JHEP

Published for SISSA by Springer

Received: February 2, 2018
Accepted: February 26, 2018
Published: March 21, 2018

Deformation of $\mathcal{N} = 4$ SYM with varying couplings via fluxes and intersecting branes

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Abstract: We study deformations of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with space-time dependent couplings by embedding probe D3-branes in supergravity backgrounds with non-trivial fluxes. The effective action on the world-volume of the D3-branes is analyzed and a map between the deformation parameters and the fluxes is obtained. As an explicit example, we consider D3-branes in a background corresponding to (p, q) 5-branes intersecting them and show that the effective theory on the D3-branes precisely agrees with the supersymmetric Janus configuration found by Gaiotto and Witten in [1]. D3-branes in an intersecting D3-brane background is also analyzed and the D3-brane effective action reproduces one of the supersymmetric configurations with $\mathrm{ISO}(1,1) \times \mathrm{SO}(2) \times \mathrm{SO}(4)$ symmetry found in our previous paper [2].

Keywords: Brane Dynamics in Gauge Theories, Gauge-gravity correspondence

ArXiv ePrint: 1801.09394

https://doi.org/10.1007/JHEP03(2018)128

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1 Introduction

As it is well-known, the effective theory on D3-branes in flat space-time becomes $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory in the field theory limit ($\alpha' \to 0$). If the background has non-trivial fluxes, the effective theory on the D3-branes will be deformed accordingly. This is one of the useful ways to obtain 4 dimensional gauge theories with less (or no) supersymmetry (SUSY). In fact, various deformations realized in this way have been investigated, for instance, in [3–6] in the context of flux compactifications. In these works, because the main motivation was to obtain a model beyond the Standard Model, the deformations were assumed to preserve 4 dimensional Poincaré symmetry ISO$(1, 3)$. One of the main purposes of this paper is to generalize the deformations to the cases where ISO$(1, 3)$ is explicitly broken. In particular, the couplings[1] in the action may depend on the space-time coordinates.

[1] In this paper, all the parameters, such as gauge couplings, Yukawa couplings, theta parameter, masses, etc., in the action are called "couplings".

In our recent paper [2], we wrote down the conditions to preserve part of the supersymmetry in deformed $\mathcal{N} = 4$ SYM with varying couplings and found various non-trivial solutions.[2] Though the motivation was in string theory, the analyses in [2] were purely field theoretical. In this paper, we try to realize such systems in string theory by putting probe D3-branes in supergravity backgrounds with fluxes and find a map between the couplings in the action of the deformed $\mathcal{N} = 4$ SYM and the fluxes in the background.

One way to obtain a theory with varying couplings is to consider a background corresponding to D-branes (or other branes) intersecting with the probe D3-branes. A typical example is a system with D3-branes embedded in a background with $[p, q]$ 7-branes that appear as codimension 2 defects in the D3-brane world-volume [10–12].[3] In this system, it is known that the complex coupling (2.9) is not a constant but depends holomorphically on a complex coordinate which is a complex combination of 2 spatial coordinates transverse to the 7-branes. Our results for the D3-brane effective action can be applied to this system as well as various other intersecting brane systems. We demonstrate it in two explicit examples; intersecting D3-(p, q)5-brane and D3-D3 systems. In the former example, we show that the effective action on the probe D3-branes precisely reproduces the action for the supersymmetric Janus configuration found in [1]. The latter example reproduces one of the supersymmetric solutions with $\mathrm{ISO}(1, 1) \times \mathrm{SO}(2) \times \mathrm{SO}(4)$ symmetry obtained in [2].

The contents of the paper is as follows. In section 2, we review the deformations of $\mathcal{N} = 4$ SYM with varying couplings that were studied in [2]. In particular, we summarize the SUSY conditions and a few explicit solutions that will be used in section 4. In section 3, we study the effective action of a stack of D3-branes in curved backgrounds with fluxes. Upon the leading order expansion with respect to α', we establish a map between the background fields and the deformation parameters of the theory in section 2. In section 4, we apply the results of the previous section to study two cases: backgrounds with (p, q) 5-branes and those with D3-branes. These two examples correspond to the realization of the deformed $\mathcal{N} = 4$ SYM studied in [1] and [2] in string theory. In section 5, we conclude the paper with various discussions on the present work and further applications. In addition, two appendices are included. In appendix A, we summarize our conventions used for the supergravity fields. In appendix B, we show the explicit calculations to obtain the effective action of D3-branes given in section 3.

2 Deformations of $\mathcal{N} = 4$ SYM with varying couplings

In this section, we review some of the results obtained in [2]. Following [2], we use a 10-dimensional notation, in which $\mathcal{N} = 4$ SYM is regarded as a dimensional reduction of 10-dimensional $\mathcal{N} = 1$ SYM. The 10-dimensional gauge field A_I $(I = 0, \ldots, 9)$ is reduced to a 4-dimensional gauge field A_μ $(\mu = 0, \ldots, 3)$ and 6 scalar fields A_A $(A = 4, \ldots, 9)$, and the 10-dimensional Majorana-Weyl spinor field Ψ describes 4 Weyl fermion fields in

[2] See, e.g., [1, 7–9] for closely related works.
[3] See also [2, 13–15] for recent related works.

