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Seesaw mirroring between light and heavy Majorana neutrinos with the help of the S_3 reflection symmetry

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Abstract: In the canonical seesaw mechanism we require the relevant neutrino mass terms to be invariant under the S_3 charge-conjugation transformations of left- and right-handed neutrino fields. Then both the Dirac mass matrix $M_{\rm D}$ and the right-handed neutrino mass matrix $M_{\rm R}$ are well constrained, so is the effective light Majorana neutrino mass matrix M_ν via the seesaw formula. We find that these mass matrices can be classified into 22 categories, among which some textures respect the well-known μ-τ permutation or reflection symmetry and flavor democracy. It is also found that there exist remarkable structural equalities or similarities between M_ν and $M_{\rm R}$, reflecting a seesaw mirroring relationship between light and heavy Majorana neutrinos. We calculate the corresponding light neutrino masses and flavor mixing parameters as well as the CP-violating asymmetries in decays of the lightest heavy Majorana neutrino, and show that only the flavored leptogenesis mechanism is possible to work for three categories of $M_{\rm D}$ and $M_{\rm R}$ in the S_3 reflection symmetry limit.

Keywords: Beyond Standard Model, Discrete Symmetries, Neutrino Physics, CP violation

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1 Introduction

The experimental discoveries of neutrino oscillations [1] have confessedly demonstrated that the standard model (SM) of particle physics is incomplete, because it cannot accommodate and explain the finite but tiny neutrino masses and significant lepton flavor mixing effects. The most canonical and popular way out is to introduce three right-handed neutrino fields $N_{\alpha{\rm R}}$ (for $\alpha = e, \mu, \tau$) and allow lepton number violation [2–6], with which the Yukawa interaction and a Majorana neutrino mass term can be written as

$$-\mathcal{L}_0 = \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}\,, \tag{1.1}$$

where $\widetilde{H} \equiv {\rm i}\sigma_2 H^*$ with H being the Higgs doublet of the SM, $\ell_{\rm L}$ denotes the left-handed lepton doublet column vector, $N_{\rm R}$ represents the right-handed neutrino column vector with the $N_{\alpha{\rm R}}$ components, and $N^{\rm c}_{\rm R} \equiv \mathcal{C}\overline{N_{\rm R}}^{\rm T}$ with T denoting the transpose and $\mathcal{C}$ being the

charge-conjugation operator. After spontaneous electroweak symmetry breaking, eq. (1.1) becomes

$$-\mathcal{L}_{\rm m} = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + \frac{1}{2} \overline{N_{\rm R}^{\rm c}} M_{\rm R} N_{\rm R} + {\rm h.c.} \; , \tag{1.2}$$

where $M_{\rm D} \equiv Y_\nu \langle H \rangle$ with $\langle H \rangle \simeq 174\,{\rm GeV}$ being the vacuum expectation value of the Higgs field. The scale of $M_{\rm R}$ can be much larger than $\langle H \rangle$ because the right-handed neutrino fields are the $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ singlets and thus have nothing to do with electroweak symmetry breaking. In this case one may integrate out the heavy degrees of freedom and then obtain an effective mass term for the three light neutrinos:

$$-\mathcal{L}_\nu = \frac{1}{2} \overline{\nu_{\rm L}} M_\nu \nu_{\rm L}^{\rm c} + {\rm h.c.} \; , \tag{1.3}$$

where $\nu_{\rm L}^{\rm c} \equiv \mathcal{C} \overline{\nu_{\rm L}}^{\rm T}$ is defined, and $M_\nu = -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^{\rm T}$ is the well-known seesaw formula [2–6] in the leading-order approximation, which naturally attributes the smallness of the scale of M_ν to the largeness of the scale of $M_{\rm R}$ as compared with the value of $\langle H \rangle$.

While the above seesaw relation can qualitatively explain why the masses of three light Majorana neutrinos m_i (i.e., the eigenvalues of M_ν) are strongly suppressed in magnitude, it unfortunately has no quantitative prediction for the values of m_i and flavor mixing parameters. To reduce the number of unknown degrees of freedom and thus enhance the predictability and testability of the seesaw mechanism, the structures of $M_{\rm D}$ and $M_{\rm R}$ need to be specified with either some empirical assumptions (e.g., texture zeros — see ref. [7] for a review) or certain flavor symmetries (e.g., A_4 and S_4 symmetries — see refs. [8–10] for recent reviews). Since the observed pattern of the 3×3 Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix V [11, 12] exhibits an approximate μ-τ permutation symmetry (i.e., $|V_{\mu i}| \simeq |V_{\tau i}|$ for $i = 1, 2, 3$), most of the larger flavor symmetry groups considered for the neutrino sector actually consist of a subgroup which allows M_ν to respect the μ-τ flavor symmetry.[1]

Of course, building a realistic neutrino mass model based on a given flavor symmetry is highly nontrivial because it is usually imperative to introduce some hypothetical gauge-singlet scalar fields (i.e., the so-called flavon fields) and make use of their vacuum expectation values to partly fix the flavor structures of massive neutrinos and charged leptons. Hence the flavor symmetry breaking is typically associated with many unknown parameters which are normally put into a hidden dustbin in most of the model-building exercises, since these new parameters are experimentally unaccessible for the time being. The variety of such models makes it practically hard to judge which flavor symmetry is closer to the truth [14].

In this situation one may follow a purely phenomenological way to focus only on the mass terms of charged leptons and neutrinos and then constrain their textures by means of certain flavor symmetries, so as to predict an acceptable flavor mixing pattern which is consistent with current neutrino oscillation data [8–10, 13]. Although there is an obvious gap between such an approach and a real neutrino mass model, the former can be regarded as a necessary or instructive step towards the latter. Considering that the underlying

[1] For a review with extensive references see ref. [13].

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flavor symmetry is most likely to manifest itself at a high energy scale far above the electroweak scale, the phenomenological approach under discussion actually fits the spirit of the bottom-up approach of model building in particle physics.

Following the same phenomenological approach, here we are interested in exploring the seesaw-induced relation between light and heavy Majorana neutrinos with the help of possible S_3 flavor symmetries. That is to say, we consider the possibility of simultaneously constraining the textures of $M_{\rm D}$ and $M_{\rm R}$ by requiring that $\mathcal{L}_{\rm m}$ in eq. (1.2) be invariant under the charge-conjugation transformations $\nu_{\rm L} \leftrightarrow \mathcal{S}_{\rm (L)} \nu^{\rm c}_{\rm L}$ and $N_{\rm R} \leftrightarrow \mathcal{S}_{\rm (R)} N^{\rm c}_{\rm R}$, where $\mathcal{S}_{\rm (L)}$ or $\mathcal{S}_{\rm (R)}$ stands for an arbitrary element belonging to an arbitrary subset of S_3 group. In this way it is easy to show that such a phenomenological requirement is equivalent to the constraints $M^*_{\rm D} = \mathcal{S}^\dagger_{\rm (L)} M_{\rm D} \mathcal{S}_{\rm (R)}$ and $M^*_{\rm R} = \mathcal{S}_{\rm (R)} M_{\rm R} \mathcal{S}_{\rm (R)}$, and therefore the structures of $M_{\rm D}$ and $M_{\rm R}$ can be strongly constrained. As a result, the structure of the light Majorana neutrino mass matrix M_ν can be partly determined via the seesaw formula $M_\nu = -M_{\rm D} M^{-1}_{\rm R} M^{\rm T}_{\rm D}$, leading to some intriguing predictions for the neutrino masses and flavor mixing parameters. In comparison with the so-called μ-τ reflection symmetry which has been used to directly constrain the form of M_ν [15], our present method can be referred to as the S_3 reflection symmetry approach. Moreover, we find that the obtained texture of M_ν is either the same as or very similar to that of $M_{\rm R}$, a remarkable consequence of our approach which is referred to as the seesaw mirroring relationship between M_ν and $M_{\rm R}$. Along this line of thought, we also examine which of the S_3-constrained textures of $M_{\rm D}$ and $M_{\rm R}$ can allow for CP violation in the lepton-number-violating decays of the lightest heavy Majorana neutrino, a necessary ingredient of the thermal leptogenesis mechanism [16] which offers a natural explanation of the observed baryon asymmetry of the Universe.

It is worth pointing out that the S_3 reflection symmetry approach under discussion is subject to the basis with the flavor eigenstates of three charged leptons being the same as their mass eigenstates (i.e., the charged-lepton mass matrix M_l is diagonal). Such a basis choice is different from the conventional model building exercises with the help of discrete flavor symmetries, in which the charged-lepton fields usually transform together with the neutrino fields under the given flavor groups [8–10]. After spontaneous flavor symmetry breaking, the charged-lepton and neutrino mass matrices are left with different residual symmetries. The basis with M_l being diagonal can always be achieved by choosing a suitable representation of the given symmetry group, but it might not be convenient from the point of view of model building. In the present work we simply assume M_l to be diagonal and make the S_3 reflection transformations only for the neutrino sector. This simple treatment allows us to directly derive the PMNS neutrino mixing matrix from the effective Majorana neutrino mass matrix M_ν via the seesaw relation, with no concern about the charged-lepton sector.

The approach and main results of this paper are expected to be instructive and useful for broadening our horizons in building realistic neutrino mass models and understanding lepton flavor mixing and CP violation. In fact, a lot of attention has been paid to applying the S_3 flavor symmetry to the quark and lepton sectors since the pioneering work done in 1978 [17, 18], and in this connection remarkable progress was made in 1996 and 1998 to predict quite large solar and atmospheric neutrino mixing angles [19–21]. Although some

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attempts have been made in combining the seesaw mechanism and the S_3 flavor symmetry (see, e.g., refs. [22–30]), our present work is different from them in several aspects:

- We constrain the structures of $M_{\rm D}$ and $M_{\rm R}$ by dictating the two neutrino mass terms in eq. (1.2) to be invariant under the S_3 reflection (i.e., charge-conjugation) transformations $\nu_{\rm L} \leftrightarrow \mathcal{S}_{\rm (L)} \nu^{\rm c}_{\rm L}$ and $N_{\rm R} \leftrightarrow \mathcal{S}_{\rm (R)} N^{\rm c}_{\rm R}$ instead of the S_3 permutation transformations $\nu_{\rm L} \leftrightarrow \mathcal{S}_{\rm (L)} \nu_{\rm L}$ and $N_{\rm R} \leftrightarrow \mathcal{S}_{\rm (R)} N_{\rm R}$. Such a new treatment makes sense because it is fully consistent with the spirit of the μ-τ reflection symmetry — a special case of the S_3 reflection symmetry under discussion, in order to produce the experimentally favored results $\theta_{23} = \pi/4$ and $\delta = 3\pi/2$ for the PMNS matrix V in its standard parametrization form [1]. In comparison, one will be left with $\delta = 0$ in the limit of the flavor democracy [19–21] or S_3 permutation symmetry [31–40].

- We carry out a systematic analysis of all the possible textures of $M_{\rm D}$ and $M_{\rm R}$ constrained by the S_3 reflection symmetry, make a classification of them, and examine whether the resulting textures of M_ν are seesaw-invariant or share the same flavor symmetry with $M_{\rm R}$ and (or) $M_{\rm D}$. Our results can therefore provide a very useful reference for further model-building exercises.

- We calculate the light neutrino masses, flavor mixing angles and CP-violting phases for each texture of M_ν in the S_3 refection symmetry limit, and examine whether the CP-violating asymmetries in decays of the lightest heavy Majorana neutrino are vanishing or not in the same limit. We find that in this case only flavored leptogenesis [41–43] is possible to work for a few textures of $M_{\rm D}$ and $M_{\rm R}$.

As many other flavor symmetries, the S_3 reflection symmetry must be broken too, so as to make M_ν fully fit current experimental data. One may discuss such symmetry breaking effects by either taking account of the renormalization-group evolution of M_ν from the seesaw scale (where the flavor symmetry is assumed to manifest itself) to the electroweak scale, or introducing some explicit symmetry breaking terms into $M_{\rm D}$ and $M_{\rm R}$ [13]. A further work of this kind depends on more technical details and empirical assumptions, and hence it is beyond the scope of the present paper and will be done elsewhere as a follow-up.

The remaining parts of this paper are organized as follows. In section 2 we first introduce the S_3 reflection transformations for left- and right-handed neutrino fields to constrain the structures of $M_{\rm D}$ and $M_{\rm R}$, and then determine the texture of M_ν with the help of the seesaw formula. All the possibilities in this connection are examined and classified. Section 3 is devoted to discussing the phenomenological consequences of M_ν, where the light neutrino masses, flavor mixing angles and CP-violating phases are calculated in a case-by-case way. In section 4 we consider both unflavored and flavored leptogenesis mechanisms and calculate the corresponding CP-violating asymmetries in decays of the lightest heavy Majorana neutrino in the S_3 reflection symmetry limit. A summary of our approach and main results, together with some discussions about extending S_3 group to A_4 group so as to illustrate the constrained textures of neutrino mass matrices in a different way, is made in section 5.

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2 Applications of the S_3 reflection symmetry

2.1 Textures of $M_{\rm D}$ and $M_{\rm R}$ under S_3 reflection symmetry

To begin with, we rewrite the mass terms in eq. (1.2) in a more concise way as follows:

$$-\mathcal{L}_{\rm m} = \frac{1}{2}\overline{\left(\nu_{\rm L} \;\; N^{\rm c}_{\rm R}\right)} \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M^{\rm T}_{\rm D} & M_{\rm R} \end{pmatrix} \begin{pmatrix} \nu^{\rm c}_{\rm L} \\ N_{\rm R} \end{pmatrix} + {\rm h.c.} \; . \tag{2.1}$$

To constrain flavor structures of the canonical seesaw mechanism, we require the neutrino mass term in eq. (2.1) to keep invariant when $\nu_{\rm L}$ and $N_{\rm R}$ transform as

$$\nu_{\rm L} \leftrightarrow \mathcal{S}_{\rm (L)} \nu^{\rm c}_{\rm L} \; , \quad N_{\rm R} \leftrightarrow \mathcal{S}_{\rm (R)} N^{\rm c}_{\rm R} \; , \tag{2.2}$$

in which $\mathcal{S}_{\rm L}$ or $\mathcal{S}_{\rm R}$ denotes an arbitrary element of G — a given subset of S_3 group, and the possibilities of both $\mathcal{S}_{\rm L} = \mathcal{S}_{\rm R}$ and $\mathcal{S}_{\rm L} \neq \mathcal{S}_{\rm R}$ are included. It is worth pointing out that only the neutrino mass term $\mathcal{L}_{\rm m}$ is dictated to be invariant under the transformations made in eq. (2.2), and hence the consequent S_3 reflection symmetry is not a *real* flavor symmetry for the whole Lagrangian of weak interactions. Instead, it only works as an effective organizing principle to simplify and constrain the structures of $M_{\rm D}$ and $M_{\rm R}$. Note that the well-known μ-τ reflection symmetry and some other *working* flavor symmetries [10, 13] were proposed in the same spirit. If such a purely phenomenological approach turns out to be compatible with current and future experimental data, it may finally be embedded into a complete flavor model of fermion masses based on a larger symmetry group.

Under the S_3 reflection transformations given in eq. (2.2), the 6×6 neutrino mass matrix in eq. (2.1) changes as follows:

$$\begin{aligned} -\mathcal{L}'_{\rm m} = {} & \frac{1}{2}\overline{\left(\nu^{\rm c}_{\rm L} \;\; N_{\rm R}\right)} \begin{pmatrix} \mathcal{S}^{\dagger}_{\rm (L)} & \\ & \mathcal{S}_{\rm (R)} \end{pmatrix} \begin{pmatrix} 0 & M_{\rm D} \\ M^{\rm T}_{\rm D} & M_{\rm R} \end{pmatrix} \begin{pmatrix} \mathcal{S}^{\dagger}_{\rm (L)} & \\ & \mathcal{S}_{\rm (R)} \end{pmatrix} \begin{pmatrix} \nu_{\rm L} \\ N^{\rm c}_{\rm R} \end{pmatrix} \\ & + \frac{1}{2}\overline{\left(\nu_{\rm L} \;\; N^{\rm c}_{\rm R}\right)} \begin{pmatrix} \mathcal{S}_{\rm (L)} & \\ & \mathcal{S}^{\dagger}_{\rm (R)} \end{pmatrix} \begin{pmatrix} 0 & M^{*}_{\rm D} \\ M^{\dagger}_{\rm D} & M^{\dagger}_{\rm R} \end{pmatrix} \begin{pmatrix} \mathcal{S}_{\rm (L)} & \\ & \mathcal{S}^{\dagger}_{\rm (R)} \end{pmatrix} \begin{pmatrix} \nu^{\rm c}_{\rm L} \\ N_{\rm R} \end{pmatrix} \; , \end{aligned} \tag{2.3}$$

where the unitarity of $\mathcal{S}_{\rm (L)}$ and $\mathcal{S}_{\rm (R)}$ has been used. It becomes transparent that the neutrino mass terms will be invariant (namely, $\mathcal{L}_{\rm m} = \mathcal{L}'_{\rm m}$) if the whole neutrino mass matrix satisfies the condition

$$\begin{pmatrix} 0 & M_{\rm D} \\ M^{\rm T}_{\rm D} & M_{\rm R} \end{pmatrix} = \begin{pmatrix} 0 & \mathcal{S}_{\rm (L)} M^{*}_{\rm D} \mathcal{S}^{\dagger}_{\rm (R)} \\ \mathcal{S}^{\dagger}_{\rm (R)} M^{\dagger}_{\rm D} \mathcal{S}_{\rm (L)} & \mathcal{S}^{\dagger}_{\rm (R)} M^{\dagger}_{\rm R} \mathcal{S}^{\dagger}_{\rm (R)} \end{pmatrix} \; . \tag{2.4}$$

This in turn means that $M_{\rm D}$ and $M_{\rm R}$ should satisfy the conditions

$$M_{\rm D} = \mathcal{S}_{\rm (L)} M^{*}_{\rm D} \mathcal{S}^{\dagger}_{\rm (R)} \; , \quad M_{\rm R} = \mathcal{S}^{\dagger}_{\rm (R)} M^{*}_{\rm R} \mathcal{S}^{\dagger}_{\rm (R)} \; . \tag{2.5}$$

Therefore, the S_3 reflection symmetry imposed on the neutrino mass terms in eq. (2.1) allows us to constrain textures of the neutrino mass matrices $M_{\rm D}$ and $M_{\rm R}$. Such a constraint can be further transferred to the effective neutrino mass matrix M_ν via the seesaw formula

$$M_\nu = -M_{\rm D} M^{-1}_{\rm R} M^{\rm T}_{\rm D} \; , \tag{2.6}$$

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from which one may examine the structural similarity between M_ν and $M_{\rm R}$. In other words, it is possible to establish a seesaw mirroring relationship between light and heavy Majorana neutrinos with the help of the S_3 reflection symmetry.

