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DBI action of real linear superfield in 4D $\mathcal{N} = 1$ conformal supergravity

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Abstract: We construct the Dirac-Born-Infeld (DBI) action of a real linear multiplet in 4D $\mathcal{N} = 1$ supergravity. Based on conformal supergravity, we derive the general condition under which the DBI action can be realized, and show that it can be constructed in the new minimal supergravity. We also generalize it to the matter coupled system.

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Contents

1 Introduction

Higher-order derivative terms play important roles in the several contexts, e.g., inflation models, modified gravity, renormalization of gravity, and so on. From a phenomenological and theoretical viewpoint, their embeddings into supersymmetry (SUSY) or supergravity (SUGRA) are also interesting. In particular, there exist many non-renormalizable terms in SUGRA and it is quite natural to consider the extension including higher-order derivative terms and the effects of them on cosmology and particle phenomenology. The higher-order derivative terms of a chiral superfield in 4D SUSY or SUGRA and their cosmological applications have been investigated so far, e.g., in refs. [1–13].

The Dirac-Born-Infeld (DBI) action [14, 15] includes such higher-order derivative terms. It was first proposed as a nonlinear generalization of Maxwell theory. The DBI action is also motivated by string theory, which is a promising candidate for a unified theory including gravity. In the context of string theory, an effective action of D-brane is described by a DBI-type action, which consists of Maxwell terms $F_{\mu\nu}$ as well as the ones of scalar fields $\partial_\mu\phi^i\partial_\nu\phi^j g_{ij}$ and a two-form $B_{\mu\nu}$ in general,

$$S_{\rm DBI} = \int d^D x\sqrt{-g}\left(1-\sqrt{\det(g_{\mu\nu}+\partial_\mu\phi^i\partial_\nu\phi^j g_{ij}+B_{\mu\nu}+F_{\mu\nu})}\right). \tag{1.1}$$

SUSY Dp-brane actions in D dimension are also important for the effective theory of superstring. With a component formalism, such actions have also been discussed in many literature. For example, in refs. [16, 17], the authors construct SUSY Dp-brane actions with local kappa symmetry based on a component formalism in 10 dimensional spacetime. In a similar way, the p-brane action in various dimensions has also been discussed in ref. [18]. In refs. [19–22], the SUSY Dp-brane in SUGRA background is constructed by considering the background super-vielbein on the brane and couplings between them.

An approach based on superfields is useful for constructing a manifestly SUSY invariant action and generalizing it. Within the formalism, such 4D $\mathcal{N}=1$ SUSY extensions of the DBI action have been known partially. The DBI action of a vector superfield, which corresponds to the case with $\phi^i = B_{\mu\nu} = 0$ in eq. (1.1), is constructed in refs. [23–27]. In particular, in refs. [24, 25], it is shown that such an action appears from the partial breaking of 4D $\mathcal{N}=2$ SUSY. Its SUGRA embedding has also been discussed in refs. [23, 26–28]. Its application to inflation models has been investigated in ref. [29]. Furthermore, in global SUSY, multiple U(1) [30, 31] and massive [32] extensions of the DBI action have been discussed. In particular, for the case with multiple U(1) vector multiplets, linear actions [33], general conditions for partial SUSY breaking [34, 35], and c-maps [36] have also been discussed.

For the DBI action of scalar fields, which corresponds to the case with $F_{\mu\nu} = B_{\mu\nu} = 0$ in eq. (1.1), its SUSY extension has been done via partially broken $\mathcal{N}=2$ SUSY theory, where the Goldstino multiplet is an $\mathcal{N}=1$ real linear superfield [25, 37, 38]. However, there has never been the SUGRA extension of the DBI action of a real linear superfield. In this paper, we discuss the embedding of the DBI action of a real linear superfield into SUGRA. The action of a chiral superfield can be found in ref. [5]. In general, it is known that the action with a chiral superfield can be rewritten in terms of the one with a real linear superfield, and vice versa (via linear-chiral duality [39]). Therefore, our action, which will be discussed in this paper, would be equivalent to that derived in ref. [5] through the duality transformation. We will discuss this point and the differences between their result and ours.

In refs. [25, 37, 38], the DBI action of a real linear multiplet is realized with a chiral multiplet, which is constrained by a specific $\mathcal{N}=1$ SUSY constraint. We will investigate the corresponding constraint which is a key for the construction of DBI action, in SUGRA. To achieve this, we use a formulation based on conformal SUGRA [40–44],[1] where one can treat off-shell SUGRA with different sets of auxiliary fields in a unified manner. Because of the restrictions on the SUGRA embedding of the $\mathcal{N}=1$ constraint, we will find that the DBI action of a real linear superfield can be realized only in the so-called new minimal formulation of SUGRA. Furthermore, we will extend the DBI action to the matter coupled version of it.

The remaining parts of this paper are organized as follows. First, we will briefly review the SUSY DBI action of a real linear superfield in section 2. There, we will find that the constraint imposed between a chiral and real linear superfield is important for

[1] We will use the superconformal tensor calculus [40–44]. See also another formulation, conformal superspace [45, 46].

the construction. Then, we will extend the constraint to that in conformal SUGRA in section 3. After a short review of conformal SUGRA, we will also review the concept of the *u-associated* derivative which is crucial for the superconformal extension. Using this *u-associated* derivative, we will complete the embedding and find that the constraint can be consistently realized in the new minimal SUGRA. With the constraint, we will construct the corresponding action in the new minimal SUGRA, and write down the bosonic component action in section 4. The linear -chiral duality and the matter coupled extension will be also discussed there. Finally, we will discuss the correspondence and differences between results in related works and ours in section 5, and summarize this paper in section 6. In appendix. A, the explicit components of the multiplet including the *u-associated* derivative are shown.

In this paper, we use the unit $M_P = 1$ where $M_P = 2.4 \times 10^{18}\,\text{GeV}$ is the reduced Planck mass, and follow the conventions of [47] in section 2 and of [48] in other parts. $a, b\cdots$ denote Minkowski indices and $\mu, \nu\cdots$ denote curved indices.

2 Review of DBI action in global SUSY

In this section, we briefly review the DBI action of a real linear superfield in global SUSY [37]. We use a chiral superfield X and a real linear superfield L which satisfy the conditions,

$$\bar{D}_{\dot{\alpha}} X = 0, \quad D^2 L = \bar{D}^2 L = 0, \tag{2.1}$$

where D_α and $\bar{D}_{\dot{\alpha}}$ are a SUSY spinor derivative and its complex conjugate. To construct the DBI action for L, we consider the following constraint between X and L,

$$X - \frac{1}{4} X \bar{D}^2 \bar{X} - \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L = 0, \tag{2.2}$$

where $\bar{X}$ is a complex conjugate of X.[2] The equation (2.2) can be solved with respect to X and we obtain

$$X = \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L + \frac{1}{2} \bar{D}^2 \left[\frac{D^\alpha L D_\alpha L \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L}{1 - \frac{1}{2} A + \sqrt{1 - A + \frac{1}{4} B^2}} \right], \tag{2.3}$$

where

$$A \equiv \frac{1}{2} \{ D^2 (\bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L) + \text{h.c.} \}, \quad B \equiv \frac{1}{2} \{ D^2 (\bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L) - \text{h.c.} \}. \tag{2.4}$$

Using this solution (2.3), we can construct the SUSY DBI action as

$$\mathcal{L} = \int d^2\theta X(L) + \text{h.c.}. \tag{2.5}$$

[2]In ref. [37], the constraint (2.2) has been obtained from the tensor multiplet in $\mathcal{N} = 2$ SUSY through partial breaking of it. Here, we do not discuss its origin and we just use the constraint as a guideline to obtain the DBI action. In section 5, we will briefly comment on the relation between the partial breaking of $\mathcal{N} = 2$ SUSY and our construction.