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4-dimensions. Ψ is a 32-component Majorana spinor satisfying the Weyl condition

$$\Gamma^{(10)}\Psi = +\Psi\,, \tag{2.1}$$

where $\Gamma^{(10)} \equiv \Gamma^{\hat{0}} \dots \Gamma^{\hat{9}}$ is the 10-dimensional chirality operator.[4] The gamma matrices $\Gamma^{\hat{I}}$ ($\hat{I} = 0, \dots, 9$) are 10-dimensional gamma matrices which are realized as 32×32 real matrices satisfying $\{\Gamma^{\hat{I}}, \Gamma^{\hat{J}}\} = 2\eta^{\hat{I}\hat{J}}$, where $\eta^{\hat{I}\hat{J}} = \mathrm{diag}(-1, +1, \dots, +1)$ is the 10-dimensional Minkowski metric.

Let us consider the following deformation of the $\mathcal{N} = 4$ SU(N) supersymmetric Yang-Mills theory:

$$\begin{aligned} S = \int d^4x\sqrt{-g}\, a\,\mathrm{tr}\Big\{ &-\frac{1}{2}\, g^{II'} g^{JJ'} F_{IJ} F_{I'J'} + i\,\overline{\Psi}\Gamma^I D_I \Psi + \frac{c}{4}\,\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \\ &- d^{IJA} F_{IJ} A_A - \frac{m^{AB}}{2} A_A A_B - i\overline{\Psi} M \Psi \Big\}\,, \end{aligned} \tag{2.2}$$

where $I, J = 0, \dots, 9$; $\mu, \nu = 0, \dots, 3$; $A, B = 4, \dots, 9$. F_{IJ} is defined as

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]\,, \tag{2.3}$$

$$F_{\mu A} = -F_{A\mu} \equiv \partial_\mu A_A + i[A_\mu, A_A] \equiv D_\mu A_A\,, \tag{2.4}$$

$$F_{AB} \equiv i[A_A, A_B]\ . \tag{2.5}$$

The covariant derivatives on the fermion field Ψ are defined as

$$D_\mu \Psi \equiv\ \partial_\mu \Psi + i[A_\mu, \Psi] + \frac{1}{4}\omega_\mu{}^{\hat{\nu}\hat{\rho}} \Gamma_{\hat{\nu}\hat{\rho}} \Psi\,, \qquad D_A \Psi \equiv\ i[A_A, \Psi]\,, \tag{2.6}$$

where the indices $\hat{\mu}, \hat{\nu} = 0, \dots, 3$ are flat indices, $\Gamma_{\hat{\nu}\hat{\rho}} \equiv \frac{1}{2}(\Gamma_{\hat{\nu}}\Gamma_{\hat{\rho}} - \Gamma_{\hat{\rho}}\Gamma_{\hat{\nu}})$ and $\omega_\mu{}^{\hat{\nu}\hat{\rho}}$ is the spin connection. We assume that the metric g_{IJ} has the form

$$ds^2 = g_{IJ} dx^I dx^J = g_{\mu\nu}(x^\rho) dx^\mu dx^\nu + \delta_{AB} dx^A dx^B\,, \tag{2.7}$$

and g^{IJ} denotes its inverse. The 4-dimensional Levi-Civita symbol $\epsilon^{\mu\nu\rho\sigma}$ is defined such that $\epsilon^{0123} = 1/\sqrt{-g}$, where $\sqrt{-g} \equiv \sqrt{-\det(g_{\mu\nu})}$. We also introduce a vielbein $e_I^{\hat{I}}$ satisfying $e_I^{\hat{I}} e_J^{\hat{J}} \eta_{\hat{I}\hat{J}} = g_{IJ}$ and its inverse $e_{\hat{I}}^I$. The gamma matrices with the curved indices are defined by $\Gamma^I = e_{\hat{I}}^I \Gamma^{\hat{I}}$.

The quantities a, c, d^{IJA}, m^{AB} are real parameters and M is a 32×32 real anti-symmetric matrix. All of them may depend on the space-time coordinates x^μ. In this paper, we call these parameters as "couplings", though m^{AB} and M are related to masses. The couplings a and c are related to the gauge coupling g_{YM} and the theta parameter θ as follows:

$$a = \frac{1}{g_{\mathrm{YM}}^2}\,, \qquad c = \frac{g_{\mathrm{YM}}^2 \theta}{8\pi^2}\ . \tag{2.8}$$

[4] In [2], the fermions are chosen to have negative chirality (minus sign in the right hand side of (2.1)). Here, we choose the chirality to be positive, in order to match the convention used in [16]. One way to relate our convention here and that in [2] is to use a transformation $x^9 \to -x^9$, which induces $\Psi^{\mathrm{here}} = \Gamma^{\hat{9}} \Psi^{\mathrm{there}}$, $A_9^{\mathrm{here}} = -A_9^{\mathrm{there}}$, $A_{I'}^{\mathrm{here}} = A_{I'}^{\mathrm{there}}$ for $I' \neq 9$, and similar sign changes for the parameters d^{IJA}, m^{AB} and m_{IJK}.

