

Top quark and Higgs boson masses in supersymmetric models

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ABSTRACT: We study the implications for bounds on the top quark pole mass m_t in models with low scale supersymmetry following the discovery of the Standard Model-like Higgs boson. In the minimal supersymmetric standard model, we find that $m_t \geq 164$ GeV, if the light CP even Higgs boson mass $m_h = 125 \pm 2$ GeV. We also explore the top quark and Higgs boson masses in two classes of supersymmetric SO(10) models with t - b - τ Yukawa coupling unification at M_{GUT} . In particular, assuming SO(10) compatible non-universal gaugino masses, setting $m_h = 125$ GeV and requiring 5% or better Yukawa unification, we obtain the result $172 \text{ GeV} \leq m_t \leq 175 \text{ GeV}$. Conversely, demanding 5% or better t - b - τ Yukawa unification and setting $m_t = 173.2$ GeV, the Higgs boson mass is predicted to lie in the range $122 \text{ GeV} \leq m_h \leq 126 \text{ GeV}$.

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1 Introduction

With the discovery by the ATLAS [1] and CMS [2] collaborations of a Standard Model (SM)-like Higgs boson with mass $m_h \simeq 125 - 126$ GeV, the particle content of the SM is fully verified, and the top quark with mass $m_t = 173.18 \pm 0.94$ GeV [3] remains the heaviest particle. The flavor structure of the SM, and, in particular, the relatively heavy top quark mass compared with other fermion masses is an open question in the SM. Likewise, as far as the SM is concerned, there is no deep reason for the Higgs boson to have mass $m_h \simeq 125 - 126$ GeV.

At the same time, one of the successes of the SM is a relatively accurate prediction of the top quark pole mass m_t from fits to the electroweak data. The top quark ‘prediction’ deduced from such fits does not change much as a result of the Higgs boson discovery due to a relatively mild dependence on the latter [4–7]. This mild dependence of the electroweak fits on the Higgs boson mass can be understood from the one loop radiative corrections to the W -boson mass. While these corrections depend quadratically on the top quark mass, there is only a mild logarithmic dependence on the SM Higgs boson mass. Employing all experimental data (including $m_h = 125.7$ GeV), fits to the electroweak data at the 95% confidence level yield $168 \text{ GeV} \lesssim m_t \lesssim 178 \text{ GeV}$, and $80.3 \text{ GeV} \lesssim M_W \lesssim 80.4 \text{ GeV}$ [4] for the W -boson mass, in good agreement with the direct measurement of these quantities. Note that while it is possible to constrain the top quark in the SM from rare decays, such constraints are not compatible with the bounds obtained from the determination of the W boson mass [4–7].

Supersymmetry remains a compelling extensions of the SM and the top quark plays a key role in several features of the minimal supersymmetric standard model (MSSM). For instance, the top quark Yukawa coupling is the dominant contributor to the mechanism of radiative electroweak symmetry breaking (REWSB). We discuss in this paper how the REWSB condition restricts the top quark mass as a function of $\tan\beta$, the ratio of the vacuum expectation values of the two MSSM Higgs doublets. The top quark also plays

a crucial role in the calculation of radiative corrections to the lightest CP even Higgs boson mass in the the MSSM. Among other things these corrections, to leading order, are proportional to the fourth power of the top quark mass. In addition, there is a logarithmic dependence on the geometric mean M_S of the stop quark masses, as well as a contribution proportional to $(A_t/M_S)^4$, where A_t is a tri-linear soft supersymmetry breaking (SSB) term [8–13]. We show in this paper that a 125 GeV SM-like Higgs boson provides strong constraints on the allowed mass range for the top quark mass. This mass interval turns out to be compatible with the range obtained from fits to the electroweak data.

In a supersymmetric SO(10) grand unified theory (GUT) with t - b - τ Yukawa unification [14–16], it was shown in [17, 18] that with suitable non-universal SSB gaugino masses at M_{GUT} the lightest CP even Higgs boson mass is predicted to lie close to 125 GeV. Such gaugino non-universality at M_{GUT} can arise, for instance, from a non-singlet F -component of the field that breaks supersymmetry. Motivated by this result we seek to provide an answer in this paper to the following question: can models compatible with t - b - τ Yukawa unification also yield a stringent constraint for the top quark mass in good agreement with the observations?

We consider two classes of SO(10) models with a minimal set of SSB parameters at M_{GUT} in which t - b - τ Yukawa unification is realized. The most well studied one has universal SSB gaugino mass terms but non-universal Higgs SSB terms, $m_{H_u}^2 \neq m_{H_d}^2$ at M_{GUT} . Here m_{H_u, H_d}^2 stand for the up/down type Higgs SSB masses². The second class of models, mentioned earlier, assumes universal Higgs SSB mass² terms, but the gaugino SSB masses at M_{GUT} are non-universal. Allowing the top quark mass to vary in the interval $0 < m_t < 220$ GeV, we scan the characteristic SSB parameter space for both classes of models and show that the t - b - τ Yukawa unification condition yields to a relatively narrow interval for the masses of the top quark and the light CP even Higgs boson. An upper bound for the top quark mass is obtained by imposing perturbativity on the top Yukawa coupling up to M_{GUT} . It is possible [19–28] to relax the universality of SSB Higgs mass² terms along with the non-universality of SSB gaugino mass terms, but we restrict ourselves here to the above two classes.