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One can check that the bosonic part of the Lagrangian (2.5) produces,

$$\mathcal{L}_B = 1 - \sqrt{1 - B \cdot B + \partial C \cdot \partial C - (B \cdot \partial C)^2}, \tag{2.6}$$

where C and B_a are a real scalar and a constrained vector satisfying $\partial^a B_a = 0$, in the real linear superfield, and we use the notation $B \cdot \partial C \equiv B^a \partial_a C$. It is known that, through the linear-chiral duality, eq. (2.6) produces the DBI action of a complex scalar, which can be interpreted as the 4D effective D3-brane action. We call eq. (2.6) the DBI action of a real linear superfield in this paper.

It is worth noting that eq. (2.3) satisfies the nilpotency condition, i.e., $X^2 = 0$, due to the Grassmann property of the SUSY spinor derivative, $\bar{D}_{\dot{\alpha}}$. This reflects the underlying Volkov-Akulov SUSY [49–53]. Instead of writing the action like eq. (2.5), we can also rewrite the same system imposing the constraint (2.2) by a chiral superfield Lagrange multiplier Λ,

$$\mathcal{L} = \int d^2\theta \left[X + \Lambda \left(X - \frac{1}{4} X \bar{D}^2 \bar{X} - \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L \right) + \tilde{\Lambda} X^2 \right] + \text{h.c.}. \tag{2.7}$$

Here we have introduced another Lagrange multiplier $\tilde{\Lambda}$, which ensures the nilpotency of X. Indeed, we need not require this condition in the Lagrangian since X satisfies $X^2 = 0$ after integrating out Λ first and solving X with respect to L, but the condition is still consistent and makes the calculation simple as far as we focus on the bosonic part of the action, as we will see in the following section.

3 Extension to 4D $\mathcal{N} = 1$ conformal SUGRA

In this section, we generalize the SUSY DBI action (2.7) discussed in section 2 to that in SUGRA.

3.1 Review of conformal SUGRA

To construct the action in SUGRA, we use conformal SUGRA formulation. Then, let us briefly review the basics of the conformal SUGRA before proceeding to the specific construction of the DBI action.

In this formulation, there are extra gauge symmetries such as dilatation, $\mathrm{U}(1)_A$ symmetry, S-SUSY and conformal boost in addition to translation, Lorentz transformation and SUSY. The commutation and anti-commutation relations are governed by the superconformal algebra and its representation Φ called a superconformal multiplet has the following components,

$$\Phi = \{\mathcal{C}, \mathcal{Z}, \mathcal{H}, \mathcal{K}, \mathcal{B}_a, \Lambda, \mathcal{D}\}, \tag{3.1}$$

where $\mathcal{Z}$ and Λ are spinors; $\mathcal{B}_a$ is a vector; the others are complex scalars. We also denote the superconformal multiplet Φ by its first component $\mathcal{C}$,

$$\Phi = \langle \mathcal{C} \rangle, \tag{3.2}$$

where $\langle \ldots \rangle$ represents the superconformal multiplet which has $\mathcal{C}$ as the first component. $\mathcal{C}$ must be invariant under the transformations of S-SUSY and conformal boost in order for $\Phi = \langle \mathcal{C} \rangle$ to be a superconformal multiplet [44].

A superconformal multiplet is characterized by the charge (w, n) under dilatation and $\mathrm{U}(1)_A$ symmetry called the Weyl weight and the chiral weight, respectively. For example, a chiral multiplet X has (w, w), in order to satisfy

$$\bar{\mathcal{D}}_{\dot{\alpha}} X = 0, \tag{3.3}$$

where $\bar{\mathcal{D}}_{\dot{\alpha}}$ is a spinor derivative [44]. For a real linear multiplet L defined by,

$$\Sigma L = \bar{\Sigma} L = 0, \tag{3.4}$$

where Σ ($\bar{\Sigma}$) is a (anti-) chiral projection operator, the values of each weight are determined as $(w, n) = (2, 0)$. We will discuss these operators, $\mathcal{D}_\alpha$ and Σ, more precisely in the following subsections.

The chiral multiplet consists of the following components, $\{z, P_L\chi, F\}$, where z and F are complex scalars and $P_L\chi$ is a chiral spinor; $P_L = (1+\gamma_5)/2$ is a left-handed projection operator. It is embedded into a general superconformal multiplet (3.1) as

$$\{z, -\sqrt{2} i P_L \chi, -F, iF, iD_a z, 0, 0\}, \tag{3.5}$$

where D_a is a superconformal covariant derivative. On the other hand, a real linear multiplet has components, $\{C, Z, B_a\}$, where C is a real scalar, Z is a Majorana spinor and B_a is a constrained vector which satisfies $D^a B_a = 0$. A real linear multiplet is embedded into a general superconformal multiplet (3.1) as

$$\{C, Z, 0, 0, B_a, -\not{D} Z, -\Box C\}, \tag{3.6}$$

where $\not{D} \equiv \gamma^a D_a$.

For later convenience, we also introduce a multiplication rule for superconformal multiplets. For a function of multiplets $f(\mathcal{C}^I)$, where I classifies different multiplets, we have

$$\begin{aligned}
\langle f(\mathcal{C}^I) \rangle = \Big[& f, f_I \mathcal{Z}^I, f_I \mathcal{H}^I - \frac{1}{4} f_{IJ} \bar{\mathcal{Z}}^J \mathcal{Z}^I, f_I \mathcal{K}^I + \frac{i}{4} f_{IJ} \bar{\mathcal{Z}}^J \gamma_5 \mathcal{Z}^I, f_I \mathcal{B}_a^I - \frac{i}{4} f_{IJ} \bar{\mathcal{Z}}^J \gamma_a \gamma_5 \mathcal{Z}^I, \\
& f_I \Lambda^I - \frac{i}{2} \gamma_5 \left(\mathcal{K}^I - \not{\mathcal{B}}^I - i\gamma_5 \not{D} \mathcal{C}^I + i\gamma_5 \mathcal{H}^I \right) f_{IJ} \mathcal{Z}^J - \frac{1}{4} \left(\bar{\mathcal{Z}}^J \mathcal{Z}^I \right) \mathcal{Z}^K f_{IJK}, \\
& f_I \mathcal{D}^I + \frac{1}{2} f_{IJ} \left(\mathcal{K}^I \mathcal{K}^J + \mathcal{H}^I \mathcal{H}^J - \mathcal{B}^{aI} \mathcal{B}_a^J - D_a \mathcal{C}^I D^a \mathcal{C}^J - 2 \bar{\mathcal{Z}}^J \Lambda^I - \bar{\mathcal{Z}}^J \not{D} \mathcal{Z}^I \right) \\
& - \frac{1}{4} f_{IJK} \bar{\mathcal{Z}}^J (\mathcal{H}^K - i\gamma_5 \mathcal{K}^K - i \not{\mathcal{B}}^K \gamma_5) \mathcal{Z}^I + \frac{1}{16} f_{IJKL} (\bar{\mathcal{Z}}^J \mathcal{Z}^I)(\bar{\mathcal{Z}}^K \mathcal{Z}^L) \Big],
\end{aligned} \tag{3.7}$$

where $f_{IJ\cdots}$ is $\partial f / \partial \mathcal{C}^I \partial \mathcal{C}^J \cdots$ and $\bar{\mathcal{Z}} \equiv \mathcal{Z}^T \hat{C}$ ($\hat{C}$ is a charge conjugation matrix).

We also need action formulas to construct a superconformal action. For a chiral multiplet $X = \{z, P_L\chi, F\}$ with its weight $(3, 3)$, there exists the so-called F-term formula [43],

$$[X]_F = \int d^4x \sqrt{-g} \mathrm{Re} \left[F + \frac{1}{\sqrt{2}} \bar{\psi}_\mu \gamma^\mu P_L \chi + \frac{1}{2} z \bar{\psi}_\mu \gamma^{\mu\nu} P_R \psi_\nu \right], \tag{3.8}$$

where ψ_μ is a gravitino. For a real multiplet $\phi = \{C, Z, H, K, B_a, \Lambda, D\}$ with its weight $(2, 0)$, we can apply the following D-term formula [43],

$$[\phi]_D = \int d^4x \sqrt{-g}\Big[D - \frac{1}{2}i\bar{\psi}\cdot\gamma\gamma_5\lambda - \frac{1}{3}CR + \frac{1}{3}(C\bar{\psi}_\mu\gamma^{\mu\rho\sigma} - i\bar{Z}\gamma^{\rho\sigma}\gamma_5)D_\rho\psi_\sigma + \frac{1}{4}\varepsilon^{abcd}\bar{\psi}_a\gamma_b\psi_c\left(B_d - \frac{1}{2}\bar{\psi}_d Z\right)\Big]. \tag{3.9}$$

Here, all the components of ϕ are real (Majorana).