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It is useful to define the complex coupling τ in terms of these quantities:

$$\tau \equiv \frac{\theta}{2\pi} + i\frac{4\pi}{g_{\rm YM}^2} = 4\pi a(c+i)\ . \tag{2.9}$$

The parameters d^{IJA} and m^{AB} exhibit the following symmetries

$$d^{IJA} = -d^{JIA}\,, \qquad d^{\mu AB} = -d^{\mu BA}\,, \qquad d^{ABC} = d^{[ABC]}\,, \qquad m^{AB} = m^{BA}\,, \tag{2.10}$$

whereas the most general form for M is given by

$$M = m_{IJK}\Gamma^{IJK}\,, \tag{2.11}$$

where m_{IJK} is a real rank-3 anti-symmetric tensor and

$$\begin{aligned}\Gamma^{IJK} &\equiv \Gamma^{[I}\Gamma^{J}\Gamma^{K]} \\ &\equiv \frac{1}{3!}(\Gamma^I\Gamma^J\Gamma^K + \Gamma^J\Gamma^K\Gamma^I + \Gamma^K\Gamma^I\Gamma^J - \Gamma^J\Gamma^I\Gamma^K - \Gamma^I\Gamma^K\Gamma^J - \Gamma^K\Gamma^J\Gamma^I)\ .\end{aligned} \tag{2.12}$$

The ansatz for the SUSY transformation is

$$\delta_\epsilon A_I = i\bar\epsilon\,\Gamma_I\Psi\,, \quad \delta_\epsilon\overline{\Psi} = \frac{1}{2}\bar\epsilon\,(-F_{IJ}\Gamma^{IJ} + A_A\overline{B}^A)\,, \tag{2.13}$$

where ϵ is the SUSY parameter represented as a 10-dimensional Majorana-Weyl spinor and $\overline{B}^A$ is a 32×32 real matrix. Both ϵ and $\overline{B}^A$ may depend on space-time.

Then, the invariance of the action (2.2) under this SUSY transformation implies the following equations:

$$0 = \bar\epsilon\, e^{I'J'K'}\Gamma_{K'}\left(\frac{1}{72}\Gamma_{I'J'}\Gamma^{IJ} + \frac{1}{4}\Gamma_{[I'}\Gamma^{[I}\delta^{J]}_{J']} - \delta^I_{I'}\delta^J_{J'}\right)\,, \tag{2.14}$$

$$0 = \bar\epsilon\left(\frac{1}{72}e^{IJK}\Gamma_{IJK} - \frac{1}{2}\Gamma^\mu\partial_\mu\log a - \left(\frac{1}{16}e_{\mu JK} - 3m_{\mu JK}\right)\Gamma^{\mu JK} - M\right), \tag{2.15}$$

$$\bar\epsilon\,\overline{B}^A = \bar\epsilon\left(F\Gamma^A + \left(-\frac{1}{4}e^{AJK} + 12m^{AJK}\right)\Gamma_{JK}\right)\,, \tag{2.16}$$

$$\partial_\mu\bar\epsilon = -\frac{1}{4}\bar\epsilon\left(F\Gamma_\mu + \left(-\frac{1}{4}e_{\mu\hat J\hat K} + 12m_{\mu\hat J\hat K} - \omega_{\mu\hat J\hat K}\right)\Gamma^{\hat J\hat K}\right)\,, \tag{2.17}$$

$$D_\mu(\bar\epsilon\,\overline{B}^A)\Gamma^\mu = \bar\epsilon\left(-2a^{-1}\Gamma_I D_\mu(a\,d^{I\mu A}) - m^{AB}\Gamma_B - \overline{B}^A\left(M + \frac{1}{2}\Gamma^\mu\partial_\mu\log a\right)\right)\,, \tag{2.18}$$

where F is a real 32×32 matrix acting on the spinor indices and

$$e^{IJK} \equiv a^{-1}\partial_\nu(ac)\,\epsilon^{\nu IJK} + 3\,d^{[IJK]} + 24\,m^{IJK}\ . \tag{2.19}$$

The condition (2.14) has a trivial solution, $e^{IJK} = 0$, which is equivalent to

$$0 = a^{-1}\partial_\mu(ac)\,\epsilon^{\mu\nu\rho\sigma} + 24\,m^{\nu\rho\sigma}\,, \tag{2.20}$$

$$0 = d^{[IJA]} + 8\,m^{IJA}\ . \tag{2.21}$$

Using the symmetries of the deformation parameters (2.10), the latter is written as

$$d^{\mu\nu A} = -24\,m^{\mu\nu A}\,, \qquad d^{\mu AB} = -12\,m^{\mu AB}\,, \qquad d^{ABC} = -8\,m^{ABC}\ . \tag{2.22}$$

Further discussions on the nature of these equations and their solutions, we refer to [2]. Let us summarize a few explicit solutions that are relevant for our discussion.

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1. $\mathrm{ISO}(1,2) \times \mathrm{SO}(3) \times \mathrm{SO}(3)$

The case with $\mathrm{ISO}(1,2) \times \mathrm{SO}(3) \times \mathrm{SO}(3)$ symmetry is analyzed by Gaiotto and Witten in [1]. (See also section 3.4 in [2]) It is a solution of the SUSY conditions (2.14)–(2.18) with the parameters depending only on x^3. $\mathrm{ISO}(1,2)$ is the Poincaré group acting on $x^{0,1,2}$ and $\mathrm{SO}(3) \times \mathrm{SO}(3)$ acts on $x^{4,5,6}$ and $x^{7,8,9}$. The metric is assumed to be flat and the non-trivial components of the couplings in the action are given as follows:[5]

$$\tau = 4\pi a(c+i) = \tau_0 + 4\pi D\, e^{i2\psi}\,, \tag{2.23}$$

$$m^{ab} = 2\left(\psi'^2 - (\psi' \cot\psi)'\right)\delta^{ab}\,, \qquad (a,b = 4,5,6)$$

$$m^{pq} = 2\left(\psi'^2 + (\psi' \tan\psi)'\right)\delta^{pq}\,, \qquad (p,q = 7,8,9) \tag{2.24}$$

$$d^{456} = \frac{2}{3}\frac{\psi'}{\sin\psi}\,, \qquad d^{789} = \frac{2}{3}\frac{\psi'}{\cos\psi}\,, \tag{2.25}$$

$$M = \frac{\psi'}{2}\Gamma^{012} - \frac{\psi'}{2\sin\psi}\Gamma^{456} - \frac{\psi'}{2\cos\psi}\Gamma^{789}\,, \tag{2.26}$$

where τ_0 and D are real constants and ψ is an arbitrary real function of x^3 with $0 < \psi < \pi/2$ assuming $D > 0$.

