η is the $SO(5, 5)$ -invariant metric, while $\{\gamma^A\}$ represent the $SO(5, 5)$ Dirac matrices. When restricting oneself to the even sector, the above set of constraints takes the following form (we furthermore set $\xi = 0$)

$$\begin{aligned}
 f_{R[MN} f_{PQ]}{}^R &= 0, & (\mathbf{35}_S^{(-2)} \oplus \mathbf{35}_C^{(-2)}) \\
 f_{MNP} \zeta^P &= 0, & (\mathbf{28}^{(+2)}) \\
 f_{MNP} f^{MNP} &= 0, & (\mathbf{1}^{(-2)}) \\
 f_{[MNP} \zeta_{Q]}|_{SD} &= 0, & (\mathbf{35}_S^{(+2)})
 \end{aligned} \tag{4.11}$$

the last two constraints not being required for the consistency of the $(1, 1)$ theory. Hence, the 8 space-filling branes found in table 11 arise here as the long weights of the $\mathbf{35}_S^{(+2)}$ of $\mathbb{R}^+ \times SO(4, 4)$.

Furthermore, a second *inequivalent* \mathbb{Z}_2 truncation is the one mentioned in the previous section giving rise to the chiral half-maximal theory with $\mathcal{N} = (2, 0)$ supersymmetry and tensorial matter. This truncation leaves the whole $SO(5, 5)$ global symmetry unbroken but it has no embedding tensor deformations. This means that, even if it has space-filling brane states available in the spectrum, we have no possibility of cancelling their charge by means of flux tadpoles.

The last step we still need to discuss within the $D = 6$ case is the one taking us from the $\mathcal{N} = (1, 1)$ to the $\mathcal{N} = (1, 0)$ theory by means of the following truncation

$$\begin{aligned}
 \mathbb{R}^+ \times SO(4, 4) & \longrightarrow \mathbb{R}^+ \times SO(4, 4), \\
 \Theta \in \underbrace{\mathbf{8}_C^{(+3)}}_{\zeta_M} \oplus \underbrace{\mathbf{8}_C^{(-1)}}_{\xi_M} \oplus \underbrace{\mathbf{56}_C^{(-1)}}_{f_{[MNP]}} & \quad \text{None} \quad , \tag{4.12}
 \end{aligned}$$

where the resulting $\mathcal{N} = (1, 0)$ theory contains tensor as well as hypermultiplets. However, no deformation parameters survive the truncation and hence no space-filling branes may be added by consistently cancelling their charge by means of flux tadpoles.

4.5 Gauged supergravities in $D = 5$

Now let us move to the five-dimensional case. Here again one can have theories with 32, 16 or 8 real supercharges. Starting from the maximal theory [52] with $E_{6(6)}$ global symmetry, the following \mathbb{Z}_2 truncation

$$\begin{aligned}
 E_{6(6)} & \longrightarrow \mathbb{R}^+ \times SO(5, 5), \\
 \Theta \in \underbrace{\overline{\mathbf{351}}}_{Z^{[AB]}} \oplus \underbrace{\mathbf{10}_{(+2)}}_{\xi_M} \oplus \underbrace{\mathbf{45}_{(-4)}}_{\zeta_{[MN]}} \oplus \underbrace{\mathbf{120}_{(+2)}}_{f_{[MNP]}} & , \tag{4.13}
 \end{aligned}$$

produces a half-maximal gauged supergravity [29] with five extra vector multiplets. Decomposing the QC's of the maximal theory and restricting ourselves to the even part, we

find the following set of QC's

$$\begin{aligned}
 \xi_M \xi^M &= 0, & (\mathbf{1}_{(+4)}) \\
 \zeta_{MN} \xi^N &= 0, & (\mathbf{10}_{(-2)}) \\
 f_{MNP} \xi^P &= 0, & (\mathbf{45}_{(+4)}) \\
 f_{R[MN} f_{PQ]}{}^R + f_{[MNP} \xi_{Q]} &= 0, & (\mathbf{210}_{(+4)}) \\
 \zeta_M{}^Q f_{NPQ} + \xi_M \zeta_{NP} &= 0, & (\mathbf{10}_{(-2)} \oplus \mathbf{120}_{(-2)} \oplus \mathbf{320}_{(-2)}) \\
 f_{MNP} f^{MNP} &= 0, & (\mathbf{1}_{(+4)}) \\
 f_{[MNP} \zeta_{QR]}|_{\text{SD}} &= 0, & (\overline{\mathbf{126}}_{(-2)})
 \end{aligned} \tag{4.14}$$

which exactly correspond to the QC of the half-maximal theory, plus the last two lines transforming in the $\mathbf{1}_{(+4)}$ and $\overline{\mathbf{126}}_{(-2)}$ as two additional ones. Therefore, the spacefilling branes of this theory are given by the 16 long weights inside the latter extra QC irreducible representation. This is once again in agreement with what found in the previous section.