Using these superconformal multiplets, the multiplication rule (3.7), and the action formulas (3.8) and (3.9), we can construct superconformal invariant actions. Finally, we fix some parts of the extra gauge symmetries by imposing the condition to one of the superconformal multiplets Φ_0 called a compensator multiplet, and obtain the Poincaré SUGRA action.

3.2 *u-associated* derivative

Now, we have prepared the tool for constructing the DBI action in SUGRA. Within the conformal SUGRA formulation, we will discuss a constraint corresponding to that in global SUSY,

$$X - \frac{1}{4}X\bar{D}^2\bar{X} - \bar{D}_{\dot{\alpha}}L\bar{D}^{\dot{\alpha}}L = 0, \tag{3.10}$$

in the following. However, it seems to be a nontrivial task to extend the term including SUSY spinor derivatives,

$$\bar{D}_{\dot{\alpha}}L\bar{D}^{\dot{\alpha}}L \tag{3.11}$$

to that in conformal SUGRA.

To treat the term (3.11) in conformal SUGRA, we need the spinor derivative defined as a superconformal operation. In ref. [44], it is pointed out that the spinor derivative in conformal SUGRA, $\mathcal{D}_\alpha$ ($\bar{\mathcal{D}}_{\dot{\alpha}}$), cannot be defined on a superconformal multiplet Φ unless Φ satisfies a specific weight condition, $w = -n$ ($w = n$). This is because $\mathcal{D}_\alpha\Phi$ ($\bar{\mathcal{D}}_{\dot{\alpha}}\Phi$) is not generically a superconformal multiplet, i.e., the first component of it is S-SUSY and conformal boost inert only when $w = -n$ ($w = n$) is satisfied. Then, it is obvious that we cannot define $\bar{\mathcal{D}}_{\dot{\alpha}}L$ as a superconformal multiplet since L has the weight with $(2, 0)$.

However, the authors in ref. [44] also proposed an improved spinor derivative operation, which can be defined on any supermultiplet. They introduced another multiplet, $\mathbf{u}$, called a *u-associated* multiplet,

$$\mathbf{u} = \{\mathcal{C}_u, \mathcal{Z}_u, \mathcal{H}_u, \mathcal{K}_u, \mathcal{B}_{au}, \Lambda_u, \mathcal{D}_u\}, \tag{3.12}$$

in order to force the first component of $\mathcal{D}_\alpha\Phi$ to be invariant under S-SUSY and conformal boost. To be specific, they defined the *u-associated* spinor derivative as

$$\mathcal{D}^{(\mathbf{u})}_\alpha\Phi = \langle(P_L\mathcal{Z})_\alpha + i(n+w)\lambda_\alpha\mathcal{C}\rangle, \quad \lambda_\alpha \equiv \frac{i(P_L\mathcal{Z}_u)_\alpha}{(w_u+n_u)\mathcal{C}_u}, \tag{3.13}$$

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where w_u and n_u are the Weyl and chiral weight of a *u-associated* multiplets, respectively. Unless $w_u + n_u = 0$, we can choose any multiplet as the *u-associated* multiplet. Then, we can define the spinor derivative for an arbitrary superconformal multiplet by this *u-associated* spinor derivative.

For our purpose, we need the *u-associated* spinor derivative acting on a real linear multiplet, $\mathcal{D}_\alpha^{(\mathbf{u})} L$. More generally, we can consider

$$\mathcal{D}_\alpha^{(\mathbf{u}_1)}(\mathbf{u}_2 L), \tag{3.14}$$

where $\mathbf{u}_1$ is a *u-associated* multiplet and $\mathbf{u}_2$ is an additional multiplet. These multiplets must satisfy $\mathbf{u}_1 \neq \mathbf{u}_2$, since $\mathcal{D}_\alpha^{(\mathbf{u})}\mathbf{u}$ is identically zero obviously from the definition (3.13).[3] Using this *u-associated* spinor derivative, eq. (3.11) can be generalized to the one in conformal SUGRA as

$$\frac{1}{\mathbf{u}_3}\bar{\mathcal{D}}^{(\mathbf{u}_1)}(\bar{\mathbf{u}}_2 L)\bar{\mathcal{D}}^{(\mathbf{u}_1)}(\bar{\mathbf{u}}_2 L), \tag{3.15}$$

where we have introduced a new multiplet $\mathbf{u}_3$,[4] for generality and omitted the spinor index, $\dot{\alpha}$, and we have also defined the conjugate of a *u-associated* derivative as $\bar{\mathcal{D}}_{\dot{\alpha}}^{\mathbf{u}}\Phi = (\mathcal{D}_\alpha^{\mathbf{u}}(\Phi)^*)^*$ following ref. [44].

Let us comment on the weight of the multiplet (3.15). The operator $\bar{\mathcal{D}}_{\dot{\alpha}}^{(\mathbf{u})}$ has the weight $(1/2, 3/2)$, then the total weight of eq. (3.15) is $(2w_2 - w_3 + 5, 2n_2 - n_3 + 3)$, where w_i and n_i with $i = 1, 2, 3$ are the Weyl and chiral weights of $\mathbf{u}_i$, respectively.

Furthermore, eq. (3.10) is a "chiral" constraint since the first and second term in eq. (3.10) are chiral multiplets. Then, we require a condition that the multiplet (3.15) is a chiral multiplet, that is,

$$\bar{\mathcal{D}}\left[\frac{1}{\mathbf{u_3}}\bar{\mathcal{D}}^{(\mathbf{u}_1)}(\bar{\mathbf{u}}_\mathbf{2} L)\bar{\mathcal{D}}^{(\mathbf{u}_1)}(\bar{\mathbf{u}}_\mathbf{2} L)\right] = 0. \tag{3.16}$$

To apply $\bar{\mathcal{D}}$ for eq. (3.15), the Weyl and chiral weight of eq. (3.15) must satisfy $w = n$ as mentioned before,

$$2w_2 - w_3 + 5 = 2n_2 - n_3 + 3. \tag{3.17}$$

The condition (3.16) implies that

$$P_R \mathcal{Z}' = 0, \tag{3.18}$$

where $P_R = (1-\gamma_5)/2$ is a right-handed projection operator and $\mathcal{Z}'$ is the second component of the multiplet (3.15). The equation (3.18) can be written explicitly as

$$\begin{aligned}&\bar{\tilde{\mathcal{Z}}}_2^c P_R \tilde{\mathcal{Z}}_2^c \left[P_R \tilde{\mathcal{Z}} + k P_R \tilde{\mathcal{Z}}_1^c - P_R \tilde{\mathcal{Z}}_3\right] + \bar{\tilde{\mathcal{Z}}} P_R \tilde{\mathcal{Z}} \left[P_R \tilde{\mathcal{Z}}_2^c + k P_R \tilde{\mathcal{Z}}_1^c - P_R \tilde{\mathcal{Z}}_3\right] \\ &\qquad - k \bar{\tilde{\mathcal{Z}}}_1^c P_R \tilde{\mathcal{Z}}_1^c \left[(1-2k)\left(P_R \tilde{\mathcal{Z}} + P_R \tilde{\mathcal{Z}}_2^c\right) + P_R \tilde{\mathcal{Z}}_3\right]\end{aligned}$$

[3]As we will discuss, we choose $\mathbf{u}_1$ and $\mathbf{u}_2$ as compensators, which become some parts of the gravity multiplet after superconformal gauge fixings. In the global SUSY expression (3.11), all the fields in the gravitational multiplet decouple from it. Therefore, it is natural to consider a possibility that a compensator appears as in eq. (3.14).