2. $\mathrm{ISO}(1,1) \times \mathrm{SO}(2) \times \mathrm{SO}(4)$

The case with $\mathrm{ISO}(1,1) \times \mathrm{SO}(2) \times \mathrm{SO}(4)$ symmetry is given in section 4.1 in [2]. Here, $\mathrm{ISO}(1,1)$, $\mathrm{SO}(2)$ and $\mathrm{SO}(4)$ act on $x^{0,1}$, $x^{4,5}$ and $x^{6,7,8,9}$, respectively, and the couplings in the action may depend on $x^{2,3}$. The metric (2.7) is assumed to be

$$ds^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta + e^\varphi \delta_{mn}dx^m dx^n + \delta_{ab}dx^a dx^b + \delta_{pq}dx^p dx^q\,, \tag{2.27}$$

where the indices are $\alpha,\beta = 0,1$; $m,n = 2,3$; $a,b = 4,5$ and $p,q = 6,7,8,9$. In this case, the complex coupling (2.9) turns out to be an arbitrary holomorphic (or anti-holomorphic) function of a complex coordinate $z \equiv \frac{1}{\sqrt{2}}(x^2 + ix^3)$ with $\mathrm{Im}\,\tau > 0$ and φ in the metric (2.27) is an arbitrary real function of $x^{2,3}$. M is of the form:

$$M = \alpha_m \Gamma^{01m} + \beta_m \Gamma^{m45}\,, \tag{2.28}$$

and α_m and β_m are determined by τ and φ as

$$\alpha_m = \frac{1}{4}\partial_m \log \mathrm{Im}\,\tau = \frac{1}{4}(\mathrm{Im}\,\tau)^{-1}\partial_n(\mathrm{Re}\,\tau)\,\epsilon^n{}_m\,, \tag{2.29}$$

$$\beta_m = \frac{s}{4}\epsilon^n{}_m\partial_n\left(\varphi - \log \mathrm{Im}\,\tau\right) + \partial_m\Lambda\,, \tag{2.30}$$

[5] Our convention is slightly different from that in [1]. The solution shown here is taken from section 3.4 in [2] with $b_0 = 2$ and $l(z) = -iz/\sqrt{2}$. We also made a transformation $x^9 \to -x^9$. (See the footnote in p.3.)

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where $s = \pm$, $\epsilon^{nm} = \epsilon^{n}{}_{m'} g^{m'm}$ is the Levi-Civita symbol for the $x^{2,3}$-plane and Λ is an arbitrary real function.[6] The non-trivial components of d^{IJA} and m^{AB} are

$$d^{nab} = -d^{anb} = -2\beta^n \epsilon^{ab} , \tag{2.31}$$

$$m^{ab} = \left(-\frac{1}{2} g^{mn} q_m q_n - g^{mn} \partial_m q_n + 8 g^{mn} \beta_m \beta_n - 4s \partial_m \beta_n \epsilon^{mn} \right) \delta^{ab} , \tag{2.32}$$

$$m^{pq} = \left(-\frac{1}{2} g^{mn} q_m q_n - g^{mn} \partial_m q_n \right) \delta^{pq} , \tag{2.33}$$

where

$$q_m \equiv \partial_m \log \mathrm{Im}\, \tau = \partial_m \log a \ . \tag{2.34}$$

3. ISO(1, 1) × SO(6)

When $\beta_m = 0$ in the previous example, the symmetry is enhanced to ISO(1, 1) × SO(6). In this case, $(\varphi - \log \mathrm{Im}\, \tau)$ is a harmonic function on $x^{2,3}$-plane satisfying

$$g^{mn} \partial_m \partial_n \left(\varphi - \log \mathrm{Im}\, \tau \right) = 0 \ . \tag{2.35}$$

This is the case studied in [10–12]. (See also section 3.3 in [2]) It is related to the effective theory on the D3-branes embedded in a 7-brane background as mentioned in the introduction.

3 D3-branes in curved backgrounds with fluxes

In this section, we study the effective action of D3-branes in curved backgrounds with fluxes and try to relate the couplings in the action (2.2) with the supergravity fields. As reviewed in appendix B.1, the effective action of Dp-branes in general backgrounds is known, at least, to the extent needed for our purpose. (See (B.1) and (B.13) for the bosonic and fermionic parts, respectively.) However, the expression of the effective action reviewed in appendix B.1 is not convenient for a direct comparison with the action (2.2) used in the field theoretical analysis.

To find a relations between couplings in (2.2) and the fluxes in the supergravity background, we expand the D3-brane effective action with respect to $\alpha' = l_s^2$ and keep only the terms that survive in the $\alpha' \to 0$ limit, assuming that the background fields are of $\mathcal{O}(\alpha'^0)$.