Explicitly, the three-dimensional unitary representations for six elements of S_3 group are

$$S^{(123)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad S^{(231)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad S^{(312)} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$
$$S^{(213)} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad S^{(132)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad S^{(321)} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{2.7}$$

These group elements can be categorized into three conjugacy classes: $\mathcal{C}_0 = \left\{S^{(123)}\right\}$, $\mathcal{C}_1 = \left\{S^{(231)}, S^{(312)}\right\}$ and $\mathcal{C}_2 = \left\{S^{(213)}, S^{(132)}, S^{(321)}\right\}$. So S_3 has one subgroup of order three, $Z_3 = \left\{S^{(123)}, S^{(231)}, S^{(312)}\right\}$, as well as three subgroups of order two, $Z_2^{(12)} = \left\{S^{(123)}, S^{(213)}\right\}$, $Z_2^{(23)} = \left\{S^{(123)}, S^{(132)}\right\}$ and $Z_2^{(31)} = \left\{S^{(123)}, S^{(321)}\right\}$. Note that S_3 group totally has $2^{3!} - 1 = 63$ non-void subsets. To characterize these subsets, we first reorder the elements of S_3 group as

$$G^6 = \left\{S^{(123)}, S^{(231)}, S^{(312)}, S^{(213)}, S^{(132)}, S^{(321)}\right\}. \tag{2.8}$$

Then an arbitrary subset can be characterized by $G^n_{i_1 i_2 \cdots i_n}$ or $G^n_{\overline{i_1 i_2 \cdots i_{6-n}}}$ (for $n = 1, 2, \cdots, 6$), in which n is the number of elements in the subset, and $i_1 i_2 \cdots i_n$ for $n \leq 3$ or $\overline{i_1 i_2 \cdots i_{6-n}}$ for $n > 3$ is the index of different subsets with the equal number of elements. What is more, $i_1, i_2, \cdots, i_n = 1, 2, \cdots, 6$ should satisfy $i_1 < i_2 < \cdots < i_n$. Note that we are making use of the index $i_1 i_2 \cdots i_n$ for $n \leq 3$ which is a reordered sequence of the order numbers of elements belonging to $G^n_{i_1 i_2 \cdots i_n}$ in G^6, and the index $\overline{i_1 i_2 \cdots i_{6-n}}$ is used for $n > 3$ which is a reordered sequence of the order numbers of elements belonging to the complement of $G^n_{\overline{i_1 i_2 \cdots i_{6-n}}}$ with respect to S_3 group in G^6. For illustration, let us give several examples to make the notation issue clear: $G^1_2 = \left\{S^{(231)}\right\}$, $G^2_{24} = \left\{S^{(231)}, S^{(213)}\right\}$, $G^3_{124} = \left\{S^{(123)}, S^{(231)}, S^{(213)}\right\}$, $G^4_{\overline{36}} = \left\{S^{(123)}, S^{(231)}, S^{(213)}, S^{(132)}\right\}$ and $G^5_{\overline{3}} = \left\{S^{(123)}, S^{(231)}, S^{(213)}, S^{(132)}, S^{(321)}\right\}$. It should also be noted that $G^3_{123} = Z_3$, $G^2_{14} = Z_2^{(12)}$, $G^2_{15} = Z_2^{(23)}$ and $G^2_{16} = Z_2^{(31)}$, and there are totally $C^n_6 = \frac{6!}{n!(6-n)!}$ different subsets $G^n_{i_1 i_2 \cdots i_n}$ or $G^n_{\overline{i_1 i_2 \cdots i_{6-n}}}$ for a given number n.

With the help of eq. (2.5), we can now obtain all the possible structures of $M_{\rm D}$ and $M_{\rm R}$ constrained by the subsets of S_3 group. Here we take set G^1_5, corresponding to $\mathcal{S}_{\rm (L)} = \mathcal{S}_{\rm (R)} = S^{(132)}$, as a typical example to do some explicit calculations. This case is particularly interesting because it actually works like the μ-τ reflection symmetry. Since set G^1_5 contains

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only a single element (i.e., $S^{(132)}$), the mass matrices $M_{\rm D}$ and $M_{\rm R}$ satisfy

$$\begin{aligned}\begin{pmatrix} \langle M_{\rm D}\rangle_{11} & \langle M_{\rm D}\rangle_{12} & \langle M_{\rm D}\rangle_{13} \\ \langle M_{\rm D}\rangle_{21} & \langle M_{\rm D}\rangle_{22} & \langle M_{\rm D}\rangle_{23} \\ \langle M_{\rm D}\rangle_{31} & \langle M_{\rm D}\rangle_{32} & \langle M_{\rm D}\rangle_{33} \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \langle M_{\rm D}\rangle^*_{11} & \langle M_{\rm D}\rangle^*_{12} & \langle M_{\rm D}\rangle^*_{13} \\ \langle M_{\rm D}\rangle^*_{21} & \langle M_{\rm D}\rangle^*_{22} & \langle M_{\rm D}\rangle^*_{23} \\ \langle M_{\rm D}\rangle^*_{31} & \langle M_{\rm D}\rangle^*_{32} & \langle M_{\rm D}\rangle^*_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} \langle M_{\rm D}\rangle^*_{11} & \langle M_{\rm D}\rangle^*_{13} & \langle M_{\rm D}\rangle^*_{12} \\ \langle M_{\rm D}\rangle^*_{31} & \langle M_{\rm D}\rangle^*_{33} & \langle M_{\rm D}\rangle^*_{32} \\ \langle M_{\rm D}\rangle^*_{21} & \langle M_{\rm D}\rangle^*_{23} & \langle M_{\rm D}\rangle^*_{22} \end{pmatrix} , \end{aligned} \tag{2.9}$$

and

$$\begin{aligned}\begin{pmatrix} \langle M_{\rm R}\rangle_{11} & \langle M_{\rm R}\rangle_{12} & \langle M_{\rm R}\rangle_{13} \\ \langle M_{\rm R}\rangle_{12} & \langle M_{\rm R}\rangle_{22} & \langle M_{\rm R}\rangle_{23} \\ \langle M_{\rm R}\rangle_{13} & \langle M_{\rm R}\rangle_{23} & \langle M_{\rm R}\rangle_{33} \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \langle M_{\rm R}\rangle^*_{11} & \langle M_{\rm R}\rangle^*_{12} & \langle M_{\rm R}\rangle^*_{13} \\ \langle M_{\rm R}\rangle^*_{12} & \langle M_{\rm R}\rangle^*_{22} & \langle M_{\rm R}\rangle^*_{23} \\ \langle M_{\rm R}\rangle^*_{13} & \langle M_{\rm R}\rangle^*_{23} & \langle M_{\rm R}\rangle^*_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} \langle M_{\rm R}\rangle^*_{11} & \langle M_{\rm R}\rangle^*_{13} & \langle M_{\rm R}\rangle^*_{12} \\ \langle M_{\rm R}\rangle^*_{13} & \langle M_{\rm R}\rangle^*_{33} & \langle M_{\rm R}\rangle^*_{23} \\ \langle M_{\rm R}\rangle^*_{12} & \langle M_{\rm R}\rangle^*_{23} & \langle M_{\rm R}\rangle^*_{22} \end{pmatrix} . \end{aligned} \tag{2.10}$$

As a result, we arrive at

$$\begin{aligned} &\langle M_{\rm D}\rangle_{11} = \langle M_{\rm D}\rangle^*_{11} , \quad \langle M_{\rm D}\rangle_{12} = \langle M_{\rm D}\rangle^*_{13} , \\ &\langle M_{\rm D}\rangle_{21} = \langle M_{\rm D}\rangle^*_{31} , \quad \langle M_{\rm D}\rangle_{22} = \langle M_{\rm D}\rangle^*_{33} , \\ &\langle M_{\rm D}\rangle_{23} = \langle M_{\rm D}\rangle^*_{32} ; \end{aligned} \tag{2.11}$$

and

$$\begin{aligned} &\langle M_{\rm R}\rangle_{11} = \langle M_{\rm R}\rangle^*_{11} , \quad \langle M_{\rm R}\rangle_{12} = \langle M_{\rm R}\rangle^*_{13} , \\ &\langle M_{\rm R}\rangle_{22} = \langle M_{\rm R}\rangle^*_{33} , \quad \langle M_{\rm R}\rangle_{23} = \langle M_{\rm R}\rangle^*_{23} . \end{aligned} \tag{2.12}$$

According to eqs. (2.11) and (2.12), the textures of $M_{\rm D}$ and $M_{\rm R}$ can be parametrized as

$$M_{\rm D} = \begin{pmatrix} A_{\rm r} & B & B^* \\ E & C & D \\ E^* & D^* & C^* \end{pmatrix} , \quad M_{\rm R} = \begin{pmatrix} a_{\rm r} & b & b^* \\ b & e & d_{\rm r} \\ b^* & d_{\rm r} & e^* \end{pmatrix} , \tag{2.13}$$

where the subscript "r" means that this element is real. Taking account of the seesaw formula in eq. (2.6), we find that the effective neutrino mass matrix M_ν has the following texture:

$$M_\nu = \begin{pmatrix} a'_{\rm r} & b' & b'^* \\ b' & e' & d'_{\rm r} \\ b'^* & d'_{\rm r} & e'^* \end{pmatrix} , \tag{2.14}$$

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where $a^{\prime}_{\rm r}$, $b^{\prime}$, $d^{\prime}_{\rm r}$ and $e^{\prime}$ are explicitly given by

$$\begin{aligned}
a^{\prime}_{\rm r} &= -\frac{1}{\det M_{\rm R}} \left\{ A^2_{\rm r} \left(|e|^2 - d^2_{\rm r}\right) + 4A_{\rm r}{\rm Re}\left[B\left(d_{\rm r}b^* - be^*\right)\right] + 2{\rm Re}\left[B^2\left(a_{\rm r}e^* - b^{*2}\right)\right] \right. \\
&\left. \qquad + 2|B|^2\left(|b|^2 - a_{\rm r}d\right) \right\} \; , \\
b^{\prime} &= -\frac{1}{\det M_{\rm R}} \left\{ A_{\rm r}E\left(|e|^2 - d^2_{\rm r}\right) + 2E{\rm Re}\left[B\left(d_{\rm r}b^* - be^*\right)\right] + A_{\rm r}C\left(d_{\rm r}b^* - be^*\right) \right. \\
&\qquad + BC\left(a_{\rm r}e^* - b^{*2}\right) + \left(B^*C + BD\right)\left(|b|^2 - ad_{\rm r}\right) + A_{\rm r}D\left(bd_{\rm r} - b^*e\right) \\
&\left. \qquad + B^*D\left(a_{\rm r}e - b^2\right) \right\} \; , \\
e^{\prime} &= -\frac{1}{\det M_{\rm R}} \left[E^2\left(|e|^2 - d^2_{\rm r}\right) + 2EC\left(d_{\rm r}b^* - be^*\right) + 2ED\left(bd_{\rm r} - b^*e\right) \right. \\
&\left. \qquad + C^2\left(a_{\rm r}e^* - b^{*2}\right) + 2CD\left(|b|^2 - a_{\rm r}d_{\rm r}\right) + D^2\left(a_{\rm r}e - b^2\right) \right] \; , \\
d^{\prime}_{\rm r} &= -\frac{1}{\det M_{\rm R}} \left\{ |E|^2\left(|e|^2 - d^2_{\rm r}\right) + 2{\rm Re}\left[\left(E^*C + ED^*\right)\left(d_{\rm r}b^* - be^*\right)\right] \right. \\
&\left. \qquad + \left(|C|^2 + |D|^2\right)\left(|b|^2 - a_{\rm r}d_{\rm r}\right) + 2{\rm Re}\left[CD^*\left(a_{\rm r}e^* - b^{*2}\right)\right] \right\} \; ,
\end{aligned} \tag{2.15}$$

with $\det M_{\rm R} = a_{\rm r}|e|^2 + 2|b|^2 d_{\rm r} - a_{\rm r}d^2_{\rm r} - 2{\rm Re}\left(b^2e^*\right)$. We see that M_ν and $M_{\rm R}$ have the same structure respecting the μ-τ reflection symmetry, and therefore there exists an interesting seesaw mirroring relationship between light and heavy Majorana neutrinos.

The other possibilities can be similarly discussed by repeating the above procedure with either sets G^n corresponding to $\mathcal{S}_{\rm (L)} = \mathcal{S}_{\rm (R)}$ or sets $G^n_{\rm L} \times G^n_{\rm R}$ including both $\mathcal{S}_{\rm (L)} = \mathcal{S}_{\rm (R)}$ and $\mathcal{S}_{\rm (L)} \neq \mathcal{S}_{\rm (R)}$ options. In table 1 we list and classify the textures of $M_{\rm D}$, $M_{\rm R}$ and M_ν corresponding to all the possible sets under consideration. For the sake of simplicity, the explicit relations between the parameters of M_ν and those of $M_{\rm D}$ and $M_{\rm R}$ have been omitted from table 1.

2.2 The seesaw mirroring structure of M_ν

Table 1 provides a classification of all the possible structures of M_ν in accordance with those of $M_{\rm D}$ and $M_{\rm R}$. For each category of M_ν, its structure is the same as or similar to the structure of $M_{\rm R}$ or $M_{\rm D}$, reflecting the seesaw mirroring feature that we have stressed.

The classification is certainly based on eqs. (2.5) and (2.6). If the textures of $M_{\rm D}$, $M_{\rm R}$ and M_ν constrained by different sets, such as $G^n_{i_1 i_2 \cdots i_n}$ (or $G^n_{i_1 i_2 \cdots i_n {\rm L}} \times G^n_{i_1 i_2 \cdots i_n {\rm R}}$) or $G^n_{\overline{i_1 i_2 \cdots i_{6-n}}}$ (or $G^n_{\overline{i_1 i_2 \cdots i_{6-n}}{\rm L}} \times G^n_{\overline{i_1 i_2 \cdots i_{6-n}}{\rm R}}$), are all the same, then they will be sorted into one group. In this way we are totally left with 22 categories of distinctive structures of the mass matrices, as listed in table 1. Note that the mass matrices belonging to categories A_1, A_2 and A_3 are actually correlated with each other via a transformation associated with $S^{(231)}$ and $S^{(312)}$. To be specific,

$$\begin{aligned}
M^{(A_2)}_{\rm D} &= S^{(231)} M^{(A_1)}_{\rm D} S^{(312)} \; , \quad M^{(A_3)}_{\rm D} = S^{(312)} M^{(A_1)}_{\rm D} S^{(231)} \; , \\
M^{(A_2)}_{\rm R} &= S^{(231)} M^{(A_1)}_{\rm R} S^{(312)} \; , \quad M^{(A_3)}_{\rm R} = S^{(312)} M^{(A_1)}_{\rm R} S^{(231)} \; , \\
M^{(A_2)}_{\nu} &= S^{(231)} M^{(A_1)}_{\nu} S^{(312)} \; , \quad M^{(A_3)}_{\nu} = S^{(312)} M^{(A_1)}_{\nu} S^{(231)} \; .
\end{aligned} \tag{2.16}$$