[4]We will refer all of $\mathbf{u}_i$ as *u-associated* multiplets.

$$
\begin{aligned}
&-2k\left[\bar{\tilde{\mathcal{Z}}}_2^c P_R \tilde{\mathcal{Z}}_1^c \left(2P_R\tilde{Z} - P_R\tilde{\mathcal{Z}}_3\right) + \bar{\tilde{Z}} P_R \tilde{\mathcal{Z}}_1^c \left(2P_R\tilde{\mathcal{Z}}_2^c - P_R\tilde{\mathcal{Z}}_3\right)\right] \\
&-2i\left[i\tilde{\mathcal{H}}_2^* + \tilde{\mathcal{K}}_2^* - k\left(i\tilde{\mathcal{H}}_1^* + \tilde{\mathcal{K}}_1^*\right)\right]\left[P_R\tilde{\mathcal{Z}}_2^c + P_R\tilde{Z} - kP_R\tilde{\mathcal{Z}}_1^c\right] - 2\bar{\tilde{\mathcal{Z}}}_2^c P_R\tilde{Z} P_R\tilde{\mathcal{Z}}_3 = 0,
\end{aligned}
\tag{3.19}
$$

where

$$\mathbf{u}_i = \{\mathcal{C}_i, \mathcal{Z}_i, \mathcal{H}_i, \mathcal{K}_i, \mathcal{B}_{ai}, \Lambda_i, \mathcal{D}_i\}, \quad (i = 1, 2, 3), \tag{3.20}$$

$$\tilde{Z} \equiv \frac{1}{C} Z, \quad \tilde{\mathcal{Z}}_i \equiv \frac{1}{C_i}\mathcal{Z}_i, \quad \tilde{\mathcal{H}}_i(\tilde{\mathcal{K}}_i) \equiv \frac{1}{C_i}\mathcal{H}_i(\mathcal{K}_i), \tag{3.21}$$

$$k \equiv \frac{w_2 + n_2 + 2}{w_1 + n_1}, \tag{3.22}$$

and "c" denotes the charge conjugation for spinors.

As a summary, we find that the superconformal realization of eq. (3.11) is the multiplet (3.15) satisfying the conditions (3.17) and (3.19).

3.3 Old minimal versus new minimal

We have found, in the previous subsection 3.2, the conditions for extending eq. (3.11) to that in conformal SUGRA. Here, we will choose a conformal compensator Φ_0 as *u-associated* multiplets, $\mathbf{u}_i$. Then, we have two choices of compensators; one of them is a chiral compensator S_0 realizing the old minimal SUGRA and the other is a real linear compensator L_0 realizing the new minimal SUGRA.[5]

Now, we will examine what forms of $\mathbf{u}_i$ with both compensators are allowed. Let us start from the old minimal SUGRA realized with a chiral compensator,

$$S_0 = \{z_0, -\sqrt{2}iP_L\chi_0, -F_0, iF_0, iD_a z_0, 0, 0\}, \tag{3.23}$$

with its weight $(1, 1)$. Here we assume that the multiplets $\mathbf{u}_i$ take the following form

$$\mathbf{u}_i = S_0^{p_i}\bar{S}_0^{q_i}, \quad (i = 1, 2, 3), \tag{3.24}$$

where p_i and q_i are the power of S_0 and $\bar{S}_0$, and satisfy $p_1 \neq 0$ since $w_1 + n_1 = (p_1 + q_1) + (p_1 - q_1) = 2p_1$ must be nonzero by a definition of the *u-associated* multiplet. Here we have to stress that eq. (3.24) is the most general form except for the case including derivative operators on a compensator,[6] which might produce higher-derivative terms of gravity. Using eq. (3.5) and the multiplication rule (3.7), the components of the multiplet

[5]We do not discuss the case of the non-minimal formulation which is realized with a complex linear compensator.

[6]For example, $S_0\Sigma\bar{S}_0$ could be considered.

in eq. (3.24) are written as

$$
\begin{aligned}
&\{\mathcal{C}_i, \mathcal{Z}_i, \mathcal{H}_i, \mathcal{K}_i, \mathcal{B}_{ai}, \Lambda_i, \mathcal{D}_i\} \\
&= \Big\{ z_0^{p_i} z_0^{*q_i}, \sqrt{2} i z_0^{p_i-1} z_0^{*q_i-1} (q_i z_0 P_R \chi_0 - p_i z_0^* P_L \chi_0), \\
&\quad z_0^{p_i-2} z_0^{*q_i-2} \left(-q_i z_0^2 z_0^* F_0^* - p_i z_0 z_0^{*2} F_0 + \frac{1}{2} q_i (q_i-1) z_0^2 \bar{\chi}_0 P_R \chi_0 + \frac{1}{2} p_i (p_i-1) z_0^{*2} \bar{\chi}_0 P_L \chi_0 \right), \\
&\quad z_0^{p_i-2} z_0^{*q_i-2} \left(-i q_i z_0^2 z_0^* F_0^* + i p_i z_0 z_0^{*2} F_0 + \frac{i}{2} q_i (q_i-1) z_0^2 \bar{\chi}_0 P_R \chi_0 - \frac{i}{2} p_i (p_i-1) z_0^{*2} \bar{\chi}_0 P_L \chi_0 \right), \\
&\quad \cdots, \cdots, \cdots \Big\},
\end{aligned}
\tag{3.25}
$$

where we have omitted the components, $\mathcal{B}_{ai}, \Lambda_i$ and $\mathcal{D}_i$, which are not necessary to evaluate eq. (3.19). One finds that eq. (3.19) cannot be satisfied by eq. (3.24) by the following reason: terms including $\mathcal{H}_i$ and $\mathcal{K}_i$ must vanish by themselves since any other terms cannot cancel them. After substituting eq. (3.25) into such a part, we obtain

$$
i\tilde{\mathcal{H}}_2^* + \tilde{\mathcal{K}}_2^* - k \left(i\tilde{\mathcal{H}}_1^* + \tilde{\mathcal{K}}_1^* \right) = 2iF_0^* z_0^{*-1} + i\bar{\chi}_0 P_R \chi_0 z_0^{*-2} (p_2^2 - p_2 p_1 - p_1 + 1).
$$

Apparently, the first term cannot be eliminated no matter how we choose the parameters p_i and q_i, and the other terms in eq. (3.19) cannot eliminate it because they do not contain F_0^*. Therefore, we find that eq. (3.24) cannot be a solution of eq. (3.19). This means that eq. (3.15) cannot be realized as a chiral constraint in the old minimal SUGRA.

Next, we examine the case in the new minimal SUGRA with a real linear compensator

$$
L_0 = \{C_0, Z_0, 0, 0, B_{0a}, -\not{D} Z_0, -\Box C_0\}
\tag{3.26}
$$

with its weight $(2, 0)$. In the same way as the old minimal case, we assume the general form of $\mathbf{u}_i$ as

$$
\mathbf{u}_i = L_0^{r_i}, \quad (i = 1, 2, 3),
\tag{3.27}
$$

whose components are

$$
\begin{aligned}
&\{\mathcal{C}_i, \mathcal{Z}_i, \mathcal{H}_i, \mathcal{K}_i, \mathcal{B}_{ai}, \Lambda_i, \mathcal{D}_i\} \\
&\quad = \left\{ C_0^{r_i}, r_i C_0^{r_i-1} Z_0, -\frac{1}{4} r_i (r_i - 1) C_0^{r_i-2} \bar{Z}_0 Z_0, \frac{i}{4} r_i (r_i - 1) C_0^{r_i-2} \bar{Z}_0 \gamma_5 Z_0, \ldots, \ldots, \ldots \right\}.
\end{aligned}
\tag{3.28}
$$

Here we have used eq. (3.6) and eq. (3.7). Then, after substituting eq. (3.28) into eq. (3.19) with the Fierz rearrangement, eq. (3.19) is summarized as

$$
(2r_2 - r_3 + 1) \left\{ C P_R Z \bar{Z}_0 P_R Z_0 + C_0 P_R Z_0 \bar{Z} P_R Z \right\} = 0.
\tag{3.29}
$$

To satisfy eq. (3.29), the coefficient must be zero,

$$
2r_2 - r_3 + 1 = 0.
\tag{3.30}
$$

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Then, we find that the chiral condition (3.19) is satisfied as long as the *u-associated* multiplets follow the condition (3.30).