We consider N D3-branes embedded in a 10 dimensional space-time parametrized by (x^μ, x^i) with $\mu = 0, 1, 2, 3$ and $i = 4, \ldots, 9$. We use the static gauge, in which the world-volume of the D3-branes is parametrized by x^μ $(\mu = 0, 1, 2, 3)$. The scalar fields, which are related to A_A $(A = 4, \ldots, 9)$ in the previous section, are denoted here as Φ^i $(i = 4, \ldots, 9)$. The scalar field Φ^i describes the position of the D3-branes in the x^i direction. Assuming that the D3-branes are placed at $x^i = 0$ when $\Phi^i = 0$, the relation between the position of D3-branes and the value of scalar fields is given by $\Phi^i = \lambda x^i$ with $\lambda \equiv 2\pi\alpha' = 2\pi l_s^2$. (See (B.9) for the precise meaning of this identification for $N > 1$.)

[6] Λ can be absorbed by a local SO(2) rotation of the $x^{4,5}$-plane. See appendix C.2 in [2].

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To simplify the analysis, we assume that (μ, i) components of the metric $g_{\mu i}$ vanish everywhere, and all the components of the Kalb-Ramond 2-form fields and all the R-R fields, except the R-R 0-form C_0, vanish at $x^i = 0$ $(i = 4, \ldots, 9)$:[7]

$$g_{\mu i} = 0\,, \quad B_2\big|_{x^i=0} = 0\,, \quad C_n\big|_{x^i=0} = 0\,, \quad (n \neq 0)\ . \tag{3.1}$$

Note that unlike in (2.7), the (i, j) component of the metric g_{ij} may have non-trivial x^μ dependence.

Here, we simply state our results on the D3-brane effective action and leave the details to appendix B. Neglecting the $\mathcal{O}(\alpha')$ terms in the action (B.1) and (B.13), we obtain:

$$\begin{aligned} S^{\rm boson}_{\rm D3} = \frac{T_3\lambda^2}{2}\int d^4x\sqrt{-g}\,{\rm tr}\Big\{ & -\frac{e^{-\phi}}{2}g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} \pm \frac{C_0}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \\ & - e^{-\phi}g^{\mu\nu}g_{ij}D_\mu\Phi^i D_\nu\Phi^j + \frac{e^{-\phi}}{2}g_{ii'}g_{jj'}[\Phi^i,\Phi^j][\Phi^{i'},\Phi^{j'}] - 2V(\Phi^i) \\ & \pm (G^R_\pm)_i{}^{\mu\nu}\Phi^i F_{\mu\nu} \mp \frac{i}{3}(G^R_\pm)_{ijk}\Phi^i[\Phi^j,\Phi^k] \mp (*_4\widetilde{F}_5)^\mu{}_{ij}\Phi^i D_\mu\Phi^j\Big\}\,, \end{aligned} \tag{3.2}$$

$$S^{\rm fermi}_{\rm D3} = \frac{T_3\lambda^2}{2}\int d^4x\,\sqrt{-g}\,e^{-\phi}\,{\rm tr}\left\{i\left(\overline{\Psi}\Gamma^\mu D_\mu\Psi + \overline{\Psi}\Gamma_k i[\Phi^k,\Psi]\right) - i\overline{\Psi}\left(M_\pm - \frac{1}{4}\omega_{\mu\hat{i}\hat{j}}\Gamma^{\mu\hat{i}\hat{j}}\right)\Psi\right\}, \tag{3.3}$$

where the upper (lower) signs correspond to the case of D3- ($\overline{\rm D3}$-) branes, D_μ denotes the 4 dimensional covariant derivative defined in (2.6) and (B.10), and $\omega_{\mu\hat{i}\hat{j}}$ is the $(\mu, \hat{i}, \hat{j})$ component of the spin connection[8] related to the vielbein $e^i_{\hat{j}}$ as

$$\omega_{\mu\hat{i}\hat{j}} = \frac{1}{2}(e^k_{\hat{i}}\partial_\mu e_{k\hat{j}} - e^k_{\hat{j}}\partial_\mu e_{k\hat{i}})\ . \tag{3.4}$$

The fluxes $(G^R_\pm)_i{}^{\mu\nu}$, $(G^R_\pm)_{ijk}$ and $(*_4\widetilde{F}_5)^\mu{}_{ij}$ in (3.2) are defined in (B.59), (B.60) and (B.54). See also appendix A for our conventions for the supergravity fields.

The quantity $M_\pm$ in (3.3) is given by

$$M_\pm \equiv \mp\frac{e^\phi}{8}\left(\frac{1}{3}(*_4F_1)_{\mu\nu\rho}\Gamma^{\mu\nu\rho} - (G^R_\pm)_{i\mu\nu}\Gamma^{i\mu\nu} + \frac{1}{3}(G^R_\pm)_{ijk}\Gamma^{ijk} - (*_4\widetilde{F}_5)_{\mu ij}\Gamma^{\mu ij}\right)\,, \tag{3.5}$$

where $(*_4F_1)_{\nu\rho\sigma}$ is defined in (B.74).