We now want to further truncate the half-maximal theory to obtain a quarter-maximal theory coupled to four hypermultiplets. This is done through

$$\begin{aligned}
 \mathbb{R}^+ \times \text{SO}(5,5) &\longrightarrow (\mathbb{R}^+)^2 \times \text{SO}(4,4), \\
 \Theta \in \underbrace{\mathbf{10}}_{\xi_M} \oplus \underbrace{\mathbf{45}}_{\zeta_{[MN]}} \oplus \underbrace{\mathbf{120}}_{f_{[MNP]}} & \quad 3 \times (\mathbf{1} \oplus \mathbf{28}), \tag{4.15}
 \end{aligned}$$

where the resulting embedding tensor can be rearranged into the *reducible* object denoted by Θ_Λ^α , where in turn, the index Λ labels the vectors, while α runs over $\text{adj}(\mathbb{R}^+) \oplus \text{adj}(\mathbb{R}^+) \oplus \text{adj}(\text{SO}(4,4))$. The QC's (4.14) then reduce to the following irreducible representations:

$$\begin{aligned}
 \mathbf{1} &\longrightarrow \mathbf{1}, & , \\
 \mathbf{10} &\longrightarrow \mathbf{1} \oplus \mathbf{1} \oplus \text{odd}, & , \\
 \mathbf{45} &\longrightarrow \mathbf{1} \oplus \mathbf{28} \oplus \text{odd}, & , \\
 \mathbf{210} &\longrightarrow \mathbf{28} \oplus \mathbf{28} \oplus \mathbf{35}_S \oplus \mathbf{35}_C \oplus \text{odd}, & , \\
 \mathbf{120} &\longrightarrow \mathbf{28} \oplus \mathbf{28} \oplus \text{odd}, & , \\
 \mathbf{320} &\longrightarrow \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{28} \oplus \mathbf{35}_V \oplus \mathbf{35}_V \oplus \text{odd}, & ,
 \end{aligned} \tag{4.16}$$

to be compared with those ones appearing in the QC's of the $\mathcal{N} = 1$ [27]

$$\Theta_\Lambda^\alpha \Theta_\Sigma^\beta f_{\alpha\beta}{}^\gamma + [t_\alpha]_\Lambda{}^\Gamma \Theta_\Sigma^\alpha \Theta_\Gamma{}^\gamma = 0, \tag{4.17}$$

where $\{f_{\alpha\beta}{}^\gamma\}$ and $\{[t_\alpha]_\Lambda{}^\Gamma\}$ are the global symmetry generators in the adjoint and vector representation, respectively. The unneeded QC irreducible representations containing the longest weights turn out to be $\mathbf{35}_S \oplus \mathbf{35}_C \oplus \mathbf{35}_V \oplus \mathbf{35}_V$, once again in perfect agreement with the analysis of section 3.

4.6 Gauged supergravities in $D = 4$

In $D = 4$, the minimal amount of supersymmetry which is consistent with Lorentz symmetry is given by four real supercharges rearranged into a single Majorana spinor. This implies the extra possibility in this case to further truncate a quarter-maximal theory down to a minimal one. The most general maximal gauged theory was studied in [28] and it turns out to enjoy $E_{7(7)}$ global symmetry. Furthermore, in [23], the truncation defined by

$$\begin{aligned}
 E_{7(7)} &\longrightarrow \mathrm{SL}(2, \mathbb{R}) \times \mathrm{SO}(6, 6) , \\
 \Theta \in &\underbrace{\mathbf{912}}_{X_{\mathcal{MNP}}} \quad \underbrace{(\mathbf{2}, \mathbf{12})}_{\xi_{\alpha M}} \oplus \underbrace{(\mathbf{2}, \mathbf{220})}_{f_{\alpha[MNP]}}, \tag{4.18}
 \end{aligned}$$

was found to yield a half-maximal theory [29] coupled to six extra vector multiplets.

The QC of the maximal theory

$$\Omega^{MQ} X_{\mathcal{MNP}} X_{QRS} = 0, \quad (\mathbf{133} \oplus \mathbf{8645}) \tag{4.19}$$

give rise to the following QC's for \mathbb{Z}_2 even objects

$$\begin{aligned}
 \xi_{\alpha M} \xi_{\beta}^M &= 0, & (\mathbf{3}, \mathbf{1}) \\
 \xi_{(\alpha}^P f_{\beta)MNP} &= 0, & (\mathbf{3}, \mathbf{66}) \\
 3 f_{\alpha R[MN} f_{\beta PQ]}^R - 2 f_{(\alpha[MNP} \xi_{\beta)Q]} &= 0, & (\mathbf{3}, \mathbf{495}) \\
 \epsilon^{\alpha\beta} (\xi_{\alpha}^P f_{\beta PMN} + \xi_{\alpha M} \xi_{\beta N}) &= 0, & (\mathbf{1}, \mathbf{66}) \\
 \epsilon^{\alpha\beta} f_{\alpha MNR} f_{\beta PQ}^R + (f \xi \text{ terms}) &= 0, & (\mathbf{1}, \mathbf{66}) \oplus (\mathbf{1}, \mathbf{2079}) \\
 f_{\alpha MNP} f_{\beta}^{MNP} &= 0, & (\mathbf{3}, \mathbf{1}) \\
 \epsilon^{\alpha\beta} f_{\alpha[MNP} f_{\beta QRS]}|_{\mathrm{SD}} &= 0, & (\mathbf{1}, \overline{\mathbf{462}})
 \end{aligned} \tag{4.20}$$

where one can recognise all the QC's of $\mathcal{N} = 4$ supergravity, plus the last two lines which therefore contain space-filling branes. In particular, the $(\mathbf{1}, \overline{\mathbf{462}})$ contains exactly the 32 long weights that we expect from the results of the previous section.