The outline for the rest of the paper is as follows. In section 2 we summarize the scanning procedure and the experimental constraints applied in our analysis. In section 3 we present the results for the SO(10) model in which t - b - τ Yukawa unification is achieved via non-universal SSB gaugino masses and universal up and down Higgs SSB mass² terms ($m_{H_u}^2 = m_{H_d}^2 = m_{10}^2$). Using the fact that the light CP even Higgs boson mass is in the interval $123 \text{ GeV} < m_h < 127 \text{ GeV}$, we show that the top quark mass lies in the range $164 \text{ GeV} < m_t < 205 \text{ GeV}$. Imposing 5% or better t - b - τ Yukawa unification, the allowed top quark mass range shrinks to $168 \text{ GeV} < m_t < 180 \text{ GeV}$. In section 4 we present results for the SO(10) model with universal SSB gaugino mass terms but which also requires that $m_{H_u}^2 < m_{H_d}^2$ [29, 30]. In this case, we find the top quark mass range $165 \text{ GeV} < m_t < 200 \text{ GeV}$. Imposing 5% or better t - b - τ Yukawa unification condition, the interval for the top quark mass is reduced to $170 \text{ GeV} < m_t < 178 \text{ GeV}$. We also consider b - τ Yukawa unification in this case and, as expected, the constraint on the top quark mass is relaxed to $168 \text{ GeV} < m_t < 200 \text{ GeV}$. Our conclusions are presented in section 5.

2 Phenomenological constraints and scanning procedure

We employ the ISAJET 7.84 package [31] to perform random scans over the parameter space. In this package, the weak scale values of gauge and third generation Yukawa couplings are evolved to M_{GUT} via the MSSM renormalization group equations (RGEs) in the \overline{DR} regularization scheme. We do not strictly enforce the unification condition $g_3 = g_1 = g_2$ at M_{GUT} , since a few percent deviation from unification can be assigned to unknown GUT-scale threshold corrections [32–34]. With the boundary conditions given at M_{GUT} , all the SSB parameters, along with the gauge and third family Yukawa couplings, are evolved back to the weak scale M_Z .

In evaluating the Yukawa couplings the SUSY threshold corrections [35] are taken into account at a common scale $M_S = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$. The entire parameter set is iteratively run between M_Z and M_{GUT} using the full 2-loop RGEs until a stable solution is obtained. To better account for the leading-log corrections, one-loop step-beta functions are adopted for the gauge and Yukawa couplings, and the SSB scalar mass parameters m_i are extracted from RGEs at appropriate scales $m_i = m_i(m_i)$. The RGE-improved 1-loop effective potential is minimized at an optimized scale M_S , which effectively accounts for the leading 2-loop corrections. Full 1-loop radiative corrections are incorporated for all sparticle masses.

In scanning the parameter space, we employ the Metropolis-Hastings algorithm as described in [36]. The data points collected all satisfy the requirement of radiative electroweak symmetry breaking (REWSB) [37–41], with the neutralino in each case being the LSP. After collecting the data, we impose the mass bounds on all the particles [42] and use the IsaTools package [43, 44] and SuperIso v2.3 [45] to implement the following phenomenological constraints:

$$\begin{aligned}
 0.8 \times 10^{-9} &\leq BR(B_s \rightarrow \mu^+ \mu^-) && \leq 6.2 \times 10^{-9} \quad (2\sigma) && [46] \\
 2.99 \times 10^{-4} &\leq BR(b \rightarrow s\gamma) && \leq 3.87 \times 10^{-4} \quad (2\sigma) && [47] \\
 0.15 &\leq \frac{BR(B_u \rightarrow \tau\nu_\tau)_{\text{MSSM}}}{BR(B_u \rightarrow \tau\nu_\tau)_{\text{SM}}} && \leq 2.41 \quad (3\sigma) && [47] \\
 0 &\leq \Delta(g-2)_\mu/2 && \leq 55.6 \times 10^{-10} && [48].
 \end{aligned}$$

We also implement the following mass bounds on the sparticle masses:

$$\begin{aligned}
 m_{\tilde{g}} &\gtrsim 1.4 \text{ TeV} && (\text{for } m_{\tilde{g}} \sim m_{\tilde{q}}) && [49, 50] \\
 m_{\tilde{g}} &\gtrsim 0.9 \text{ TeV} && (\text{for } m_{\tilde{g}} \ll m_{\tilde{q}}) && [49, 50] \\
 M_A &\gtrsim 700 \text{ GeV} && (\text{for } \tan\beta \simeq 48) && [51].
 \end{aligned}$$