The potential $V(\Phi^i)$ in (3.2) has two contributions:

$$V(\Phi^i) \equiv \lambda^{-2}(V_{\rm DBI}(\Phi^i) \pm V_{\rm CS}(\Phi^i))\,, \tag{3.6}$$

where

$$V_{\rm DBI}(\Phi^i) \equiv e^{-\phi}\left(1 + \lambda v_i^{\rm DBI}\Phi^i + \frac{\lambda^2}{2}m_{ij}^{\rm DBI}\Phi^i\Phi^j\right)\,, \tag{3.7}$$

$$V_{\rm CS}(\Phi^i) \equiv \lambda v_i^{\rm CS}\Phi^i + \frac{\lambda^2}{2}m_{ij}^{\rm CS}\Phi^i\Phi^j\,, \tag{3.8}$$

[7] It is generically possible to choose a gauge such that $B_2|_{x^i=0} = 0$ and $C_n|_{x^i=0} = 0$ $(n \neq 0)$ (at least locally) provided the components $H_{\mu\nu\rho}$ and $F_{\mu\nu\rho}$ vanish at $x^i = 0$. Obviously, the reason for considering a non-vanishing C_0 is that we want to capture the theta parameter θ in the SYM action (2.2).

[8] The hatted indices are the flat indices as in the previous section. We assume $e^i_{\hat{\mu}} = 0$ and $e^\mu_{\hat{i}} = 0$ without loss of generality under the assumption (3.1).

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with

$$v_i^{\rm DBI} \equiv -\partial_i\phi + \frac{1}{2}g^{\mu\nu}\partial_i g_{\mu\nu} = \partial_i \log\left(\sqrt{-g}\, e^{-\phi}\right) , \tag{3.9}$$

$$\begin{aligned} m_{ij}^{\rm DBI} &\equiv v_i^{\rm DBI} v_j^{\rm DBI} - \partial_i\partial_j\phi + \frac{1}{2}\left(g^{\mu\nu}\partial_i\partial_j g_{\nu\mu} - g^{\mu\mu'}\partial_i g_{\mu'\nu'}g^{\nu'\nu}\partial_j g_{\nu\mu} - g^{\mu\mu'}g^{\nu\nu'}H_{i\mu'\nu'}H_{j\nu\mu}\right) \\ &= \frac{1}{\sqrt{-g}\, e^{-\phi}}\partial_i\partial_j\left(\sqrt{-g}\, e^{-\phi}\right) + \frac{1}{2}H_i{}^{\mu\nu}H_{j\mu\nu} , \end{aligned} \tag{3.10}$$

$$v_i^{\rm CS} \equiv -(*_4\widetilde{F}_5)_i , \tag{3.11}$$

$$m_{ij}^{\rm CS} \equiv -\frac{1}{2}\left(\partial_i(*_4\widetilde{F}_5)_j + \frac{1}{2}(*_4\widetilde{F}_3)_j{}^{\mu\nu}H_{\mu\nu i} + (i \leftrightarrow j)\right) , \tag{3.12}$$

and $(*_4\widetilde{F}_5)_j$ and $(*_4\widetilde{F}_3)_j{}^{\mu\nu}$ are defined in (B.54).

All the supergravity fields and their derivatives in the action (3.2) and (3.3) are evaluated at $x^i = 0$. The first term in $V_{\rm DBI}$ (3.7) can be discarded in the comparison with the field theory results, because it doesn't depend on Φ^i. If we require $\Phi^i = 0$ and $A_\mu = 0$ to be a solution of the equations of motion, the linear term in (3.6) has to vanish:

$$0 = e^{-\phi}v_i^{\rm DBI} \pm v_i^{\rm CS} , \tag{3.13}$$

which is the condition that the force due to NS-NS and R-R fields cancel each other. In this case, we can safely take the $l_s \to 0$ limit.

Since the metric used in the action (2.2) is assumed to be of the form (2.7), we introduce a new metric

$$\bar{g}_{\mu\nu} \equiv e^{2\omega}g_{\mu\nu} , \quad \bar{g}_{AB} \equiv \delta_{AB} , \quad \bar{g}_{\mu A} \equiv 0 , \tag{3.14}$$

where $\mu, \nu = 0, \ldots, 3$; $A, B = 4, \ldots, 9$ and ω is a real function. Here, we put a factor $e^{2\omega}$ in the 4-dimensional metric, because it is often convenient to make a Weyl transformation to get a metric $\bar{g}_{IJ}$ that can be identified with g_{IJ} used in the previous section.[9] In addition, we redefine the scalar fields as

$$A^A \equiv e^{-\omega}e_i^A\Phi^i , \tag{3.15}$$

where e_i^A is the vielbein for the transverse space $e_i^{\hat{i}}$ with the identification $A = \hat{i} = 4, \ldots, 9$. so that the kinetic term can be written as in (2.2) with the metric $\bar{g}_{IJ}$ defined in (3.14).

Then, discarding the total derivative terms, the bosonic part of the D3 brane action (3.2) becomes

$$\begin{aligned} S_{\rm D3}^{\rm boson} = \frac{T_3\lambda^2}{2}\int d^4x\sqrt{-\bar{g}}\,{\rm tr}\Big\{ &-\frac{e^{-\phi}}{2}\bar{g}^{IJ}\bar{g}^{KL}F_{IK}F_{JL} \pm \frac{C_0}{4}\bar{\epsilon}^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \pm e^{-3\omega}(G_\pm^R)_A{}^{\mu\nu}A^A F_{\mu\nu} \\ &\mp \frac{i}{3}e^{-\omega}(G_\pm^R)_{ABC}A^A[A^B, A^C] + \left(\mp(*_4\widetilde{F}_5)_{\mu AB} + 2e^{-\phi}\omega_{\mu AB}\right)\bar{g}^{\mu\nu}A^A D_\nu A^B \\ &- \widehat{m}_{AB}A^A A^B - 2e^{-4\omega}V(\Phi^i)\Big\} , \end{aligned} \tag{3.16}$$

[9] See appendix C.1 in [2] for useful formulas for the Weyl transformation.