To further truncate to the quarter-maximal theories, we perform the following truncation [53]

$$\begin{aligned}
 \mathrm{SL}(2, \mathbb{R}) \times \mathrm{SO}(6, 6) &\longrightarrow (\mathrm{SL}(2, \mathbb{R}))^3 \times \mathrm{SO}(4, 4) , \\
 \Theta \in &\underbrace{(\mathbf{2}, \mathbf{12})}_{\xi_{\alpha M}} \oplus \underbrace{(\mathbf{2}, \mathbf{220})}_{f_{\alpha[MNP]}} \quad (\mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{28}), \tag{4.21}
 \end{aligned}$$

where once more we may regroup the embedding tensor of the $\mathcal{N} = 2$ theory into the object Θ_{Λ}^A , where Λ takes values in the $(\mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{1})$, while the index A spans the whole adjoint representation of the global symmetry group.

The $\mathcal{N} = 4$ QC irreducible representations break into

$$\begin{aligned}
(\mathbf{3}, \mathbf{1}) &\longrightarrow (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \quad , \\
(\mathbf{3}, \mathbf{66}) &\longrightarrow (\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{28}) \oplus \text{odd} \quad , \\
(\mathbf{3}, \mathbf{495}) &\longrightarrow (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{35}_S) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{35}_C) \\
&\quad \oplus (\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{28}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{28}) \oplus \text{odd} \quad , \\
(\mathbf{1}, \mathbf{66}) &\longrightarrow (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{28}) \oplus \text{odd} \quad , \\
(\mathbf{1}, \mathbf{2079}) &\longrightarrow (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{1}) \\
&\quad \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{28}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{28}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{28}) \\
&\quad \oplus (\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{28}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{350}) \\
&\quad \oplus (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{35}_V) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{35}_V) \oplus \text{odd} \quad .
\end{aligned} \tag{4.22}$$

The QC's demanded for consistency of the $\mathcal{N} = 2$ theory read [27]

$$\begin{aligned}
\Theta_\Lambda^\alpha \Theta_\Sigma^\beta f_{\alpha\beta}{}^\gamma + [t_\alpha]_\Lambda^\Gamma \Theta_\Sigma^\alpha \Theta_\Gamma^\gamma &= 0 \quad , \\
\Theta^{\Lambda[\alpha} \Theta_\Lambda^{\beta]} &= 0 \quad ,
\end{aligned} \tag{4.23}$$

where $\{f_{\alpha\beta}{}^\gamma\}$ and $\{[t_\alpha]_\Lambda^\Gamma\}$ are the global symmetry generators in the adjoint and vector representation, respectively. The unneeded QC irreducible representations containing the longest weights turn out to be $(\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{35}_S) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{35}_C) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{35}_V) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{35}_V)$, once again in perfect agreement with the analysis performed in the previous section.

A last step that can be discussed here in four dimensions is the one further breaking supersymmetry to $\mathcal{N} = 1$. This truncation is concretely realised as follows

$$\begin{aligned}
(\text{SL}(2, \mathbb{R}))^3 \times \text{SO}(4, 4) &\longrightarrow (\text{SL}(2, \mathbb{R}))^7 \quad , \\
\Theta \in (\mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{28}) &\quad (\mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2}) \quad .
\end{aligned} \tag{4.24}$$

However, the minimal theory one ends up with is purely coupled to chiral multiplets and hence it possesses *no vector fields*. As a consequence, the obtained supergravity model will reorganise the embedding tensor deformations surviving the above truncation into massive deformations not associated with any gauging. In particular, in this model all of them may be interpreted as parameters inducing a holomorphic superpotential in the seven complex scalar fields. Due to this, we do not have any QC's required for consistency and we do expect all of them to be sourced by space-filling branes. This class of theories will be studied in detail in the next section, where these parameters will be interpreted as the $2^7 = 128$ generalised fluxes coming from an orbifold compactification of type IIB string theory.

To conclude this section, we quickly comment on the three-dimensional case. The embedding tensor of the maximal theory belongs to the $\mathbf{1} \oplus \mathbf{3875}$ of $E_{8(8)}$ [24], while the quadratic constraint belongs to the $\mathbf{3875} \oplus \mathbf{147250}$ [54]. By truncating the theory to the half-maximal one with symmetry $\text{SO}(8, 8)$, one can show that the embedding tensor is truncated to the one of the half-maximal theory [25], while the quadratic constraints are truncated to the quadratic constraints of the half-maximal theory plus extra constraints,

and the highest-dimensional representation of such remaining constraints is the **6435**, in agreement with the results of the previous section (see table 11). Similarly, one can study how the further truncations to the theories with eight and four supersymmetries give patterns for the quadratic constraints in agreement with table 11. Moreover, the theory with four supersymmetries in three dimensions can be further truncated to the minimal theory. Just as in the 4D minimally supersymmetric case, we expect no quadratic constraint to be present here and hence all highest weights surviving the branching of the QC's will correspond to exotic space-filling brane states. Effective descriptions of this type can be e.g. obtained by compactifying M-theory on Joyce 8-manifolds of Spin(7) holonomy. Internal manifolds of this type admit an orbifold limit where they are described as T^8/Γ , where the discrete symmetry Γ can be e.g. \mathbb{Z}_2^4 . These M-theory backgrounds are also known to have perturbative corners given by type IIA orientifolds of Joyce 7-manifolds of G_2 holonomy [55], or heterotic strings on such G_2 -manifolds [56]. However, we leave a careful analysis of all these features of the three-dimensional case as a future project.

