

A scale-invariant Higgs sector and structure of the vacuum

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ABSTRACT: In view of the current status of measured Higgs boson properties, we consider a question whether only the Higgs self-interactions can deviate significantly from the Standard-Model (SM) predictions. This may be possible if the Higgs effective potential is irregular at the origin. As an example we investigate an extended Higgs sector with singlet scalar(s) and classical scale invariance. We develop a perturbative formulation necessary to analyze this model in detail. The behavior of a phenomenologically valid potential in the perturbative regime is studied around the electroweak scale. We reproduce known results: the Higgs self-interactions are substantially stronger than the SM predictions, while the Higgs interactions with other SM particles are barely changed. We further predict that the interactions of singlet scalar(s), which is a few to several times heavier than the Higgs boson, tend to be fairly strong. If probed, these features will provide vivid clues to the structure of the vacuum. We also examine Veltman's condition for the Higgs boson mass.

KEYWORDS: Higgs Physics, Spontaneous Symmetry Breaking, Beyond Standard Model

ARXIV EPRINT: [1503.02819](https://arxiv.org/abs/1503.02819)

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1 Introduction

It appears that completion of the standard model (SM) of particle physics, as it stands, has been achieved with the discovery of the Higgs boson at the LHC experiments [1, 2]. In particular, the two parameters of the Higgs potential,

$$V_{\text{SM}}(H) = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2, \tag{1.1}$$

namely, the vacuum expectation value (VEV) of the Higgs field $v_H = \mu_H/\sqrt{\lambda_H} = 246$ GeV and the Higgs boson mass $m_h = \sqrt{2}\mu_H \approx 126$ GeV, are now determined. Here, H denotes the Higgs doublet field $H = (H^+, H^0)^T$. This means that all the parameters of the SM have been determined. After the discovery, investigations of properties of the Higgs boson are being performed rapidly, such as measurements of its spin, CP , and couplings with various SM particles. Up to now, there are no evident contradictions with the SM predictions. Accuracies of the measurements are improving, and identification with the SM Higgs boson is becoming more likely. These features, however, do not readily lead to the conclusion that the SM is confirmed altogether. As an important piece, confirmation

of the self-interactions of the Higgs boson is still missing, which is indispensable to unveil the structure of the Higgs potential.

There are some phenomenological and theoretical problems in the Higgs sector of the SM as of today. For instance, there is a huge hierarchy between the electroweak and Planck scales, which generates the “Naturalness problem”. Furthermore, even if we permit fine tuning of parameters, we face a problem that the vacuum of the Higgs potential is unstable or metastable at a high-energy scale.¹ These problems give us motivations to consider extensions of the SM Higgs sector.

In view of the present status, it is an interesting question whether it is possible, with such an extension, that only the Higgs self-interactions are significantly different from the SM predictions, while other properties of the Higgs boson are barely affected. Naively, one may expect that deviations of the Higgs self-couplings from the SM values can be expressed as effects of higher-dimensional operators, which are suppressed by a cut-off scale Λ as $(v_H/\Lambda)^n$. It is based on a general model-independent argument in the case that the Higgs effective potential can be expanded as a polynomial in the Higgs field H . Since the cut-off scale may be of order a few TeV scale or higher, one may expect that the deviation is already constrained to be quite small.

One direction to evade such an argument is to consider models with non-decoupling effects to Higgs self-interactions. There are many examples [3, 4]. Here, we would like to consider a different possibility. In various physics of spontaneous symmetry breakdown (including condensed matter physics), there appear effective potentials which cannot be expanded in polynomials in the field variables, namely effective potentials which are irregular at the origin. For example, the theory of superconductivity at $T = 0$ gives such an effective potential, and in certain strongly interacting systems such singular potentials are also expected to appear (although they are difficult to compute reliably). As an example which is perturbatively computable within relativistic quantum field theory, we consider an extended Higgs sector with classical scale invariance. In this case, typically the Higgs effective potential takes a form $\sim \lambda\phi^4 [\ln(\phi^2/v_H^2) - 1/2]$, which is irregular at the origin via the Coleman-Weinberg (CW) mechanism [5], where $\phi = \sqrt{2} \text{Re } H^0$. If this potential is expanded about the VEV in terms of the physical Higgs field $h = \phi - v_H$, the expansion generates powers of h higher than the quartic power, and they arise in powers of h/v_H without being suppressed by a cut-off scale. At the same time, it is expected that the triple and quartic couplings of h can have order-unity deviations from the SM values.

In recent years, CW-type potentials have been studied extensively as models of an extended Higgs sector, with different motivations. For instance, classical scale invariance as a solution to the hierarchy problem has become a hot subject. These are models which become scaleless at a new physics scale M (e.g. Planck scale), $\mu_H^2(M) = 0$. In these models, typically the scale invariance is broken radiatively by the CW mechanism, which (in steps) leads to a generation of the electroweak scale as the Higgs VEV [6–13], or generates scales involving nonzero VEVs of other scalar fields [14–28]. Even without explicit

¹The scale of the vacuum instability is very sensitive to the top quark mass and depends on our precise knowledge of the top quark mass in the future.

realizations, classical scale invariance is often mentioned as a possibility or guideline for an underlying mechanism.

The purpose of this paper is to explore physics in the vicinity of our vacuum, characterized by the effective potential of an extended Higgs sector. As a minimal extension of the SM Higgs sector, we consider a scale-invariant Higgs potential with a real singlet scalar field which belongs to the N representation of a global $O(N)$ symmetry. Though there have been several works in which the same or similar extensions are considered, we re-analyze the model by examining the validity range of perturbative calculations of the effective potential in detail. We can obtain a phenomenologically valid potential within the validity range of a perturbative analysis. We clarify its behavior around the electroweak scale. We also examine Veltman’s condition [29] for the Higgs mass, which is a criterion for judging a “Naturality” of the scalar potential by examining the coefficient of the quadratic divergence.

Some of previous works on similar models are subject to possible instability of their predictions when higher-order perturbative corrections are included. Such instability was already pointed out by the original CW paper, and we re-examine this condition. Other works, which are legitimate with respect to the perturbative validity, use the framework of Gildener-Weinberg (GW) [30] to analyze the effective potential. In this framework, one concentrates on a one-dimensional subspace of the configuration space of the effective potential, namely that corresponding to the physical Higgs direction, and examines the potential shape on that subspace. On the other hand, we analyze the effective potential in a way closer to the original CW approach, which enables to clarify a global structure of the potential shape in the configuration space.

Ref. [9] analyzed a gauged and non-gauged version of a scale-invariant model, and the latter is close to the one we examine. Phenomenologically we reproduce similar aspects, hence we state the differences of our analysis in comparison. As mentioned above, ref. [9] uses the GW framework to analyze the properties of the effective potential. Instead we present a formulation which enables analysis of global properties of the effective potential. As a result, for instance, we are able to analyze the structure of the potential in every direction around the vacuum in the configuration space and predict the interactions among the physical scalar particles.

This paper is organized as follows. In section 2 we explain our model. In section 3, we develop a theoretical framework needed for a perturbative analysis of the model. We give results of our numerical analysis in section 4. Conclusions and discussion are given in section 5.

2 Model and effective Higgs potential

2.1 Lagrangian

The scale-invariant limit of the SM has long been excluded experimentally. Hence, to impose classical scale invariance, we need to extend the Higgs sector. As a minimal extension, we consider a scale-invariant extension of the SM with an additional real singlet scalar field with a Higgs-portal coupling. We consider the case where the singlet scalar field is in the

fundamental representation of a global $O(N)$ group: $\vec{S} = (S_1, \dots, S_N)^T$. The Lagrangian is given by

$$\mathcal{L} = [\mathcal{L}_{\text{SM}}]_{\mu_H \rightarrow 0} + \frac{1}{2}(\partial_\mu \vec{S})^2 - \lambda_{\text{HS}}(H^\dagger H)(\vec{S} \cdot \vec{S}) - \frac{\lambda_S}{4}(\vec{S} \cdot \vec{S})^2, \quad (2.1)$$

where the real singlet field \vec{S} interacts with itself and the Higgs doublet field H via the self-interaction and portal interaction with the coupling constants λ_S and λ_{HS} , respectively.