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Cases	$M_{\rm D}$	$M_{\rm R}$	M_ν	Sets
A_1	$\begin{pmatrix} A_{\rm r} & B & B^* \\ E & C & D \\ E^* & D^* & C^* \end{pmatrix}$	$\begin{pmatrix} a_{\rm r} & b & b^* \\ b & e & d_{\rm r} \\ b^* & d_{\rm r} & e^* \end{pmatrix}$	$\begin{pmatrix} a'_{\rm r} & b' & b'^* \\ b' & e' & d'_{\rm r} \\ b'^* & d'_{\rm r} & e'^* \end{pmatrix}$	G^1_5, $G^1_{5\rm L}\times G^1_{5\rm R}$
A_2	$\begin{pmatrix} C & D & E \\ D^* & C^* & E^* \\ B & B^* & A_{\rm r} \end{pmatrix}$	$\begin{pmatrix} e & d_{\rm r} & b \\ d_{\rm r} & e^* & b^* \\ b & b^* & a_{\rm r} \end{pmatrix}$	$\begin{pmatrix} e' & d'_{\rm r} & b' \\ d'_{\rm r} & e'^* & b'^* \\ b' & b'^* & a'_{\rm r} \end{pmatrix}$	G^1_4, $G^1_{4\rm L}\times G^1_{4\rm R}$
A_3	$\begin{pmatrix} C^* & E^* & D^* \\ B^* & A_{\rm r} & B \\ D & E & C \end{pmatrix}$	$\begin{pmatrix} e^* & b^* & d_{\rm r} \\ b^* & a_{\rm r} & b \\ d_{\rm r} & b & e \end{pmatrix}$	$\begin{pmatrix} e'^* & b'^* & d'_{\rm r} \\ b'^* & a'_{\rm r} & b' \\ d'_{\rm r} & b' & e' \end{pmatrix}$	G^1_6, $G^1_{6\rm L}\times G^1_{6\rm R}$
B_1	$\begin{pmatrix} A_{\rm r} & B_{\rm r} & B_{\rm r} \\ E_{\rm r} & C_{\rm r} & D_{\rm r} \\ E_{\rm r} & D_{\rm r} & C_{\rm r} \end{pmatrix}$	$\begin{pmatrix} a_{\rm r} & b_{\rm r} & b_{\rm r} \\ b_{\rm r} & e_{\rm r} & d_{\rm r} \\ b_{\rm r} & d_{\rm r} & e_{\rm r} \end{pmatrix}$	$\begin{pmatrix} a'_{\rm r} & b'_{\rm r} & b'_{\rm r} \\ b'_{\rm r} & e'_{\rm r} & d'_{\rm r} \\ b'_{\rm r} & d'_{\rm r} & e'_{\rm r} \end{pmatrix}$	G^2_{15}
B_2	$\begin{pmatrix} C_{\rm r} & D_{\rm r} & E_{\rm r} \\ D_{\rm r} & C_{\rm r} & E_{\rm r} \\ B_{\rm r} & B_{\rm r} & A_{\rm r} \end{pmatrix}$	$\begin{pmatrix} e_{\rm r} & d_{\rm r} & b_{\rm r} \\ d_{\rm r} & e_{\rm r} & b_{\rm r} \\ b_{\rm r} & b_{\rm r} & a_{\rm r} \end{pmatrix}$	$\begin{pmatrix} e'_{\rm r} & d'_{\rm r} & b'_{\rm r} \\ d'_{\rm r} & e'_{\rm r} & b'_{\rm r} \\ b'_{\rm r} & b'_{\rm r} & a'_{\rm r} \end{pmatrix}$	G^2_{14}
B_3	$\begin{pmatrix} C_{\rm r} & E_{\rm r} & D_{\rm r} \\ B_{\rm r} & A_{\rm r} & B_{\rm r} \\ D_{\rm r} & E_{\rm r} & C_{\rm r} \end{pmatrix}$	$\begin{pmatrix} e_{\rm r} & b_{\rm r} & d_{\rm r} \\ b_{\rm r} & a_{\rm r} & b_{\rm r} \\ d_{\rm r} & b_{\rm r} & e_{\rm r} \end{pmatrix}$	$\begin{pmatrix} e'_{\rm r} & b'_{\rm r} & d'_{\rm r} \\ b'_{\rm r} & a'_{\rm r} & b'_{\rm r} \\ d'_{\rm r} & b'_{\rm r} & e'_{\rm r} \end{pmatrix}$	G^2_{16}
C	$\begin{pmatrix} A_{\rm r} & B & B^* \\ B^* & A_{\rm r} & B \\ B & B^* & A_{\rm r} \end{pmatrix}$	$\begin{pmatrix} a_{\rm r} & b_{\rm r} & b_{\rm r} \\ b_{\rm r} & a_{\rm r} & b_{\rm r} \\ b_{\rm r} & b_{\rm r} & a_{\rm r} \end{pmatrix}$	$\begin{pmatrix} a'_{\rm r} & b'_{\rm r} & b'_{\rm r} \\ b'_{\rm r} & a'_{\rm r} & b'_{\rm r} \\ b'_{\rm r} & b'_{\rm r} & a'_{\rm r} \end{pmatrix}$	G^2_{45}, G^2_{46} G^2_{56}, G^3_{456}
D	$\begin{pmatrix} A_{\rm r} & B_{\rm r} & B_{\rm r} \\ B_{\rm r} & A_{\rm r} & B_{\rm r} \\ B_{\rm r} & B_{\rm r} & A_{\rm r} \end{pmatrix}$	$\begin{pmatrix} a_{\rm r} & b_{\rm r} & b_{\rm r} \\ b_{\rm r} & a_{\rm r} & b_{\rm r} \\ b_{\rm r} & b_{\rm r} & a_{\rm r} \end{pmatrix}$	$\begin{pmatrix} a'_{\rm r} & b'_{\rm r} & b'_{\rm r} \\ b'_{\rm r} & a'_{\rm r} & b'_{\rm r} \\ b'_{\rm r} & b'_{\rm r} & a'_{\rm r} \end{pmatrix}$	G^3_{145}, G^3_{146} G^3_{156}, $G^4_{\overline{23}}$
E_1	$\begin{pmatrix} A_{\rm r} & B_{\rm r} & B_{\rm r} \\ B_{\rm r} & A_{\rm r} & B_{\rm r} \\ B_{\rm r} & B_{\rm r} & A_{\rm r} \end{pmatrix}$	$\begin{pmatrix} a_{\rm r} & b_{\rm r} & b_{\rm r} \\ b_{\rm r} & b_{\rm r} & a_{\rm r} \\ b_{\rm r} & a_{\rm r} & b_{\rm r} \end{pmatrix}$	$\begin{pmatrix} a'_{\rm r} & b'_{\rm r} & b'_{\rm r} \\ b'_{\rm r} & b'_{\rm r} & a'_{\rm r} \\ b'_{\rm r} & a'_{\rm r} & b'_{\rm r} \end{pmatrix}$	G^2_{25}, G^2_{35}, G^3_{125} G^3_{135}, G^3_{235}, $G^4_{\overline{46}}$
E_2	$\begin{pmatrix} A_{\rm r} & B_{\rm r} & B_{\rm r} \\ B_{\rm r} & A_{\rm r} & B_{\rm r} \\ B_{\rm r} & B_{\rm r} & A_{\rm r} \end{pmatrix}$	$\begin{pmatrix} b_{\rm r} & a_{\rm r} & b_{\rm r} \\ a_{\rm r} & b_{\rm r} & b_{\rm r} \\ b_{\rm r} & b_{\rm r} & a_{\rm r} \end{pmatrix}$	$\begin{pmatrix} b'_{\rm r} & a'_{\rm r} & b'_{\rm r} \\ a'_{\rm r} & b'_{\rm r} & b'_{\rm r} \\ b'_{\rm r} & b'_{\rm r} & a'_{\rm r} \end{pmatrix}$	G^2_{24}, G^2_{34}, G^3_{124} G^3_{134}, G^3_{234}, $G^4_{\overline{56}}$
E_3	$\begin{pmatrix} A_{\rm r} & B_{\rm r} & B_{\rm r} \\ B_{\rm r} & A_{\rm r} & B_{\rm r} \\ B_{\rm r} & B_{\rm r} & A_{\rm r} \end{pmatrix}$	$\begin{pmatrix} b_{\rm r} & b_{\rm r} & a_{\rm r} \\ b_{\rm r} & a_{\rm r} & b_{\rm r} \\ a_{\rm r} & b_{\rm r} & b_{\rm r} \end{pmatrix}$	$\begin{pmatrix} b'_{\rm r} & b'_{\rm r} & a'_{\rm r} \\ b'_{\rm r} & a'_{\rm r} & b'_{\rm r} \\ a'_{\rm r} & b'_{\rm r} & b'_{\rm r} \end{pmatrix}$	G^2_{26}, G^2_{36}, G^3_{126} G^3_{136}, G^3_{236}, $G^4_{\overline{45}}$

Table 1. All the possible structures of $M_{\rm D}$ and $M_{\rm R}$ constrained by sets G^n or $G^n_{\rm L}\times G^n_{\rm R}$ in the S_3 reflection symmetry limit, and the consequent structures of M_ν via the seesaw formula. The subscript "r" of a given matrix element means that this element is real.

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Cases	$M_{\rm D}$	$M_{\rm R}$	M_ν	Sets
F	$\begin{pmatrix} A_{\rm r} & B_{\rm r} & C_{\rm r} \\ C_{\rm r} & A_{\rm r} & B_{\rm r} \\ B_{\rm r} & C_{\rm r} & A_{\rm r} \end{pmatrix}$	$\begin{pmatrix} a_{\rm r} & b_{\rm r} & e_{\rm r} \\ b_{\rm r} & e_{\rm r} & a_{\rm r} \\ e_{\rm r} & a_{\rm r} & b_{\rm r} \end{pmatrix}$	$\begin{pmatrix} a'_{\rm r} & b'_{\rm r} & e'_{\rm r} \\ b'_{\rm r} & e'_{\rm r} & a'_{\rm r} \\ e'_{\rm r} & a'_{\rm r} & b'_{\rm r} \end{pmatrix}$	G^1_2, G^1_3, G^2_{12} G^2_{13}, G^2_{23}, G^3_{123} $G^1_{2\rm L}\times G^1_{2\rm R}$, $G^1_{3\rm L}\times G^1_{3\rm R}$
H_1	$\begin{pmatrix} A_{\rm r} & B_{\rm r} & B_{\rm r} \\ D_{\rm r} & C_{\rm r} & C_{\rm r} \\ D_{\rm r} & C_{\rm r} & C_{\rm r} \end{pmatrix}$	$\begin{pmatrix} a_{\rm r} & b_{\rm r} & b_{\rm r} \\ b_{\rm r} & e_{\rm r} & d_{\rm r} \\ b_{\rm r} & d_{\rm r} & e_{\rm r} \end{pmatrix}$	$\begin{pmatrix} a'_{\rm r} & b'_{\rm r} & b'_{\rm r} \\ b'_{\rm r} & e'_{\rm r} & e'_{\rm r} \\ b'_{\rm r} & e'_{\rm r} & e'_{\rm r} \end{pmatrix}$	$G^2_{15\rm L}\times G^2_{15\rm R}$
H_2	$\begin{pmatrix} C_{\rm r} & C_{\rm r} & D_{\rm r} \\ C_{\rm r} & C_{\rm r} & D_{\rm r} \\ B_{\rm r} & B_{\rm r} & A_{\rm r} \end{pmatrix}$	$\begin{pmatrix} e_{\rm r} & d_{\rm r} & b_{\rm r} \\ d_{\rm r} & e_{\rm r} & b_{\rm r} \\ b_{\rm r} & b_{\rm r} & a_{\rm r} \end{pmatrix}$	$\begin{pmatrix} e'_{\rm r} & e'_{\rm r} & b'_{\rm r} \\ e'_{\rm r} & e'_{\rm r} & b'_{\rm r} \\ b'_{\rm r} & b'_{\rm r} & a'_{\rm r} \end{pmatrix}$	$G^2_{14\rm L}\times G^2_{14\rm R}$
H_3	$\begin{pmatrix} C_{\rm r} & D_{\rm r} & C_{\rm r} \\ B_{\rm r} & A_{\rm r} & B_{\rm r} \\ C_{\rm r} & D_{\rm r} & C_{\rm r} \end{pmatrix}$	$\begin{pmatrix} e_{\rm r} & b_{\rm r} & d_{\rm r} \\ b_{\rm r} & a_{\rm r} & b_{\rm r} \\ d_{\rm r} & b_{\rm r} & e_{\rm r} \end{pmatrix}$	$\begin{pmatrix} e'_{\rm r} & b'_{\rm r} & e'_{\rm r} \\ b'_{\rm r} & a'_{\rm r} & b'_{\rm r} \\ e'_{\rm r} & b'_{\rm r} & e'_{\rm r} \end{pmatrix}$	$G^2_{16\rm L}\times G^2_{16\rm R}$
I_1	$A_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} a_{\rm r} & b_{\rm r} & b_{\rm r} \\ b_{\rm r} & b_{\rm r} & a_{\rm r} \\ b_{\rm r} & a_{\rm r} & b_{\rm r} \end{pmatrix}$	$a'_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	$G^2_{25\rm L}\times G^2_{25\rm R}$, $G^2_{35\rm L}\times G^2_{35\rm R}$ $G^3_{125\rm L}\times G^3_{125\rm R}$, $G^3_{135\rm L}\times G^3_{135\rm R}$ $G^3_{235\rm L}\times G^3_{235\rm R}$, $G^4_{\overline{46}\rm L}\times G^4_{\overline{46}\rm R}$
I_2	$A_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} b_{\rm r} & a_{\rm r} & b_{\rm r} \\ a_{\rm r} & b_{\rm r} & b_{\rm r} \\ b_{\rm r} & b_{\rm r} & a_{\rm r} \end{pmatrix}$	$a'_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	$G^2_{24\rm L}\times G^2_{24\rm R}$, $G^2_{34\rm L}\times G^2_{34\rm R}$ $G^3_{124\rm L}\times G^3_{124\rm R}$, $G^3_{134\rm L}\times G^3_{134\rm R}$ $G^3_{234\rm L}\times G^3_{234\rm R}$, $G^4_{\overline{56}\rm L}\times G^4_{\overline{56}\rm R}$
I_3	$A_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} b_{\rm r} & b_{\rm r} & a_{\rm r} \\ b_{\rm r} & a_{\rm r} & b_{\rm r} \\ a_{\rm r} & b_{\rm r} & b_{\rm r} \end{pmatrix}$	$a'_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	$G^2_{26\rm L}\times G^2_{26\rm R}$, $G^2_{36\rm L}\times G^2_{36\rm R}$ $G^3_{126\rm L}\times G^3_{126\rm R}$, $G^3_{136\rm L}\times G^3_{136\rm R}$ $G^3_{236\rm L}\times G^3_{236\rm R}$, $G^4_{\overline{45}\rm L}\times G^4_{\overline{45}\rm R}$
J	$A_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} a_{\rm r} & b_{\rm r} & e_{\rm r} \\ b_{\rm r} & e_{\rm r} & a_{\rm r} \\ e_{\rm r} & a_{\rm r} & b_{\rm r} \end{pmatrix}$	$a'_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	$G^2_{12\rm L}\times G^2_{12\rm R}$, $G^2_{13\rm L}\times G^2_{13\rm R}$ $G^2_{23\rm L}\times G^2_{23\rm R}$, $G^3_{123\rm L}\times G^3_{123\rm R}$
K	$A_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} a_{\rm r} & b_{\rm r} & b_{\rm r} \\ b_{\rm r} & a_{\rm r} & b_{\rm r} \\ b_{\rm r} & b_{\rm r} & a_{\rm r} \end{pmatrix}$	$a'_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	$G^2_{45\rm L}\times G^2_{45\rm R}$, $G^2_{46\rm L}\times G^2_{46\rm R}$ $G^2_{56\rm L}\times G^2_{56\rm R}$, $G^3_{145\rm L}\times G^3_{145\rm R}$ $G^3_{146\rm L}\times G^3_{146\rm R}$, $G^3_{156\rm L}\times G^3_{156\rm R}$ $G^3_{456\rm L}\times G^3_{456\rm R}$, $G^4_{\overline{23}\rm L}\times G^4_{\overline{23}\rm R}$
L	$\begin{pmatrix} a_{\rm r} & b_{\rm r} & b_{\rm r} \\ b_{\rm r} & a_{\rm r} & b_{\rm r} \\ b_{\rm r} & b_{\rm r} & a_{\rm r} \end{pmatrix}$	$A_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	———	All $G^n_{i_1 i_2\cdots i_n}$ or $G^n_{\overline{i_1 i_2\cdots i_{6-n}}}$ that are not listed above
N	$A_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	$A_{\rm r}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	———	$G^n_{i_1 i_2\cdots i_n\rm L}\times G^n_{i_1 i_2\cdots i_n\rm R}$ or $G^n_{\overline{i_1 i_2\cdots i_{6-n}}\rm L}\times G^n_{\overline{i_1 i_2\cdots i_{6-n}}\rm R}$ that are not listed above

Table 1. Continued.

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We find that the same correlations exist for $M_{\rm D}$, $M_{\rm R}$ and M_ν in categories B_i, E_i, H_i and I_i (for $i = 1, 2, 3$). In fact, eq. (2.16) for categories A_i and similar relations of this kind for other categories can be understood as follows.

1. The corresponding sets in categories X_1, X_2 and X_3 (for $X = A$, B, E, H or I) contain $S^{(132)}$, $S^{(213)}$ and $S^{(321)}$, respectively. The other possible elements (i.e., $S^{(123)}$, $S^{(231)}$, $S^{(312)}$) contained by X_1 are simultaneously contained by X_2 and X_3.

2. The three-dimensional representation of S_3 group in eq. (2.7) is a unitary representation, and hence $\mathcal{S}^\dagger_{\rm (L)} = \mathcal{S}^{-1}_{\rm (L)}$ and $\mathcal{S}^\dagger_{\rm (R)} = \mathcal{S}^{-1}_{\rm (R)}$ hold.

3. Since $S^{(132)}$, $S^{(213)}$ and $S^{(321)}$ belong to one conjugacy class $\mathcal{C}_2$, they can be connected with one another by one element of S_3 group. Namely, $S^{(213)} = S^{(231)} S^{(132)} [S^{(231)}]^{-1}$, $S^{(321)} = S^{(312)} S^{(132)} [S^{(312)}]^{-1}$ and $S^{(213)} = S^{(312)} S^{(321)} [S^{(312)}]^{-1}$.

4. The conjugacy class $\mathcal{C}_1$ containing elements $S^{(231)}$ and $S^{(312)}$ is a self reciprocal class, and the subgroup Z_3 is an Abelian group. As a result, $S^{(231)} = [S^{(312)}]^{-1}$, $S^{(312)} = [S^{(231)}]^{-1}$, $[S^{(231)}]^2 = S^{(312)}$ and $[S^{(312)}]^2 = S^{(231)}$ hold, and the three elements of Z_3 commute with one another (i.e., $\left[S^{(123)}, S^{(231)}\right] = \left[S^{(123)}, S^{(312)}\right] = \left[S^{(312)}, S^{(231)}\right] = 0$).

These properties, together with eqs. (2.5) and (2.6), allow us to easily obtain eq. (2.16) and other similar relations. Such relations are very helpful in the sense that once the result for one case is achieved, the results for the other two cases can be conveniently figured out in no need of repeating the relevant calculations.

The notation $G^n_{\rm L} \times G^n_{\rm R}$ in table 1 means that the left-handed fields $\nu_{\rm L}$ and the right-handed fields $N_{\rm R}$ can transform with different elements of G^n, corresponding to $\mathcal{S}_{\rm L}$ for $\nu_{\rm L}$ and $\mathcal{S}_{\rm R}$ for $N_{\rm R}$ shown in eq. (2.2), where $\mathcal{S}_{\rm L}$ and $\mathcal{S}_{\rm R}$ can be either identical or different.[2] Note that sets G^1_1 and $G^1_{1{\rm L}} \times G^1_{1{\rm R}}$ are trivial in the sense that they only restrict all the elements of a given mass matrix to be real. That is why for categories L and N listed in table 1 the corresponding sets do not include G^1_1 and $G^1_{1{\rm L}} \times G^1_{1{\rm R}}$. In these two cases the heavy Majorana neutrino mass matrix $M_{\rm R}$ has a democracy texture of rank one, and thus its determinant is zero, making the seesaw formula in eq. (2.6) does not work anymore.

It is obvious that if a set contains element $S^{(123)}$, then $M_{\rm D}$, $M_{\rm R}$ and M_ν will all be real. Eq. (2.5) tells us that the structure of $M_{\rm D}$ is constrained by both $\mathcal{S}_{\rm L}$ and $\mathcal{S}_{\rm R}$, and that of $M_{\rm R}$ is constrained only by $\mathcal{S}_{\rm R}$. As a result, $M_{\rm D}$ is constrained more strictly in the case associated with $G^n_{\rm L} \times G^n_{\rm R}$ than in the case associated with G^n, but the constraints on $M_{\rm R}$ in these two situations are the same. In fact, G^1 and $G^1_{\rm L} \times G^1_{\rm R}$ are identical and thus lead to the same textures for relevant mass matrices.