5 IIB on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, fluxes and Bianchi identities

As explained at the end of the previous section, performing three \mathbb{Z}_2 truncations on a maximal gauged supergravity theory in four dimensions yields an $\mathcal{N} = 1$ supergravity model where the supergravity multiplet is coupled to seven chiral multiplets. The scalar sector of the theory contains seven complex fields spanning the coset space $(\text{SL}(2, \mathbb{R})/\text{SO}(2))^7$ which are usually denoted by $\Phi^\alpha \equiv (S, T_i, U_i)$ with $i = 1, 2, 3$. The kinetic Lagrangian follows from the Kähler potential

$$K = -\log(-i(S - \bar{S})) - \sum_{i=1}^3 \log(-i(T_i - \bar{T}_i)) - \sum_{i=1}^3 \log(-i(U_i - \bar{U}_i)), \quad (5.1)$$

yielding

$$\mathcal{L}_{\text{kin}} = \frac{\partial S \partial \bar{S}}{(-i(S - \bar{S}))^2} + \sum_{i=1}^3 \left(\frac{\partial T_i \partial \bar{T}_i}{(-i(T_i - \bar{T}_i))^2} + \frac{\partial U_i \partial \bar{U}_i}{(-i(U_i - \bar{U}_i))^2} \right). \quad (5.2)$$

The presence of fluxes induces a scalar potential V for the scalar fields which is given in terms of the above Kähler potential and a holomorphic superpotential W by

$$V = e^K \left(-3|W|^2 + K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} \right), \quad (5.3)$$

where $K^{\alpha\bar{\beta}}$ is the inverse of the aforementioned Kähler metric and D_α denotes the Kähler-covariant derivative.

As already mentioned earlier, the superpotential W is induced by the 128 deformation parameters surviving the truncation of the **912** of $E_{7(7)}$ with respect to \mathbb{Z}_2^3 . The form of such superpotential is given by the most general polynomial in (S, T_i, U_i) without any mixed terms, i.e.

$$W \sim 1 + \dots + S T_1 T_2 T_3 U_1 U_2 U_3, \quad (5.4)$$

which precisely includes 128 terms, each of which is induced by its own deformation parameter.

In what follows we will review how the above class of minimal supergravities arise from orbifold compactifications of type IIB string theory down to four dimensions. Before moving to that analysis though, let us note that the consistency of the $\mathcal{N} = 1$ theory *only* requires all massive deformations of this type to be arranged into a holomorphic superpotential. In our specific case this requirement is automatically fulfilled by any polynomial function of the type given in (5.4), while no further QC's on its coefficients are needed. This means that any QC in terms of the superpotential couplings surviving the \mathbb{Z}_2^3 truncation is expected to be relaxed in our compactification yielding minimal supersymmetry by means of space-filling branes.

As previously anticipated, the above minimal supergravity models with seven chiral multiplets arise from dimensional reductions of type IIB string theory on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with O3/O7-planes.¹¹ The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold acts on the six internal coordinates precisely as described below equation (3.21). What further realises a supersymmetry breaking down to a minimal amount is a \mathbb{Z}_2 flipping the sign of all the coordinates on the T^6 . There need to be O3-planes located at each fixed point of this involution, while a triplet of O7-planes are placed at fixed points of those involutions obtained by combining this last \mathbb{Z}_2 with the three non-trivial generators of the orbifold group.

Adopting the type IIB language, the seven complex scalars of the $\mathcal{N} = 1$ model have the following physical interpretation

$$\begin{aligned} S &\leftrightarrow \text{Axiodilaton} & , \\ T_i &\leftrightarrow \text{Kähler moduli} & , \\ U_i &\leftrightarrow \text{Complex structure moduli} . \end{aligned} \tag{5.5}$$

The first understanding of the mechanism that perturbatively induces a superpotential from fluxes in this context is due to [58], where a superpotential of the form

$$W_{\text{GVW}} = \underbrace{P_F(U_i)}_{F \text{ flux}} + S \underbrace{P_H(U_i)}_{H \text{ flux}} , \tag{5.6}$$

where P_F & P_H are cubic polynomials in the complex structure moduli controlled by R-R and NS-NS three-form fluxes with different legs in internal space. Superpotentials of this type were found to describe special type IIB Minkowski backgrounds [59] with no-scale symmetry due to the absence of T -dependence in W .