2.2 Effective potential up to one-loop level

The one-loop effective potential in the Landau gauge, renormalized in the $\overline{\text{MS}}$ -scheme, is given by

$$V_{\text{eff}}(\phi, \varphi) = V_{\text{tree}}(\phi, \varphi) + V_{1\text{-loop}}(\phi, \varphi), \quad (2.2)$$

$$V_{\text{tree}}(\phi, \varphi) = \frac{\lambda_H}{4}\phi^4 + \frac{\lambda_{\text{HS}}}{2}\phi^2\varphi^2 + \frac{\lambda_S}{4}\varphi^2, \quad (2.3)$$

$$V_{1\text{-loop}}(\phi, \varphi) = \sum_i \frac{n_i}{4(4\pi)^2} M_i^4(\phi, \varphi) \left[\ln \frac{M_i^2(\phi, \varphi)}{\mu^2} - c_i \right]. \quad (2.4)$$

Here, the expectation values of the scalar fields in the presence of source J are given by

$$\langle H \rangle_J = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad \langle \vec{S} \rangle_J = (\varphi, 0, \dots, 0)^T; \quad \phi, \varphi \in \mathbb{R}. \quad (2.5)$$

The index i denotes the internal particle in the loop, and their parameters are given by²

$$\begin{aligned} W \text{ bosons: } n_W &= 6, \quad M_W^2 = \frac{1}{4}g^2\phi^2, \quad c_W = \frac{5}{6}; \\ Z \text{ boson: } n_Z &= 3, \quad M_Z^2 = \frac{1}{4}(g^2 + g'^2)\phi^2, \quad c_Z = \frac{5}{6}; \\ \text{massive scalar bosons: } n_\pm &= 1, \quad M_\pm^2 = F_\pm^2, \quad c_\pm = \frac{3}{2}; \\ \text{NG bosons of the Higgs field: } n_{\text{NG}} &= 3, \quad M_{\text{NG}}^2 = \lambda_H\phi^2 + \lambda_{\text{HS}}\varphi^2, \quad c_{\text{NG}} = \frac{3}{2}; \\ \text{NG bosons of the singlet field: } n_{\text{NG}} &= N - 1, \quad M_{\text{NG}}^2 = \lambda_{\text{HS}}\phi^2 + \lambda_S\varphi^2, \quad c_{\text{NG}} = \frac{3}{2}; \\ \text{up-type quarks } (F = u, c, t): n_F &= -4N_C, \quad M_F^2 = \frac{1}{2} \left(y_{FF}^{(\text{U})} \right)^2 \phi^2, \quad c_F = \frac{3}{2}; \\ \text{down-type quarks } (f = d, s, b): n_f &= -4N_C, \quad M_f^2 = \frac{1}{2} \left(y_{ff}^{(\text{D})} \right)^2 \phi^2, \quad c_f = \frac{3}{2}; \\ \text{charged leptons } (f = e, \mu, \tau): n_f &= -4, \quad M_f^2 = \frac{1}{2} \left(y_{ff}^{(\text{E})} \right)^2 \phi^2, \quad c_f = \frac{3}{2}, \end{aligned} \quad (2.6)$$

²Strictly speaking, the terminology ‘‘Nambu-Goldstone (NG) bosons’’ for the internal particles is inadequate except at the vacuum configuration and depends on how the symmetries are broken by the vacuum. More precisely, ‘‘NG bosons’’ represent the scalar modes which are orthogonal to the radial directions of H and S .

and F_{\pm} are defined by

$$F_{\pm}(\phi, \varphi) = \frac{3\lambda_{\text{H}} + \lambda_{\text{HS}}}{2}\phi^2 + \frac{\lambda_{\text{HS}} + 3\lambda_{\text{S}}}{2}\varphi^2 \pm \sqrt{\left(\frac{3\lambda_{\text{H}} - \lambda_{\text{HS}}}{2}\phi^2 + \frac{\lambda_{\text{HS}} - 3\lambda_{\text{S}}}{2}\varphi^2\right)^2 + 4\lambda_{\text{HS}}^2\phi^2\varphi^2}. \quad (2.7)$$

Hereafter, we neglect all the Yukawa couplings except the top Yukawa coupling, $y_{\text{t}} = y_{33}^{(\text{U})} \sim \mathcal{O}(1)$, since other Yukawa couplings are fairly small. It is customary to denote the summation over i together with particle's statistical factor as supertrace ‘‘STr,’’ which we also use below.

2.3 Renormalization group analysis

We can extend the applicability range of the effective potential by a renormalization-group (RG) improvement. According to the general formulation [31, 32], the $(L + 1)$ -loop beta functions and anomalous dimensions can be used to improve the L -loop effective potential. Here, we obtain the leading-logarithmic (LL) potential by improving the tree-level potential by the one-loop beta functions and anomalous dimensions. It is argued in [31, 32] that, within the range where $\ln(M_i^2/\mu^2)$ are not too large, combining the one-loop effective potential with the one-loop beta functions and anomalous dimensions gives a better approximation. Hence, we obtain an improved-next-to-leading-order (improved-NLO) potential by combining them. Comparing NLO, LL and improved-NLO potentials, we can examine validity (stability) of the predictions in the vicinity of the vacuum.

The beta functions and anomalous dimensions are defined by

$$\beta_X = \mu \frac{dX}{d\mu}, \quad \gamma_A = \frac{\mu}{A} \frac{dA}{d\mu}, \quad (2.8)$$

respectively, where X is a coupling constant and A is a field, both renormalized at scale μ . The one-loop beta functions and anomalous dimensions of the model are given by

$$\beta_{\lambda_{\text{H}}} = \frac{1}{(4\pi)^2} \left[\frac{9}{8}g^4 + \frac{3}{4}g^2g'^2 + \frac{3}{8}g'^4 - 6y_{\text{t}}^4 - 4 \left(\frac{9}{4}g^2 + \frac{3}{4}g'^2 - 3y_{\text{t}}^2 \right) \lambda_{\text{H}} + 24\lambda_{\text{H}}^2 + 2N\lambda_{\text{HS}}^2 \right], \quad (2.9)$$

$$\beta_{\lambda_{\text{HS}}} = \frac{1}{(4\pi)^2} \lambda_{\text{HS}} \left[-2 \left(\frac{9}{4}g^2 + \frac{3}{4}g'^2 - 3y_{\text{t}}^2 \right) + 12\lambda_{\text{H}} + 8\lambda_{\text{HS}} + 2(N + 2)\lambda_{\text{S}} \right], \quad (2.10)$$

$$\beta_{\lambda_{\text{S}}} = \frac{1}{(4\pi)^2} (8\lambda_{\text{HS}}^2 + 2(N + 8)\lambda_{\text{S}}^2), \quad (2.11)$$

$$\gamma_{\phi} = \frac{1}{(4\pi)^2} \left(\frac{9}{4}g^2 + \frac{3}{4}g'^2 - 3y_{\text{t}}^2 \right), \quad (2.12)$$

$$\gamma_{\varphi} = 0. \quad (2.13)$$

The beta functions which do not include λ_{HS} or λ_{S} are suppressed.

We obtain the improved potentials by taking the renormalization scale as $t = \ln(\sqrt{\phi^2 + \varphi^2}/v_H)$.

$$V_{\text{eff}}^{(\text{LL})}(\phi, \varphi) = \frac{\lambda_H(t)}{4}\phi^4(t) + \frac{\lambda_{\text{HS}}(t)}{2}\phi^2(t)\varphi^2(t) + \frac{\lambda_S(t)}{4}\varphi^4(t), \quad (2.14)$$

$$V_{\text{eff}}^{(\text{imp-NLO})}(\phi, \varphi) = \frac{\lambda_H(t)}{4}\phi^4(t) + \frac{\lambda_{\text{HS}}(t)}{2}\phi^2(t)\varphi^2(t) + \frac{\lambda_S(t)}{4}\varphi^4(t) + \sum_i \frac{n_i}{4(4\pi)^2} M_i^4(\phi(t), \varphi(t)) \left[\ln \frac{M_i^2(\phi(t), \varphi(t))}{\mu^2(t)} - c_i \right], \quad (2.15)$$

where

$$\mu(t) = v_H e^t, \quad (2.16)$$

$$\phi(t) = \xi_\phi(t)\phi, \quad (2.17)$$

$$\varphi(t) = \xi_\varphi(t)\varphi, \quad (2.18)$$

$$\xi_i(t) = \exp \left[\int_0^t \gamma_i(t') dt' \right]. \quad (2.19)$$

3 Perturbatively valid parameter region

In a previous work [16] a parameter region has been searched assuming

$$\lambda_H \sim \lambda_{\text{HS}} < 1. \quad (3.1)$$

In that case, however, the Coleman-Weinberg mechanism does not work properly, since the one-loop corrections cannot compete against the tree-level terms in such a parameter region if the order counting in perturbation theory is legitimate. (This is in analogy to the case of pure scalar ϕ^4 field theory considered in the original CW paper [5].) As a result, the renormalization scale considered in [16] is about $10^5 v_H$, where the large logarithmic corrections invalidate a perturbative analysis. In this section, we reconsider the parameter region, in which the results are perturbatively valid and the CW mechanism works properly.

As mentioned in the Introduction, there is a framework of analysis to find a vacuum of a one-loop effective potential and to compute particle contents of scalar bosons systematically, which was introduced by Gildener and Weinberg [30]. In that framework we obtain the following general results. A perturbatively valid vacuum appears on the ray of a flat direction of the tree-level potential; a light scalar boson “scalon” (corresponding to the Higgs boson in our context) appears in addition to heavy scalar bosons; interactions among the scalon and the other scalar bosons are derived. Although it is a useful framework, we consider that our framework presented below is advantageous to analyze more global features of the potential, including the vicinity of the vacuum.

3.1 Order counting in perturbative expansion

We investigate an order counting among the coupling constants, with which the CW mechanism is expected to work in the perturbative regime. To realize the CW mechanism, the

tree-level and one-loop contributions to the effective potential should be comparable and compete with each other. Parametrically this requires a relation

$$|\lambda_H| \sim \frac{N \lambda_{HS}^2}{(4\pi)^2} - \frac{N_C y_t^4}{(4\pi)^2} \ll 1 \quad (3.2)$$

to be satisfied. (This is in analogy to the case of massless scalar QED considered in the original CW paper [5], in which $\lambda \sim \frac{e^4}{(4\pi)^2} \ll 1$ is required as a consistent parameter region.) Note that we take y_t into account in eq. (3.2) since the top quark gives the dominant one-loop contribution among the SM particles which couple to the Higgs particle. According to eq. (3.2), we consider that λ_H and λ_{HS}^2 (as well as y_t^4) are naively counted as the same order quantities in perturbative expansions.³ It follows that the relation between λ_H and λ_{HS} should read

$$|\lambda_H| \ll |\lambda_{HS}|. \quad (3.3)$$

Thus, eq. (3.2) and eq. (3.3) indicate that λ_H^2 and λ_{HS}^2 need to be counted as different orders, although they both belong to the one-loop contributions. Furthermore, λ_{HS} needs to be large, at least of order $\sqrt{N_C/N} y_t \approx 1.7 N^{-1/2}$, in order to beat the top quark negative contribution, for stabilizing the vacuum.