3 SO(10) GUT with non universal gauginos masses

One of the main motivations of supersymmetric SO(10) GUT, in addition to gauge coupling unification, is matter unification. The spinor representation of SO(10) unifies all matter fermions of a given family in a single multiplet (16_i), which also contains the right handed neutrino (ν_R). Another virtue of SO(10) is that, in principle, the two MSSM Higgs doublets

can be accommodated in a single ten dimensional (10_H) representation, which then yields the following Yukawa couplings

$$Y_{ij} 16_i 16_j 10_H. \tag{3.1}$$

Here $i, j = 1, 2, 3$ stand for family indices and the $SO(10)$ indices have been omitted for simplicity. For the third generation quarks and leptons, the interaction in eq. (3.1) yields the following Yukawa coupling unification condition at M_{GUT} [14–16]

$$Y_t = Y_b = Y_\tau = Y_{\nu_\tau}. \tag{3.2}$$

In gravity mediated supersymmetry breaking scenario [52–55], implementing eq. (3.2), in particular $Y_t = Y_b = Y_\tau$ at M_{GUT} , can place significant constraints on the supersymmetric spectrum [56]. These constraints depend on the particular boundary conditions for sparticle SSB masses chosen at M_{GUT} . If the SSB gaugino mass terms are assumed to be universal at M_{GUT} , $m_{H_u}^2(M_{GUT}) < m_{H_d}^2(M_{GUT})$ is required in order for Yukawa coupling unification to be consistent with radiative electroweak symmetry breaking (REWSB). This splitting may arise, for example, via a D -term contribution to all scalar masses, or it can be generated via “Just-So” splitting [29, 30]. The results of this scenario are presented in section 4.

Alternatively, it is possible to achieve t - b - τ Yukawa coupling unification consistent with REWSB by assuming the gaugino SSB mass terms to be non-universal at M_{GUT} . In this case, non-universality among the Higgs SSB mass terms is not needed.

It has been pointed out [57–59] that non-universal MSSM gaugino masses at M_{GUT} can arise from non-singlet F -terms, compatible with the underlying GUT symmetry. The SSB gaugino masses in supergravity [52–55] can arise, say, from the following dimension five operator:

$$-\frac{F^{ab}}{2M_P} \lambda^a \lambda^b + \text{c.c.} . \tag{3.3}$$

Here λ^a is the two-component gaugino field, F^{ab} denotes the F -component of the field which breaks SUSY, and the indices a, b run over the adjoint representation of the gauge group. The resulting gaugino mass matrix is $\langle F^{ab} \rangle / M_P$, where the supersymmetry breaking parameter $\langle F^{ab} \rangle$ transforms as a singlet under the MSSM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$.

If F transforms as a 54 or 210 dimensional representation of $SO(10)$ [57–59], one obtains the following relation among the MSSM gaugino masses at M_{GUT} :

$$M_3 : M_2 : M_1 = 2 : -3 : -1, \tag{3.4}$$

where M_1, M_2, M_3 denote the gaugino masses corresponding to $U(1), SU(2)_L$ and $SU(3)_c$, respectively. In order to obtain the correct sign for the desired contribution to $(g-2)_\mu$, we choose $M_1 > 0, M_2 > 0$ and $M_3 < 0$. Notice that, in general, if F^{ab} transforms non trivially under $SO(10)$, the SSB terms such as the trilinear couplings and scalar mass terms are not necessarily universal at M_{GUT} . However, we can assume, consistent with $SO(10)$ gauge symmetry, that the coefficients associated with terms that violate the $SO(10)$ -invariant form are suitably small, except for the gaugino term in eq. (3.3). We also assume that D -term contributions to the SSB terms are much smaller compared with contributions from fields with non-zero auxiliary F -terms.

Employing the boundary condition from eq. (3.4), one can define the MSSM gaugino masses at M_{GUT} in terms of the mass parameter $M_{1/2}$:

$$\begin{aligned} M_1 &= M_{1/2} \\ M_2 &= 3M_{1/2} \\ M_3 &= -2M_{1/2}. \end{aligned} \tag{3.5}$$

Note that M_2 and M_3 have opposite signs which is important in implementing Yukawa coupling unification as shown in [19–22]. In order to quantify Yukawa coupling unification, following [60], we define the quantity $R_{tb\tau}$ as,

$$R_{tb\tau} = \frac{\max(Y_t, Y_b, Y_\tau)}{\min(Y_t, Y_b, Y_\tau)}. \tag{3.6}$$

We have performed random scans in the fundamental parameter space as follows:

$$\begin{aligned} 0 &\leq m_{16} \leq 10 \text{ TeV} \\ 0 &\leq M_{1/2} \leq 5 \text{ TeV} \\ 0 &\leq m_{10} \leq 10 \text{ TeV} \\ -3 &\leq A_0/m_{16} \leq 2 \text{ TeV} \\ 1.1 &\leq \tan \beta \leq 60 \\ 0 &\leq m_t \leq 220 \text{ GeV} \\ \mu &> 0. \end{aligned} \tag{3.7}$$

Here m_{16} is the universal SSB mass for MSSM sfermions, m_{10} is the universal SSB mass term for the up/down MSSM Higgs doublets, $M_{1/2}$ is the gaugino mass parameter, $\tan\beta$ is the ratio of the vacuum expectation values (VEVs) of the two MSSM Higgs doublets, A_0 is the universal SSB trilinear scalar interaction (with corresponding Yukawa couplings factored out), m_t denotes the top-quark mass and $\mu > 0$ sets the sign for the bi-linear SSB Higgs mixing term whose absolute value is fixed by requiring REWSB.

In figure 1, we present our results in the m_t - $\tan\beta$ and m_t - M_S planes. *Gray* points are consistent with REWSB and neutralino LSP. *Blue* points form a subset of the *gray* ones and satisfy sparticle mass bounds and other constraints described in section 2. *Orange* points belong to a subset of *blue* points and satisfy the lightest CP even Higgs boson mass bound $123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}$.