In [60] it was argued, on the basis of string dualities, that the superpotential in (5.6) should be generalised to contain new fluxes which are named *non-geometric*. Subsequently, in [61] the set of generalised fluxes was further enlarged to include the complete set of objects closed under perturbative and non-perturbative string dualities. The corresponding superpotential reads

$$\begin{aligned} W = & \underbrace{P_F(U_i)}_{F \text{ flux}} + S \underbrace{P_H(U_i)}_{H \text{ flux}} + \sum_k T_k \underbrace{P_Q^{(k)}(U_i)}_{Q \text{ flux}} + S \sum_k T_k \underbrace{P_P^{(k)}(U_i)}_{P \text{ flux}} \\ & + T_1 T_2 T_3 \left(\underbrace{P_{F'}(U_i)}_{F' \text{ flux}} + S \underbrace{P_{H'}(U_i)}_{H' \text{ flux}} \right) + \sum_k T_i T_j \left(\underbrace{P_{Q'}^{(k)}(U_i)}_{Q' \text{ flux}} + S \underbrace{P_{P'}^{(k)}(U_i)}_{P' \text{ flux}} \right) , \end{aligned} \tag{5.7}$$

¹¹See for instance [57] for a review on orientifold models with fluxes.

where P_F , P_H , $P_Q^{(k)}$ and $P_P^{(k)}$ are cubic polynomials in the complex structure moduli given by

$$\begin{aligned}
 P_F(U_i) &= a_0 - \sum_i a_1^{(i)} U_i + \sum_i a_2^{(i)} \frac{U_1 U_2 U_3}{U_i} - a_3 U_1 U_2 U_3 \quad , \\
 P_H(U_i) &= -b_0 + \sum_i b_1^{(i)} U_i - \sum_i b_2^{(i)} \frac{U_1 U_2 U_3}{U_i} + b_3 U_1 U_2 U_3 \quad , \\
 P_Q^{(k)}(U_i) &= c_0^{(k)} + \sum_i c_1^{(ik)} U_i - \sum_i c_2^{(ik)} \frac{U_1 U_2 U_3}{U_i} - c_3^{(k)} U_1 U_2 U_3 \quad , \\
 P_P^{(k)}(U_i) &= -d_0^{(k)} - \sum_i d_1^{(ik)} U_i + \sum_i d_2^{(ik)} \frac{U_1 U_2 U_3}{U_i} + d_3^{(k)} U_1 U_2 U_3 \quad ,
 \end{aligned}
 \tag{5.8}$$

while $P_{F'}$, $P_{H'}$, $P_{Q'}^{(k)}$ and $P_{P'}^{(k)}$ are cubic polynomials in the complex structure moduli given by

$$\begin{aligned}
 P_{F'}(U_i) &= a'_3 - \sum_i a'_2^{(i)} U_i + \sum_i a'_1^{(i)} \frac{U_1 U_2 U_3}{U_i} - a'_0 U_1 U_2 U_3 \quad , \\
 P_{H'}(U_i) &= -b'_3 + \sum_i b'_2^{(i)} U_i - \sum_i b'_1^{(i)} \frac{U_1 U_2 U_3}{U_i} + b'_0 U_1 U_2 U_3 \quad , \\
 P_{Q'}^{(k)}(U_i) &= c'_3{}^{(k)} + \sum_i c'_2{}^{(ik)} U_i - \sum_i c'_1{}^{(ik)} \frac{U_1 U_2 U_3}{U_i} - c'_0{}^{(k)} U_1 U_2 U_3 \quad , \\
 P_{P'}^{(k)}(U_i) &= -d'_3{}^{(k)} - \sum_i d'_2{}^{(ik)} U_i + \sum_i d'_1{}^{(ik)} \frac{U_1 U_2 U_3}{U_i} + d'_0{}^{(k)} U_1 U_2 U_3 \quad .
 \end{aligned}
 \tag{5.9}$$

Note that the superpotential in (5.7) exactly comprises the aforementioned 128 terms coming from the \mathbb{Z}_2^3 truncation of the embedding tensor of maximal gauged supergravity in four dimensions.

Once we understood these orbifold compactifications of type IIB with fluxes as \mathbb{Z}_2^3 truncations of maximal gauged supergravities in $D = 4$, we would now like to interpret all the QC irreducible representations accordingly as quadratic conditions for the fluxes which may be sourced by space-filling objects in string theory. By applying our prescription, we can identify all the space-filling branes as the longest weights of the QC irreducible representations of the $\mathcal{N} = 2$ theory decomposed by performing the last \mathbb{Z}_2 truncation. All of the other long weights contained in the rest of the \mathbb{Z}_2 even QC irreducible representations which were unneeded in the two previous steps of the truncation are interpreted as *Bianchi Identities* (BI), i.e. consistency constraints of the background itself. To be more explicit, the BI's can be seen as conditions for the absence of extra space-filling objects whose negative-tension counterparts realise the background itself. Mathematically, those quadratic constraints can be interpreted as conditions enforcing the closure of the flux-twisted exterior derivative operator.