We derive a systematic approximation of the effective potential eq. (2.4), taking into account the above order counting. First, the contributions from the NG bosons of the Higgs field on the right-hand-side of eq. (2.4) can be written as follows:

$$\begin{aligned} & \frac{3}{64\pi^2} (\lambda_H \phi^2 + \lambda_{HS} \varphi^2)^2 \left[\ln \left(\frac{\lambda_H \phi^2 + \lambda_{HS} \varphi^2}{\mu^2} \right) - \frac{3}{2} \right] \\ & \simeq \frac{3}{64\pi^2} (\lambda_{HS} \varphi^2)^2 \left[\ln \left(\frac{\lambda_{HS} \varphi^2}{\mu^2} \right) - \frac{3}{2} \right]. \end{aligned} \quad (3.4)$$

Secondly, $F_{\pm}(\phi, \varphi)$ in the fourth and fifth terms in eq. (2.4) can be written as follows:

$$\begin{aligned} F_{\pm}(\phi, \varphi) & \simeq \frac{\lambda_{HS}}{2} \phi^2 + \frac{\lambda_{HS} + 3\lambda_S}{2} \varphi^2 \\ & \pm \sqrt{\left[-\frac{\lambda_{HS}}{2} \phi^2 + \frac{\lambda_{HS} - 3\lambda_S}{2} \varphi^2 \right]^2 + 4\lambda_{HS}^2 \phi^2 \varphi^2} \\ & \equiv F_{\pm\text{app}}(\phi, \varphi), \end{aligned} \quad (3.5)$$

where we have defined $F_{\pm\text{app}}(\phi, \varphi)$ as the approximate form of $F_{\pm}(\phi, \varphi)$. Then we substitute eqs. (3.4) and (3.5) to the expression of the effective potential eq. (2.4).

We also have to apply eq. (3.2) or eq. (3.3) to the tree-level potential in eq. (2.3). In particular, eq. (3.2) can be interpreted as follows. Although the term proportional to λ_H is tree-level, after taking into account the above order counting of the coupling constants, this term should be regarded as next-to-leading order (NLO), in contrast to the term

³We do not consider a fine cancellation between the λ_{HS}^2 and y_t^4 contributions.

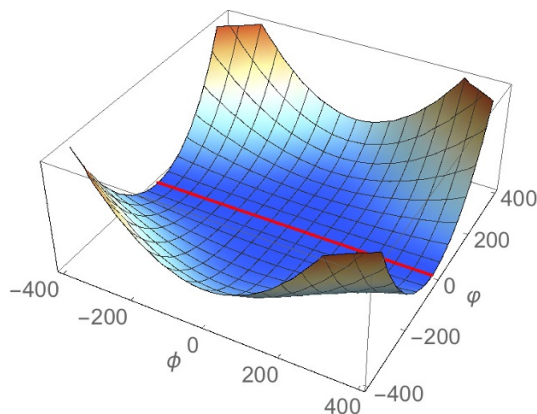


Figure 1. The LO effective potential V_{LO} as a function of ϕ and φ . The red line shows the ϕ axis, which composes the minima of the potential.

proportional to λ_{HS} , which is at the LO. For this reason, the LO contributions to the effective potential in the above order counting is given by

$$V_{\text{LO}} = \frac{\lambda_{\text{HS}}}{2} \phi^2 \varphi^2 + \frac{\lambda_{\text{S}}}{4} \varphi^4. \quad (3.6)$$

We show V_{LO} in figure 1. At LO, the potential is flat along the ϕ axis, which composes the minima of this potential.⁴ Thus, at LO, the vacuum is not determined uniquely. At every vacuum the Higgs boson is massless, while the singlet scalars are massive.

The NLO contributions are given by

$$\begin{aligned} V_{\text{NLO}} = & \frac{\lambda_{\text{H}}}{4} \phi^4 + \frac{F_{+\text{app}}^2(\phi, \varphi)}{64\pi^2} \left[\ln \left(\frac{F_{+\text{app}}(\phi, \varphi)}{\mu^2} \right) - \frac{3}{2} \right] \\ & + \frac{3}{64\pi^2} (\lambda_{\text{HS}} \varphi^2)^2 \left[\ln \left(\frac{\lambda_{\text{HS}} \varphi^2}{\mu^2} \right) - \frac{3}{2} \right] \\ & + \frac{N-1}{64\pi^2} (\lambda_{\text{HS}} \phi^2 + \lambda_{\text{S}} \varphi^2)^2 \left[\ln \left(\frac{\lambda_{\text{HS}} \phi^2 + \lambda_{\text{S}} \varphi^2}{\mu^2} \right) - \frac{3}{2} \right] \\ & - \frac{4N_{\text{C}}}{64\pi^2} M_{\text{t}}^4(\phi) \left[\ln \left(\frac{M_{\text{t}}^2(\phi)}{\mu^2} \right) - \frac{3}{2} \right] \\ & + \frac{6}{64\pi^2} M_{\text{W}}^4(\phi) \left[\ln \left(\frac{M_{\text{W}}^2(\phi)}{\mu^2} \right) - \frac{5}{6} \right] \\ & + \frac{3}{64\pi^2} M_{\text{Z}}^4(\phi) \left[\ln \left(\frac{M_{\text{Z}}^2(\phi)}{\mu^2} \right) - \frac{5}{6} \right]. \end{aligned} \quad (3.7)$$

We explain the reason for omitting $F_{-\text{app}}$ in the next subsection.

As seen above, at every vacuum of V_{LO} , $\varphi = 0$ and the φ -direction becomes a massive mode. Hence, from consistency of the perturbative expansion, we expect that $\varphi = 0$ holds also at the vacuum of $V_{\text{LO}} + V_{\text{NLO}}$. According to the general argument on the potential

⁴If λ_{S} is small and of the same order as λ_{H} , $\lambda_{\text{S}} \varphi^4$ should be counted as NLO. In this case, both ϕ and φ axes become the flat minima of V_{LO} . Other features, especially the results presented in the next section, are hardly affected, if the global minimum of $V_{\text{LO}} + V_{\text{NLO}}$ is on the ϕ axis.

with a flat direction at LO, with an appropriate choice of the couplings $(\lambda_H, \lambda_{HS}, \lambda_S)$, $V_{LO} + V_{NLO}$ exhibits a minimum on the ϕ -axis by the CW mechanism, and the Higgs boson becomes massive at NLO. Hence, the Higgs boson is generally expected to be much lighter than the singlet scalars. Thus, the SM gauge group is broken by the vacuum as required $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$, while the $O(N)$ global symmetry is unbroken. [There appears no NG mode with respect to the $O(N)$ group.]

3.2 Comment on the Hessian matrix

A special prescription is needed for computing the CW potential, in the case that the minimum of the effective potential is not determined uniquely at the LO of the perturbative expansion, such as in eq. (3.6).

Generally the arguments of logarithms in a one-loop effective potential include the eigenvalues m_i^2 of the Hessian matrix of the tree-level scalar potential. The Hessian matrix is a matrix whose elements are given by $\frac{\partial^2 V_{tree}}{\partial x_i \partial x_j}$, where x_i denotes a scalar field. Therefore, the eigenvalues m_i^2 at the potential minimum coincide with the mass-squared eigenvalues of the scalar fields at tree level. In the case that the tree-level potential has a unique minimum, all the eigenvalues of the Hessian matrix at the minimum are positive, $m_i^2 > 0$, and $m_i^4 \ln m_i^2$ in the one-loop effective potential are well-defined.⁵

Since in our case V_{LO} is flat along the ϕ -axis, m_ϕ^2 (the eigenvalue in the direction of ϕ) can be negative at configuration points (ϕ, φ) infinitesimally away from the ϕ -axis. Instead, if we determine the potential minimum of $V_{LO} + V_{NLO}$ and compute the Hessian matrix of $V_{LO} + V_{NLO}$ in the vicinity of the minimum, all the eigenvalues are positive. Thus, a naive perturbative treatment is inappropriate in computing quantum fluctuations in the vicinity of a vacuum in the case that there is a negative eigenvalue.

Here we adopt a prescription⁶ that (only) the eigenvalue m_ϕ^2 cannot be determined by V_{LO} , and that this eigenvalue should be determined by $V_{LO} + V_{NLO}$, whose effect through $m_\phi^4 \ln m_\phi^2$ should be included in V_{NNLO} .⁷ Since we do not compute V_{NNLO} , in practice we simply neglect its contribution. The other eigenvalue m_φ^2 determined by V_{LO} is positive. In eq. (2.6), the two eigenvalues are given by $F_\pm(\phi, \varphi)$. The eigenvalues of the modes orthogonal to these radial modes in the scalar sector are composed by those of the three degenerate modes of H and those of the $N - 1$ degenerate modes of \vec{S} , all of which are non-negative; see eqs. (2.6) and (3.4).⁸ We consider that these should be included in the computation of V_{NLO} . Based on these considerations, we omit the contribution of F_{-app} (the eigenvalue in the direction ϕ) in eq. (3.7).

⁵For massless modes, such as NG bosons of the Higgs field, we define the values of $m_i^4 \ln m_i^2$ in the limit $m_i^2 \rightarrow 0$, i.e., zero. See eq. (3.4).