It is interesting to note that REWSB can arise even for m_t values as low as 10 GeV, but $\tan\beta$ values are then constrained to be in a narrow range. For a given top quark mass we have a well defined $\tan\beta$ interval from the requirement of REWSB. For instance, from the m_t - $\tan\beta$ plane, one sees that in order to have REWSB with $m_t \approx 10 \text{ GeV}$, the value of $\tan\beta$ should be around 3 or so. Applying all the collider and B-physics constraints except the lightest Higgs boson mass bound, we obtain for the top quark mass the bound $125 \text{ GeV} \lesssim m_t \lesssim 208 \text{ GeV}$ (*blue* points in figure 1). Applying next the Higgs boson mass bound $123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}$, the top quark is expected to lie in the interval $164 \text{ GeV} \lesssim m_t \lesssim 205 \text{ GeV}$. Similar observations can be made from the m_t - M_S plane. It

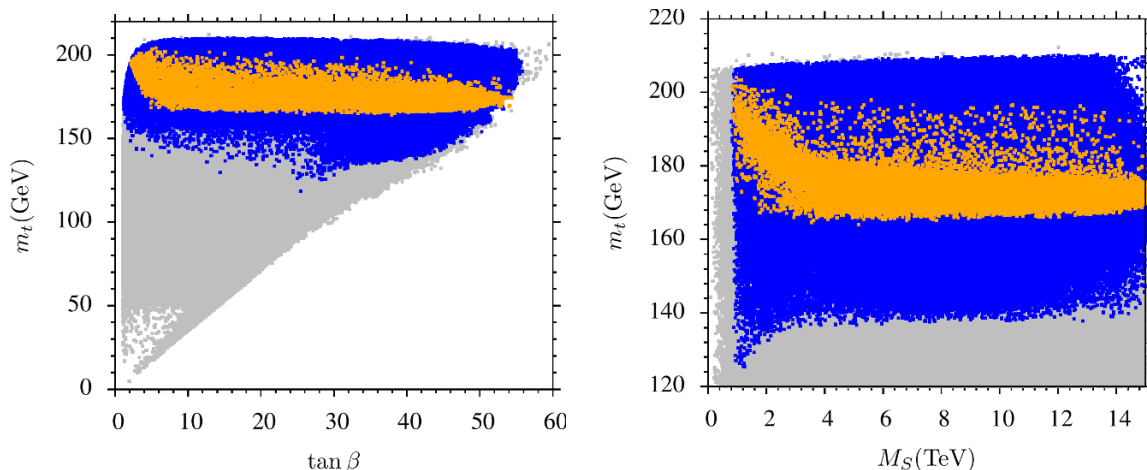


Figure 1. Plots in the m_t - $\tan\beta$ and m_t - M_S planes. *Gray* points are consistent with REWSB and neutralino LSP. *Blue* points form a subset of the *gray* and satisfy sparticle mass bounds and other constraints described in section 2. *Orange* points belong to a subset of *blue* points and satisfy the lightest CP even Higgs boson mass bound $123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}$.

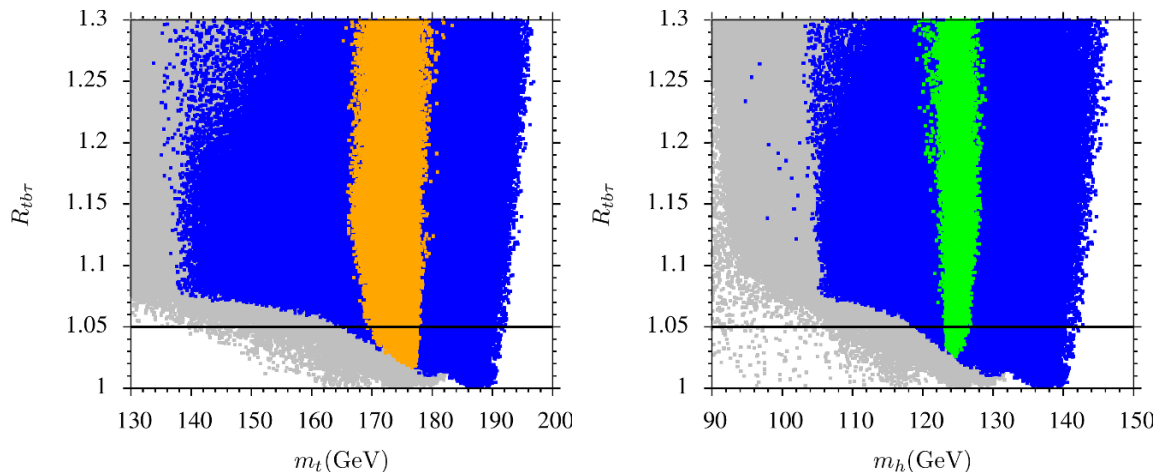


Figure 2. Plots in the $R_{tb\tau}$ - m_t and $R_{tb\tau}$ - m_h planes. Color coding is same as in figure 1. The *green* points in the $R_{tb\tau}$ - m_h plane form a subset of *blue* points and satisfy the bound $172.3 \text{ GeV} \leq m_t \leq 174.1 \text{ GeV}$.

is evident from this plane that no matter how heavy the stop mass, the top quark cannot be lighter than 164 GeV in low scale supersymmetry when light CP-even Higgs boson is in this the interval $123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}$. This lower bound is very close to the values obtained from fits to the electroweak data [4–7].