By following the above prescription, let us now proceed to identify the space-filling brane states and the BI's for type IIB compactifications on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$. The truncation

net reads¹²

$$\begin{array}{ccc}
 E_{7(7)} & & \\
 \downarrow \mathbb{Z}_2 & & \\
 \text{SL}(2, \mathbb{R}) \times \text{SO}(6, 6) & \rightarrow & (\mathbf{1}, \overline{\mathbf{462}}) \\
 \downarrow \mathbb{Z}_2 & & \\
 \text{SL}(2, \mathbb{R})_U^3 \times \text{SO}(4, 4) & \rightarrow & (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{35}_S) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{35}_C) \oplus \\
 & & \oplus (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{35}_V) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{35}_V) \\
 \downarrow \mathbb{Z}_2 & & \\
 \text{SL}(2, \mathbb{R})^7 & &
 \end{array} \tag{5.10}$$

where the irreducible representations pulled out on the right of the above diagram will precisely give rise to the BI's of our theory, upon truncation down to $\text{SL}(2, \mathbb{R})^7$, while the space-filling branes will be captured by those QC irreducible representations surviving the triple discrete truncation, i.e.

$$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{350}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{28}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{3}, \mathbf{28}) \oplus (\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{28}) \oplus \text{shorter weights},$$

of $\text{SL}(2, \mathbb{R})_U^3 \times \text{SO}(4, 4)$. Further decomposition down to $\text{SL}(2, \mathbb{R})^7$ yields the following space-filling branes

$$\begin{array}{ll}
 (\mathbf{3}_{T_i}, \mathbf{3}_{T_j}, \mathbf{3}_{T_k}) & (1 \text{ irrep}) , \\
 (\mathbf{3}_S, \mathbf{3}_{T_i}, \mathbf{3}_{T_j}) & (3 \text{ irrep's}) , \\
 (\mathbf{3}_S, \mathbf{3}_{U_i}, \mathbf{3}_{U_j}) & (3 \text{ irrep's}) , \\
 (\mathbf{3}_{T_i}, \mathbf{3}_{U_j}, \mathbf{3}_{U_k}) & (3 \text{ irrep's}) , \\
 (\mathbf{3}_{T_i}, \mathbf{3}_{U_i}, \mathbf{3}_{U_j}) & (6 \text{ irrep's}) ,
 \end{array} \tag{5.11}$$

which precisely contain the $16 \times 2^3 = 2^7 = 128$ space-filling branes we needed from the previous analysis, while for the BI's one finds

$$\begin{array}{ll}
 (\mathbf{3}_{T_i}, \mathbf{3}_{T_j}, \mathbf{3}_{U_j}) & (6 \text{ irrep's}) , \\
 (\mathbf{3}_S, \mathbf{3}_{T_i}, \mathbf{3}_{U_j}) & (6 \text{ irrep's}) ,
 \end{array} \tag{5.12}$$

containing $12 \times 2^3 = 96$ BI's in total. Note that this perfectly matches the results of [30], to which the present analysis can be regarded as an independent check.

A further physical comment concerning the possibility of relaxing the constraints. As already stated above, each and every quadratic condition for type IIB fluxes appearing in (5.11) can be relaxed by adding the correspondent space-filling (exotic) brane sourcing the associated flux tadpole. All those branes are consistently preserving the same four real supercharges as the very background. On the other hand, it becomes very natural to ask about a similar possibility for the BI's in (5.12). Indeed, just as any other algebraic constraint, these are not needed for consistency. Their physical interpretation is that of

¹²The consistent identification for the different $\text{SL}(2, \mathbb{R})$ labels is $\text{SL}(2, \mathbb{R})^7 \equiv \text{SL}(2, \mathbb{R})_{U_1} \times \text{SL}(2, \mathbb{R})_{U_2} \times \text{SL}(2, \mathbb{R})_{U_3} \times \text{SL}(2, \mathbb{R})_{T_1} \times \text{SL}(2, \mathbb{R})_{T_2} \times \text{SL}(2, \mathbb{R})_{T_3} \times \text{SL}(2, \mathbb{R})_S$.

enforcing the condition for closure of the flux-twisted exterior derivative operator defined on our CY background. Our intuition seems to suggest that the original CY will be deformed into an G -structure manifold as an effect of the backreaction of fluxes to the background geometry. In this context, the twisted exterior derivative operator will now receive torsion-induced contributions. Following the philosophy of [62], part of the contributions to the internal curvature can be interpreted as the presence of space-filling KK-monopoles and T-duals thereof.

6 Conclusions

In this paper we have considered the supergravity theories that arise as sequences of \mathbb{Z}_2 truncations of the maximal theories. We have determined in all cases the 1/2-BPS space-filling branes that preserve the supersymmetry of the truncated theory and the representations of the symmetry of such theory to which they belong. We have then discussed all the possible gaugings of these theories as described in terms of the embedding tensor. We have shown that for any theory, among the representations of the quadratic constraint on the embedding tensor that survive the truncation but are not needed for supersymmetry, the highest-dimensional ones are precisely those of the 1/2-BPS space-filling branes that preserve the same supersymmetry of the truncated theory. This can be naturally interpreted as the fact that these quadratic constraints after the truncation become tadpole conditions for such branes.

We point out that the number of different \mathbb{Z}_2 truncations of a given supergravity theory, that from a group-theory point of view is given by the number of different ways in which the symmetry of the truncated theory can be embedded in the symmetry of the original one, is also given by the number of vector central charges of the supersymmetry algebra. This rather intriguing result shows once again the deep relation between supersymmetry and group theory.