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where $\bar{\epsilon}^{\mu\nu\rho\sigma}$ is the Levi-Civita symbol with $\bar{\epsilon}^{0123} = 1/\sqrt{-\bar{g}}$, supergravity fields with indices A, B, C are defined as $(G^R_\pm)_{ABC} \equiv (G^R_\pm)_{ijk} e^i_A e^j_B e^k_C$, etc., and $\widehat{m}_{AB}$ is defined as

$$\widehat{m}_{AB} \equiv \frac{1}{2}\left(e^{-\phi}\bar{g}^{\mu\nu} E_{\mu A' A} E_\nu{}^{A'}{}_B - \frac{1}{\sqrt{-\bar{g}}}\partial_\nu\left(\sqrt{-\bar{g}}\, e^{-\phi}\bar{g}^{\mu\nu} E_{\mu AB}\right) \pm e^{-2\omega}(*_4\widetilde{F}_5)^\mu{}_{AA'} E_\mu{}^{A'}{}_B \right) + (A \leftrightarrow B) \tag{3.17}$$

with

$$E_{\mu AB} \equiv e^{-\omega} e_{iA}\partial_\mu(e^\omega e^i_B) = \frac{1}{2}\partial_\mu g^{ij} e_{iA} e_{jB} + \partial_\mu\omega\delta_{AB} + \omega_{\mu AB}\ . \tag{3.18}$$

The fermionic part (3.3) is rewritten as

$$S^{\rm fermi}_{\rm D3} = \frac{T_3\lambda^2}{2}\int d^4x\sqrt{-\bar{g}}\, e^{-\phi}\,\mathrm{tr}\left\{ i(\overline{\widehat{\Psi}}\widehat{\Gamma}^\mu D_\mu\widehat{\Psi} + \overline{\widehat{\Psi}}\widehat{\Gamma}_A i[A^A,\widehat{\Psi}]) - i\overline{\widehat{\Psi}}\left(\widehat{M}_\pm - \frac{1}{4}\omega_{\mu AB}\widehat{\Gamma}^{\mu AB}\right)\widehat{\Psi}\right\}, \tag{3.19}$$

where we have defined

$$\begin{aligned}\widehat{M}_\pm &\equiv e^{-\omega} M_\pm \\ &= \mp\frac{e^\phi}{8}\left(\frac{e^{2\omega}}{3}(*_4F_1)_{\mu\nu\rho}\widehat{\Gamma}^{\mu\nu\rho} - e^\omega (G^R_\pm)_{A\mu\nu}\widehat{\Gamma}^{A\mu\nu} + \frac{e^{-\omega}}{3}(G^R_\pm)_{ABC}\widehat{\Gamma}^{ABC} - (*_4\widetilde{F}_5)_{\mu AB}\widehat{\Gamma}^{\mu AB}\right),\end{aligned} \tag{3.20}$$

and

$$\widehat{\Gamma}^\mu \equiv e^{-\omega}\Gamma^\mu\,, \quad \widehat{\Gamma}^A \equiv e^A_i\Gamma^i\,, \quad \widehat{\Psi} \equiv e^{-\frac{3}{2}\omega}\Psi\ . \tag{3.21}$$

Here, $\widehat{\Gamma}^I$ $(I = 0, 1, \ldots, 9)$ are the gamma matrices satisfying

$$\{\widehat{\Gamma}^I, \widehat{\Gamma}^J\} = 2\bar{g}^{IJ}\ . \tag{3.22}$$

Now, we can readily find the correspondence between the couplings and the supergravity fields. By comparing the action (2.2) with (3.16) and (3.19), assuming (3.13), we obtain

$$a = \frac{T_3\lambda^2}{2}e^{-\phi}\,, \quad c = \pm e^\phi C_0\,, \qquad d^{\mu\nu A} = \mp e^{-3\omega+\phi}(G^R_\pm)^{A\mu\nu}\ , \tag{3.23}$$

$$d_{\mu AB} = \mp\frac{e^\phi}{2}(*_4\widetilde{F}_5)_{\mu AB} + \omega_{\mu AB}\,, \qquad d^{ABC} = \pm\frac{e^{-\omega+\phi}}{3}(G^R_\pm)_{ABC}\,, \tag{3.24}$$

$$m_{\mu\nu\rho} = \mp\frac{e^{2\omega+\phi}}{24}(*_4F_1)_{\mu\nu\rho}\,, \qquad m_{\mu\nu A} = \pm\frac{e^{\omega+\phi}}{24}(G^R_\pm)_{A\mu\nu}\,, \tag{3.25}$$

$$m_{\mu AB} = \pm\frac{e^\phi}{24}(*_4\widetilde{F}_5)_{\mu AB} - \frac{1}{12}\omega_{\mu AB}\,, \qquad m_{ABC} = \mp\frac{e^{-\omega+\phi}}{24}(G^R_\pm)_{ABC}\,, \tag{3.26}$$

and

$$m_{AB} = 2(\widehat{m}_{AB} + e^{-2\omega}m^{\rm DBI}_{AB} \pm e^{-2\omega+\phi}m^{\rm CS}_{AB})\ . \tag{3.27}$$

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Note that the first equation of (3.25) can be written as

$$m_{\mu\nu\rho} = \mp\frac{e^{2\omega+\phi}}{24}\epsilon^{\mu}{}_{\nu\rho\sigma}\partial_\mu C_0 = \mp\frac{e^{\phi}}{24}\bar{\epsilon}^{\mu}{}_{\nu\rho\sigma}\partial_\mu C_0 \ . \tag{3.28}$$

Then, the relations (3.23)–(3.27) imply (2.20) and (2.22), which is equivalent to the condition $e^{IJK} = 0$ that solves one of the SUSY condition (2.14) as discussed in the previous section.