⁶This corresponds to the following prescription for computing V_{eff} by the background field method. We determine the propagator of the relevant quantum field not by the LO vertices alone but also by including one-loop self-energy corrections. Note that both LO vertices and one-loop self-energy corrections are dependent on the background fields (ϕ, φ) and determined from V_{LO} and V_{NLO} , respectively.

⁷This is consistent in the vicinity of the ϕ -axis since m_ϕ^2 is an NLO quantity.

⁸At the vacuum, the three modes orthogonal to the radial mode of H are identified with the NG modes, whose eigenvalues vanish to all orders, while all the N modes of \vec{S} become degenerate, since the $O(N)$ symmetry is unbroken.

An analysis for a consistent treatment of the NG modes has been performed recently [33], which is similar in spirit to the above argument. We will further develop the above method for including higher-order corrections consistently in our future work.

3.3 Relation between λ_H and λ_{HS} at $\mu = v_H$

To study the effective potential in the vicinity of the vacuum, a natural choice of the renormalization scale would be $\mu = v_H$, where $v_H = 246 \text{ GeV}$ is the VEV of the Higgs field. Here, we set $\mu = v_H$ and $\varphi = 0$ and examine a relation among the scalar couplings ($\lambda_H, \lambda_{HS}, \lambda_S$). (Other couplings are fixed to the SM values.) As a result we obtain a relation between λ_H and λ_{HS} .

Setting $\varphi = 0$, the effective potential takes a form

$$V_{\text{eff}}(\phi, \varphi = 0) = C_1 \phi^4 + C_2 \phi^4 \ln\left(\frac{\phi}{\mu}\right), \quad (3.8)$$

where C_1 and C_2 are constants dependent on λ_H, λ_{HS} and independent of λ_S . This shows that both mass and VEV of the Higgs boson are independent of λ_S . [This is not the case if we use RG-improved potentials, since terms including both λ_S and $\ln(\sqrt{\phi^2 + \varphi^2}/v_H)$ are resummed.]

The potential minimum is determined by

$$\left. \frac{\partial}{\partial \phi} V_{\text{eff}}(\phi, \varphi = 0) \right|_{\phi=v_H} = 0, \quad (3.9)$$

from which we obtain v_H as a function of λ_H, λ_{HS} and μ . Then we set $\mu = v_H$:

$$v_H = f(\lambda_H, \lambda_{HS}; \mu) \Big|_{\mu=v_H}. \quad (3.10)$$

Since v_H is the only dimensionful parameter, the right-hand-side is proportional to v_H and we can divide both sides by v_H . This gives a relation between λ_H and λ_{HS} , which are renormalized at $\mu = v_H$. The relation is shown in figure 2, obtained by solving eq. (3.10) numerically.⁹ Note that λ_{HS} should be of order $\sqrt{N_C/N} y_t \approx 1.7 N^{-1/2}$ or larger (see section 3.1), while it should be smaller than order 4π to ensure perturbativity.

4 Results of analysis

4.1 Phenomenologically valid parameters

Using the effective potential $V_{\text{LO}} + V_{\text{NLO}}$ obtained in the previous section, we search for a phenomenologically valid parameter region for the couplings ($\lambda_H, \lambda_{HS}, \lambda_S$). We require that the observed mass and VEV of the Higgs boson are reproduced.

⁹If we neglect the gauge couplings (and all the Yukawa couplings except y_t), we obtain a simple relation:

$$\lambda_H = -\frac{1}{16\pi^2} [y_t^4 (3 + 3 \ln 2 - 6 \ln y_t) + N \lambda_{HS}^2 (\ln \lambda_{HS} - 1)]. \quad (3.11)$$

This gives a good approximation of the numerical result.

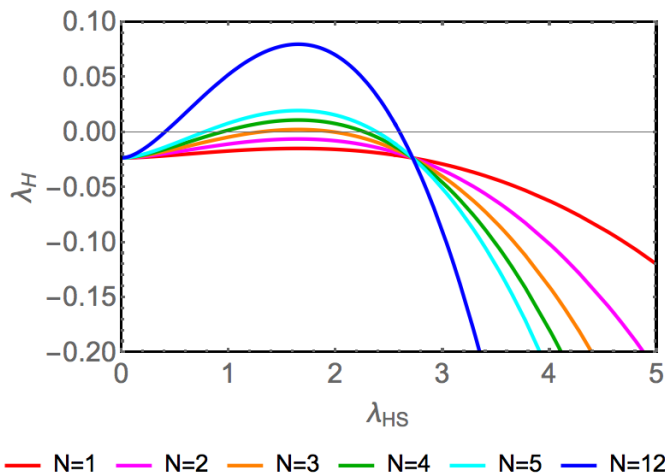


Figure 2. The relation between λ_H and λ_{HS} obtained at the vacuum ($\varphi = 0$) with a choice of the renormalization scale $\mu = v_H$, for various N . [Eq. (3.10)].

N	1	2	3	4	5	12
$\lambda_H(v_H)$	-0.11	-0.055	-0.025	-0.0045	0.012	0.075
$\lambda_{HS}(v_H)$	4.8	3.4	2.8	2.4	2.1	1.4

Table 1. Values of λ_H and λ_{HS} at $\mu = v_H$ fixed by $m_h = 126$ GeV and $v_H = 246$ GeV, in the case $V_{\text{eff}} = V_{\text{LO}} + V_{\text{NLO}}$.

The analysis in section 3.3 shows that, as long as we choose a renormalization scale $\mu \simeq v_H$ such that a perturbative analysis is valid close to the vacuum, the couplings λ_H and λ_{HS} are related, as demonstrated in figure 2. The ratio of the Higgs boson mass and the VEV, m_h/v_H , varies along each line shown in the figure. Thus, if we fix $m_h = 126$ GeV and $v_H = 246$ GeV, the values of λ_H and λ_{HS} are fixed. The values of λ_H and λ_{HS} for various N are shown in table 1. These values are consistent with our order estimate eq. (3.2). If we use RG-improved potentials, λ_H and λ_{HS} are no longer fixed, since they depend on the value of λ_S .

Let us present results of our analysis for the case $N = 1$. Table 2 shows our predictions, together with representative input parameters. We compare the three different approximations of the effective potential: (I) $V_{\text{LO}} + V_{\text{NLO}}$, (II) $V_{\text{eff}}^{(\text{LL})}$, and (III) $V_{\text{eff}}^{(\text{imp-NLO})}$. Here, $V_{\text{eff}}^{(\text{LL})}$ and $V_{\text{eff}}^{(\text{imp-NLO})}$ are defined similarly to eqs. (2.14) and (2.15) from V_{LO} and $V_{\text{LO}} + V_{\text{NLO}}$ with the scale choice $t = \ln(\sqrt{\phi^2 + \varphi^2}/v_H)$, respectively. As mentioned, in the case (I), λ_H and λ_{HS} are fixed by m_h and v_H , while λ_S can be taken as a free input parameter. Reflecting this feature, even in the cases (II) and (III), the values of λ_H and λ_{HS} are tightly constrained, while λ_S can be taken fairly freely. For this reason, we take λ_S as the input parameter for all three cases. In the table we take $\lambda_S = 0.10$ as an example. In all cases there exist λ_H and λ_{HS} which reproduce the mass and VEV of the Higgs boson. Furthermore, in accord with the argument in section 3.1, the mass of the singlet scalar is predicted to be several times larger than the Higgs boson mass in each case. We find that

	$N = 1$		
	(I)	(II)	(III)
μ	$v_H = 246$ [GeV]		
$y_t(v_H)$	0.919		
$g(v_H)$	0.644		
$g'(v_H)$	0.359		
$\lambda_H(v_H)$	-0.11	-0.059	-0.082
$\lambda_{HS}(v_H)$	4.8	4.5	4.3
$\lambda_S(v_H)$	0.10	0.10	0.10
v_H [GeV]	246		
m_h [GeV]	126		
$\langle\varphi\rangle$ [GeV]	0	0	0
m_s [GeV]	556	527	524
$\sin\theta_{\text{mix}}$	0	0	0
Landau pole [TeV]	3.5	4.1	4.7

Table 2. Predictions of our model, together with some representative input parameters, for $N = 1$ in three different approximations of the effective potentials: (I) $V_{\text{LO}} + V_{\text{NLO}}$, (II) $V_{\text{eff}}^{(\text{LL})}$, and (III) $V_{\text{eff}}^{(\text{imp-NLO})}$. In each case, a parameter set for $(\lambda_H, \lambda_{HS}, \lambda_S)$ is chosen such that the mass and VEV of the Higgs boson are reproduced.

the predicted masses of the singlet scalar are consistent with each other within about 5% accuracy. The differences may be taken as a reference for the stability of our predictions. An undesirable feature is that the locations of the Landau pole are close and in the several TeV region. This originates from the large value of λ_{HS} at the electroweak scale $\mu \simeq v_H$ (see section 3.1).

Dependences of the predictions on λ_S is as follows. If we raise the value of λ_S , the locations of the Landau pole are even lowered, since the couplings $\lambda_H, \lambda_{HS}, \lambda_S$ increase with the renormalization scale as they influence each other. The mass of the singlet scalars are barely dependent on λ_S , since only the fourth derivative in the φ direction of the effective potential is affected at LO. The values of λ_H and λ_{HS} do not change very much with λ_S . For instance, if we take $\lambda_S = 0.5$ and 1.0 , the Landau pole appears at 3.2TeV and 2.8TeV respectively, while the mass of the singlet scalar changes little.