In figure 2 we present the results in the $R_{tb\tau}$ - m_t and $R_{tb\tau}$ - m_h planes. The color coding is the same as in figure 1. The *green* points in the $R_{tb\tau}$ - m_h plane form a subset of *blue* points and satisfy the bound $172.3 \text{ GeV} \leq m_t \leq 174.1 \text{ GeV}$.

The horizontal lines in the two planes shown in figure 2 correspond to 5% t - b - τ Yukawa coupling unification ($R_{tb\tau} = 1.05$). From the $R_{tb\tau}$ - m_t plane, we see that without requiring any constraints other than REWSB, t - b - τ Yukawa unification better than 5% predicts that

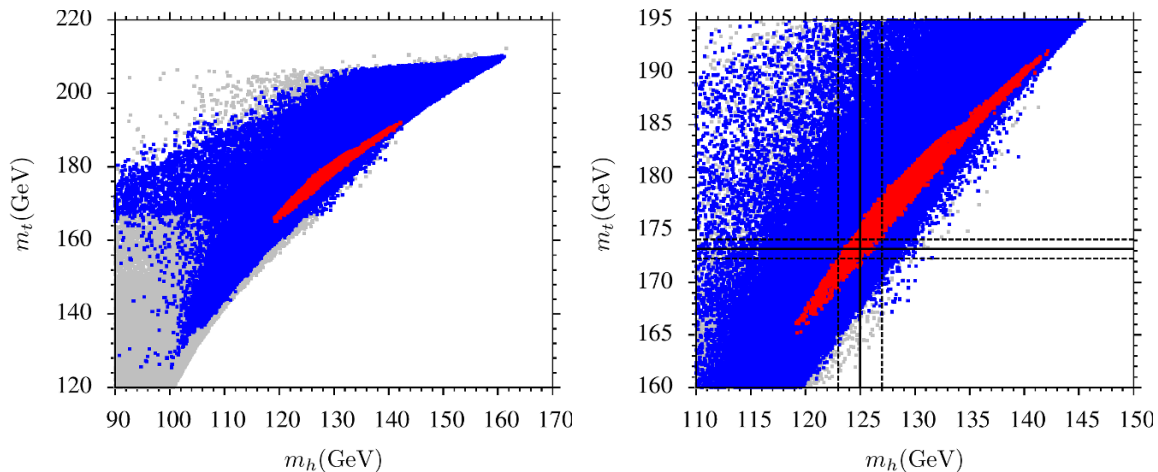


Figure 3. Plots in the m_t - m_h plane. The right panel is just a zoomed-in version of the left panel. The color coding is the same as in figure 1 with the addition of *red* points which form a subset of *blue* points and satisfy $R_{tb\tau} < 1.05$.

the top quark cannot be lighter than 140 GeV. The top quark mass is further constrained by imposing the phenomenological constraints mentioned in section 2. Even without accounting for the bound on the light CP even Higgs mass (*blue* points), the top quark mass is found to lie in the range $166 \text{ GeV} < m_t < 192 \text{ GeV}$. While it is not evident from figure 2, of the constraints on the top quark mass implemented in *blue*, the most severe comes from observation of the decay $B_s \rightarrow \mu^+ \mu^-$. It is well known that in low scale supersymmetry models, this flavor-changing decay receives contributions from the exchange of the pseudoscalar Higgs boson A [61, 62], and its branching ratio is proportional to $(\tan\beta)^6/m_A^4$. Since t - b - τ Yukawa unification happens for large $\tan\beta$ (≈ 47) and it prefers relatively small values for the CP odd pseudoscalar mass ($m_A < 2 \text{ TeV}$) in this model [17, 18], the top quark mass is severely constrained by the $B_s \rightarrow \mu^+ \mu^-$ decay. Applying the light CP even Higgs mass bound, the top quark is predicted to have a mass in the narrow window $170 \text{ GeV} \leq m_t \leq 178 \text{ GeV}$.

Next let us consider the constraints on the Higgs boson mass. From the $R_{tb\tau}$ - m_h plane, taking into account the collider and B-physics bounds (but excluding the top quark mass bound), t - b - τ YU better than 5% requires that the light CP-even Higgs boson mass $m_h > 119 \text{ GeV}$. After imposing the 1σ top quark mass bound, the model predicts that the Higgs mass lies in the range $122 \text{ GeV} \leq m_h \leq 126 \text{ GeV}$, in good agreement with the current experimental observations.

We display the correlation between the top quark and Higgs boson masses in the presence of Yukawa unification, in the m_t - m_h plane in figure 3. The right panel is a zoomed-in version of the left panel in this figure. The color coding is the same as in figure 1, with the addition of *red* points which form a subset of *blue* points and satisfy $R_{tb\tau} < 1.05$.

The sharp edge towards the right in the m_t - m_h plane shows that one requires a heavy top quark in order to obtain a Higgs boson mass of 125 GeV. We also observe that requiring 5% or better Yukawa unification makes the Higgs and top quark masses strongly correlated.

Requiring Yukawa unification along with a 125 GeV Higgs boson mass, one predicts the top quark mass to be in the interval $172 \text{ GeV} \leq m_t \leq 175 \text{ GeV}$. Conversely, requiring Yukawa unification along with the top quark mass to be at its experimentally observed central value, the Higgs boson mass is predicted to be in the range $124 \text{ GeV} \leq m_h \leq 126 \text{ GeV}$.