Noting that $w_i = 2r_i$ and $n_i = 0$ in the ansatz (3.27), the weight condition (3.17) which the chiral multiplet should obey is now reduced to

$$2r_2 - r_3 + 1 = 0. \tag{3.31}$$

This is nothing but eq. (3.30) which is satisfied automatically.

Therefore, we conclude that one can make a multiplet in eq. (3.15) a chiral one with the real linear compensator if eq. (3.30) is satisfied. Here and hereafter, we focus on the case of the new minimal SUGRA with $r_1 = r_3 = 1$ and $r_2 = 0$ for simplicity. In this case, the multiplet in eq. (3.15) becomes

$$\frac{1}{L_0}\bar{\mathcal{D}}^{(L_0)}L\bar{\mathcal{D}}^{(L_0)}L. \tag{3.32}$$

We present the components of this chiral multiplet (3.32) explicitly in appendix A.

3.4 Embedding the constraint into conformal SUGRA

Let us consider the remaining terms, X and $X\bar{D}^2\bar{X}$ in eq. (3.10). For X, we just regard it as a superconformal chiral multiplet with the weight (w, w). In order to extend the second one, $X\bar{D}^2\bar{X}$, to a superconformal multiplet, we replace it with $X\Sigma\bar{X}$, where Σ is a chiral projection operator in conformal SUGRA. However, Σ cannot always be applied for any multiplet Φ in the same way as the spinor derivative $\mathcal{D}$. It can be applied only when Φ satisfies the following weight condition,

$$w_\Phi = n_\Phi + 2. \tag{3.33}$$

Therefore, we compensate the weight of $\bar{X}$, which has the weight $(w, -w)$, by the real linear compensator multiplet L_0^s, where s is the power of L_0,

$$X\Sigma\left(\frac{1}{L_0^s}\bar{X}\right). \tag{3.34}$$

Here, the term, $\frac{1}{L_0^s}\bar{X}$, has the weight $(-2s+w, -w)$. According to eq. (3.33), s must satisfy the condition,

$$s = w - 1. \tag{3.35}$$

Taking into account this condition and the fact that Σ raises the weight by (1,3), eq. (3.34) has the weight $(3, 3)$, which is correct for a chiral multiplet. Since the total weight of eq. (3.34) must be the same as the first term X, the value of w is determined as

$$w = 3. \tag{3.36}$$

Then, we find $s = 2$ from eq. (3.35), and eq. (3.34) becomes

$$X\Sigma\left(\frac{1}{L_0^2}\bar{X}\right). \tag{3.37}$$

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Finally, the weight of the multiplet in eq. (3.15) with that in eq. (3.27) is $(3,3)$ as long as eq. (3.31) is satisfied, then eq. (3.32) is automatically satisfied.

Therefore, we find the complete embedding of a global SUSY expression (3.10),

$$X+\frac{1}{2}X\Sigma\left(\frac{1}{L_0^2}\bar{X}\right)+\frac{1}{4L_0}\bar{\mathcal{D}}^{(L_0)}L\bar{\mathcal{D}}^{(L_0)}L=0, \tag{3.38}$$

where X is a chiral multiplet with $(3,3)$, L is a real linear multiplet with $(2,0)$, and L_0 is a real linear compensator with $(2,0)$.

4 Component action

In this section, we derive the DBI action based on the constraint (3.38) in the new minimal SUGRA.

4.1 Minimal action

We first consider the minimal extension of the action (2.6). The action corresponding to eq. (2.7) is expected to be

$$S=[2X]_F+\left[2\Lambda\left\{X+\frac{1}{2}X\Sigma\left(\frac{\bar{X}}{L_0^2}\right)+\frac{1}{4L_0}\bar{\mathcal{D}}^{(L_0)}L\bar{\mathcal{D}}^{(L_0)}L\right\}\right]_F+[\tilde{\Lambda}X^2]_F+\left[\frac{3}{2}L_0V_R\right]_D, \tag{4.1}$$

where $V_R\equiv\log\frac{L_0}{S\bar{S}}$, S is a chiral multiplet with $(1,1)$, and we have assigned the weights of the Lagrange multiplier chiral multiplet Λ to $(0,0)$ and also $\tilde{\Lambda}$ to $(-3,-3)$ in such a way that the total weight is equal to $(3,3)$. The last term in eq. (4.1) is responsible for the kinetic term of the gravitational multiplet. Note that this term is invariant under the transformation $S\to Se^{i\Theta}$ where Θ is a chiral multiplet with the weight $(0,0)$ since $[L_0(\Theta+\bar{\Theta})]_D\equiv 0$ by the nature of a real linear multiplet. Due to this additional gauge invariance, we have gauge degrees of freedom other than superconformal ones. After imposing the gauge fixing condition for this additional gauge symmetry as $S=\{1,0,0\}$, the bosonic part of (4.1) is given by

$$\begin{aligned}S_B=\int d^4x\sqrt{-g}\Bigg[\Bigg(&F_X(1+\Lambda)-\frac{|F_X|^2\Lambda}{C_0^2}-\frac{\Lambda}{4C_0}(B_a-i\hat{D}_aC)^2\\&+\frac{C\Lambda}{2C_0^2}(B_a-i\hat{D}_aC)(B_0^a-i\hat{D}^aC_0)-\frac{C^2\Lambda}{4C_0^3}(B_{0a}-i\hat{D}_aC_0)^2+\text{h.c.}\Bigg)\\&-\frac{3}{2}\hat{\Box}C_0\log C_0-\frac{3}{2}\hat{\Box}C_0-\frac{3}{4C_0}(B_0\cdot B_0+\hat{D}C_0\cdot\hat{D}C_0)+3A\cdot B_0\Bigg],\end{aligned} \tag{4.2}$$

where Λ and F_X are a scalar component of the chiral multiplet Λ and an auxiliary field of X, and $\hat{D}_\mu$ is a superconformal covariant derivative only including bosonic fields, for example,

$$\hat{D}_\mu C=\partial_\mu C-2b_\mu C, \tag{4.3}$$

where b_μ is the gauge field of dilatation. The third term in eq. (4.1), $\tilde{\Lambda}X^2$, imposes the nilpotency condition for X. Thanks to this, we can drop the scalar component of the chiral multiplet X since the first scalar component can be represented as a fermion bilinear after solving $X^2 = 0$. That is why, we have inserted this term into the action from the beginning. Integrating out the gauge field of $\mathrm{U}(1)_A$ symmetry A_μ, we obtain

$$B_{0a} = 0. \tag{4.4}$$

To eliminate the dilatation symmetry and conformal boost symmetry, we impose the following D-gauge and K-gauge conditions,

$$C_0 = 1, \qquad b_\mu = 0. \tag{4.5}$$

These conditions simplify the action (4.2), which becomes

$$\begin{aligned} S_B = \int d^4x\sqrt{-g}\bigg[\frac{1}{2}R + \Big(F_X(1+\Lambda) - |F_X|^2\Lambda \\ - \frac{\Lambda}{4}(B\cdot B - 2iB\cdot\partial C - \partial C\cdot\partial C) + \text{h.c.}\Big)\bigg]. \end{aligned} \tag{4.6}$$

Then, eliminating the auxiliary field F_X leads to

$$S_B = \int d^4x\sqrt{-g}\bigg[\frac{1}{2}R + \frac{1}{2\lambda}\Big((\lambda+1)^2 + \chi^2\Big) - \frac{1}{2}(B\cdot B - \partial C\cdot\partial C)\lambda - B\cdot\partial C\chi\bigg], \tag{4.7}$$

where $\lambda = \mathrm{Re}\Lambda$ and $\chi = \mathrm{Im}\Lambda$. Finally, we obtain the following conditions from the E.O.Ms for λ and χ,

$$\frac{\chi}{\lambda} = B\cdot\partial C, \tag{4.8}$$
$$\frac{1}{\lambda^2} = 1 - (B\cdot\partial C)^2 - B\cdot B + \partial C\cdot\partial C. \tag{4.9}$$

Substituting them into the action (4.7), we obtain the on-shell DBI action of a real linear multiplet,

$$S_B = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + 1 - \sqrt{1 - B\cdot B + \partial C\cdot\partial C - (B\cdot\partial C)^2}\right]. \tag{4.10}$$

This is almost the same form as eq. (2.6) except for that our action (4.10) is formulated in curved background.