To see the shape of the effective potential, we show in figure 3 (a) the contour plot of the potential in case (I) of table 2. Since the potential is symmetric under $\phi \rightarrow -\phi$ or $\varphi \rightarrow -\varphi$, we show only the upper-right part of the configuration space. As expected there is a global minimum along the ϕ -axis. There is also a shallower local minimum along the φ -axis, which is generated by the CW mechanism of a competition between $\lambda_S\varphi^4$ and $\lambda_{HS}^2\varphi^4 \ln(\lambda_{HS}\varphi^2/\mu^2)$ terms.¹⁰ In cases (II) and (III) of table 2, we obtain qualitatively similar contour plots.

¹⁰This local minimum on the φ -axis is about ten times shallower than the global minimum. It becomes even shallower if the value of λ_S is larger. We will not be concerned about the minimum on the φ -axis in our analysis as long as it is not a global minimum.

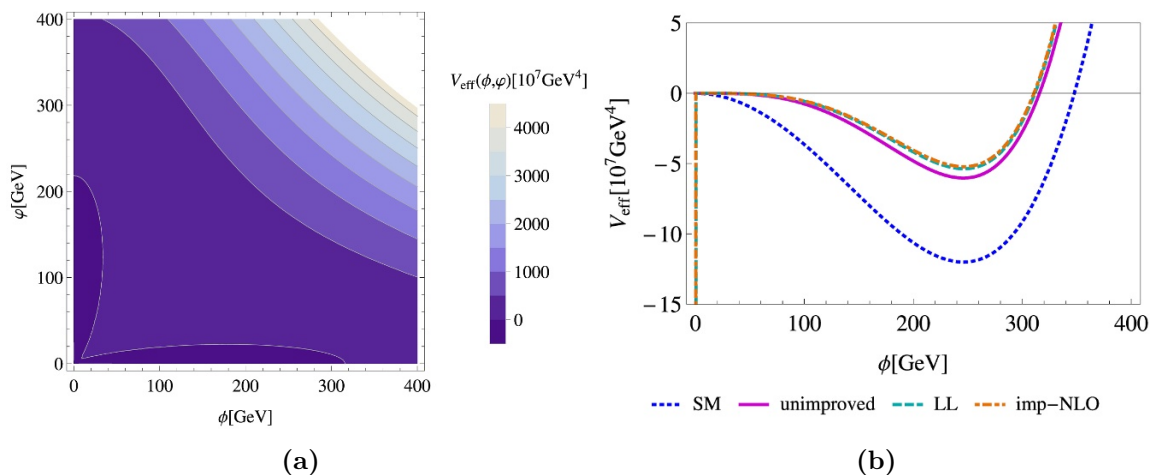


Figure 3. (a) Contour plot of the unimproved effective potential $V_{\text{LO}} + V_{\text{NLO}}$. (b) Comparison of the effective potentials on the ϕ -axis ($\varphi = 0$), in the approximations (I) $V_{\text{LO}} + V_{\text{NLO}}$, (II) $V_{\text{eff}}^{(\text{LL})}$, and (III) $V_{\text{eff}}^{(\text{imp-NLO})}$. The effective potential for the SM is also shown.

In figure 3 (b) we show the effective potentials on the ϕ -axis (i.e., for $\varphi = 0$) for all three cases, together with the effective potential of the SM. We see that differences between the three approximations are small. In particular, the difference between the LL approximation (II) and the improved-NLO approximation (III) is hardly visible. These features show good stability of our predictions in the vicinity of the vacuum, and that they are within the validity range of perturbation theory.

There is a significant difference between our effective potential and that of the SM. In comparison to the SM, the minimum of our potential is shallow, although the values of v_{H} and m_{h} are common. The clear difference of the potential shapes indicates that the higher derivatives of the potentials at the minimum are appreciably different. Namely, we anticipate that the Higgs self-interactions of our model are appreciably different from those of the SM.

Next we examine the cases $N > 1$. In general, we expect that the location of the Landau pole is raised as compared to the $N = 1$ case. This is because the required value of λ_{HS} to overwhelm the top-loop contribution decreases with N as $1/\sqrt{N}$, see section 3.1. One may confirm this tendency in table 1. We show the cases $N = 4$ and $N = 12$ in table 3 with the input $\lambda_{\text{S}} = 0.10$ at $\mu = v_{\text{H}}$. As expected the positions of the Landau pole are raised up to order a few tens TeV for these N . The corresponding shapes of the effective potentials are displayed in figures 4.

In figure 5 are shown the plots of the phenomenologically favored region for the portal coupling λ_{HS} and the Higgs quartic coupling λ_{H} . We show the results for the case (I), which are independent of λ_{S} . The other parameters are fixed to the SM values. These figures show how the parameters of the model under consideration are constrained in the current status. As we discussed, currently λ_{S} is barely constrained.

	$N = 4$			$N = 12$		
	(I)	(II)	(III)	(I)	(II)	(III)
$\lambda_H(v_H)$	-0.0045	-0.061	-0.0005	0.075	-0.063	0.082
$\lambda_{HS}(v_H)$	2.4	2.3	2.4	1.4	1.4	1.4
$\lambda_S(v_H)$	0.10	0.10	0.10	0.10	0.10	0.10
$\langle\varphi\rangle$ [GeV]	0	0	0	0	0	0
m_s [GeV]	378	378	375	285	293	286
$\sin\theta_{\text{mix}}$	0	0	0	0	0	0
Landau pole [TeV]	16	19	17	28	37	26

Table 3. Predictions of our model for $N = 4$ and $N = 12$ in three different approximations of the effective potentials: (I) $V_{\text{LO}} + V_{\text{NLO}}$, (II) $V_{\text{eff}}^{(\text{LL})}$, and (III) $V_{\text{eff}}^{(\text{imp-NLO})}$. In each case, a parameter set for $(\lambda_H, \lambda_{HS}, \lambda_S)$ is chosen such that the mass and VEV of the Higgs boson are reproduced.

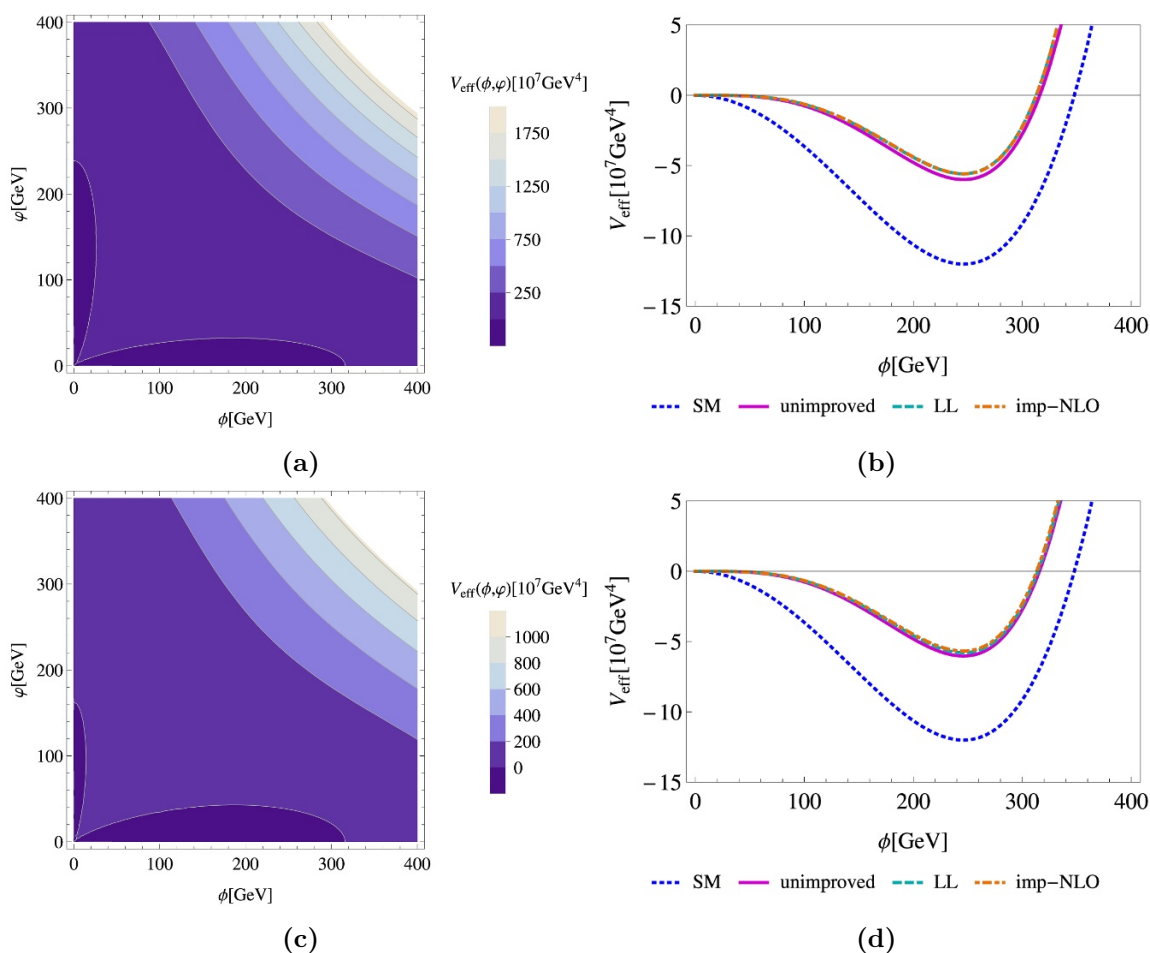


Figure 4. Same as figures 3 but for (a)(b) $N = 4$, and (c)(d) $N = 12$, corresponding to the parameters of table 3.