Of course, the structure of M_ν is in general different from that of $M_{\rm R}$. But as shown in table 1, M_ν and $M_{\rm R}$ *do* share the same texture for categories A_i to F, in which the structures of relevant mass matrices are dominated by sets G^n and $G^1_{\rm L} \times G^1_{\rm R}$. As for

[2] Without invoking any confusion, we have omitted the subscript $i_1 i_2 \cdots i_n$ (or $\overline{i_1 i_2 \cdots i_{6-n}}$) of set $G^n_{i_1 i_2 \cdots i_n}$ (or $G^n_{\overline{i_1 i_2 \cdots i_{6-n}}}$) here and hereafter for the sake of simplicity.

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categories H_i to K, in which sets $G^n_{\rm L} \times G^n_{\rm R}$ (for $n = 2, 3, \cdots, 6$) dominate, the structure of M_ν is quite similar to that of $M_{\rm R}$ or to a combination of the structures of $M_{\rm D}$ and $M_{\rm R}$. In these cases $M_{\rm D}$ is constrained more strictly than $M_{\rm R}$, and hence it possesses a much simpler texture which dominates the texture pattern of M_ν via the seesaw formula in eq. (2.6). Especially in categories I_i to K, the mass matrices M_ν and $M_{\rm D}$ exactly share the same democracy texture. To characterize the relationship between light and heavy Majorana neutrinos in the seesaw framework under consideration, we refer to the structural equality or similarity between M_ν and $M_{\rm R}$ as a seesaw mirroring relationship.

Another thing that deserves attention is that the mass matrices constrained by any one of $S^{(123)}$, $S^{(231)}$ and $S^{(312)}$ must be real. In other words, $M_{\rm D}$ and $M_{\rm R}$ will be real if the corresponding set in a given category contains one of the above three elements. Only categories A_1, A_2, A_3 and C, in which $S^{(123)}$, $S^{(231)}$ and $S^{(312)}$ are not involved, give rise to complex $M_{\rm D}$ and $M_{\rm R}$. Among them, only categories A_1, A_2 and A_3 allow us to obtain the complex textures of M_ν via the seesaw formula. This observation means that in the S_3 reflection symmetry limit there are only four possibilities to accommodate CP violation in the lepton-number-violating decays of heavy Majorana neutrinos, and only three possibilities to accommodate CP violation in the effective light neutrino mass matrix M_ν.

At this point it is also worth mentioning that S_3 is not a symmetry of the Lagrangian in the neutrino sector. Although we have considered the subgroups and subsets of S_3 group, they are mainly used as a tool to constrain and classify possible structures of the neutrino mass matrices. From the phenomenological point of view, our strategy is expected to be helpful for understanding the neutrino flavor structures under S_3 symmetry and providing a reference about which larger group should be introduced and which representations should be determined when doing a realistic model-building exercise. We admit that a larger flavor symmetry group may not have a direct connection with S_3, but the latter is likely to play an indirect but suggestive role in bridging an underlying flavor symmetry and a phenomenologically favored pattern of M_ν. Since S_3 is so simple and instructive in reflecting the possible interchange among three flavor families, it should be qualified as a good bottom-up example in probing what is behind tiny neutrino masses and significant flavor mixing effects.

3 Neutrino masses and flavor mixing patterns

Now we proceed to calculate the light neutrino masses and flavor mixing parameters for each of the textures of M_ν listed in table 1 in the basis where the flavor eigenstates of three charged leptons are identical with their mass eigenstates. Although some of the flavor mixing patterns derived from M_ν in the S_3 reflection symmetry limit are expected to be far away from the observed pattern of the PMNS matrix, it remains instructive to see their salient features from a phenomenological point of view.

Since M_ν is symmetric, it can be diagonalized by a unitary transformation matrix V as follows: $V^\dagger M_\nu V^* = \widehat{M_\nu}$, where $\widehat{M_\nu} \equiv {\rm Diag}\{m_1, m_2, m_3\}$ with m_i (for $i = 1, 2, 3$) being the neutrino masses. In the chosen flavor basis V is just the PMNS matrix which describes

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the effects of neutrino mixing and CP violation, and its standard parametrization form is

$$V = P_l \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\mathrm{i}\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{\mathrm{i}\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{\mathrm{i}\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{\mathrm{i}\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{\mathrm{i}\delta} & c_{13}c_{23} \end{pmatrix} P_\nu \,, \tag{3.1}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ with $ij = 12, 13, 23$, $P_l = \mathrm{Diag}\left\{e^{\mathrm{i}\phi_e}, e^{\mathrm{i}\phi_\mu}, e^{\mathrm{i}\phi_\tau}\right\}$ contains three unphysical phases which can be absorbed by rephasing the charged-lepton fields, and $P_\nu = \mathrm{Diag}\left\{e^{\mathrm{i}\rho}, e^{\mathrm{i}\sigma}, 1\right\}$ contains two physical Majorana phases. Therefore, a diagonalization of the effective Majorana neutrino mass matrix M_ν allows us to determine three neutrino masses m_i, three flavor mixing angles θ_{ij} and three CP-violating phases δ, ρ and σ. In the following we do such exercises by examining all the textures of M_ν listed in table 1.

3.1 Categories A_i

In category A_1 the light neutrino mass matrix M_ν satisfies the μ-τ reflection symmetry, which naturally predicts the phenomenologically favored results $\theta_{23} = \pi/4$ and $\delta = -\pi/2$ [13, 15]. It is therefore interesting to reproduce this texture from the canonical seesaw mechanism in the S_3 reflection symmetry limit. To be specific, the μ-τ reflection symmetry structure of M_ν in this case leads us to

$$\begin{aligned} &\theta_{23}^{A_1} = \frac{\pi}{4} \,, \qquad \delta^{A_1} = \pm\frac{\pi}{2} \,, \qquad \rho^{A_1}, \sigma^{A_1} = 0 \text{ or } \frac{\pi}{2} \,, \\ &\phi_e^{A_1} = 0 \text{ or } \frac{\pi}{2} \,, \quad \phi_\mu^{A_1} + \phi_\tau^{A_1} = 2\phi_e^{A_1} \pm \pi \,. \end{aligned} \tag{3.2}$$

In addition, the other two flavor mixing angles and the three neutrino masses in category A_1 can be expressed as follows:

$$\begin{aligned} \tan\theta_{13}^{A_1} &= \left| \frac{\mathrm{Im}(e'')}{\sqrt{2}\mathrm{Re}(b'')} \right| \,, \\ \tan 2\theta_{12}^{A_1} &= \frac{2\sqrt{2}\cos 2\theta_{13}^{A_1}\mathrm{Re}(b'')}{c_{13}^{A_1}\left\{\left[\mathrm{Re}(e'') - d''\right]\cos 2\theta_{13}^{A_1} - \left[\mathrm{Re}(e'') + d''\right]{s_{13}^{A_1}}^2 - a''{c_{13}^{A_1}}^2\right\}} \,, \\ m_1 &= \left| -d'' - \frac{\sqrt{2}\mathrm{Re}(b'')}{c_{13}^{A_1}\sin 2\theta_{12}^{A_1}} + \frac{\left[a'' + \mathrm{Re}(e'') + d''\right]{c_{13}^{A_1}}^2}{2\cos 2\theta_{13}^{A_1}} \right| \,, \\ m_2 &= \left| -d'' + \frac{\sqrt{2}\mathrm{Re}(b'')}{c_{13}^{A_1}\sin 2\theta_{12}^{A_1}} + \frac{\left[a'' + \mathrm{Re}(e'') + d''\right]{c_{13}^{A_1}}^2}{2\cos 2\theta_{13}^{A_1}} \right| \,, \\ m_3 &= \frac{a''{s_{13}^{A_1}}^2 + \left[\mathrm{Re}(e'') + d''\right]{c_{13}^{A_1}}^2}{\cos 2\theta_{13}^{A_1}} \,, \end{aligned} \tag{3.3}$$

where $a'' = a'_{\mathrm{r}}\exp\left(-2\mathrm{i}\phi_e^{A_1}\right)$, $b'' = b'\exp\left[-\mathrm{i}\left(\phi_e^{A_1} + \phi_\mu^{A_1}\right)\right]$, $e'' = e'\exp\left(-2\mathrm{i}\phi_\mu^{A_1}\right)$ and $d'' = d'_{\mathrm{r}}\exp\left[-\mathrm{i}\left(\phi_\mu^{A_1} + \phi_\tau^{A_1}\right)\right]$.

For categories A_2 and A_3, the corresponding textures of M_ν are related to that in category A_1 via eq. (2.16). One may therefore choose the same order of three mass eigenvalues

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and then establish similar correlations among the three PMNS matrices of categories A_1, A_2 and A_3 with the help of eq. (2.16):

$$V^{A_2} = S^{(231)} V^{A_1} \; , \quad V^{A_3} = S^{(312)} V^{A_1} \; . \tag{3.4}$$

As a consequence, the relevant flavor mixing parameters in categories A_2 and A_3 can be respectively related to those of category A_1 as follows:

$$\begin{aligned}
\tan\theta_{23}^{A_2} &= \frac{1}{\sqrt{2}\tan\theta_{13}^{A_1}} \; , \\
\sin\theta_{13}^{A_2} &= \frac{1}{\sqrt{2}}\cos\theta_{13}^{A_1} \; , \\
\cos 2\theta_{12}^{A_2} &= -\frac{1-\sin^2\theta_{13}^{A_1}}{1+\sin^2\theta_{13}^{A_1}}\cos 2\theta_{12}^{A_1} \; , \\
\sin\delta^{A_2} &= \frac{\cos\theta_{12}^{A_1}\sin\theta_{12}^{A_1}\cos^2\theta_{13}^{A_1}\sin\theta_{13}^{A_1}}{2\cos\theta_{12}^{A_2}\sin\theta_{12}^{A_2}\cos^2\theta_{13}^{A_2}\sin\theta_{13}^{A_2}\cos\theta_{23}^{A_2}\sin\theta_{23}^{A_2}}\sin\delta^{A_1} \; , \\
\phi_e^{A_2} &= \phi_\mu^{A_1} + \delta^{A_2} \; , \; \phi_\mu^{A_2} = \phi_\tau^{A_1} \; , \; \phi_\tau^{A_2} = \phi_e^{A_1} - \delta^{A_1} \; , \\
\rho^{A_2} &= \varphi_1 + \rho^{A_1} - \delta^{A_2} \; , \; \sigma^{A_2} = \varphi_2 + \sigma^{A_1} - \delta^{A_2} \; ;
\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}
\tan\theta_{23}^{A_3} &= \sqrt{2}\tan\theta_{13}^{A_1} \; , \\
\sin\theta_{13}^{A_3} &= \frac{1}{\sqrt{2}}\cos\theta_{13}^{A_1} \; , \\
\cos 2\theta_{12}^{A_3} &= -\frac{1-\sin^2\theta_{13}^{A_1}}{1+\sin^2\theta_{13}^{A_1}}\cos 2\theta_{12}^{A_1} \; , \\
\sin\delta^{A_3} &= \frac{\cos\theta_{12}^{A_1}\sin\theta_{12}^{A_1}\cos^2\theta_{13}^{A_1}\sin\theta_{13}^{A_1}}{2\cos\theta_{12}^{A_3}\sin\theta_{12}^{A_3}\cos^2\theta_{13}^{A_3}\sin\theta_{13}^{A_3}\cos\theta_{23}^{A_3}\sin\theta_{23}^{A_3}}\sin\delta^{A_1} \; , \\
\phi_e^{A_3} &= \phi_\tau^{A_1} + \delta^{A_3} \; , \; \phi_\mu^{A_3} = \phi_e^{A_1} - \delta^{A_1} \; , \; \phi_\tau^{A_3} = \phi_\mu^{A_1} \; , \\
\rho^{A_3} &= \pi - \varphi_1 + \rho^{A_1} - \delta^{A_3} \; , \; \sigma^{A_3} = \pi - \varphi_2 + \sigma^{A_1} - \delta^{A_3} \; ,
\end{aligned} \tag{3.6}$$

where $\sin\varphi_1 = \mp\cos\theta_{12}^{A_1}\sin\theta_{13}^{A_1}/\sqrt{1-\cos^2\theta_{12}^{A_1}\cos^2\theta_{13}^{A_1}}$, $\cos\varphi_1 = -\sin\theta_{12}^{A_1}/\sqrt{1-\cos^2\theta_{12}^{A_1}\cos^2\theta_{13}^{A_1}}$, $\sin\varphi_2 = \mp\sin\theta_{12}^{A_1}\sin\theta_{13}^{A_1}/\sqrt{1-\sin^2\theta_{12}^{A_1}\cos^2\theta_{13}^{A_1}}$ and $\cos\varphi_2 = \cos\theta_{12}^{A_1}/\sqrt{1-\sin^2\theta_{12}^{A_1}\cos^2\theta_{13}^{A_1}}$ with the "$\mp$" signs corresponding to $\delta^{A_1} = \pm\pi/2$. The analytical results of three neutrino masses in these two categories are formally the same as those given in eq. (3.3), but of course the relevant flavor mixing parameters need to be substituted with the ones obtained in eq. (3.5) or (3.6). It is obvious that none of the flavor mixing angles and CP-violating phases in categories A_2 and A_3 take special values, and this simply means that the standard parametrization of V is not the best choice for these two cases. One may therefore consider to choose another parametrization of V which can automatically reveal the S_3 reflection symmetry hidden in M_ν in categories A_2 and A_3.

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3.2 Categories B_i

In category B_1 the structure of M_ν possesses the μ-τ permutation symmetry,[3] which naturally predicts $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ in the standard parametrization of V [44–47]. The whole pattern of V in this case is found to be

$$V^{B_1} = \begin{pmatrix} \mp\dfrac{\left(-a'_{\rm r}+e'_{\rm r}+d'_{\rm r}+\sqrt{\Delta}\right)}{\sqrt{8b'^2_{\rm r}+\left(-a'_{\rm r}+e'_{\rm r}+d'_{\rm r}+\sqrt{\Delta}\right)^2}} & \pm\dfrac{\left(a'_{\rm r}-e'_{\rm r}-d'_{\rm r}+\sqrt{\Delta}\right)}{\sqrt{8b'^2_{\rm r}+\left(a'_{\rm r}-e'_{\rm r}-d'_{\rm r}+\sqrt{\Delta}\right)^2}} & 0 \\ \dfrac{2|b'_{\rm r}|}{\sqrt{8b'^2_{\rm r}+\left(-a'_{\rm r}+e'_{\rm r}+d'_{\rm r}+\sqrt{\Delta}\right)^2}} & \dfrac{2|b'_{\rm r}|}{\sqrt{8b'^2_{\rm r}+\left(a'_{\rm r}-e'_{\rm r}-d'_{\rm r}+\sqrt{\Delta}\right)^2}} & -\dfrac{1}{\sqrt{2}} \\ \dfrac{2|b'_{\rm r}|}{\sqrt{8b'^2_{\rm r}+\left(-a'_{\rm r}+e'_{\rm r}+d'_{\rm r}+\sqrt{\Delta}\right)^2}} & \dfrac{2|b'_{\rm r}|}{\sqrt{8b'^2_{\rm r}+\left(a'_{\rm r}-e'_{\rm r}-d'_{\rm r}+\sqrt{\Delta}\right)^2}} & \dfrac{1}{\sqrt{2}} \end{pmatrix} \tag{3.7}$$

with two sign options corresponding to the positive or negative sign of $b'_{\rm r}$. It is easy to see that $\left|V^{B_1}_{\mu i}\right| = \left|V^{B_1}_{\tau i}\right|$ holds (for $i = 1, 2, 3$), a clear reflection of the μ-τ permutation symmetry. Note, however, that the structure of $M^{B_1}_\nu$ shown in table 1 is just a particular example which respects the μ-τ permutation symmetry but has no complex elements. A general form of M_ν in the μ-τ permutation symmetry limit must have a structure similar to $M^{B_1}_\nu$, but it should contain some complex elements [13].

Given the texture of $M^{B_1}_\nu$ in table 1 and the pattern of V^{B_1} in eq. (3.7), it is straightforward to calculate the neutrino masses and flavor mixing parameters. We obtain

$$\begin{aligned} m_1 &= \frac{1}{2}\left(a'_{\rm r}+e'_{\rm r}+d'_{\rm r}-\sqrt{\Delta}\right) , \\ m_2 &= \frac{1}{2}\left(a'_{\rm r}+e'_{\rm r}+d'_{\rm r}+\sqrt{\Delta}\right) , \\ m_3 &= e'_{\rm r}-d'_{\rm r} ; \end{aligned} \tag{3.8}$$

and

$$\begin{aligned} &\theta^{B_1}_{23} = \frac{\pi}{4} , \qquad \theta^{B_1}_{13} = 0 , \qquad \theta^{B_1}_{12} = \arccos\left(\frac{2\sqrt{2}|b'_{\rm r}|}{\sqrt{8b'^2_{\rm r}+\left(a'_{\rm r}-e'_{\rm r}-d'_{\rm r}+\sqrt{\Delta}\right)^2}}\right) , \\ &\delta^{B_1} \in [0, 2\pi) , \qquad \rho^{B_1} = 0 , \qquad \sigma^{B_1} = \pi , \qquad \phi^{B_1}_e = 0 \text{ or } \pi , \qquad \phi^{B_1}_\mu = \pi , \qquad \phi^{B_1}_\tau = 0 , \end{aligned} \tag{3.9}$$

where $\Delta = 8b'^2_{\rm r} + (-a'_{\rm r} + e'_{\rm r} + d'_{\rm r})^2$. Note that m_1, m_2 or m_3 in eq. (3.8) may be negative, but a minus sign can always be absorbed into three unphysical phases and two Majorana phases.

Analogous to categories A_1, A_2 and A_3, the neutrino masses and flavor mixing parameters of B_1, B_2 and B_3 are also correlated with one another via eq. (2.16). In fact,

[3] That is, the light Majorana neutrino mass term is invariant under the permutation transformations $\nu_e \leftrightarrow \nu_e$ and $\nu_\mu \leftrightarrow \nu_\tau$, and thus the structure of M_ν gets constrained.