Before closing this subsection, let us discuss the linear-chiral duality. It is known that the action of a real linear multiplet can be rewritten in terms of that of a chiral multiplet. However, in the case with the action including derivative terms such as eq. (4.1), it is nontrivial to take this duality transformation in a manifestly SUSY way.[7] Then, we focus only on the bosonic part (4.10) and discuss this duality at the component level of bosonic part.

[7] In global SUSY, the dual action has been obtained at the level of superfield in ref. [38].

We start from the following Lagrangian which is the relevant part in the action (4.10),

$$\mathcal{L} = 1 - \sqrt{1 - B \cdot B + \partial C \cdot \partial C - (B \cdot \partial C)^2}. \tag{4.11}$$

To rewrite this Lagrangian (4.11) in terms of the complex scalar of a chiral multiplet, we first relax the constraint on the vector field B_a. We impose it by the E.O.M for a scalar field ℓ, that is, we use

$$\mathcal{L} = 1 - \sqrt{1 - B \cdot B + \partial C \cdot \partial C - (B \cdot \partial C)^2} + B \cdot \partial \ell, \tag{4.12}$$

where B_a is now an unconstrained vector. The Lagrangian (4.12) is equivalent to the original one (4.11) since the variation with respect to ℓ leads to the constraint, $\partial_a B^a = 0$. Instead of ℓ, varying with respect to B_a gives

$$\partial^a \ell + (\partial^a C B \cdot \partial C + B^a)\{1 - B \cdot B + \partial C \cdot \partial C - (B \cdot \partial C)^2\}^{-1/2} = 0. \tag{4.13}$$

Our task is now to solve this equation (4.13) with respect to B_a. By taking scalar products of eq. (4.13) with $B_a, \partial_a C$ and $\partial_a \ell$, we obtain three independent equations and can solve them with respect to B^2, $B \cdot \partial C$, and $B \cdot \partial \ell$. The solutions are

$$B^2 = \frac{(\partial \ell)^2 (1 + (\partial C)^2)^2 - (\partial C \cdot \partial \ell)^2 (2 + (\partial C)^2)}{Y^2}, \tag{4.14}$$

$$B \cdot \partial C = -\frac{\partial C \cdot \partial \ell}{Y}, \tag{4.15}$$

$$B \cdot \partial \ell = \frac{-(\partial \ell)^2 (1 + (\partial C)^2) + (\partial C \cdot \partial \ell)^2}{Y}, \tag{4.16}$$

where

$$Y \equiv \{(1 + (\partial C)^2)(1 + (\partial \ell)^2) - (\partial C \cdot \partial \ell)^2\}^{1/2}. \tag{4.17}$$

Substituting these solutions into the Lagrangian (4.12), we obtain the dual action,

$$\begin{aligned} \mathcal{L} &= 1 - \sqrt{1 + (\partial C)^2 + (\partial \ell)^2 + (\partial C)^2 (\partial \ell)^2 - (\partial C \cdot \partial \ell)^2} \\ &= 1 - \sqrt{1 + \partial \phi \cdot \partial \bar{\phi} - \frac{1}{4} (\partial \phi)^2 (\partial \bar{\phi})^2 + \frac{1}{4} (\partial \phi \cdot \partial \bar{\phi})^2}, \end{aligned} \tag{4.18}$$

where we have defined a complex scalar $\phi = \ell + iC$. The Lagrangian (4.18) can be written as the DBI form

$$\mathcal{L} = 1 - \sqrt{\det \left(g_{ab} + \frac{1}{2} \partial_a \phi \partial_b \bar{\phi} \right)}. \tag{4.19}$$

This Lagrangian (4.19) agrees with the one constructed in ref. [5] using a chiral multiplet directly.

4.2 Matter coupled extension

Finally, we discuss the matter coupled DBI action given by

$$S = [2f(\Phi^I)X]_F + \left[2\Lambda\left\{X + \frac{1}{2}X\Sigma\left(\frac{\bar{X}}{M(L_0,\Phi^I,\bar{\Phi}^{\bar{J}})}\right) + \frac{1}{4L_0}\bar{\mathcal{D}}^{(L_0)}L\bar{\mathcal{D}}^{(L_0)}L\right\}\right]_F$$
$$+ [\mathcal{F}(L_0,\Phi^I,\bar{\Phi}^{\bar{J}})]_D + [\tilde{\Lambda}X^2]_F, \tag{4.20}$$

where Φ^I ($\bar{\Phi}^{\bar{J}}$) is a (anti-) chiral matter multiplet; $f(\Phi)$ is a holomorphic function of Φ^I with $(0,0)$; $M(L_0,\Phi^I,\bar{\Phi}^{\bar{J}})$ and $\mathcal{F}(L_0,\Phi^I,\bar{\Phi}^{\bar{J}})$ are real functions of $\Phi^I,\bar{\Phi}^{\bar{J}}$ and L_0 with $(4,0)$ and $(2,0)$, respectively. Note that we have omitted superpotential term $[W(\Phi^I)]_F$, where $W(\Phi^I)$ is a holomorphic function of Φ^I with the weight $(w,n) = (3,3)$, since the term is irrelevant to the following discussion. Taking into account the nilpotency condition on X, the bosonic component of the action (4.20) is given by

$$S_B = \int d^4x\sqrt{-g}\Bigg[\bigg(F_X(f+\Lambda) - \frac{\Lambda|F_X|^2}{M} - \frac{\Lambda}{4C_0}(B_a - i\hat{D}_aC)^2$$
$$+ \frac{C\Lambda}{2C_0^2}(B_a - i\hat{D}_aC)(B_0^a - i\hat{D}^aC_0) - \frac{C^2\Lambda}{4C_0^3}(B_0^a - i\hat{D}^aC_0)^2 + \text{h.c.}\bigg) + \mathcal{L}_m\Bigg], \tag{4.21}$$

where

$$\mathcal{L}_m = -\frac{1}{3}(\mathcal{F} - \mathcal{F}_{C_0}C_0)R(b) + \frac{1}{2}\mathcal{F}_{C_0C_0}(\hat{D}C_0\cdot\hat{D}C_0 - B_0\cdot B_0)$$
$$+ 2\mathcal{F}_{I\bar{J}}(F^I\bar{F}^{\bar{J}} - \hat{D}\Phi^I\cdot\hat{D}\bar{\Phi}^{\bar{J}}) + \left(-i\mathcal{F}_{C_0I}B_0\cdot\hat{D}\Phi^I + \text{h.c.}\right). \tag{4.22}$$

In the above expression, Φ^I ($\bar{\Phi}^{\bar{J}}$) and F^I ($\bar{F}^{\bar{J}}$) represent the scalar and auxiliary components of the (anti-) chiral matter multiplet, and subscripts denote the derivative with respect to the corresponding scalar. $R(b)$ becomes a Ricci scalar when $b_\mu = 0$ is imposed as the K-gauge condition.