Although the analysis in this paper was performed uniquely in terms of the branes of the IIB theory, it would be interesting to reinterpret this from the point of view of IIA and also from the point of view of M-theory. In particular, the analysis of the tadpole conditions for all the possible fluxes that can be included in the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold was performed in [30] also in the case of the IIA O6 orientifold, and this could then be compared with the techniques developed in this paper.

Moreover, one can extend the three-dimensional case to include a more detailed analysis. In particular, the minimally supersymmetric gauged theories turn out to have fewer constraints coming from consistency and supersymmetry [63] and hence a further truncation down to two real supercharges could be a very valuable venue for string model-building. Three-dimensional theories with minimal supersymmetry can be obtained by compactifying M-theory on Joyce 8-manifolds of Spin(7) holonomy, and internal manifolds of this type admit an orbifold limit where they are described as T^8/Γ , where the discrete symmetry Γ can be e.g. \mathbb{Z}_2^4 , so that our techniques can be applied to this case. One can also use the methods presented in this paper to investigate the duality relations with perturbative corners given by type IIA orientifolds of Joyce 7-manifolds of G_2 holonomy [55], or heterotic

strings on such G_2 -manifolds [56]. We leave a careful analysis of all these features of the three-dimensional case as a future project.

Finally, we stress again that the vast majority of the branes discussed in this paper are exotic, in the sense that they do not have a clear higher dimensional origin. It would be of extreme interest to get any understanding of the dynamics of these objects, that in our analysis must be included for symmetry arguments. This would dramatically improve our understanding of string theory and our ability to construct models.

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A $D = 8$ spinor conventions

In this appendix we discuss in detail all the spinor conventions that we have adopted in section 2. All the fermions of the eight-dimensional maximal theory are spinors of $SO(3) \times SO(2)$. We have denoted with σ_m ($m = 1, 2, 3$) the Pauli matrices of $SO(3)$. The gamma matrices of $SO(2)$ are the first two Pauli matrices, that we denote with τ_a ($a = 1, 2$), while the third Pauli matrix is the chirality matrix of $SO(2)$,

$$\tau_a \tau_b = i \epsilon_{ab} \tau_3 \quad . \quad (A.1)$$

As eq. (2.4) shows, all the spinors are chiral with respect to $\gamma_9 \tau_3$. They also satisfy the Majorana condition in eq. (2.5), with $C = C_8 \sigma_2 \tau_1$ as in eq. (2.6). The matrix C_8 is defined in eq. (2.7), and commutes with γ_9 . On the other hand, the matrix τ_1 anticommutes with τ_3 , and hence the matrix C and $\gamma_9 \tau_3$ anticommute, so that the chirality conditions in eq. (2.4) and the Majorana condition in eq. (2.5) are compatible. We call the Majorana conditions in eq. (2.5) ‘symplectic’ because C_8 is symmetric and satisfies the condition in eq. (2.7) with the minus sign, so that the condition $\Psi = C_8 \bar{\Psi}^T$ would not be consistent. This is standard in the supergravity literature.

We now want to discuss the reality properties and the properties under Majorana flip of the various fermionic bilinears that can be constructed, and in particular of the ones that occur in section 2. We make use of the identities

$$\begin{aligned} C^\dagger \gamma_\mu C &= -\gamma_\mu^T \\ C^\dagger \sigma_m C &= -\sigma_m^T \\ C^\dagger \tau_a C &= \tau_a^T \\ C^\dagger \tau_3 C &= -\tau_3 \quad , \end{aligned} \quad (A.2)$$

bilinear	reality property	flip
$\bar{\psi}\chi$	real	even
$\bar{\psi}\gamma_\mu\chi$	imaginary	odd
$\bar{\psi}\sigma_m\chi$	imaginary	odd
$\bar{\psi}\tau_a\chi$	real	even
$\bar{\psi}\tau_3\chi$	imaginary	odd
$\bar{\psi}\gamma_{\mu\nu}\chi$	real	odd
$\bar{\psi}\gamma_\mu\sigma_m\chi$	real	even
$\bar{\psi}\gamma_\mu\tau_a\chi$	imaginary	odd
$\bar{\psi}\gamma_\mu\tau_3\chi$	real	even
$\bar{\psi}\sigma_m\tau_a\chi$	imaginary	odd
$\bar{\psi}\sigma_m\tau_3\chi$	real	even
$\bar{\psi}\gamma_{\mu\nu\rho}\chi$	imaginary	even
$\bar{\psi}\gamma_{\mu\nu}\sigma_m\chi$	imaginary	even
$\bar{\psi}\gamma_{\mu\nu}\tau_a\chi$	real	odd
$\bar{\psi}\gamma_{\mu\nu}\tau_3\chi$	imaginary	even
$\bar{\psi}\gamma_\mu\sigma_m\tau_a\chi$	real	even
$\bar{\psi}\gamma_\mu\sigma_m\tau_3\chi$	imaginary	odd
$\bar{\psi}\gamma_{\mu\nu\rho\sigma}\chi$	real	even
$\bar{\psi}\gamma_{\mu\nu\rho}\sigma_m\chi$	real	odd
$\bar{\psi}\gamma_{\mu\nu\rho}\tau_a\chi$	imaginary	even
$\bar{\psi}\gamma_{\mu\nu\rho}\tau_3\chi$	real	odd
$\bar{\psi}\gamma_{\mu\nu}\sigma_m\tau_a\chi$	imaginary	even
$\bar{\psi}\gamma_{\mu\nu}\sigma_m\tau_3\chi$	real	odd
$\bar{\psi}\gamma_{\mu\nu\rho\sigma}\sigma_m\chi$	imaginary	odd
$\bar{\psi}\gamma_{\mu\nu\rho\sigma}\tau_a\chi$	real	even
$\bar{\psi}\gamma_{\mu\nu\rho}\sigma_m\tau_a\chi$	real	odd
$\bar{\psi}\gamma_{\mu\nu\rho}\sigma_m\tau_3\chi$	imaginary	even
$\bar{\psi}\gamma_{\mu\nu\rho\sigma}\sigma_m\tau_a\chi$	imaginary	odd