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the three neutrino masses for both categories B_2 and B_3 are the same as those given by eq. (3.8), and the flavor mixing parameters in these two cases are found to be

$$\theta_{23}^{B_2} = \frac{\pi}{2}\,, \quad \theta_{13}^{B_2} = \frac{\pi}{4}\,, \quad \theta_{12}^{B_2} = \arccos\left(\frac{2\sqrt{2}|b'_{\rm r}|}{\sqrt{8b'^2_{\rm r} + \left(-a'_{\rm r} + e'_{\rm r} + d'_{\rm r} + \sqrt{\Delta}\right)^2}}\right),$$
$$\rho^{B_2} = \sigma^{B_2} = \pi - \delta^{B_2} = -\phi_e^{B_2} = -\phi_\tau^{B_2} \text{ or } \pi - \phi_\tau^{B_2}\,, \quad \phi_\mu^{B_2} = 0\,; \tag{3.10}$$

and

$$\theta_{23}^{B_3} = 0\,,\ \theta_{13}^{B_3} = \frac{\pi}{4}\,, \quad \theta_{12}^{B_3} = \arccos\left(\frac{2\sqrt{2}|b'_{\rm r}|}{\sqrt{8b'^2_{\rm r} + \left(-a'_{\rm r} + e'_{\rm r} + d'_{\rm r} + \sqrt{\Delta}\right)^2}}\right),$$
$$\rho^{B_3} = \sigma^{B_3} = -\delta^{B_3} = -\phi_e^{B_3} = -\phi_\mu^{B_3} \text{ or } -\pi - \phi_\mu^{B_3}\,, \quad \phi_\tau^{B_3} = \pi\,, \tag{3.11}$$

respectively. Needless to say, these two possibilities are strongly disfavored by current neutrino oscillation data [48].

3.3 Categories C and D

The structures of M_ν in categories C and D are exactly the same, and thus their phenomenological consequences are also the same. In particular, they lead us to the well-known tri-bimaximal neutrino mixing pattern [49–51],

$$V = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \tag{3.12}$$

To be explicit, the results for neutrino masses and flavor mixing parameters are

$$m_1 = a'_{\rm r} - b'_{\rm r}\,, \quad m_2 = a'_{\rm r} + 2b'_{\rm r}\,, \quad m_3 = a'_{\rm r} - b'_{\rm r}\,, \tag{3.13}$$

and

$$\theta_{23} = \frac{\pi}{4}\,, \qquad \theta_{13} = 0\,, \qquad \theta_{12} = \arctan\left(\frac{1}{\sqrt{2}}\right),$$
$$\delta \in [0, 2\pi)\,, \qquad \rho = \pi\,, \qquad \sigma = 0\,, \quad \phi_e = 0\,, \quad \phi_\mu = 0\,, \quad \phi_\tau = \pi\,. \tag{3.14}$$

So far many model-building exercises have been done in this connection to introduce small perturbations to M_ν and consequently small corrections to V in eq. (3.12), so as to arrive at a better fit of current experimental data [35–40].

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3.4 Categories E_i

In category E_1 the PMNS matrix reads

$$V^{E_1} = \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{\mathrm{i}}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & -\frac{\mathrm{i}}{\sqrt{2}} \end{pmatrix} , \tag{3.15}$$

which is also the tri-bimaximal flavor mixing pattern with some trivial phases. A straightforward calculation allows us to obtain the neutrino masses for category E_1:

$$m_1 = a'_{\rm r} - b'_{\rm r} , \quad m_2 = a'_{\rm r} + 2b'_{\rm r} , \quad m_3 = a'_{\rm r} - b'_{\rm r} , \tag{3.16}$$

and the same result is true for categories E_2 and E_3. The corresponding flavor mixing parameters are found to be

$$\begin{aligned} &\theta_{23}^{E_1} = \frac{\pi}{4} , && \theta_{13}^{E_1} = 0 , && \theta_{12}^{E_1} = \arctan\left(\frac{1}{\sqrt{2}}\right) , \\ &\delta^{E_1} \in [0, 2\pi) , && \rho^{E_1} = \frac{3\pi}{2} , && \sigma^{E_1} = \frac{\pi}{2} , \\ &\phi_e^{E_1} = \frac{\pi}{2} , && \phi_\mu^{E_1} = \frac{\pi}{2} , && \phi_\tau^{E_1} = \frac{3\pi}{2} \end{aligned} \tag{3.17}$$

in category E_1;

$$\begin{aligned} &\theta_{23}^{E_2} = \frac{\pi}{2} , \quad \theta_{13}^{E_2} = \frac{\pi}{4} , \quad \theta_{12}^{E_2} = \arctan\left(\sqrt{2}\right) , \\ &\rho^{E_2} = \sigma^{E_2} = \frac{\pi}{2} - \delta^{E_2} = \pi - \phi_e^{E_2} = -\phi_\tau^{E_2} , \quad \phi_\mu^{E_2} = \frac{3\pi}{2} \end{aligned} \tag{3.18}$$

in category E_2; and

$$\begin{aligned} &\theta_{23}^{E_3} = 0 , \quad \theta_{13}^{E_3} = \frac{\pi}{4} , \quad \theta_{12}^{E_3} = \arctan\left(\sqrt{2}\right) , \\ &\rho^{E_3} = \sigma^{E_3} = \frac{3\pi}{2} - \delta^{E_3} = \pi - \phi_e^{E_3} = \pi - \phi_\mu^{E_3} , \quad \phi_\tau^{E_3} = \frac{\pi}{2} \end{aligned} \tag{3.19}$$

in category E_3. One can see that the latter two cases are strongly disfavored by current neutrino oscillation data [48].

3.5 Category F

In this category of M_ν the PMNS matrix is given by

$$V = \begin{pmatrix} \mathrm{i}\frac{-a'_{\rm r} + b'_{\rm r} + \lambda}{a'_{\rm r} - e'_{\rm r}} \sqrt{\frac{-2b'_{\rm r} + a'_{\rm r} + e'_{\rm r} + 2\lambda}{6\lambda}} & \frac{-a'_{\rm r} + b'_{\rm r} - \lambda}{a'_{\rm r} - e'_{\rm r}} \sqrt{\frac{2b'_{\rm r} - a'_{\rm r} - e'_{\rm r} + 2\lambda}{6\lambda}} & \frac{1}{\sqrt{3}} \\ \mathrm{i}\frac{-b'_{\rm r} + e'_{\rm r} - \lambda}{a'_{\rm r} - e'_{\rm r}} \sqrt{\frac{-2b'_{\rm r} + a'_{\rm r} + e'_{\rm r} + 2\lambda}{6\lambda}} & \frac{-b'_{\rm r} + e'_{\rm r} + \lambda}{a'_{\rm r} - e'_{\rm r}} \sqrt{\frac{2b'_{\rm r} - a'_{\rm r} - e'_{\rm r} + 2\lambda}{6\lambda}} & \frac{1}{\sqrt{3}} \\ \mathrm{i}\sqrt{\frac{-2b'_{\rm r} + a'_{\rm r} + e'_{\rm r} + 2\lambda}{6\lambda}} & \sqrt{\frac{2b'_{\rm r} - a'_{\rm r} - e'_{\rm r} + 2\lambda}{6\lambda}} & \frac{1}{\sqrt{3}} \end{pmatrix} \tag{3.20}$$

with $\lambda = \sqrt{a_{\rm r}'^2 + b_{\rm r}'^2 + e_{\rm r}'^2 - a_{\rm r}' b_{\rm r}' - a_{\rm r}' e_{\rm r}' - b_{\rm r}' e_{\rm r}'}$. The masses of three light neutrinos are

$$m_1 = m_2 = \lambda \,, \quad m_3 = a_{\rm r}' + b_{\rm r}' + e_{\rm r}' \,, \tag{3.21}$$

and the corresponding flavor mixing parameters are found to be

$$\begin{aligned} \theta_{23} &= \frac{\pi}{4} \,, \quad \theta_{13} = \arccos\left(\frac{2}{\sqrt{6}}\right) , \\ \theta_{12} &= \arctan\left(\frac{a-b+\lambda}{a-b-\lambda}\sqrt{\frac{2b_{\rm r}' - a_{\rm r}' - e_{\rm r}' + 2\lambda}{-2b_{\rm r}' + a_{\rm r}' + e_{\rm r}' + 2\lambda}}\right) , \\ \delta &= 0 \text{ or } \pi \,, \quad \rho = \frac{\pi}{2} - \delta \,, \quad \sigma = \phi_e = \delta \,, \quad \phi_\mu = \phi_\tau = 0 \,. \end{aligned} \tag{3.22}$$

This case turns out to be strongly disfavored by current experimental data.

3.6 Categories H_i

For categories H_1, H_2 and H_3, the corresponding neutrino masses and flavor mixing parameters can easily be obtained from categories B_1, B_2 and B_3 by taking $d_{\rm r}' = e_{\rm r}'$. In this way one is left with $m_3 = 0$, corresponding to the inverted neutrino mass hierarchy. Here let us focus on the normal neutrino mass hierarchy. The PMNS matrix is found to be

$$V^{H_1} = \begin{pmatrix} 0 & \mp\dfrac{\left(-a_{\rm r}' + 2e_{\rm r}' + \sqrt{\Delta'}\right)}{\sqrt{8b_{\rm r}'^2 + \left(-a_{\rm r}' + 2e_{\rm r}' + \sqrt{\Delta'}\right)^2}} & \pm\dfrac{\left(a_{\rm r}' - 2e_{\rm r}' + \sqrt{\Delta'}\right)}{\sqrt{8b_{\rm r}'^2 + \left(a_{\rm r}' - 2e_{\rm r}' + \sqrt{\Delta'}\right)^2}} \\ -\dfrac{1}{\sqrt{2}} & \dfrac{2|b_{\rm r}'|}{\sqrt{8b_{\rm r}'^2 + \left(-a_{\rm r}' + 2e_{\rm r}' + \sqrt{\Delta'}\right)^2}} & \dfrac{2|b_{\rm r}'|}{\sqrt{8b_{\rm r}'^2 + \left(a_{\rm r}' - 2e_{\rm r}' + \sqrt{\Delta'}\right)^2}} \\ \dfrac{1}{\sqrt{2}} & \dfrac{2|b_{\rm r}'|}{\sqrt{8b_{\rm r}'^2 + \left(-a_{\rm r}' + 2e_{\rm r}' + \sqrt{\Delta'}\right)^2}} & \dfrac{2|b_{\rm r}'|}{\sqrt{8b_{\rm r}'^2 + \left(a_{\rm r}' - 2e_{\rm r}' + \sqrt{\Delta'}\right)^2}} \end{pmatrix} , \tag{3.23}$$

and the neutrino masses are given by

$$\begin{aligned} m_1 &= 0 \,, \\ m_2 &= \frac{1}{2}\left(a_{\rm r}' + 2e_{\rm r}' - \sqrt{\Delta'}\right) , \\ m_3 &= \frac{1}{2}\left(a_{\rm r}' + 2e_{\rm r}' + \sqrt{\Delta'}\right) , \end{aligned} \tag{3.24}$$

where $\Delta' = 8b_{\rm r}'^2 + \left(-a_{\rm r}' + 2e_{\rm r}'\right)^2$. The results of m_i in eq. (3.24) are also valid for categories H_2 and H_3. To be explicit, the flavor mixing parameters in these three cases are obtained below:

$$\begin{aligned} \theta_{23}^{H_1} &= \frac{\pi}{4} \,, \quad \theta_{12}^{H_1} = \frac{\pi}{2} \,, \quad \theta_{13}^{H_1} = \arccos\left(\frac{2\sqrt{2}|b_{\rm r}'|}{\sqrt{8b_{\rm r}'^2 + \left(a_{\rm r}' - 2e_{\rm r}' + \sqrt{\Delta'}\right)^2}}\right) , \\ \delta^{H_1} &= \pi - \sigma^{H_1} = \phi_e^{H_1} \text{ or } \phi_e^{H_1} + \pi \,, \quad \rho^{H_1} = \phi_\mu^{H_1} = \phi_\tau^{H_1} = 0 \end{aligned} \tag{3.25}$$

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for category H_1;

$$\begin{aligned}
\theta_{23}^{H_2} &= \arctan\left(\frac{2|b'_{\rm r}|}{a'_{\rm r}-2e'_{\rm r}+\sqrt{\Delta'}}\right), \\
\theta_{12}^{H_2} &= \arctan\left(\frac{2\sqrt{2}|b'_{\rm r}|}{\sqrt{8b'^2_{\rm r}+\left(-a'_{\rm r}+2e'_{\rm r}+\sqrt{\Delta'}\right)^2}}\right), \\
\theta_{13}^{H_2} &= \arcsin\left(\frac{2|b'_{\rm r}|}{\sqrt{8b'^2_{\rm r}+\left(a'_{\rm r}-2e'_{\rm r}+\sqrt{\Delta'}\right)^2}}\right), \\
\delta^{H_2} &= \sigma^{H_2} = \phi_e^{H_2} = \phi_\mu^{H_2} = 0\,, \quad \rho^{H_2} = \pi\,, \quad \phi_\tau^{H_2} = 0 \text{ or } -\pi
\end{aligned} \tag{3.26}$$

for category H_2; and

$$\begin{aligned}
\theta_{23}^{H_3} &= \arctan\left(\frac{a'_{\rm r}-2e'_{\rm r}+\sqrt{\Delta'}}{2|b'_{\rm r}|}\right), \\
\theta_{12}^{H_3} &= \arctan\left(\frac{2\sqrt{2}|b'_{\rm r}|}{\sqrt{8b'^2_{\rm r}+\left(-a'_{\rm r}+2e'_{\rm r}+\sqrt{\Delta'}\right)^2}}\right), \\
\theta_{13}^{H_3} &= \arcsin\left(\frac{2|b'_{\rm r}|}{\sqrt{8b'^2_{\rm r}+\left(a'_{\rm r}-2e'_{\rm r}+\sqrt{\Delta'}\right)^2}}\right), \\
\delta^{H_3} &= \rho^{H_3} = \sigma^{H_3} = \phi_e^{H_3} = \pi\,, \quad \phi_\tau^{H_3} = 0\,, \quad \phi_\mu^{H_3} = 0 \text{ or } -\pi
\end{aligned} \tag{3.27}$$

for category H_3, respectively. Note that the flavor mixing angles in the latter two cases satisfy the relations $\tan\theta_{12}^{H_2}\tan\theta_{23}^{H_2} = \sin\theta_{13}^{H_2}$ and $\tan\theta_{12}^{H_3} = \tan\theta_{23}^{H_3}\sin\theta_{13}^{H_3}$.

3.7 Categories I_i, J and K

In these five categories the textures of M_ν are all democratic, and thus the corresponding PMNS matrix is of the form

$$V = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \tag{3.28}$$

corresponding to a special neutrino mass spectrum with $m_1 = m_2 = 0$ and $m_3 = 3a'_{\rm r}$. To be explicit, the pattern of V in eq. (3.28) leads us to

$$\begin{aligned}
\theta_{23} = \frac{\pi}{4}\,, \quad \theta_{13} = \arccos\left(\frac{2}{\sqrt{6}}\right), \quad \theta_{12} = \frac{\pi}{6}\,, \\
\delta = \rho = \sigma = \phi_e = \phi_\mu = \phi_\tau = 0\,,
\end{aligned} \tag{3.29}$$

which are strongly disfavored by current neutrino oscillation data.

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4 Leptogenesis in the S_3 symmetry limit

Now we examine whether the leptogenesis mechanism [16],[4] which can provide a natural way to account for the observed matter-antimatter asymmetry of the Universe [53], works or not in the S_3 reflection symmetry limit under discussion. According to this mechanism, the lepton-number-violating, CP-violating and out-of-equilibrium decays of heavy Majorana neutrinos N_i may result in a lepton-antilepton asymmetry in the early Universe, and the latter can subsequently be converted to the wanted baryon-antibaryon asymmetry through the B-L conserving sphaleron process [54, 55]. Here what we are concerned with are the CP-violating asymmetries between the decay modes $N_i \to \ell_\alpha + H$ and their CP-conjugate processes $N_i \to \bar{\ell}_\alpha + \overline{H}$, usually denoted as $\epsilon_{i\alpha}$ (for $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$), because they will finally determine the strength of baryogenesis via leptogenesis. Assuming that the masses of three heavy Majorana neutrinos are hierarchical (i.e., $M_1 \ll M_2 < M_3$), it has been shown that only the CP-violating asymmetries $\epsilon_{1\alpha}$ survive and contribute to the lepton-antilepton asymmetry. In this case the expression of $\epsilon_{1\alpha}$ is given by [42, 56]

$$\begin{aligned}\epsilon_{1\alpha} &= \frac{\Gamma\left(N_i \to \ell_\alpha + H\right) - \Gamma\left(N_i \to \bar{\ell}_\alpha + \overline{H}\right)}{\sum\limits_\alpha \left[\Gamma\left(N_i \to \ell_\alpha + H\right) + \Gamma\left(N_i \to \bar{\ell}_\alpha + \overline{H}\right)\right]} \\ &= \frac{1}{8\pi v^2 \left(\widetilde{M}_{\rm D}^\dagger \widetilde{M}_{\rm D}\right)_{11}} \sum_{j\neq 1} \left\{ {\rm Im}\left[\left(\widetilde{M}_{\rm D}^*\right)_{\alpha 1} \left(\widetilde{M}_{\rm D}\right)_{\alpha j} \left(\widetilde{M}_{\rm D}^\dagger \widetilde{M}_{\rm D}\right)_{1j}\right] \times \mathcal{F}\left(\frac{M_j^2}{M_1^2}\right) \right. \\ &\quad \left. + {\rm Im}\left[\left(\widetilde{M}_{\rm D}^*\right)_{\alpha 1} \left(\widetilde{M}_{\rm D}\right)_{\alpha j} \left(\widetilde{M}_{\rm D}^\dagger \widetilde{M}_{\rm D}\right)^*_{1j}\right] \times \mathcal{G}\left(\frac{M_j^2}{M_1^2}\right) \right\} , \end{aligned} \tag{4.1}$$

where $\widetilde{M}_{\rm D} = M_{\rm D} U_{\rm R}^*$ with $U_{\rm R}$ being the unitary matrix used to diagonalize $M_{\rm R}$ (i.e., $U_{\rm R}^\dagger M_{\rm R} U_{\rm R}^* = \widehat{M}_N = {\rm Diag}\{M_1, M_2, M_3\}$), M_i (for $i = 1, 2, 3$) stands for the mass of the heavy Majorana neutrino N_i, $v \equiv \langle H \rangle \approx 174{\rm GeV}$ is the vacuum expectation value of the Higgs field, $\mathcal{F}(x)$ and $\mathcal{G}(x)$ are the loop functions defined as $\mathcal{F}(x) = \sqrt{x}\left\{(2-x)/(1-x) + (1+x)\ln\left[x/(1+x)\right]\right\}$ and $\mathcal{G}(x) = 1/(1-x)$, respectively. If all the interactions in the period of leptogenesis are blind to lepton flavors, then only the total CP-violating asymmetry ϵ_1 is relevant,

$$\epsilon_1 = \sum_\alpha \epsilon_{1\alpha} = \frac{1}{8\pi v^2 \left(\widetilde{M}_{\rm D}^\dagger \widetilde{M}_{\rm D}\right)_{11}} \sum_{j\neq 1} {\rm Im}\left[\left(\widetilde{M}_{\rm D}^\dagger \widetilde{M}_{\rm D}\right)^2_{1j}\right] \times \mathcal{F}\left(\frac{M_j^2}{M_1^2}\right) . \tag{4.2}$$

In the literature ϵ_1 and $\epsilon_{1\alpha}$ correspond to the so-called "unflavored" and "flavored" leptogenesis. In view of table 1, it is obvious that only categories A_i (for $i = 1, 2, 3$) and C are likely to lead us to nonzero ϵ_1 or $\epsilon_{1\alpha}$, and thus we are going to calculate them in the following.