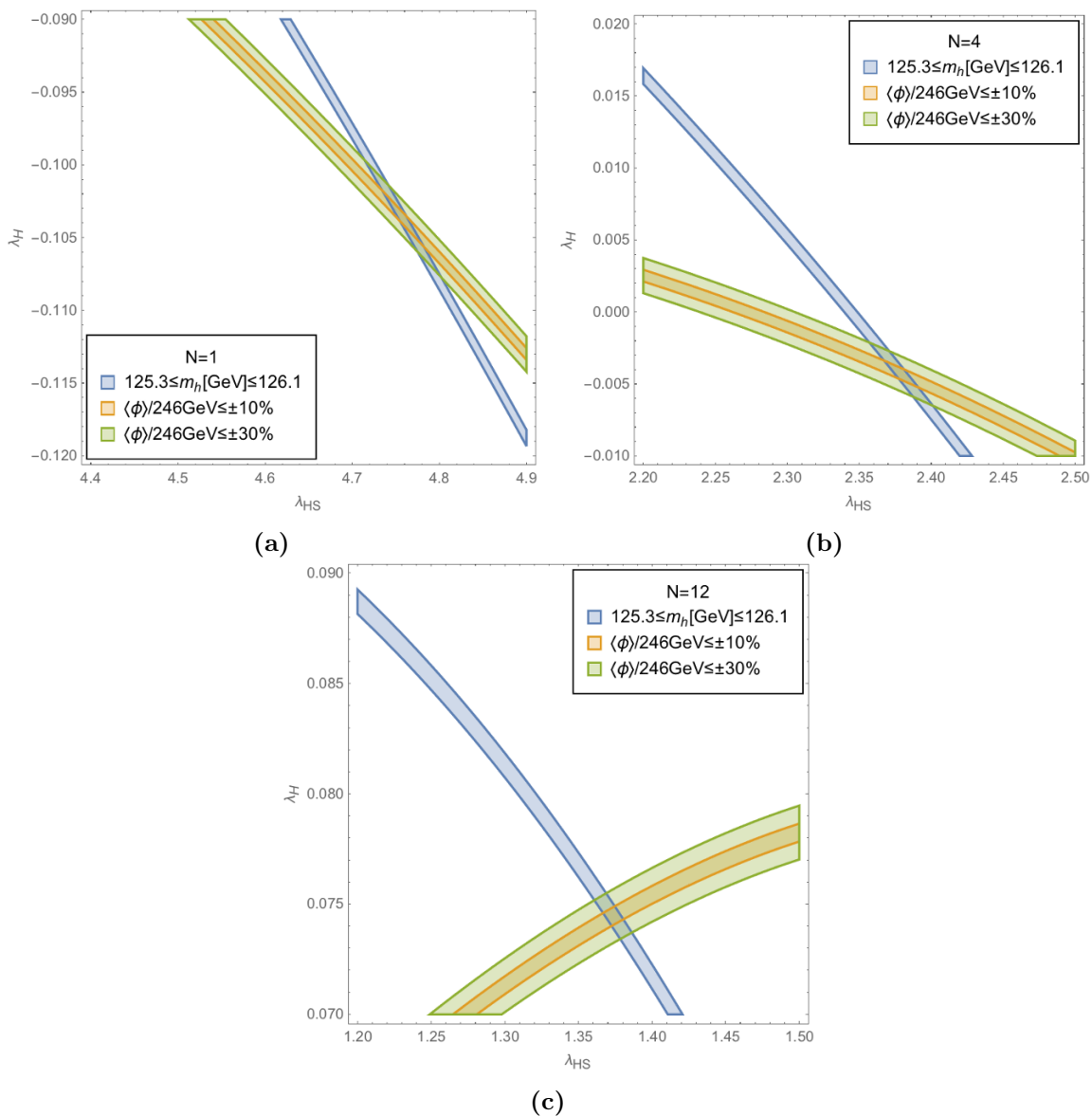


Figure 5. Plots for phenomenologically favoured region of the portal coupling λ_{HS} and the Higgs quartic coupling λ_H for (a) $N = 1$, (b) 4 and (c) 12, corresponding to the experimental data [34]. λ_S is set as 0.1, although the results are fairly insensitive to λ_S .

	$N = 1$			$N = 4$			$N = 12$		
	(I)	(II)	(III)	(I)	(II)	(III)	(I)	(II)	(III)
$\lambda_{\text{hhh}}/\lambda_{\text{hhh}}^{(\text{SM})}$	1.7	1.8	1.8	1.7	1.7	1.7	1.7	1.6	1.7
$\lambda_{\text{hhhh}}/\lambda_{\text{hhhh}}^{(\text{SM})}$	3.7	4.3	4.5	3.7	3.2	3.4	3.7	2.8	3.1
λ_{hss}	11.4	10.2	10.2	5.02	5.02	4.96	2.80	2.95	2.83
λ_{hhss}	14	13	13	5.6	5.7	5.7	3.0	3.2	3.1
λ_{ssss}	–	6.5	–	–	1.9	–	–	0.9	–

Table 4. Coupling constants among the scalar particles, corresponding to the parameters of tables 2 and 3. The coupling constants are defined in eq. (4.2).

4.2 Interactions among physical scalar particles

4.2.1 Our results

Let us expand the effective potential about the vacuum after rewriting $\phi^2 \rightarrow H^\dagger H$, $\varphi^2 \rightarrow \vec{S} \cdot \vec{S}$ and

$$\begin{aligned}
 H &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_H + h \end{pmatrix}, \\
 \vec{S} &= (v_S + s_1, s_2, \dots, s_N)^T \\
 &= (s_1, s_2, \dots, s_N)^T \equiv \vec{s}.
 \end{aligned}
 \tag{4.1}$$

Then the effective potential takes a form

$$\begin{aligned}
 V_{\text{eff}} &= \text{const.} + \frac{1}{2} m_h^2 h^2 + \frac{1}{2} m_s^2 \vec{s} \cdot \vec{s} + \frac{\lambda_{\text{hhh}}}{3!} v_H h^3 + \frac{\lambda_{\text{hhhh}}}{4!} h^4 \\
 &+ \frac{\lambda_{\text{hss}}}{2} v_H h \vec{s} \cdot \vec{s} + \frac{\lambda_{\text{hhss}}}{4} h^2 \vec{s} \cdot \vec{s} + \frac{\lambda_{\text{ssss}}}{4!} (\vec{s} \cdot \vec{s})^2 + \dots,
 \end{aligned}
 \tag{4.2}$$

where only up to dimension-four interactions are shown explicitly.

We show the values of the coupling constants among the scalar particles in table 4. They correspond to the parameters of tables 2 and 3. In the case $N = 1$, the triple self-coupling of the Higgs boson λ_{hhh} turns out to be larger than the SM value by a factor 1.7–1.8, while the quartic self-coupling λ_{hhhh} is larger by a factor 3.7–4.5. The range of each value shows the level of accuracy of our prediction. (In general higher derivatives of the potential have larger uncertainties.) These values are barely dependent on the input value for λ_S , since the effective potential in case (I) is independent of λ_S on the ϕ -axis, and the dependence is weak in the cases (II) and (III).

For $N = 1$, the coupling constants involving the singlet scalars are large and of order ten. They become even larger if the input value for λ_S is taken to be larger. One may wonder if perturbation theory is valid with such large coupling constants. We remind the reader that the coupling constants in the original Lagrangian are not so large, and that our predictions have been tested to be within the perturbative regime in the vicinity of the

scale of the vacuum. These large couplings are considered to be a typical feature of the present model and need to be tested by future experiments.

For larger N , the Higgs triple self-coupling is barely dependent on N , while the quartic coupling decreases slightly with N . The N dependences of the couplings involving the singlet scalars are more evident. Generally we obtain smaller couplings for larger N . This originates from high sensitivities of these couplings on λ_{HS} , which reduces with N as $\sim 1/\sqrt{N}$, and can be regarded as a characteristic feature of our potential.

In the cases (I) and (III) the values of λ_{SSSS} are not shown in the table, for the following reason. If we compute the fourth derivative of V_{NLO} with respect to \vec{S} and set $h = |\vec{s}| = 0$, the contribution of the NG modes of H diverges. This does not happen in the case (II) since V_{NLO} is not involved. The divergence originates from an infra-red region, $k \sim 0$ in $\int d^4k/(k^2)^2$, and is an artifact of setting all the external momenta to zero. In physical amplitudes, momenta flowing into the quartic vertex are almost always non-zero, which regularize the IR divergence in the loop integral. Since the other couplings have similar values for the cases (I), (II), (III), we expect that the values of λ_{SSSS} for the case (II) would give reasonable estimates of the quartic vertex appropriate for physical amplitudes.

4.2.2 Comparison with results using Gildener-Weinberg's framework

Let us compare our results with those using the conventional GW framework.

We show in table 5(a)–(c) comparisons of the couplings among the physical Higgs boson and singlet scalar(s), including their self-couplings, as defined in eq. (4.2). The effective potential in the GW framework coincides with our effective potential after setting $\varphi = 0$. Hence, the Higgs self-couplings are the same in both analyses. [We list the values for case (I) of our analysis as the corresponding values in the GW framework.]