While there is a few GeV theoretical error in the calculation of the Higgs mass [63], the strong correlation between the Higgs boson mass, the top quark mass and Yukawa unification seems to be quite compelling in this class of models.

4 SO(10) GUT with universal gauginos masses

In this section we consider t - b - τ Yukawa unification in a supersymmetric SO(10) GUT model with universal SSB gauginos masses and “Just so” Higgs mass splitting [29, 30]. This GUT scale boundary condition is commonly known as the non-universal Higgs mass (NUHM2) model. The SSB parameters include m_{16} , $M_{1/2}$, m_{H_d} , m_{H_u} , A_0 , $\tan\beta$. This model predicts a heavy sfermion spectrum ($m_{16} \gtrsim 20 \text{ TeV}$) but relatively light gaugino masses [60]. For instance, the gluinos in this scenario are not heavier than 3 TeV or so [64], which can be tested at the LHC. We next investigate the allowed range for the top quark and Higgs boson masses in this scenario.

We have performed random scans for the following ranges of parameters:

$$\begin{aligned}
 0 &\leq m_{16} \leq 21 \text{ TeV} \\
 0 &\leq M_{1/2} \leq 5 \text{ TeV} \\
 0 &\leq m_{H_d} \leq 27 \text{ TeV} \\
 0 &\leq m_{H_u} \leq 25 \text{ TeV} \\
 -36 &\leq A_0/m_{16} \leq 3 \text{ TeV} \\
 1.1 &\leq \tan\beta \leq 60 \\
 0 &\leq m_t \leq 220 \text{ GeV} \\
 \mu &> 0.
 \end{aligned}
 \tag{4.1}$$

In figure 4, we present results in the m_t - $\tan\beta$ and m_t - M_S planes. The color coding is the same as in figure 1

In contrast to the SO(10) model discussed in section 3, one can see from the m_t - $\tan\beta$ plane that there is no restriction in this version of SO(10) on either the top quark mass or the value of $\tan\beta$ from the point of view of REWSB. After applying sparticle mass bounds and constraints from B-physics, the lower bound on the top quark mass is dramatically increased (*blue* points in figure 4). The lower bound for *blue* points in figure 4 appears to be higher compared to the bounds obtained from figure 1. However, this is likely due to a lack of statistics, as indicated in the m_t - M_S plane, figure 4. Indeed, we have collected far less data in this case compared to the model presented in section 3. A more exhaustive study will likely fill the regions around the isolated *blue* points in m_t - $\tan\beta$ plane. The main reason for a lack of extensive statistics for this model is that we are mainly interested in the correlation between the top quark and Higgs boson masses, and this is amply illustrated by

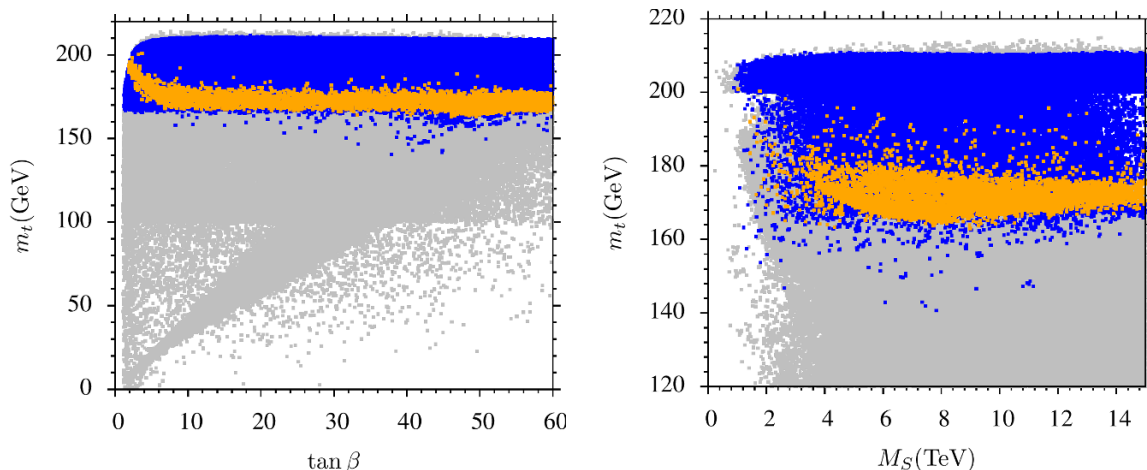


Figure 4. Plots in the m_t - $\tan\beta$ and m_t - M_S planes. Color coding same as in figure 1.