Table 12. The reality properties and the properties under Majorana flip of various fermionic bilinears. The analogous properties for all the other bilinears can be derived from those in this table using the duality relations in eq. (A.3).

the first of which is the same as eq. (2.7). A Majorana spinor Ψ , satisfying eq. (2.5), also satisfies $\bar{\Psi} = -\Psi^T C^\dagger$. As a consequence, for instance the bilinear $\bar{\psi}\sigma_m\tau_a\chi$ is equal to $-\bar{\chi}\sigma_m\tau_a\psi$, which means that the bilinear is odd under Majorana flip. By complex conjugation the bilinear goes to $\bar{\chi}\sigma_m\tau_a\psi$, which is minus the bilinear itself because of Majorana flip. As a consequence, the bilinear is purely imaginary. One can easily generalise this to get the reality conditions and the Majorana-flip properties for all the bilinears. We give a summary of the properties of the various bilinears in table 12.

We now want to derive the duality relations among different bilinears. Starting from the definition of the γ_9 matrix given in eq. (2.3), by multiple contractions from the left with gamma matrices one gets

$$\gamma_{\mu_1 \dots \mu_m} = -\frac{(-)^{\lfloor \frac{n+1}{2} \rfloor} i}{n!} \epsilon_{\mu_1 \dots \mu_m \nu_1 \dots \nu_n} \gamma^{\nu_1 \dots \nu_n} \gamma_9 \quad m+n=8 \quad . \quad (\text{A.3})$$

Using this relation and the chirality properties in eq. (2.4), the properties of the bilinears that contain the matrix $\gamma_{\mu_1 \dots \mu_m}$ are related to those of the bilinears that contain the matrix $\gamma_{\nu_1 \dots \nu_n} \tau_3$. In particular from the bilinears given in table 12 one can derive all the others.

Given a spinor Ψ satisfying $\gamma_9 \tau_3 \Psi = \Psi$, for the particular case of $m=n=4$ one gets

$$\gamma_{\mu_1 \dots \mu_4} \Psi = -\frac{i}{4!} \epsilon_{\mu_1 \dots \mu_4 \nu_1 \dots \nu_4} \gamma^{\nu_1 \dots \nu_4} \tau_3 \Psi \quad . \quad (\text{A.4})$$

Contracting from the left with τ_a and using eq. (A.1), one then obtains the self-duality condition

$$\gamma_{\mu_1 \dots \mu_4} \tau_a \Psi = -\frac{1}{4!} \epsilon_{\mu_1 \dots \mu_4 \nu_1 \dots \nu_4} \epsilon_{ab} \gamma^{\nu_1 \dots \nu_4} \tau_b \Psi \quad . \quad (\text{A.5})$$

In general, in eight dimensions one can impose on a doublet of 4-forms $X_{\mu_1 \dots \mu_4 a}$ the self-duality condition

$$X_{\mu_1 \dots \mu_4 a} = \frac{\alpha}{4!} \epsilon_{\mu_1 \dots \mu_4 \nu_1 \dots \nu_4} \epsilon_{ab} X^{\nu_1 \dots \nu_4 b} \quad , \quad (\text{A.6})$$

where α can be either 1 or -1 . In particular, eq. (A.5) corresponds to the case $\alpha = -1$. On the other hand, in section 2 we have shown that from the field-strength $F_{\mu\nu\rho\sigma}^A$ of the 3-form potential $A_{\mu\nu\rho}^A$ one can construct the composite quantity $F_{\mu\nu\rho\sigma}^A V_{Aa}$ that satisfies the self-duality relation in eq. (2.24), corresponding again to the case $\alpha = -1$ in eq. (A.6). In general, if $X_{\mu_1 \dots \mu_4 a}$ and $Y_{\mu_1 \dots \mu_4 a}$ satisfy eq. (A.6) with the same α , one can prove that the following identities hold:

$$X_{\mu\nu\rho\sigma a} Y^{\mu\nu\rho\sigma a} = 0 \quad X_{\mu\nu\rho\sigma a} Y^{\mu\nu\rho\sigma b} \epsilon_{ab} = 0 \quad . \quad (\text{A.7})$$

These relations have been used to prove the closure of the supersymmetry algebra on the 3-forms in section 2.

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