[4] For a review, see e.g. ref. [52].

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4.1 Unflavored leptogenesis

4.1.1 Categories A_i

Let us first consider category A_1, and then turn to categories A_2 and A_3. In category A_1 the three mass matrices all respect the μ-τ reflection symmetry, so it is easy to calculate their corresponding mass eigenvalues and flavor mixing parameters. To be more specific, the unitary matrix $U_{\rm R}$ used to diagonalize $M_{\rm R}$ can be decomposed as $U_{\rm R} = P_1^{\rm R}\widetilde{U}^{\rm R}P_2^{\rm R}$, where $\widetilde{U}^{\rm R} = O_{23}\tilde{O}_{13}O_{12}$ is of the same form as the standard parameterization shown in eq. (3.1), and $P_1^{\rm R} = {\rm Diag}\left\{e^{{\rm i}\phi_1^{\rm R}}, e^{{\rm i}\phi_2^{\rm R}}, e^{{\rm i}\phi_3^{\rm R}}\right\}$ and $P_2^{\rm R} = {\rm Diag}\left\{e^{{\rm i}\rho^{\rm R}}, e^{{\rm i}\sigma^{\rm R}}, 1\right\}$ are the diagonal phase matrices. Then we obtain

$$\begin{aligned} &\theta_{23}^{\rm R} = \frac{\pi}{4} && \delta^{\rm R} = \pm\frac{\pi}{2}\,, && \rho^{\rm R}, \sigma^{\rm R} = 0 \text{ or } \frac{\pi}{2}\,, \\ &\phi_1^{\rm R} = 0 \text{ or } \frac{\pi}{2}\,, && \phi_2^{\rm R} + \phi_3^{\rm R} = 2\phi_1^{\rm R} \pm \pi\,. \end{aligned} \tag{4.3}$$

One can see that all the phase parameters take very special values.

We proceed to calculate the elements $\left(\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}\right)_{1j}$ which appear in eq. (4.2). The Hermitian matrix $\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}$ can be rewritten as

$$\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D} = U_{\rm R}^{\rm T}M_{\rm D}^{\dagger}M_{\rm D}U_{\rm R}^{*} = U_{\rm R}^{\rm T}U_{23}U_{23}^{\dagger}M_{\rm D}^{\dagger}M_{\rm D}U_{23}U_{23}^{\dagger}U_{\rm R}^{*} = U_{\rm R}^{\prime\rm T}HU_{\rm R}^{\prime *}\,, \tag{4.4}$$

in which

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \dfrac{\rm i}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\ 0 & \dfrac{-\rm i}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{pmatrix}\,, \tag{4.5}$$

and

$$H = U_{23}^{\dagger}M_{\rm D}^{\dagger}M_{\rm D}U_{23} = \begin{pmatrix} A^{\prime} & B^{\prime} & C^{\prime} \\ B^{\prime} & E^{\prime} & D^{\prime} \\ C^{\prime} & D^{\prime} & F^{\prime} \end{pmatrix} \tag{4.6}$$

is a real symmetric matrix whose elements are given by

$$\begin{aligned} A^{\prime} &= A_{\rm r}^2 + 2|E|^2\,, \\ B^{\prime} &= -\sqrt{2}{\rm Im}\left(A_{\rm r}B + E^{*}C + ED^{*}\right)\,, \\ C^{\prime} &= \sqrt{2}{\rm Re}\left(A_{\rm r}B + E^{*}C + ED^{*}\right)\,, \\ D^{\prime} &= -{\rm Im}\left(B^2 + 2CD^{*}\right)\,, \\ E^{\prime} &= |B|^2 + |C|^2 + |D|^2 - {\rm Re}\left(B^2 + 2CD^{*}\right)\,, \\ F^{\prime} &= |B|^2 + |C|^2 + |D|^2 + {\rm Re}\left(B^2 + 2CD^{*}\right)\,, \end{aligned} \tag{4.7}$$

and finally

$$U_{\rm R}^{\prime} = U_{23}^{T}U_{\rm R} = \begin{pmatrix} \eta & 0 & 0 \\ 0 & {\rm i}x & {\rm i}y \\ 0 & y & x \end{pmatrix} \begin{pmatrix} c_{13}^{\rm R}c_{12}^{\rm R} & c_{13}^{\rm R}s_{12}^{\rm R} & s_{13}^{\rm R}e^{-{\rm i}\delta^{\rm R}} \\ -s_{12}^{\rm R} & c_{12}^{\rm R} & 0 \\ -s_{13}^{\rm R}c_{12}^{\rm R}e^{{\rm i}\delta^{\rm R}} & -s_{13}^{\rm R}s_{12}^{\rm R}e^{{\rm i}\delta^{\rm R}} & c_{13}^{\rm R} \end{pmatrix} \begin{pmatrix} e^{{\rm i}\rho^{\rm R}} & 0 & 0 \\ 0 & e^{{\rm i}\sigma^{\rm R}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4.8}$$

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with $c_{ij}^{\rm R}=\cos\theta_{ij}^{\rm R}$, $s_{ij}^{\rm R}=\sin\theta_{ij}^{\rm R}$, and $x=\mathrm{i}\sin\phi_2^{\rm R}$ and $y=\cos\phi_2^{\rm R}$ for $\eta=1$ (i.e., $\phi_1^{\rm R}=0$) or $x=\cos\phi_2^{\rm R}$ and $y=\mathrm{i}\sin\phi_2^{\rm R}$ for $\eta=\mathrm{i}$ (i.e., $\phi_1^{\rm R}=\pi/2$). With the help of eqs. (4.6) and (4.8), eq. (4.4) can be expressed as

$$\left(\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}\right)_{1j}=\sum_{k=1}^{3}W_k\begin{pmatrix}c_{13}^{\rm R}c_{12}^{\rm R} & c_{13}^{\rm R}s_{12}^{\rm R} & s_{13}^{\rm R}e^{\mathrm{i}\delta^{\rm R}}\\ -s_{12}^{\rm R} & c_{12}^{\rm R} & 0\\ -s_{13}^{\rm R}c_{12}^{\rm R}e^{-\mathrm{i}\delta^{\rm R}} & -s_{13}^{\rm R}s_{12}^{\rm R}e^{-\mathrm{i}\delta^{\rm R}} & c_{13}^{\rm R}\end{pmatrix}_{kj}P_{jj}\,, \tag{4.9}$$

in which $P=\mathrm{Diag}\left\{1,e^{\mathrm{i}(\rho^{\rm R}-\sigma^{\rm R})},e^{\mathrm{i}\rho^{\rm R}}\right\}$ and

$$W_k=\left[\begin{pmatrix}c_{13}^{\rm R}c_{12}^{\rm R} & -s_{12}^{\rm R} & -s_{13}^{\rm R}c_{12}^{\rm R}e^{\mathrm{i}\delta^{\rm R}}\\ c_{13}^{\rm R}s_{12}^{\rm R} & c_{12}^{\rm R} & -s_{13}^{\rm R}s_{12}^{\rm R}e^{\mathrm{i}\delta^{\rm R}}\\ s_{13}^{\rm R}e^{-\mathrm{i}\delta^{\rm R}} & 0 & c_{13}^{\rm R}\end{pmatrix}\begin{pmatrix}\eta & 0 & 0\\ 0 & \mathrm{i}x & y\\ 0 & \mathrm{i}y & x\end{pmatrix}H\begin{pmatrix}\eta^* & 0 & 0\\ 0 & -\mathrm{i}x^* & -\mathrm{i}y^*\\ 0 & y^* & x^*\end{pmatrix}\right]_{1k}\,. \tag{4.10}$$

Concretely,

$$\begin{aligned}W_1&=A'c_{12}^{\rm R}c_{13}^{\rm R}-ps_{12}-qs_{13}^{\rm R}c_{12}^{\rm R}e^{\mathrm{i}\delta^{\rm R}}\,,\\ W_2&=p^*c_{12}^{\rm R}c_{13}^{\rm R}-t_1s_{12}-rs_{13}^{\rm R}c_{12}^{\rm R}e^{\mathrm{i}\delta^{\rm R}}\,,\\ W_3&=q^*c_{12}^{\rm R}c_{13}^{\rm R}-r^*s_{12}-t_2s_{13}^{\rm R}c_{12}^{\rm R}e^{\mathrm{i}\delta^{\rm R}}\,,\end{aligned} \tag{4.11}$$

where

$$\begin{aligned}p&=\eta^*(\mathrm{i}xB'+yC')\,,\\ q&=\eta^*(\mathrm{i}yB'+xC')\,,\\ t_1&=|x|^2E'+|y|^2F'-2D'\mathrm{Im}\,(xy^*)\,,\\ t_2&=|x|^2F'+|y|^2E'+2D'\mathrm{Im}\,(xy^*)\,,\\ r&=x^*yE'+xy^*F'-\mathrm{i}D'\left(|x|^2-|y|^2\right)\,,\end{aligned} \tag{4.12}$$

with t_1 and t_2 being real. Taking account of eqs. (4.9)–(4.12), we obtain

$$\begin{aligned}\left(\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}\right)_{11}&=A'^2c_{12}^{\rm R2}c_{13}^{\rm R2}-2s_{12}^{\rm R}c_{12}^{\rm R}c_{13}^{\rm R}\mathrm{Re}(p)-2\mathrm{i}e^{\mathrm{i}\delta^R}s_{13}^{\rm R}c_{13}^{\rm R}c_{12}^{\rm R2}\mathrm{Im}(q)\\ &\quad+2\mathrm{i}e^{\mathrm{i}\delta^R}s_{13}^{\rm R}s_{12}^{\rm R}c_{12}^{\rm R}\mathrm{Im}(r)+t_1s_{12}^{\rm R2}+t_2s_{13}^{\rm R2}c_{12}^{\rm R2}\,,\\ \left(\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}\right)_{12}&=e^{\mathrm{i}\left(\rho^{\rm R}-\sigma^{\rm R}\right)}\left[A's_{12}^{\rm R}c_{12}^{\rm R}c_{13}^{\rm R2}-2s_{12}^{\rm R2}c_{13}^{\rm R}\mathrm{Re}(p)+p^*c_{13}^{\rm R}\right.\\ &\quad-2\mathrm{i}e^{\mathrm{i}\delta^R}s_{13}^{\rm R}c_{13}^{\rm R}s_{12}^{\rm R}c_{12}^{\rm R}\mathrm{Im}(q)+2\mathrm{i}e^{\mathrm{i}\delta^R}s_{13}^{\rm R}s_{12}^{\rm R2}\mathrm{Im}(r)\\ &\quad\left.-re^{\mathrm{i}\delta^R}s_{13}^{\rm R}-t_1s_{12}^{\rm R}c_{12}^{\rm R}+t_2s_{13}^{\rm R2}s_{12}^{\rm R}c_{12}^{\rm R}\right]\,,\\ \left(\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}\right)_{13}&=e^{\mathrm{i}\rho^{\rm R}}\left[A'e^{\mathrm{i}\delta^R}s_{13}^{\rm R}c_{13}^{\rm R}c_{12}^{\rm R2}-pe^{\mathrm{i}\delta^R}s_{13}^{\rm R}c_{12}^{\rm R}-t_2e^{\mathrm{i}\delta^R}s_{13}^{\rm R}c_{13}^{\rm R}c_{12}^{\rm R}\right.\\ &\quad\left.-r^*s_{12}^{\rm R}c_{13}^{\rm R}+qc_{12}^{\rm R}-2\mathrm{i}c_{12}^{\rm R}c_{13}^{\rm R2}\mathrm{Im}(q)\right]\,.\end{aligned} \tag{4.13}$$

In view of eqs. (4.3) and (4.12) together with the definitions of η, x and y, it is apparent that p, t_1 and t_2 are real; q and r are purely imaginary; $\mathrm{i}e^{\mathrm{i}\delta^R}=\pm1$, $e^{\mathrm{i}\left(\rho^{\rm R}-\sigma^{\rm R}\right)}=1$ or $\pm\mathrm{i}$

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and $e^{\mathrm{i}\rho^{\mathrm{R}}} = 1$ or i. We are therefore left with

$$\mathrm{Im}\left[\left(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\right)^{2}_{12}\right] = \mathrm{Im}\left[\left(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\right)^{2}_{13}\right] = 0\,, \quad \epsilon_1 = 0\,. \tag{4.14}$$

In other words, there is no CP violation at all in N_1 decays for category A_1.

If the three heavy Majorana neutrinos have the same mass hierarchy in categories A_1, A_2 and A_3, then the expressions of three eigenvalues of $M_{\mathrm{R}}^{A_i}$ are of the same form, and therefore eq. (2.16) leads us to

$$U_{\mathrm{R}}^{A_2} = S^{(231)} U_{\mathrm{R}}^{A_1}\,, \quad U_{\mathrm{R}}^{A_3} = S^{(312)} U_{\mathrm{R}}^{A_1}\,. \tag{4.15}$$

With the help of eqs. (2.16) and (4.15), we find

$$\begin{aligned}
\widetilde{M}_{\mathrm{D}}^{A_2\dagger}\widetilde{M}_{\mathrm{D}}^{A_2} &= U_{\mathrm{R}}^{A_2 T} M_{\mathrm{D}}^{A_2\dagger} M_{\mathrm{D}}^{A_2} U_{\mathrm{R}}^{A_2 *} \\
&= U_{\mathrm{R}}^{A_1 T} S^{(312)} S^{(231)} M_{\mathrm{D}}^{A_1\dagger} S^{(312)} S^{(231)} M_{\mathrm{D}}^{A_1} S^{(312)} S^{(231)} U_{\mathrm{R}}^{A_1 *} \\
&= U_{\mathrm{R}}^{A_1 T} M_{\mathrm{D}}^{A_1\dagger} M_{\mathrm{D}}^{A_1} U_{\mathrm{R}}^{A_1 *} \\
&= \widetilde{M}_{\mathrm{D}}^{A_1\dagger}\widetilde{M}_{\mathrm{D}}^{A_1}\,, \\
\widetilde{M}_{\mathrm{D}}^{A_3\dagger}\widetilde{M}_{\mathrm{D}}^{A_3} &= U_{\mathrm{R}}^{A_3 T} M_{\mathrm{D}}^{A_3\dagger} M_{\mathrm{D}}^{A_3} U_{\mathrm{R}}^{A_3 *} \\
&= U_{\mathrm{R}}^{A_1 T} S^{(231)} S^{(312)} M_{\mathrm{D}}^{A_1\dagger} S^{(231)} S^{(312)} M_{\mathrm{D}}^{A_1} S^{(231)} S^{(312)} U_{\mathrm{R}}^{A_1 *} \\
&= U_{\mathrm{R}}^{A_1 T} M_{\mathrm{D}}^{A_1\dagger} M_{\mathrm{D}}^{A_1} U_{\mathrm{R}}^{A_1 *} \\
&= \widetilde{M}_{\mathrm{D}}^{A_1\dagger}\widetilde{M}_{\mathrm{D}}^{A_1}\,.
\end{aligned} \tag{4.16}$$

This result in turn means

$$\epsilon_1^{A_2} = \epsilon_1^{A_3} = \epsilon_1^{A_1} = 0\,. \tag{4.17}$$

We conclude that in the S_3 reflection symmetry limit there is no way to realize unflavored leptogenesis for categories A_i. This conclusion will change when the lepton flavor effects are taken into account.

4.1.2 Category *C*

In this case the three eigenvalues of M_{R} are given by $a_{\mathrm{r}} - b_{\mathrm{r}}$, $a_{\mathrm{r}} - b_{\mathrm{r}}$ and $a_{\mathrm{r}} + 2b_{\mathrm{r}}$, respectively. For simplicity, let us assume $a_{\mathrm{r}} > 0$ and $b_{\mathrm{r}} < 0$, such that $M_1 = a_{\mathrm{r}} + 2b_{\mathrm{r}} \ll M_2 = M_3 = a_{\mathrm{r}} - b_{\mathrm{r}}$ can be satisfied. The corresponding unitary matrix U_{R} is

$$U_{\mathrm{R}} = \begin{pmatrix} \dfrac{1}{\sqrt{3}} & -\dfrac{2}{\sqrt{6}} & 0 \\ \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{6}} & -\dfrac{1}{\sqrt{2}} \end{pmatrix}\,. \tag{4.18}$$

Consequently,

$$\widetilde{M}_{\mathrm{D}} = M_{\mathrm{D}} U_{\mathrm{R}}^{*} = \begin{pmatrix} \dfrac{1}{\sqrt{3}}\left(A_{\mathrm{r}} + 2\mathrm{Re}B\right) & -\dfrac{1}{\sqrt{6}}\left(2A_{\mathrm{r}} - 2\mathrm{Re}B\right) & \sqrt{2}\ \mathrm{i}\ \mathrm{Im}B \\ \dfrac{1}{\sqrt{3}}\left(A_{\mathrm{r}} + 2\mathrm{Re}B\right) & \dfrac{1}{\sqrt{6}}\left(A_{\mathrm{r}} - 2B^{*} + B\right) & \dfrac{1}{\sqrt{2}}\left(A_{\mathrm{r}} - B\right) \\ \dfrac{1}{\sqrt{3}}\left(A_{\mathrm{r}} + 2\mathrm{Re}B\right) & \dfrac{1}{\sqrt{6}}\left(A_{\mathrm{r}} + B^{*} - 2B\right) & -\dfrac{1}{\sqrt{2}}\left(A_{\mathrm{r}} - B^{*}\right) \end{pmatrix}\,. \tag{4.19}$$

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Then it is straightforward for us to obtain

$$\left(\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}\right)_{11} = \left(A_{\rm r}+2{\rm Re}B\right)^2 \, , \quad \left(\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}\right)_{12} = 0 \, , \quad \left(\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}\right)_{13} = 0 \, . \tag{4.20}$$

As a result,

$$\epsilon_{1\alpha} = 0 \, , \quad \epsilon_1 = \sum_{\alpha} \epsilon_{1\alpha} = 0 \, , \tag{4.21}$$

where α runs over e, μ and τ. Therefore, there is no way for both unflavored and flavored leptogenesis to work in category C.