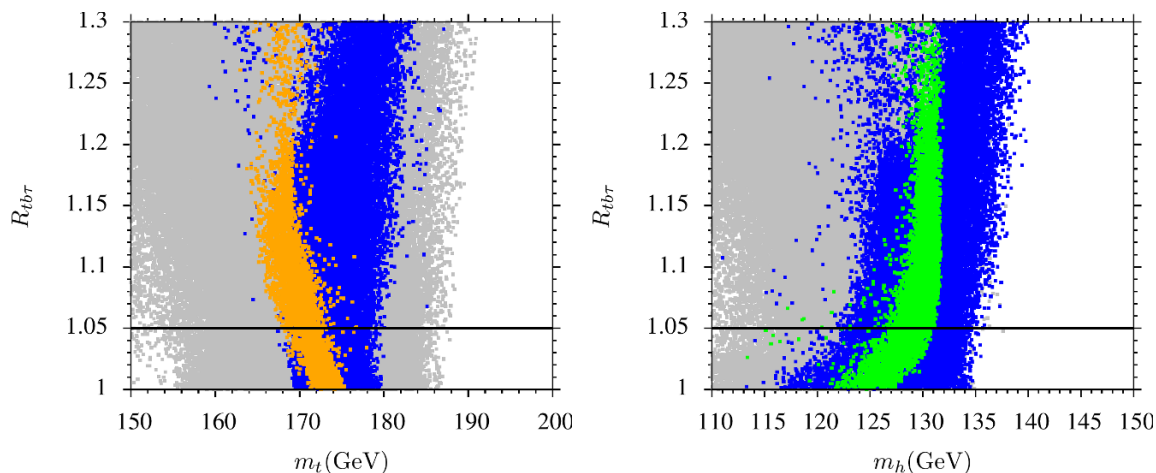


Figure 5. Plots in $R_{tb\tau}$ - m_t and $R_{tb\tau}$ - m_h planes. The color coding is the same as figure 2. *Green* points in the $R_{tb\tau}$ - m_t plane belong to a subset of *blue* points and satisfy the top quark mass bound $172.3 \text{ GeV} \leq m_t \leq 174.1 \text{ GeV}$.

the data that we have collected as is obvious by focusing on the *orange* points in figure 4. As a reminder to the reader, the *orange* points form a subset of the *blue* ones and require the Higgs boson to have a mass in the range $123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}$. This constraint on m_h requires, in turn, that m_t is restricted to lie in the range $164 \text{ GeV} < m_t < 200 \text{ GeV}$.

In figure 5 we show the results in the $R_{tb\tau}$ - m_t and $R_{tb\tau}$ - m_h planes. The color coding is the same as in figure 4. The *green* points in the $R_{tb\tau}$ - m_t plane belong to a subset of *blue* points and satisfy the 1σ top quark mass bound $172.3 \text{ GeV} \leq m_t \leq 174.1 \text{ GeV}$.

As shown in figure 5, requiring REWSB and t - b - τ Yukawa unification better than 5% requires that m_t is heavier than 140 GeV and lighter than 188 GeV. The top quark mass gets confined to the range $166 \text{ GeV} < m_t < 180 \text{ GeV}$ after applying the collider and B-physics constraints on the data from section 2 (excluding the Higgs boson mass bound). This bound is virtually unchanged after applying the experimental bound on the Higgs

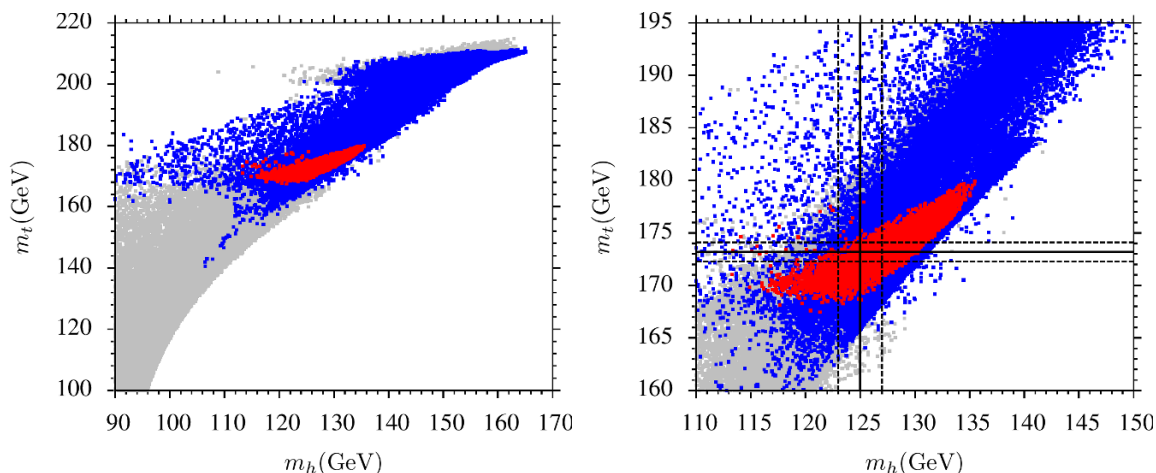


Figure 6. Plots in m_t - m_h plane. Color coding same as figure 3.

mass (*orange* points in the figure). The most severe constraint which gives rise to the *blue* region comes from the decay $b \rightarrow s\gamma$. This is because in low scale supersymmetry the dominant contribution to the $b \rightarrow s\gamma$ branching ratio is proportional to $\mu A_t \tan\beta$, while Yukawa unification with universal gaugino masses at M_{GUT} requires $A_t \gtrsim 15$ TeV [60]. The SUSY contribution to $BR(B_s \rightarrow \mu^+ \mu^-)$ is suppressed in this case because, as pointed out in [64], $m_A > 3$ TeV in this scenario.

The results in the m_t - m_h plane in figure 6 show the correlation between the top quark and Higgs boson masses in the presence of Yukawa unification. This figure is similar in spirit to figure 3 and shares its color coding.