Before setting superconformal gauge conditions, we integrate out the auxiliary field F_X and the Lagrange multiplier Λ. We can easily solve the E.O.M for F_X and obtain

$$S_B = \int d^4x\sqrt{-g}\Bigg[\frac{M}{2\lambda}\left\{(\lambda+p)^2 + (\chi+q)^2\right\} - \frac{\lambda}{2C_0}(B\cdot B - \hat{D}C\cdot\hat{D}C)$$
$$- \frac{\chi}{C_0}B\cdot\hat{D}C + \frac{C\lambda}{C_0^2}(B_0\cdot B - \hat{D}C_0\cdot\hat{D}C) + \frac{C\chi}{C_0^2}(B_0\cdot\hat{D}C + B\cdot\hat{D}C_0)$$
$$- \frac{C^2\lambda}{2C_0^3}(B_0\cdot B_0 - \hat{D}C_0\cdot\hat{D}C_0) - \frac{C^2\chi}{C_0^3}B_0\cdot\hat{D}C_0 + \mathcal{L}_m\Bigg], \tag{4.23}$$

where $\lambda = \mathrm{Re}\Lambda$, $\chi = \mathrm{Im}\Lambda$, $p = \mathrm{Re}f$, and $q = \mathrm{Im}f$. Note that, at this stage, the matter Lagrangian $\mathcal{L}_m$ is not affected by the DBI sector. Next, we eliminate λ and χ by using their E.O.Ms, which are given by

$$-\frac{M}{2\lambda^2}\left\{(\lambda+p)^2 + (\chi+q)^2\right\} + \frac{M}{\lambda}(\lambda+p) + \mathcal{A} = 0, \tag{4.24}$$
$$\frac{M}{\lambda}(\chi+q) + \mathcal{B} = 0, \tag{4.25}$$

where

$$\mathcal{A} \equiv -\frac{1}{2C_0}(B \cdot B - \hat{D}C \cdot \hat{D}C) + \frac{C}{C_0^2}(B_0 \cdot B - \hat{D}C_0 \cdot \hat{D}C) - \frac{C^2}{2C_0^3}(B_0 \cdot B_0 - \hat{D}C_0 \cdot \hat{D}C_0), \tag{4.26}$$

$$\mathcal{B} \equiv -\frac{1}{C_0}B \cdot \hat{D}C + \frac{C}{C_0^2}(B_0 \cdot \hat{D}C + B \cdot \hat{D}C_0) - \frac{C^2}{C_0^3}B_0 \cdot \hat{D}C_0. \tag{4.27}$$

Solutions for them are

$$\lambda|_{\text{sol}}^{-1} = \frac{1}{p}\sqrt{1 + \frac{2\mathcal{A}}{M} - \frac{\mathcal{B}^2}{M^2}}, \tag{4.28}$$

$$\chi|_{\text{sol}} = -q - \frac{\lambda|_{\text{sol}}}{M}\mathcal{B}. \tag{4.29}$$

Substituting the above solutions into the action (4.23), we obtain a relatively simple form

$$S_B = \int d^4x\sqrt{-g}\left[Mp\left(1 - \sqrt{1 + \frac{2\mathcal{A}}{M} - \frac{\mathcal{B}^2}{M^2}}\right) - q\mathcal{B} + \mathcal{L}_m\right]. \tag{4.30}$$

The remaining issue is the elimination of auxiliary fields B_0^a and A_a. However, it is difficult to do it because of the presence of nonlinear terms of B_0^a contained in the first term in eq. (4.30). In addition, $\mathcal{L}_m$ has $A_a A^a$ as well as mixing terms between B_0^a and A_a in general cases. Therefore, integration of those auxiliary fields is technically difficult and we cannot obtain the complete on-shell action.[8]

Although a general case is difficult to complete the remaining task, we can continue our discussion for the following special case. Let us consider the following choice of $\mathcal{F}(L_0, \Phi^I, \bar{\Phi}^{\bar{J}})$,

$$\mathcal{F} = L_0 \log\left(\frac{L_0 G(\Phi^i, \bar{\Phi}^{\bar{j}})}{S\bar{S}}\right), \tag{4.31}$$

where Φ^i is a matter chiral multiplet with its weight $(0,0)$, $G(\Phi^i, \bar{\Phi}^{\bar{j}})$ is a real function of Φ^i and $\bar{\Phi}^{\bar{j}}$, and S is a chiral multiplet with $(1,1)$. This action is also invariant under the transformation $S \to Se^{i\Theta}$ in the same way as the last term in eq. (4.1), which characterizes the new minimal SUGRA.

We use the D-gauge condition to make the Ricci scalar term canonical. From eq. (4.22), we can find an appropriate D-gauge choice [54]

$$\mathcal{F} - \mathcal{F}_{C_0}C_0 = -\frac{3}{2}. \tag{4.32}$$

As the choice of the additional gauge, we set $\mathcal{F}_{C_0} = 0$ [54]. Then, we can solve these gauge conditions with respect to C_0 and S and obtain

$$S\bar{S} = \frac{3}{2}eG, \tag{4.33}$$

$$C_0 = \frac{3}{2}. \tag{4.34}$$

Using the K-gauge, we also set a condition $b_\mu = 0$.

[8]The general matter coupled system in the new minimal SUGRA not including higher-order derivative terms can be found in ref. [54].

Under these conditions, $\mathcal{L}_m$ becomes

$$\mathcal{L}_m = \frac{1}{2}R + 2\mathcal{F}_{i\bar{j}}(F^i\bar{F}^{\bar{j}} - \partial_a\Phi^i\partial^a\bar{\Phi}^{\bar{j}}) - \frac{1}{2}B_0^aB_{0a} + (-i\mathcal{F}_{C_0i}B_0^a\partial_a\Phi^i + \text{h.c.}) + (iB_0^a\partial_a\log S + \text{h.c.}) + 2B_0^aA_a, \tag{4.35}$$

where A_a is the $\text{U}(1)_A$ gauge field mentioned above. We find that the E.O.M for A_a gives a constraint $B_0^a = 0$ and the difficulty due to the nonlinear term of B_0^a is circumvented in this case. This result is irrelevant to other parts of the action (4.30) since they do not contain terms of A_a. F^i can be eliminated by their E.O.Ms, and we finally obtain the following on-shell action,

$$S_B = \int d^4x\sqrt{-g}\left[Mp\left(1 - \sqrt{1 + \frac{2\mathcal{A}}{M} - \frac{\mathcal{B}^2}{M^2}}\right) - q\mathcal{B} + \frac{1}{2}R - 2\mathcal{F}_{i\bar{j}}\partial_a\Phi^i\partial^a\bar{\Phi}^{\bar{j}}\right], \tag{4.36}$$

with

$$\mathcal{A} = \frac{1}{3}(\partial C\cdot\partial C - B\cdot B), \quad \mathcal{B} = -\frac{2}{3}B\cdot\partial C. \tag{4.37}$$

Here, the real function M should be understood as $M|_{C_0=3/2}$. Note that, in this case, we cannot add superpotential terms of Φ^i by the following reason: to obtain the constraint $B_0^a = 0$, we assumed that only S has the weight $(w, n) = (1, 1)$ and a special form of $\mathcal{F}$ giving $\mathcal{F}_{S\bar{S}} = 0$, otherwise such a constraint does not appear. For the superconformal invariance, the superpotential W should have $(3, 3)$. From the weight condition, a possible form is $W = S^3g(\Phi^i)$ but this term is forbidden by the symmetry under $S \to Se^{i\Theta}$ which the D-term part $[\mathcal{F}]_D$ has. Therefore, we cannot add any superpotential terms of matter multiplets.

5 Relation between our results and other works

Here, we comment on the differences between ours and the results in ref. [5], in which the DBI action of a chiral multiplet is constructed in the old minimal SUGRA. As we mentioned before, the DBI action of a real linear multiplet can be rewritten in terms of a chiral multiplet through the linear-chiral duality and the whole action of a chiral multiplet is obtained in global SUSY in terms of superfield [38]. The authors of ref. [5] embedded the dual chiral multiplet action into the old minimal SUGRA. On the other hand, our starting point is the action of a real linear multiplet, more precisely, the constraint (2.2) imposed upon it. This constraint has its origin in the tensor multiplet of $\mathcal{N} = 2$ SUSY [25, 37, 38]. Indeed, in global SUSY case, the real linear multiplet corresponds to a Goldstino multiplet for the broken SUSY. From such a viewpoint, our construction is important since it makes the connection with the partial breaking of $\mathcal{N} = 2$ SUSY much clearer .