In conventional analyses using the GW framework, the couplings involving the singlet scalars are derived using the tree-level interactions (by substituting the VEV of the Higgs boson appropriately). Therefore, loop-induced effects are not included. In our case, since the portal coupling λ_{HS} is large, its loop-induced effects can be large. These effects are most enhanced for $N = 1$, where λ_{HS} is the largest; see table 5(a). In particular there is a large enhancement of the singlet quartic self-coupling λ_{SSSS} , if λ_{S} is small and the effect of λ_{HS} -induced loop contribution is much larger. (We list the case $\lambda_{\text{S}} = 0.1$ in the table.) Note that, although λ_{S} is practically a free parameter, a smaller value is preferred with regard to the Landau pole. For completeness, we show the λ_{S} dependences of our predictions in table 6. We see that only λ_{SSSS} depends considerably on λ_{S} , and the dependence is roughly consistent with that of $6\lambda_{\text{S}} + \text{const.}$, as anticipated.

In view of the large portal coupling, certainly it is sensible to include the loop-induced effects in computing the interactions involving the singlet scalars. In this case, we need to set up a formulation with proper account of order counting, as described in the previous section. It inevitably requires departure from the analysis in a one-dimensional subspace, i.e., the GW formulation.

The differences between our results and those of the GW framework tend to decrease for larger N , since the portal coupling becomes smaller. This tendency can be confirmed in tables 5(b)(c).

$N = 1$ case	λ_{hhh}	λ_{hhhh}	λ_{hss}	λ_{hhss}	λ_{ssss}	
SM prediction	0.78	0.78	none	none	none	
GW's framework	1.3	2.9	$2\lambda_{HS} = 9.6$	$2\lambda_{HS} = 9.6$	$6\lambda_S = 0.6$	
our analysis	(I)	1.3	2.9	11.4	13.8	–
	(II)	1.4	3.4	10.2	13.0	6.5
	(III)	1.4	3.6	10.2	13.5	–

(a)

$N = 4$ case	λ_{hhh}	λ_{hhhh}	λ_{hss}	λ_{hhss}	λ_{ssss}	
SM prediction	0.78	0.78	none	none	none	
GW's framework	1.3	2.9	$2\lambda_{HS} = 4.8$	$2\lambda_{HS} = 4.8$	$6\lambda_S = 0.6$	
our analysis	(I)	1.3	2.9	5.0	5.6	–
	(II)	1.3	2.5	5.0	5.7	1.9
	(III)	1.4	2.7	5.0	5.7	–

(b)

$N = 12$ case	λ_{hhh}	λ_{hhhh}	λ_{hss}	λ_{hhss}	λ_{ssss}	
SM prediction	0.78	0.78	none	none	none	
GW's framework	1.3	2.9	$2\lambda_{HS} = 2.8$	$2\lambda_{HS} = 2.8$	$6\lambda_S = 0.6$	
our analysis	(I)	1.3	2.9	2.8	3.0	–
	(II)	1.3	2.2	3.0	3.2	0.92
	(III)	1.3	2.4	2.8	3.1	–

(c)

Table 5. Comparisons between the results derived by our analysis and those derived by GW's method for $N = 1, 4$ and 12 cases in (a), (b) and (c), respectively.

λ_S	0.1	0.3	0.5	1.0	2.0
λ_{hss} (I)	11.4	11.5	11.5	11.7	11.9
λ_{hss} (II)	10.2	10.2	10.3	10.3	10.4
λ_{hss} (III)	10.2	10.3	10.3	10.4	10.4
λ_{ssss} (II)	6.53	7.91	9.32	12.8	19.7

Table 6. λ_S dependences of the couplings involving the singlet scalar bosons.

4.3 Veltman’s condition for the Higgs mass

Veltman’s condition is a condition for the quadratic divergence to vanish in the radiative correction to the Higgs potential. Generally the one-loop effective potential for the Higgs boson in a cut-off regularization scheme is given by

$$V_{1\text{-loop}}(\phi) = \frac{1}{64\pi^2} \text{STr} \left[\Lambda^4 \left(\ln \Lambda^2 - \frac{1}{2} \right) + 2M^2(\phi)\Lambda^2 + M^4(\phi) \left(\ln \frac{M^2(\phi)}{\Lambda^2} - \frac{1}{2} \right) \right] + \text{c.t.}, \quad (4.3)$$

where Λ is a UV regulator, and $M^2(\phi) (\ll \Lambda^2)$ is the mass-squared matrix. The first term is a cosmological constant term, which we subtract by the counter term (with tremendous fine-tuning, but we will not be concerned about it here). The second term is a quadratically divergent term. It can either be subtracted (fine-tuning), which makes the theory unnatural, or vanish if

$$\frac{1}{2} \frac{\partial^2}{\partial(\delta\phi_i)\partial(\delta\phi_j)} \left[\text{STr} M^2 \left(\langle \vec{\phi} \rangle + \delta\vec{\phi} \right) \right]_{\delta\vec{\phi} \rightarrow \vec{0}} = 0 \quad \text{for each } i, j \quad (4.4)$$

holds at a scale μ_0 , which makes the theory natural. Here, $\delta\vec{\phi}$ denotes a fluctuation field vector around the VEV $\langle \vec{\phi} \rangle$.

In the SM, the coefficient of the quadratically divergent term is given by

$$\begin{aligned} \frac{1}{2} \frac{\partial^2}{\partial h^2} \left[\text{STr} M^2(v_H + h) \right]_{\mu=v_H, h \rightarrow 0} &= \frac{9g^2(v_H)}{4} + \frac{3g'^2(v_H)}{4} - 6y_t^2(v_H) + 6\lambda_H(v_H) \\ &\simeq -4.1. \end{aligned} \quad (4.5)$$

Thus, Veltman’s condition is violated at order unity and fine-tuning is necessary if the cut-off scale is much higher than the electroweak scale.

Here, we examine whether the fine-tuning is tamed comparatively in the scale-invariant model. The coefficient of Λ^2 is expressed by the matrix

$$\begin{aligned} &\frac{1}{2} \frac{\partial^2}{\partial(\delta\phi_i)\partial(\delta\phi_j)} \left[\text{STr} M^2 \left(\langle \vec{\phi} \rangle + \delta\vec{\phi} \right) \right]_{\delta\vec{\phi} \rightarrow \vec{0}} \\ &= \begin{pmatrix} \frac{9g^2(v_H)}{4} + \frac{3g'^2(v_H)}{4} - 6y_t^2(v_H) + 6\lambda_H(v_H) + N\lambda_{\text{HS}}(v_H) & 0 \\ 0 & (4\lambda_{\text{HS}}(v_H) + (N+2)\lambda_S(v_H))\mathbf{1}_{N \times N} \end{pmatrix}. \end{aligned} \quad (4.6)$$

It is diagonal, since there is no mixing between the Higgs boson and the singlet scalar boson. Let us moderately examine only the coefficient of the Higgs boson for various N . The hh component of the above matrix takes values 0.76, 5.6, 12.8 for $N = 1, 4, 12$, respectively. Instead, we can examine the constraint by Veltman’s condition on the parameters of the model, see figures 6(a)–(c), which also combine the phenomenologically valid regions plotted in figures 5. As can be seen, the fine-tuning is tamed particularly for $N = 1$, which may be a good feature of the model.¹¹

¹¹Since the cut-off scale, given by the Landau pole, is at several TeV scale in the $N = 1$ case, there will not be a serious fine-tuning problem below this scale.