The sharp right edge in the m_t - m_h plane is retained in this version of the SO(10) model, and it shows that one needs a heavy top quark in order to obtain a 125 GeV Higgs boson. We also note that 5% or better Yukawa unification (red points) makes the Higgs and top quark mass interdependence stronger as in the SO(10) model of section 3. Requiring the Higgs boson to have a mass of 125 GeV yields a top quark mass in the interval $168 \text{ GeV} \leq m_t \leq 177 \text{ GeV}$. However, fixing the top quark mass in this case does not yield a sharp prediction for the Higgs boson mass. For instance, for the measured central value of m_t , the Higgs boson mass lies in the range $113 \text{ GeV} \leq m_h \leq 131 \text{ GeV}$.

It is interesting to consider b - τ Yukawa unification which may be more natural with non-universal SSB mass terms in the Higgs sector. In figure 7 we present results in the m_t - m_h plane with brown points signifying 5% or better b - τ Yukawa unification. The rest of the color coding is the same as in figure 1. We can see two distinct brown regions corresponding to b - τ Yukawa unification in this figure. The island-like region is nothing but a somewhat larger region than the *red* region of figure 6. It is larger because b - τ Yukawa unification is less restrictive than t - b - τ Yukawa unification. Unlike t - b - τ Yukawa unification, b - τ Yukawa unification is realized for both large and small $\tan\beta$ values. The region formed by the isolated brown points in figure 7 corresponds to the case of small $\tan\beta$. As pointed out in [65–67], b - τ Yukawa unification for small $\tan\beta$ values requires that the sfermion SSB mass terms are larger than 5 TeV or so. Moreover, the non-universal SSB sfermion mass terms at M_{GUT} are need to be based on their SU(5) representation. While we do

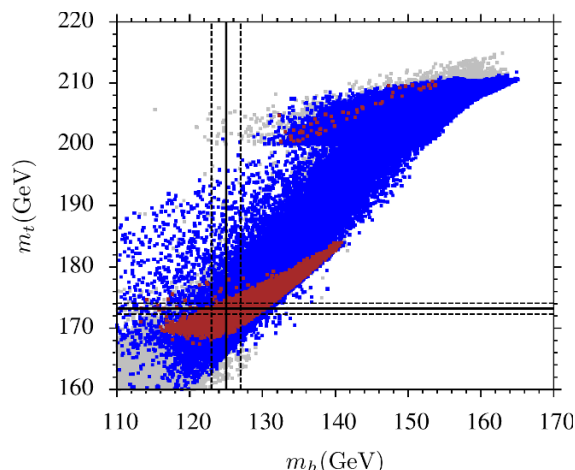


Figure 7. Plots in m_t - m_h plane. The color coding is the same as figure 1. The brown points form a subset of *blue* points and represent 5% or better b - τ Yukawa unification.

not have non-universal SSB mass terms characteristic of SU(5), the top quark mass is free in our analysis. As a result, b - τ Yukawa unification does occur for small $\tan\beta$ values, but it occurs in an experimentally unacceptable region for the top quark mass, namely 200–210 GeV.

5 Conclusions

We have tried to understand the top quark mass from the requirement that t - b - τ Yukawa coupling unification occurs at M_{GUT} . For this, we consider two SO(10) GUT models, with one model having universal SSB gaugino masses but non-universal SSB Higgs mass terms. In the second example we have universal Higgs SSB mass terms but non-universal SSB gaugino masses. We have also considered the correlation between the Higgs boson and top quark masses. The upper bound $m_t \sim 210$ GeV or so on the top quark mass comes from requiring perturbativity of the top quark Yukawa coupling up to scale M_{GUT} . Radiative electroweak symmetry breaking imposes a lower bound on the top quark mass, which depends on $\tan\beta$. In the model with non-universal SSB gaugino masses, applying all the collider and B-physics constraints including the bound on the Higgs boson mass yields the interval $164 \text{ GeV} \lesssim m_t \lesssim 205 \text{ GeV}$. Further, imposing Yukawa coupling unification narrows this mass range, namely $172 \text{ GeV} \lesssim m_t \lesssim 175 \text{ GeV}$.

For the model with non-universal SSB Higgs mass terms, imposing the collider bounds including the bound from the Higgs boson mass, one arrives at a similar range for the top quark mass as the previously quoted result. However, the allowed top quark mass range after imposing Yukawa coupling unification is somewhat more relaxed than in the previous case, to wit $168 \text{ GeV} \lesssim m_t \lesssim 177 \text{ GeV}$, which is quite consistent with the experimental data. Furthermore requiring b - τ instead of t - b - τ Yukawa unification in this model relaxes the top quark range, $167 \text{ GeV} \lesssim m_t \lesssim 182 \text{ GeV}$, for large $\tan\beta$ values. The small $\tan\beta$ scenario in this model predicts a top quark mass in the neighborhood of 200 GeV or so, and is, therefore, disfavored.

The correlation between the Higgs boson mass and the top quark mass is also very interesting, particularly in the model with non-universal SSB gaugino mass terms. By imposing t - b - τ Yukawa coupling unification and requiring the top quark mass to be close to its observed central value, the Higgs boson mass is founded to lie in the range $124 \text{ GeV} \lesssim m_h \lesssim 126 \text{ GeV}$. This correlation between m_t and m_h from Yukawa unification is not as strong in the model with non-universal SSB Higgs mass terms. In this case we obtain the result $113 \text{ GeV} \lesssim m_h \lesssim 131 \text{ GeV}$.

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