Although the ways of construction are different, our action would realize their result. Indeed, at the bosonic component level, we have found the correspondence between the result in ref. [5] and ours. However, we also found that the action cannot be realized in the old minimal SUGRA when we do not consider the case including higher-derivative terms of a chiral compensator, which may contradict the result of ref. [5]. Unlike the DBI action of a real linear multiplet, that of a vector multiplet can be constructed in both of the old and new minimal SUGRA [28]. The difference originates from the necessity of *u-associated* derivatives in the DBI action of a real linear multiplet. For a vector superfield case, we can construct the DBI action only with the chiral projection operator Σ, which does not require *u-associated* multiplet to make the operand superfield a primary superfield [44–46]. It is interesting to explore these reasons and we expect that the direct derivation of the constraint (2.2) and also DBI action from $\mathcal{N}=2$ SUGRA are necessary to understand this issue, which would be our future work.[9]

6 Summary

In this paper, we have discussed superconformal generalization of a DBI action of a real linear superfield known in global SUSY.

To achieve this, we have focused on the constraint (2.2) between a chiral multiplet and a real linear multiplet, which comes from the partial breaking of 4D $\mathcal{N}=2$ SUSY [37]. However, it is a nontrivial task to embed this constraint into conformal SUGRA due to the existence of the SUSY spinor derivative, which in general, cannot be applied for arbitrary multiplets in conformal SUGRA. Instead of using an original spinor derivative, we have adopted the *u-associated* spinor derivative, proposed in ref. [44]. We obtained the condition (3.17) and (3.19) by requiring that the corresponding constraint (3.15) in conformal SUGRA becomes a chiral constraint. Surprisingly, we have found that these conditions can be realized only in the new minimal formulation of SUGRA when we choose the general power function of compensator as the *u-associated* multiplet. Then, we have derived the condition (3.30) which *u-associated* multiplets must satisfy.

After embedding the constraint into the new minimal SUGRA, we have shown the component action which is formulated in curved spacetime. We have also discussed the linear-chiral duality at the level of bosonic components and rewritten the action from a complex scalar field of a chiral multiplet. Finally, we have constructed the action where matter multiplets are directly coupled to the DBI sector. Due to the appearance of non-linear terms for vector field B_{0a}, we have restricted the discussion to the special form of matter function (4.31) and derived the bosonic action (4.36).

In this paper, we have shown that the DBI action of a real linear multiplet cannot be realized in the old minimal SUGRA as a naive embedding of the constraint (2.2), which may contradict the result of ref. [5]. The duality relation between the old and new minimal SUGRA [54] is generically not obvious when there exist higher-derivative terms. For example, the non-minimal coupling of gravity is realized only in new minimal SUGRA [4] as in the case of the DBI action we discussed here. Such an issue may be revealed with the help of deep understanding of SUGRA system with higher-order derivative terms.

To investigate our model further, we need the direct derivation of the constraint from $\mathcal{N}=2$ SUGRA. And also, the remaining part in eq. (1.1), i.e., a term including $B_{\mu\nu}$, and possible combinations of the Maxwell, scalar and 2-form parts have not been constructed. We leave them for future work.

[9]For the DBI action of a vector multiplet, such attempts have been recently discussed [55]. There, the partial breaking of $\mathcal{N}=2$ SUSY in some $\mathcal{N}=1$ SUSY background has been discussed.

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A The components of *u-associated* spinor derivative multiplet

Here we show the explicit component form of

$$\frac{1}{L_0}\bar{\mathcal{D}}^{(L_0)}L\bar{\mathcal{D}}^{(L_0)}L. \tag{A.1}$$

As we have seen in section 3, eq. (A.1) is a chiral multiplet with weight $(3,3)$. The components of this multiplet, $\{z', P_L\chi', F'\}$, are

$$\begin{aligned}
z' &= \frac{C^2}{C_0}\left(\bar{\tilde{Z}} - \bar{\tilde{Z}}_0\right) P_R \left(\tilde{Z} - \tilde{Z}_0\right), && \text{(A.2)}\\
P_L\chi' &= \frac{\sqrt{2}C^2}{C_0} P_L\Big[\left(\not{\tilde{B}} - i\not{D}\tilde{C} - \not{\tilde{B}}_0 + i\not{D}\tilde{C}_0\right)\left(\tilde{Z} - \tilde{Z}_0\right) - \frac{3i}{2}\tilde{Z}_0\bar{\tilde{Z}}_0 P_R\tilde{Z}_0 \\
&\quad - \frac{i}{2}\tilde{Z}_0\bar{\tilde{Z}}P_R\tilde{Z} + \frac{i}{4}\gamma^a\tilde{Z}_0\bar{\tilde{Z}}\gamma_a\gamma_5\tilde{Z} + i\tilde{Z}\bar{\tilde{Z}}_0 P_R\tilde{Z}_0 - \frac{i}{2}\gamma^a\tilde{Z}\bar{\tilde{Z}}_0\gamma_a\gamma_5\tilde{Z}_0\Big], && \text{(A.3)}\\
F' &= \frac{C^2}{C_0}\Big[-\left(\tilde{B}_a - iD_a\tilde{C}\right)^2 + 2\left(\tilde{B}_a - iD_a\tilde{C}\right)\left(\tilde{B}^a - iD^a\tilde{C}\right) - \left(\tilde{B}_{0a} - iD_{0a}\tilde{C}\right)^2 \\
&\quad + i\bar{\tilde{Z}}_0\gamma_5\left(\not{\tilde{B}} - i\not{D}\tilde{C}\right)\left(\tilde{Z} - \tilde{Z}_0\right) + \frac{i}{2}\bar{\tilde{Z}}\gamma_5\left(\not{\tilde{B}}_0 - i\not{D}\tilde{C}_0\right)\tilde{Z} \\
&\quad - 2i\bar{\tilde{Z}}\gamma_5\left(\not{\tilde{B}}_0 - i\not{D}\tilde{C}_0\right)\tilde{Z}_0 + \frac{3i}{2}\bar{\tilde{Z}}_0\gamma_5\left(\not{\tilde{B}}_0 - i\not{D}\tilde{C}_0\right)\tilde{Z}_0 \\
&\quad + 2\left(\bar{\tilde{Z}} - \bar{\tilde{Z}}_0\right)P_R\not{D}\left(\tilde{Z} - \tilde{Z}_0\right) + \frac{1}{2}\bar{\tilde{Z}}_0 P_R\tilde{Z}_0\bar{\tilde{Z}}\tilde{Z} + \frac{1}{2}\bar{\tilde{Z}}P_R\tilde{Z}\bar{\tilde{Z}}_0\tilde{Z}_0 \\
&\quad + 2\bar{\tilde{Z}}P_R\tilde{Z}_0\bar{\tilde{Z}}\tilde{Z}_0 - 3\bar{\tilde{Z}}P_R\tilde{Z}_0\bar{\tilde{Z}}_0\tilde{Z}_0 - 3\bar{\tilde{Z}}\tilde{Z}_0\bar{\tilde{Z}}_0 P_R\tilde{Z}_0 + \frac{1}{2}\bar{\tilde{Z}}_0 P_R\tilde{Z}_0\bar{\tilde{Z}}_0\tilde{Z}_0\Big], && \text{(A.4)}
\end{aligned}$$

where the fields with $\tilde{}$ are divided by the first components of the multiplet they belong to, in the same way as eq. (3.21), and the superconformal derivative D_a is understood to act only on the numerator but not on the denominator, e.g., $D^a\tilde{C} \equiv D^aC/C = D^a\log C$.

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