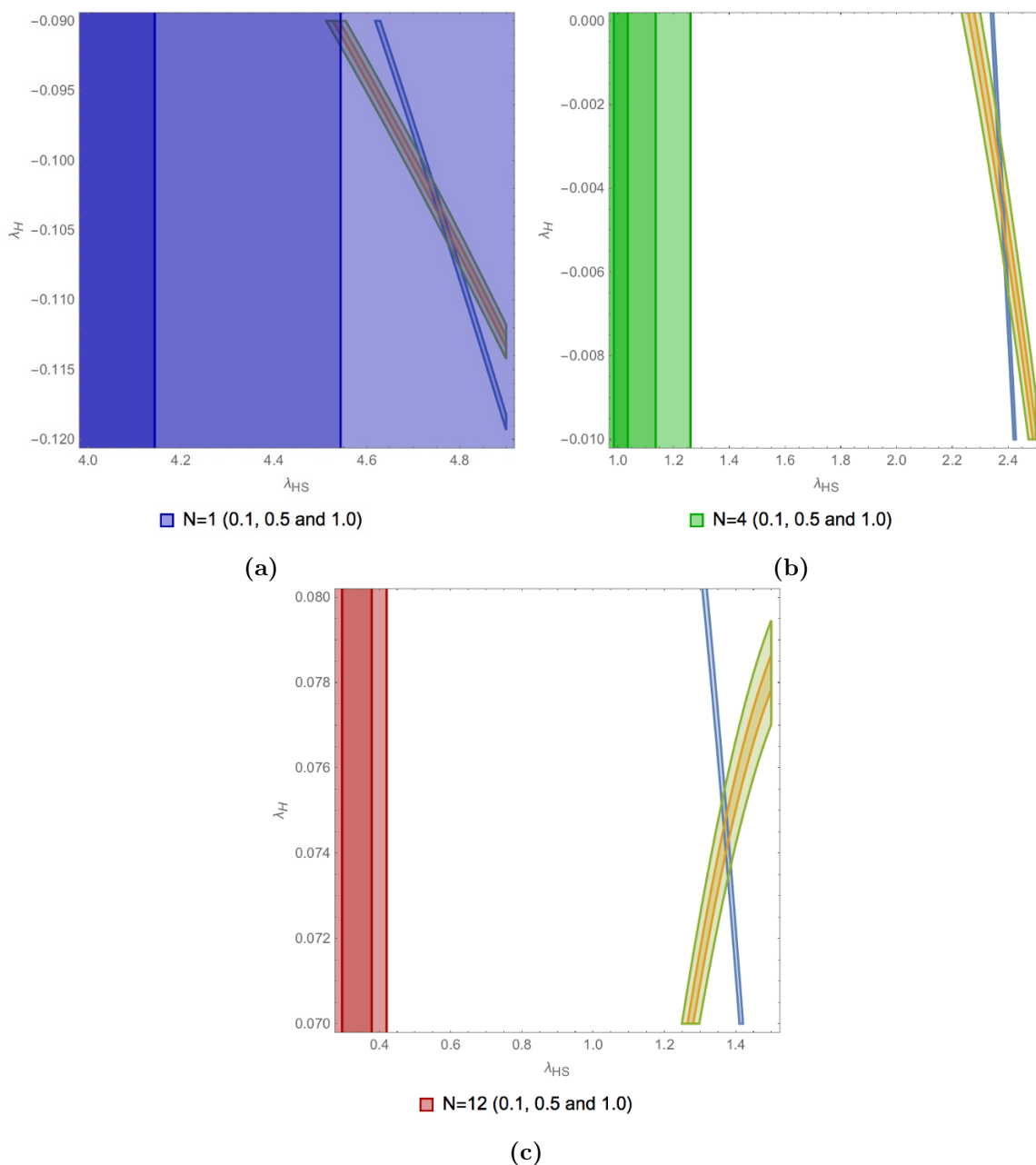


Figure 6. Favoured regions of Veltman’s condition for the Higgs mass for $N = 1$ [(a)], $N = 4$ [(b)] and $N = 12$ [(c)]. We combine the plot in each case with the respective plot of figures 5. The regions in the *darkest*, *darker* and *light* colors correspond to the magnitudes of the coefficient of the quadratic divergence [given by the hh component of eq. (4.6)] smaller than 0.1, 0.5 and 1.0, respectively.

On the other hand, the quadratic divergence for the singlet component is significant. Although λ_S is a free parameter, it is expected to be positive semi-definite at the electroweak scale in order to stabilize the vacuum at LO.¹² One way to remedy this problem is to couple right-handed Majorana neutrinos to the singlet scalars, as advocated in [11]. Here, we do not pursue such possibilities and leave it as an open question.

5 Conclusions and discussion

Up to now experimental data show that properties of the Higgs boson are consistent with the SM predictions. It is an intriguing question, with respect to the structure of the vacuum, whether only the Higgs self-interactions not measured so far can deviate significantly from the SM predictions. A possible scenario is that the Higgs effective potential is irregular at the origin. In this analysis we studied an extension of the Higgs sector with classical scale invariance as an example of such potentials. As a minimal model, we considered the SM Higgs sector in the scale-invariant limit, coupled via a portal interaction to an N -plet of real scalars under a global $O(N)$ symmetry (singlet under the SM gauge group). We analyzed the electroweak symmetry breaking by the CW mechanism using RG and examined interactions among the scalar particles which probe the structure of the potential around the vacuum.

We computed the effective potential in the configuration space of the Higgs and singlet scalar fields (ϕ, φ) . The input parameters are the Higgs self-coupling λ_H , portal coupling λ_{HS} , and self-coupling of the singlet scalars λ_S at tree level. In order to obtain perturbatively valid predictions, we find that parametrically $|\lambda_H| \ll |\lambda_{HS}|$ needs to be satisfied. Furthermore, λ_{HS} needs to be large, of order $\sqrt{N_C/N} y_t \approx 1.7N^{-1/2}$, in order to overwhelm the contribution of the top quark loop. We developed a special perturbative formulation for our model in order to analyze the (ϕ, φ) space, since a naive treatment fails. The consistent parameter region of the couplings $(\lambda_H, \lambda_{HS}, \lambda_S)$ is found which gives a global minimum of the effective potential at $(\phi, \varphi) = (v_H, 0)$ with the Higgs VEV $v_H = 246$ GeV and the Higgs boson mass $m_h = 126$ GeV. The potential is stable against RG improvements, showing validity of the perturbative predictions. λ_H and λ_{HS} are almost fixed, while $\lambda_S (> 0)$ can be taken fairly freely. The SM gauge group is broken as in the SM, while the $O(N)$ symmetry is unbroken. (Hence, there is no NG boson.) The Higgs boson and singlet scalars do not mix at the vacuum. The mass of the N degenerate singlet scalars arises at LO of the perturbative expansion, whereas the mass of the Higgs boson is generated at NLO. Hence, generally the singlet scalars are much heavier than the Higgs boson. For $N = 1$ and $\lambda_S = 0.1$ the mass is about 500 GeV, and it becomes lighter for larger N and heavier for larger λ_S .

Since the coupling λ_{HS} should be large in order to beat the top loop contribution, the Landau pole appears at a few to a few tens TeV (the position of the Landau pole is higher for larger N and smaller for larger λ_S). Hence, the cut-off scale of this model is considered to be around this scale. For instance, this feature conflicts with a scenario which imposes

¹²Since one-loop effects induced by the portal coupling are large, there may be a consistent region where λ_S is negative and still the vacuum is stabilized.

a classically scale-invariant boundary condition at the Planck scale as a possible solution to the naturalness problem. Even without such a motivation, it can be a serious drawback of this model that the Landau pole is located so close to the electroweak scale. These features are consistent with the results of the analysis given in [9], while we presented more detailed analyses.

We computed the triple and quartic (self-)couplings of the Higgs and singlet scalar particles at the vacuum. Computation of the interactions involving the singlet scalars is a unique aspect of the use of our formulation, since the portal coupling is large and loop-induced effects tend to be large. We obtain the Higgs triple and quartic self-couplings which are larger than the SM values by factors 1.6–1.8 and 2.8–4.5, respectively. The triple coupling is hardly dependent on N , while the quartic coupling decreases slightly with N . Both of them are barely dependent on λ_S . According to the studies [35, 36], we naively expect that the triple coupling can be detected at 2σ level at a future ILC, with an integrated luminosity of 1.1 ab^{-1} at a centre-of-mass energy $\sqrt{s} = 500 \text{ GeV}$ and 140 fb^{-1} at $\sqrt{s} = 1 \text{ TeV}$. The couplings involving the singlet scalars are fairly large, of order ten, in the case of small N , reflecting the irregularity of the potential at the origin and a large value of λ_{HS} . If probed, these large couplings provide fairly vivid clues to the structure of the effective potential in the vicinity of the vacuum.

It is difficult to detect signals of this model at the current LHC experiments. Since there is no mixing between the Higgs and singlet scalar particles, the couplings of the Higgs boson with other particles are unchanged from the SM values at tree level. The singlet scalar particles interact with the SM particles only through the Higgs boson. Furthermore, the singlet scalars are heavy and can be produced only in pairs due to the $O(N)$ symmetry (or Z_2 symmetry for $N = 1$). These features make it difficult to detect the singlet scalars directly at the LHC experiments. The production cross sections of the singlet scalars at the LHC are expected to be very small and it would be difficult to detect them, although a further detailed analysis is needed. On the other hand, it would be difficult to probe the extended Higgs sector from loop effects in various precision measurements. These appear as higher-order effects to the Higgs effects, and since the Higgs effects themselves are small, we expect that detection of an anomaly is non-trivial.

From the cosmological point of view, there is a possibility that the singlet scalar boson(s) is a part of dark matter. The singlet boson is stable due to $O(N)$ or Z_2 symmetry. Since the couplings between singlet(s) and Higgs λ_{hss} and λ_{hhss} are very large, the annihilation cross section is also large, which decreases its relic abundance. In $N > 1$ cases, the total relic abundance is the sum over each individual singlet s_i . Then typically the scalar couplings decrease and the total relic abundance increases with N . Our naive estimation shows that singlet(s) can be a dark matter, whose abundance is less than around 1% of the total dark matter relic abundance [37]. We are currently preparing a further detailed study.

We also examined Veltman's condition (vanishing of the quadratic divergence) for the Higgs mass. We find that for $N = 1$ the fine-tuning is relaxed compared to the SM, by the effect of the large portal coupling which cancels against the top-quark loop effects. Since the current level of the fine-tuning in the SM indicates that the natural scale of the cut-off

is quite close to the electroweak scale, this may be a good tendency of the model. The relaxation of fine-tuning is not significant for $N > 1$.

A possible scenario to avoid the Landau pole near the electroweak scale is to promote the global $O(N)$ symmetry to a gauge symmetry (the group can also be replaced by another non-abelian group). An appropriate gauge-symmetric extension pushes up the location of the Landau pole, driven by the asymptotically-free nature, and in an extreme case, up to the Planck scale in the context of a radiative electroweak symmetry breaking scenario [9].

Since the Higgs self-interactions by higher powers of the Higgs field are not suppressed and scalar interactions are large, one may suspect that our model belongs to one of the strongly-interacting Higgs sector, which can be analyzed, e.g. using a non-linear sigma model. We note that it is not a strongly-interacting model, at least around the electroweak scale. We reemphasize that our predictions are well within the perturbative regime, and the usual loop expansion with only renormalizable interactions gives stable predictions with only a few input parameters. To realize a phenomenologically valid scenario, a slightly unusual order-counting is employed, as discussed in section 3.

Acknowledgments

The works of K.E. and Y.S. were supported in part by Tohoku University Institute for International Advanced Research and Education and by Grant-in-Aid for scientific research No. 23540281 from MEXT, Japan, respectively.

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