4.2 Flavored leptogenesis

In the unflavored leptogenesis case as discussed above, the Yukawa interactions of charged leptons are not taken into account, since the equilibrium temperature of heavy Majorana neutrinos is assumed to be high enough that such interactions cannot distinguish one lepton flavor from another. In other words, all the relevant Yukawa interactions are blind to lepton flavors. When the equilibrium temperature is lower, however, it is possible that the Yukawa interactions of charged leptons become faster than the (inverse) decays of N_i or equivalently comparable to the expansion rate of the Universe. In this case the flavor effects must be taken into consideration [41, 42].

Here we focus on the possibility that the equilibrium temperature T lies in the range 10^9 GeV $< T < 10^{12}$ GeV, in which the τ lepton can be in thermal equilibrium and thus are distinguishable from the e and μ flavors. In this case both the CP-violating asymmetries and washout effects involving the τ flavor should be treated separately [57, 58]. It is then possible to achieve successful leptogenesis provided $\epsilon_{1\alpha} \neq 0$ holds, even though the total CP-violating asymmetry ϵ_1 is vanishing or vanishingly small.

We have shown in eq. (4.21) that both $\epsilon_{1\alpha}$ and ϵ_1 are vanishing in category C, and thus it is impossible to realize either unflavored or flavored leptogenesis in this case in the S_3 reflection symmetry limit. In the following we calculate the flavor-dependent CP-violating asymmetries $\epsilon_{1\alpha}$ for categories A_i by using eq. (4.1), to examine whether flavored leptogenesis has a chance to work or not in this case.

Given $M_{\rm D}$ and $U_{\rm R}$ in category A_1, a lengthy but straightforward calculation leads us to

$$\begin{aligned}
\left(\widetilde{M}_{\rm D}^{*}\right)_{e1}\left(\widetilde{M}_{\rm D}\right)_{e2} = {} & e^{{\rm i}(\rho^{\rm R}-\sigma^{\rm R})}\left\{A_{\rm r}^2 s_{12}^{\rm R}c_{12}^{\rm R}c_{13}^{\rm R2} + \sqrt{2}A_{\rm r}c_{13}^{\rm R}\cos 2\theta_{12}^{\rm R}{\rm Re}\left(\eta^* B e^{-{\rm i}\phi_2^{\rm R}}\right)\right. \\
& +2\ {\rm i}\ e^{{\rm i}\delta^{\rm R}}s_{13}^{\rm R}\cos 2\theta_{12}^{\rm R}{\rm Re}\left(\eta^* B e^{-{\rm i}\phi_2^{\rm R}}\right){\rm Im}\left(\eta^* B e^{-{\rm i}\phi_2^{\rm R}}\right) \\
& +2s_{13}^{\rm R2}s_{12}^{\rm R}c_{12}^{\rm R}\left[{\rm Im}\left(\eta^* B e^{-{\rm i}\phi_2^{\rm R}}\right)\right]^2 - 2s_{12}^{\rm R}c_{12}^{\rm R}\left[{\rm Re}\left(\eta^* B e^{-{\rm i}\phi_2^{\rm R}}\right)\right]^2 \\
& \left.+2\sqrt{2}\ {\rm i}\ e^{{\rm i}\delta^{\rm R}}A_{\rm r}s_{13}^{\rm R}c_{13}^{\rm R}s_{12}^{\rm R}c_{12}^{\rm R}{\rm Im}\left(\eta^* B e^{-{\rm i}\phi_2^{\rm R}}\right)\right\} \, , \\
\left(\widetilde{M}_{\rm D}^{*}\right)_{e1}\left(\widetilde{M}_{\rm D}\right)_{e3} = {} & e^{{\rm i}\rho^{\rm R}}\left\{A_{\rm r}^2 s_{13}^{\rm R}c_{13}^{\rm R}c_{12}^{\rm R}e^{{\rm i}\delta^{\rm R}} - 2e^{{\rm i}\delta^{\rm R}}s_{13}^{\rm R}c_{13}^{\rm R}c_{12}^{\rm R}\left[{\rm Im}\left(\eta^* B e^{-{\rm i}\phi_2^{\rm R}}\right)\right]^2\right. \\
& \left.- 2\ {\rm i}\ s_{12}^{\rm R}c_{13}^{\rm R}{\rm Re}\left(\eta^* B e^{-{\rm i}\phi_2^{\rm R}}\right){\rm Im}\left(\eta^* B e^{-{\rm i}\phi_2^{\rm R}}\right)\right.
\end{aligned}$$

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$$+\sqrt{2}\ \mathrm{i}\ A_{\mathrm{r}}\cos 2\theta_{13}^{\mathrm{R}}c_{12}^{\mathrm{R}}\mathrm{Im}\left(\eta^{*}Be^{-\mathrm{i}\phi_{2}^{\mathrm{R}}}\right)$$
$$-\sqrt{2}\ A_{\mathrm{r}}s_{13}^{\mathrm{R}}s_{12}^{\mathrm{R}}e^{\mathrm{i}\delta^{\mathrm{R}}}\mathrm{Re}\left(\eta^{*}Be^{-\mathrm{i}\phi_{2}^{\mathrm{R}}}\right)\Big\}\,, \tag{4.22}$$

and

$$\begin{aligned}\left(\widetilde{M}_{\mathrm{D}}^{*}\right)_{\mu1}\left(\widetilde{M}_{\mathrm{D}}\right)_{\mu2} &= e^{\mathrm{i}\left(\rho^{\mathrm{R}}-\sigma^{\mathrm{R}}\right)}\Big\{|E|^{2}s_{12}^{\mathrm{R}}c_{12}^{\mathrm{R}}c_{13}^{\mathrm{R}2}-\frac{1}{2}|z_{1}+z_{2}|^{2}s_{12}^{\mathrm{R}}c_{12}^{\mathrm{R}}+\frac{1}{2}|z_{1}-z_{2}|^{2}s_{13}^{\mathrm{R}2}s_{12}^{\mathrm{R}}c_{12}^{\mathrm{R}}\\ &\quad-\sqrt{2}\ s_{12}^{\mathrm{R}2}c_{13}^{\mathrm{R}}\mathrm{Re}\left[E^{*}\left(z_{1}+z_{2}\right)\right]+\sqrt{2}\ \mathrm{i}\ e^{\mathrm{i}\delta^{\mathrm{R}}}s_{13}^{\mathrm{R}}c_{13}^{\mathrm{R}}s_{12}^{\mathrm{R}}c_{12}^{\mathrm{R}}\mathrm{Im}\left[E^{*}\left(z_{1}-z_{2}\right)\right]\\ &\quad-\mathrm{i}\ e^{\mathrm{i}\delta^{\mathrm{R}}}s_{13}^{\mathrm{R}}\cos 2\theta_{12}^{\mathrm{R}}\mathrm{Im}\left(z_{1}^{*}z_{2}\right)+\frac{1}{\sqrt{2}}\ c_{13}^{\mathrm{R}}E^{*}\left(z_{1}+z_{2}\right)\\ &\quad-\frac{1}{2}s_{13}^{\mathrm{R}}e^{\mathrm{i}\delta^{\mathrm{R}}}\left(|C|^{2}-|D|^{2}\right)\Big\}\,,\\ \left(\widetilde{M}_{\mathrm{D}}^{*}\right)_{\mu1}\left(\widetilde{M}_{\mathrm{D}}\right)_{\mu3} &= e^{\mathrm{i}\rho^{\mathrm{R}}}\Big\{|E|^{2}s_{13}^{\mathrm{R}}c_{13}^{\mathrm{R}}c_{12}^{\mathrm{R}}e^{\mathrm{i}\delta^{\mathrm{R}}}-\frac{1}{2}\ s_{12}^{\mathrm{R}}c_{13}^{\mathrm{R}}\left(|C|^{2}-|D|^{2}\right)+\mathrm{i}\ s_{12}^{\mathrm{R}}c_{13}^{\mathrm{R}}\mathrm{Im}\left(z_{1}^{*}z_{2}\right)\\ &\quad-\sqrt{2}\ \mathrm{i}\ s_{13}^{\mathrm{R}2}c_{12}^{\mathrm{R}}\mathrm{Im}\left[E^{*}\left(z_{1}-z_{2}\right)\right]-\frac{1}{2}e^{\mathrm{i}\delta^{\mathrm{R}}}s_{13}^{\mathrm{R}}c_{13}^{\mathrm{R}}c_{12}^{\mathrm{R}}|z_{1}-z_{2}|^{2}\\ &\quad+\frac{1}{\sqrt{2}}\ c_{12}^{\mathrm{R}}E^{*}\left(z_{1}-z_{2}\right)-\frac{1}{\sqrt{2}}\ e^{\mathrm{i}\delta^{\mathrm{R}}}s_{13}^{\mathrm{R}}s_{12}^{\mathrm{R}}E\left(z_{1}^{*}+z_{2}^{*}\right)\Big\}\,,\end{aligned} \tag{4.23}$$

as well as

$$\begin{aligned}\left(\widetilde{M}_{\mathrm{D}}^{*}\right)_{\tau1}\left(\widetilde{M}_{\mathrm{D}}\right)_{\tau2} &= e^{\mathrm{i}\left(\rho^{\mathrm{R}}-\sigma^{\mathrm{R}}\right)}\Big\{|E|^{2}s_{12}^{\mathrm{R}}c_{12}^{\mathrm{R}}c_{13}^{\mathrm{R}2}-\frac{1}{2}|z_{1}+z_{2}|^{2}s_{12}^{\mathrm{R}}c_{12}^{\mathrm{R}}+\frac{1}{2}|z_{1}-z_{2}|^{2}s_{13}^{\mathrm{R}2}s_{12}^{\mathrm{R}}c_{12}^{\mathrm{R}}\\ &\quad-\sqrt{2}\ s_{12}^{\mathrm{R}2}c_{13}^{\mathrm{R}}\mathrm{Re}\left[E^{*}\left(z_{1}+z_{2}\right)\right]+\sqrt{2}\ \mathrm{i}\ e^{\mathrm{i}\delta^{\mathrm{R}}}s_{13}^{\mathrm{R}}c_{13}^{\mathrm{R}}s_{12}^{\mathrm{R}}c_{12}^{\mathrm{R}}\mathrm{Im}\left[E^{*}\left(z_{1}-z_{2}\right)\right]\\ &\quad-\mathrm{i}\ e^{\mathrm{i}\delta^{\mathrm{R}}}s_{13}^{\mathrm{R}}\cos 2\theta_{12}^{\mathrm{R}}\mathrm{Im}\left(z_{1}^{*}z_{2}\right)+\frac{1}{2}s_{13}^{\mathrm{R}}e^{\mathrm{i}\delta^{\mathrm{R}}}\left(|C|^{2}-|D|^{2}\right)\\ &\quad+\frac{1}{\sqrt{2}}\ c_{13}^{\mathrm{R}}E\left(z_{1}^{*}+z_{2}^{*}\right)\Big\}\,,\\ \left(\widetilde{M}_{\mathrm{D}}^{*}\right)_{\tau1}\left(\widetilde{M}_{\mathrm{D}}\right)_{\tau3} &= e^{\mathrm{i}\rho^{\mathrm{R}}}\Big\{|E|^{2}s_{13}^{\mathrm{R}}c_{13}^{\mathrm{R}}c_{12}^{\mathrm{R}}e^{\mathrm{i}\delta^{\mathrm{R}}}+\frac{1}{2}s_{12}^{\mathrm{R}}c_{13}^{\mathrm{R}}\left(|C|^{2}-|D|^{2}\right)+\mathrm{i}\ s_{12}^{\mathrm{R}}c_{13}^{\mathrm{R}}\mathrm{Im}\left(z_{1}^{*}z_{2}\right)\\ &\quad-\sqrt{2}\ \mathrm{i}\ s_{13}^{\mathrm{R}2}c_{12}^{\mathrm{R}}\mathrm{Im}\left[E^{*}\left(z_{1}-z_{2}\right)\right]-\frac{1}{2}e^{\mathrm{i}\delta^{\mathrm{R}}}s_{13}^{\mathrm{R}}c_{13}^{\mathrm{R}}c_{12}^{\mathrm{R}}|z_{1}-z_{2}|^{2}\\ &\quad-\frac{1}{\sqrt{2}}\ c_{12}^{\mathrm{R}}E\left(z_{1}^{*}-z_{2}^{*}\right)-\frac{1}{\sqrt{2}}\ e^{\mathrm{i}\delta^{\mathrm{R}}}s_{13}^{\mathrm{R}}s_{12}^{\mathrm{R}}E^{*}\left(z_{1}+z_{2}\right)\Big\}\,,\end{aligned} \tag{4.24}$$

where $\eta=1$ (or $-\mathrm{i}$) for $\phi_{1}^{\mathrm{R}}=0$ (or $\pi/2$), and

$$z_{1}=\eta^{*}Ce^{-\mathrm{i}\phi_{2}^{\mathrm{R}}}\,,\quad z_{2}=\eta De^{\mathrm{i}\phi_{2}^{\mathrm{R}}}\,. \tag{4.25}$$

With the help of eq. (4.13) and eqs. (4.22)–(4.25), we further obtain

$$\begin{aligned}&\mathrm{Im}\left[\left(\widetilde{M}_{\mathrm{D}}^{*}\right)_{e1}\left(\widetilde{M}_{\mathrm{D}}\right)_{e2}\left(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\right)_{12}\right]=0,\\ &\mathrm{Im}\left[\left(\widetilde{M}_{\mathrm{D}}^{*}\right)_{e1}\left(\widetilde{M}_{\mathrm{D}}\right)_{e3}\left(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\right)_{13}\right]=0,\\ &\mathrm{Im}\left[\left(\widetilde{M}_{\mathrm{D}}^{*}\right)_{\mu1}\left(\widetilde{M}_{\mathrm{D}}\right)_{\mu2}\left(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\right)_{12}\right]=\eta_{1}\left|\left(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\right)_{12}\right|\mathrm{Im}\left[\frac{1}{\sqrt{2}}c_{13}^{\mathrm{R}}E^{*}(z_{1}+z_{2})\right.\\ &\qquad\left.-\frac{1}{2}s_{13}^{\mathrm{R}}e^{\mathrm{i}\delta^{\mathrm{R}}}\left(|C|^{2}-|D|^{2}\right)\right],\end{aligned}$$

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$$
\begin{aligned}
\mathrm{Im}\Big[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{\mu 1}\big(\widetilde{M}_{\mathrm{D}}\big)_{\mu 3}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{13}\Big] &= \eta_2\Big|\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{13}\Big|\mathrm{Re}\Big[\frac{1}{\sqrt{2}}\,c_{12}^{\mathrm{R}}E^{*}(z_1-z_2) \\
&\quad -\frac{1}{\sqrt{2}}\,e^{\mathrm{i}\delta^{\mathrm{R}}}s_{13}^{\mathrm{R}}s_{12}^{\mathrm{R}}E(z_1^{*}+z_2^{*})-\frac{1}{2}s_{12}^{\mathrm{R}}c_{13}^{\mathrm{R}}\big(|C|^2-|D|^2\big)\Big], \\
\mathrm{Im}\Big[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{\tau 1}\big(\widetilde{M}_{\mathrm{D}}\big)_{\tau 2}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{12}\Big] &= \eta_1\Big|\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{12}\Big|\mathrm{Im}\Big[\frac{1}{\sqrt{2}}\,c_{13}^{\mathrm{R}}E(z_1^{*}+z_2^{*}) \\
&\quad +\frac{1}{2}s_{13}^{\mathrm{R}}e^{\mathrm{i}\delta^{\mathrm{R}}}\big(|C|^2-|D|^2\big)\Big], \\
\mathrm{Im}\Big[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{\tau 1}\big(\widetilde{M}_{\mathrm{D}}\big)_{\tau 3}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{13}\Big] &= \eta_2\Big|\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{13}\Big|\mathrm{Re}\Big[-\frac{1}{\sqrt{2}}\,c_{12}^{\mathrm{R}}E(z_1^{*}-z_2^{*}) \\
&\quad -\frac{1}{\sqrt{2}}\,e^{\mathrm{i}\delta^{\mathrm{R}}}s_{13}^{\mathrm{R}}s_{12}^{\mathrm{R}}E^{*}(z_1+z_2) \\
&\quad +\frac{1}{2}s_{12}^{\mathrm{R}}c_{13}^{\mathrm{R}}\big(|C|^2-|D|^2\big)\Big], \qquad (4.26)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathrm{Im}\Big[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{e1}\big(\widetilde{M}_{\mathrm{D}}\big)_{e2}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{12}^{*}\Big] &= 0, \\
\mathrm{Im}\Big[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{e1}\big(\widetilde{M}_{\mathrm{D}}\big)_{e3}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{13}^{*}\Big] &= 0, \\
\mathrm{Im}\Big[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{\mu 1}\big(\widetilde{M}_{\mathrm{D}}\big)_{\mu 2}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{12}^{*}\Big] &= \eta_3\Big|\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{12}\Big|\mathrm{Im}\Big[\frac{1}{\sqrt{2}}\,c_{13}^{\mathrm{R}}E^{*}(z_1+z_2) \\
&\quad -\frac{1}{2}s_{13}^{\mathrm{R}}e^{\mathrm{i}\delta^{\mathrm{R}}}\big(|C|^2-|D|^2\big)\Big], \\
\mathrm{Im}\Big[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{\mu 1}\big(\widetilde{M}_{\mathrm{D}}\big)_{\mu 3}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{13}^{*}\Big] &= \eta_4\Big|\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{13}\Big|\mathrm{Re}\Big[\frac{1}{\sqrt{2}}\,c_{12}^{\mathrm{R}}E^{*}(z_1-z_2) \\
&\quad -\frac{1}{\sqrt{2}}\,e^{\mathrm{i}\delta^{\mathrm{R}}}s_{13}^{\mathrm{R}}s_{12}^{\mathrm{R}}E(z_1^{*}+z_2^{*})-\frac{1}{2}s_{12}^{\mathrm{R}}c_{13}^{\mathrm{R}}\big(|C|^2-|D|^2\big)\Big], \\
\mathrm{Im}\Big[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{\tau 1}\big(\widetilde{M}_{\mathrm{D}}\big)_{\tau 2}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{12}^{*}\Big] &= \eta_3\Big|\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{12}\Big|\mathrm{Im}\Big[\frac{1}{\sqrt{2}}\,c_{13}^{\mathrm{R}}E(z_1^{*}+z_2^{*}) \\
&\quad +\frac{1}{2}s_{13}^{\mathrm{R}}e^{\mathrm{i}\delta^{\mathrm{R}}}\big(|C|^2-|D|^2\big)\Big], \\
\mathrm{Im}\Big[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{\tau 1}\big(\widetilde{M}_{\mathrm{D}}\big)_{\tau 3}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{13}^{*}\Big] &= \eta_4\Big|\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{13}\Big|\mathrm{Re}\Big[-\frac{1}{\sqrt{2}}\,c_{12}^{\mathrm{R}}E(z_1^{*}-z_2^{*}) \\
&\quad -\frac{1}{\sqrt{2}}\,e^{\mathrm{i}\delta^{\mathrm{R}}}s_{13}^{\mathrm{R}}s_{12}^{\mathrm{R}}E^{*}(z_1+z_2) \\
&\quad +\frac{1}{2}s_{12}^{\mathrm{R}}c_{13}^{\mathrm{R}}\big(|C|^2-|D|^2\big)\Big], \qquad (4.27)
\end{aligned}
$$

where $\eta_i = \pm 1$ (for $i = 1, 2, 3, 4$). Combining eqs. (4.26) and (4.27), we arrive at

$$
\begin{aligned}
\mathrm{Im}\left[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{\mu 1}\big(\widetilde{M}_{\mathrm{D}}\big)_{\mu 2}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{12}\right] &= -\mathrm{Im}\left[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{\tau 1}\big(\widetilde{M}_{\mathrm{D}}\big)_{\tau 2}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{12}\right] \neq 0\,, \\
\mathrm{Im}\left[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{\mu 1}\big(\widetilde{M}_{\mathrm{D}}\big)_{\mu 3}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{13}\right] &= -\mathrm{Im}\left[\big(\widetilde{M}_{\mathrm{D}}^{*}\big)_{\tau 1}\big(\widetilde{M}_{\mathrm{D}}\big)_{\tau 3}\big(\widetilde{M}_{\mathrm{D}}^{\dagger}\widetilde{M}_{\mathrm{D}}\big)_{13}\right] \neq 0\,,
\end{aligned}
$$

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$$\text{Im}\left[\left(\widetilde{M}_{\rm D}^{*}\right)_{\mu 1}\left(\widetilde{M}_{\rm D}\right)_{\mu 2}\left(\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}\right)^{*}_{12}\right] = -\text{Im}\left[\left(\widetilde{M}_{\rm D}^{*}\right)_{\tau 1}\left(\widetilde{M}_{\rm D}\right)_{\tau 2}\left(\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}\right)^{*}_{12}\right] \neq 0\,,$$
$$\text{Im}\left[\left(\widetilde{M}_{\rm D}^{*}\right)_{\mu 1}\left(\widetilde{M}_{\rm D}\right)_{\mu 3}\left(\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}\right)^{*}_{13}\right] = -\text{Im}\left[\left(\widetilde{M}_{\rm D}^{*}\right)_{\tau 1}\left(\widetilde{M}_{\rm D}\right)_{\tau 3}\left(\widetilde{M}_{\rm D}^{\dagger}\widetilde{M}_{\rm D}\right)^{*}_{13}\right] \neq 0\,. \quad (4.28)$$

As a result,

$$\epsilon_{1e} = 0\,, \quad \epsilon_{1\mu} = -\epsilon_{1\tau} \neq 0\,, \quad \epsilon_1 = \sum_\alpha \epsilon_{1\alpha} = 0\,. \quad (4.29)$$

It is therefore possible to realize μ- or τ-flavored leptogenesis in this case, at least in principle. Since a realistic example of this kind needs to include proper S_3 reflection symmetry breaking effects, it will be studied elsewhere.

Finally, if the mass hierarchies of three heavy Majorana neutrinos in categories A_2 and A_3 are the same as that in category A_1, then one can get

$$\left(\widetilde{M}_{\rm D}^{A_2 *}\right)_{\alpha 1}\left(\widetilde{M}_{\rm D}^{A_2}\right)_{\alpha j} = \left(\widetilde{M}_{\rm D}^{A_1 *}\right)_{\beta 1}\left(\widetilde{M}_{\rm D}^{A_1}\right)_{\beta j}\,,$$
$$\left(\widetilde{M}_{\rm D}^{A_3 *}\right)_{\gamma 1}\left(\widetilde{M}_{\rm D}^{A_3}\right)_{\gamma j} = \left(\widetilde{M}_{\rm D}^{A_1 *}\right)_{\lambda 1}\left(\widetilde{M}_{\rm D}^{A_1}\right)_{\lambda j}\,, \quad (4.30)$$

and therefore

$$\epsilon^{A_2}_{1\alpha} = \epsilon^{A_1}_{1\beta}\,, \quad \epsilon^{A_3}_{1\gamma} = \epsilon^{A_1}_{1\lambda}\,, \quad (4.31)$$

where $\alpha\beta = e\mu$, $\mu\tau$ and τe; $\gamma\lambda = e\tau$, μe and $\tau\mu$; and $j = 2$ or 3. As a result,

$$\epsilon^{A_2}_{1\tau} = 0\,, \qquad \epsilon^{A_2}_{1e} = -\epsilon^{A_2}_{1\mu} \neq 0\,, \qquad \epsilon^{A_2}_{1} = \sum_\alpha \epsilon^{A_2}_{1\alpha} = 0\,;$$
$$\epsilon^{A_3}_{1\mu} = 0\,, \qquad \epsilon^{A_3}_{1\tau} = -\epsilon^{A_3}_{1e} \neq 0\,, \qquad \epsilon^{A_3}_{1} = \sum_\alpha \epsilon^{A_3}_{1\alpha} = 0\,, \quad (4.32)$$

for categories A_2 and A_3.

5 Some further discussions

In this work we have made a new attempt to specify the flavor structures associated with the canonical seesaw mechanism, so as to promote its predictability and testability. What we have done is to require the relevant neutrino mass terms to be invariant under the S_3 reflection transformations of both left- and right-handed neutrino fields. This treatment allows us to constrain the Dirac mass matrix $M_{\rm D}$ and the right-handed neutrino mass matrix $M_{\rm R}$ to some extent, and the effective light Majorana neutrino mass matrix M_ν is in turn constrained through the seesaw relation. We find that the structures of $M_{\rm D}$, $M_{\rm R}$ and M_ν can be classified into 22 categories, among which some structures respect the well-known μ-τ symmetry and (or) flavor democracy. In particular, we find that the texture of M_ν may be either the same as or similar to that of $M_{\rm R}$, and this property reflects a seesaw mirroring relationship between light and heavy Majorana neutrinos. To be specific, we have calculated the light neutrino masses and flavor mixing parameters for all the textures of M_ν, and examined whether the CP-violating asymmetries in decays of

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the lightest heavy Majorana neutrino are vanishing or not in the S_3 reflection symmetry limit. Our calculations show that only the flavored leptogenesis mechanism is possible to work for categories A_1, A_2 and A_3 listed in table 1.

One might wonder whether some different neutrino mixing patterns and related leptogenesis can be obtained in our approach if S_3 symmetry group is extended to S_4 or A_4. The answer is affirmative. To illustrate, let us briefly discuss the situation associated with A_4 group in our framework. It is well known that A_4 group is defined as the even permutation of four objects and has twelve elements being divided into four classes. So A_4 group has four irreducible representations — three inequivalent one-dimensional representations ($\underline{\mathbf{1}}$, $\underline{\mathbf{1}}'$ and $\underline{\mathbf{1}}''$) and one three-dimensional representation ($\underline{\mathbf{3}}$). Now that we work in the basis where M_l is diagonal, it is more interesting for us to consider the three-dimensional unitary representation of A_4 group, which has been used in refs. [8, 59] rather than refs. [60, 61]. In this representation the two generators of A_4, denoted as S and T, are given by

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} , \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} , \tag{5.1}$$

where $\omega = \exp(\mathrm{i}2\pi/3)$. Then all the elements of A_4 can be presented by the following twelve 3×3 matrices: $\mathbf{1}$, S, T, ST, TS, T^2, ST^2, T^2S, STS, TST, TST^2 and T^2ST.

In a way similar to the S_3 reflection transformations, we may require the neutrino mass term in eq. (2.1) to keep invariant under the transformations made in eq. (2.2) with $S_{(\mathrm{L})}$ or $S_{(\mathrm{R})}$ being an arbitrary element of the given subset of A_4 group. In this case we are left with the same form of the constraints on M_{D} and M_{R} as obtained in eq. (2.5). One may systematically categorize all the possible structures of neutrino mass matrices as we have done in table 1 for S_3 group, but for A_4 group it seems unnecessary to do so because in most cases the A_4-induced constraints are so strong that the resultant textures of neutrino mass matrices are trivial and disinteresting. If a case with the given subset having more than one element is considered, for example, it will be unable to result in any CP violation in both light and heavy neutrino sectors. Some cases with only one element may also lead to trivial results, and those more interesting cases usually involve many unknown parameters. Here we only show a simple example of this kind which allows us to obtain the textures of neutrino mass matrices different from those listed in table 1, together with a nonzero CP-violating asymmetry ϵ_1 in the lightest heavy Majorana neutrino decays. It is the case where the subset only contains element S, and in this case M_{D} and M_{R} constrained by eq. (2.5) are

$$\begin{aligned} M_{\mathrm{D}} &= \begin{pmatrix} 2A_{\mathrm{r}} & 2B_{\mathrm{r}} & 2B_{\mathrm{r}} \\ 2B_{\mathrm{r}} & A_{\mathrm{r}}+B_{\mathrm{r}} & A_{\mathrm{r}}+B_{\mathrm{r}} \\ 2B_{\mathrm{r}} & A_{\mathrm{r}}+B_{\mathrm{r}} & A_{\mathrm{r}}+B_{\mathrm{r}} \end{pmatrix} + \mathrm{i}C_{\mathrm{r}} \begin{pmatrix} 2 & 2 & 2 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix} , \\ M_{\mathrm{R}} &= \begin{pmatrix} a_{\mathrm{r}} & b_{\mathrm{r}} & e_{\mathrm{r}} \\ b_{\mathrm{r}} & d_{\mathrm{r}} & a_{\mathrm{r}}+e_{\mathrm{r}}-d_{\mathrm{r}} \\ e_{\mathrm{r}} & a_{\mathrm{r}}+e_{\mathrm{r}}-d_{\mathrm{r}} & b_{\mathrm{r}}-e_{\mathrm{r}}+d_{\mathrm{r}} \end{pmatrix} , \end{aligned} \tag{5.2}$$

where $\mathrm{Re}\left[\langle M_{\mathrm{D}}\rangle_{12}\right] = \mathrm{Re}\left[\langle M_{\mathrm{D}}\rangle_{13}\right] = \mathrm{Re}\left[\langle M_{\mathrm{D}}\rangle_{21}\right]$, $\mathrm{Re}\left[\langle M_{\mathrm{D}}\rangle_{22}\right] = \left(\mathrm{Re}\left[\langle M_{\mathrm{D}}\rangle_{11}\right] + \mathrm{Re}\left[\langle M_{\mathrm{D}}\rangle_{12}\right]\right)/2$, $\mathrm{Im}\left[\langle M_{\mathrm{D}}\rangle_{11}\right] = \mathrm{Im}\left[\langle M_{\mathrm{D}}\rangle_{12}\right] = \mathrm{Im}\left[\langle M_{\mathrm{D}}\rangle_{13}\right]$,

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$\mathrm{Im}\left[\langle M_{\rm D}\rangle_{21}\right] = -\mathrm{Im}\left[\langle M_{\rm D}\rangle_{11}\right]/2$ and $\mathrm{Im}\left[M_{\rm R}\right] = 0$ have been assumed to reduce the number of free parameters. The light Majorana neutrino mass matrix M_ν turns out to have the form

$$M_\nu = \begin{pmatrix} 2a'_{\rm r} - b'_{\rm r} & b'_{\rm r} & b'_{\rm r} \\ b'_{\rm r} & a'_{\rm r} & a'_{\rm r} \\ b'_{\rm r} & a'_{\rm r} & a'_{\rm r} \end{pmatrix} + {\rm i}e'_{\rm r} \begin{pmatrix} 4 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & -2 & -2 \end{pmatrix} , \tag{5.3}$$

and the explicit relations between the parameters of M_ν and those of $M_{\rm D}$ and $M_{\rm R}$ can easily be derived. We see that this example leads to a texture of M_ν which respects the μ-τ permutation symmetry. Diagonalizing M_ν allows us to obtain the masses of three light neutrinos and their flavor mixing parameters. For the inverted neutrino mass hierarchy,[5] the results are

$$m_1 = \frac{1}{2}\left(-3b'_{\rm r} + \Delta\right) , \quad m_2 = \frac{1}{2}\left(3b'_{\rm r} + \Delta\right) , \quad m_3 = 0 \tag{5.4}$$

and

$$V = \begin{pmatrix} \pm\frac{2}{\sqrt{6}} \frac{-4a'_{\rm r} + b'_{\rm r} - \Delta - 6{\rm i}e'_{\rm r}}{t} & \pm\frac{1}{\sqrt{3}} \frac{4a'_{\rm r} - b'_{\rm r} + \Delta + 12{\rm i}e'_{\rm r}}{t} & 0 \\ \pm\frac{1}{\sqrt{6}} \frac{4a'_{\rm r} - b'_{\rm r} + \Delta - 12{\rm i}e'_{\rm r}}{t} & \pm\frac{1}{\sqrt{3}} \frac{4a'_{\rm r} - b'_{\rm r} + \Delta - 6{\rm i}e'_{\rm r}}{t} & -\frac{1}{\sqrt{2}} \\ \pm\frac{1}{\sqrt{6}} \frac{4a'_{\rm r} - b'_{\rm r} + \Delta - 12{\rm i}e'_{\rm r}}{t} & \pm\frac{1}{\sqrt{3}} \frac{4a'_{\rm r} - b'_{\rm r} + \Delta - 6{\rm i}e'_{\rm r}}{t} & \frac{1}{\sqrt{2}} \end{pmatrix} , \tag{5.5}$$

where $\Delta = \sqrt{(-4a'_{\rm r} + b'_{\rm r})^2 + 72e'^2_{\rm r}}$ and $t = \sqrt{(4a'_{\rm r} - b'_{\rm r} + \Delta)^2 + 72e'^2_{\rm r}}$, and the "$\pm$" signs correspond to the sign of $e'_{\rm r}$. It is obvious that V can be regarded as a variation of the tri-bimaximal flavor mixing pattern, and the equalities $|V_{\mu i}| = |V_{\tau i}|$ hold (for $i = 1, 2, 3$). To be explicit,

$$\begin{aligned} &\theta_{13} = 0 , && \theta_{23} = \frac{\pi}{4} , \quad \theta_{12} = \arctan\left[\frac{1}{\sqrt{2}}\sqrt{\frac{\left(4a'_{\rm r} - b'_{\rm r} + \Delta\right)^2 + 144e'^2_{\rm r}}{\left(4a'_{\rm r} - b'_{\rm r} + \Delta\right)^2 + 36e'^2_{\rm r}}}\right] , \\ &\delta \in [0, 2\pi) , && \rho = -\varphi_2 \text{ or } -\varphi_2 + \pi , \quad \sigma = -\varphi_1 - \pi \text{ or } -\varphi_1 , \\ &\phi_e = \varphi_1 + \varphi_2 + \pi , \quad && \phi_\mu = \pi , \qquad \phi_\tau = 0 . \end{aligned} \tag{5.6}$$

where $\tan\varphi_1 = 6e'_{\rm r}/\left(4a'_{\rm r} - b'_{\rm r} + \Delta\right)$ and $\tan\varphi_2 = 12e'_{\rm r}/\left(4a'_{\rm r} - b'_{\rm r} + \Delta\right)$.

Given eq. (5.2), a straightforward calculation leads us to the conclusion $\epsilon_1 \neq 0$ in the basis where $M_{\rm R}$ is diagonal (i.e., $M_{\rm R} = \widehat{M}_N = \mathrm{Diag}\{d_{\rm r} - e_{\rm r} - \Delta', d_{\rm r} - e_{\rm r} + \Delta', a_{\rm r} + b_{\rm r} + e_{\rm r}\}$ with $\Delta' = \sqrt{a_{\rm r}^2 - a_{\rm r}b_{\rm r} + b_{\rm r}^2 - 2a_{\rm r}d_{\rm r} + b_{\rm r}d_{\rm r} + d_{\rm r}^2 + a_{\rm r}e_{\rm r} - 2b_{\rm r}e_{\rm r} - d_{\rm r}e_{\rm r} + e_{\rm r}^2}$). It is therefore possible to realize both unflavored and flavored leptogenesis in this case.

Finally, let us make another remark. Although the S_3 reflection symmetry helps a lot in determining the flavor structures in the seesaw mechanism, it must be broken so as to make the relevant phenomenological results fit current experimental data to a good or acceptable degree of accuracy. A further work along this line of thought will be done somewhere else. All in all, we expect that our structural classification and discussions in the S_3 reflection symmetry limit will be useful for phenomenological studies of neutrino mass

[5] The normal hierarchy case will not be discussed here, because it yields $\theta_{12} = \pi/2$ and thus disfavored.

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generation, lepton flavor mixing, CP violation and leptogenesis when specific symmetry breaking effects and more accurate experimental data are taken into account. The same idea and similar analyses can be extended and applied to some other seesaw mechanisms.

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