In the following section, we are going to check these identifications by explicitly inserting some particular backgrounds in the effective action for the D3-branes and comparing with the supersymmetric deformations of the $\mathcal{N} = 4$ SYM reviewed in section 2.

4 Examples

4.1 (p, q) 5-branes and Gaiotto-Witten solution

In this subsection, we consider D3-branes embedded in a background with (p, q) 5-branes.[10] The brane configuration is summarized as

	0	1	2	3	4	5	6	7	8	9
(probe) D3	o	o	o	o						
(p, q) 5	o	o	o		o	o	o			

(4.1)

The effective action on the D3-brane world-volume can be written down by using (3.2) and (3.3). As we will soon see, because the (p, q) 5-branes are not extended along the x^3-direction, the gauge coupling and the theta parameter of the D3-brane action depend on the coordinate x^3. This brane configuration is related to the supersymmetric Janus configurations considered in [1]. We will show that the action obtained by using (3.2) and (3.3) is indeed consistent with that obtained in [1], which provides a consistency check of our results in section 3.

In this subsection, the letters for the indices are chosen as $\alpha, \beta = 0, 1, 2$; $a, b, c = 4, 5, 6$ and $p, q, r = 7, 8, 9$. Let us consider n (p, q) 5-branes placed at $x^3 = 0$, $x^p = x_0^p$ $(p = 7, 8, 9)$. The supergravity solution corresponding to the (p, q) 5-branes can be obtained by applying $\mathrm{SL}(2, \mathbb{Z})$ duality to the D5-brane solution. Its explicit form is[11]

$$ds_E^2 = h(r)^{-\frac{1}{4}}(\eta_{\alpha\beta}dx^\alpha dx^\beta + \delta_{ab}dx^a dx^b) + h(r)^{\frac{3}{4}}((dx^3)^2 + \delta_{pq}dx^p dx^q)\,, \tag{4.2}$$

$$e^{-\phi} = \frac{\rho}{p^2 g_s^{-1} h(r)^{1/2} + (q + p\chi_0)^2 g_s h(r)^{-1/2}}\,, \qquad C_0 = \frac{pq(1 - h(r)) + \rho\chi_0 g_s}{p^2 g_s^{-1} h(r) + (q + p\chi_0)^2 g_s}\,, \tag{4.3}$$

$$H_3 = 2np\, l_s^2 \epsilon_3\,, \qquad F_3 = 2nq\, g_s l_s^2 \epsilon_3\,, \tag{4.4}$$

where ds_E^2 denotes the line element in the Einstein frame, χ_0 is a constant,

$$h(r) \equiv 1 + \frac{n\sqrt{\rho g_s}\, l_s^2}{r^2}\,, \quad \rho \equiv p^2 g_s^{-1} + (q + p\chi_0)^2 g_s\,, \quad r^2 \equiv (x^3)^2 + \sum_{p=7,8,9}(x^p - x_0^p)^2\,. \tag{4.5}$$

[10] Here, p and q are relativley prime integers and a (p, q) 5-brane is a bound state of p NS5-brane and q D5-brane.

[11] See, e.g., [17–19].

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ϵ_3 in (4.4) is the volume form of the unit S^3 embedded in the $\mathbb{R}^4$ parametrized by $x^{3,7,8,9}$ with its center at the position of the (p,q) 5-brane. ϵ_3 can be written explicitly as

$$\epsilon_3 = \sin^2\theta \sin\phi_1 d\theta \wedge d\phi_1 \wedge d\phi_2 \,, \tag{4.6}$$

where (θ, ϕ_1, ϕ_2) are the coordinates on the unit S^3 with $0 \le \theta \le \pi$, $0 \le \phi_1 \le \pi$ and $0 \le \phi_2 \le 2\pi$, related to $x^{3,7,8,9}$ as

$$\begin{aligned} x^3 &= r\cos\theta \,, \\ x^7 - x_0^7 &= r\sin\theta\cos\phi_1 \,, \\ x^8 - x_0^8 &= r\sin\theta\sin\phi_1\cos\phi_2 \,, \\ x^9 - x_0^9 &= r\sin\theta\sin\phi_1\sin\phi_2 \,. \end{aligned} \tag{4.7}$$

Note that H_3 and F_3 in (4.4) can be written as

$$H_{p'q'r'} = \frac{p}{\sqrt{\rho g_s}} \varepsilon_{p'q'r'}{}^{s'} \partial_{s'} h(r) \,, \quad F_{p'q'r'} = q\sqrt{\frac{g_s}{\rho}} \varepsilon_{p'q'r'}{}^{s'} \partial_{s'} h(r) \,, \tag{4.8}$$

where $p', q', r', s' = 3, 7, 8, 9$ and $\varepsilon_{p'q'r'}{}^{s'} = \varepsilon_{p'q'r't'}\delta^{t's'}$ is the Levi-Civita symbol for the flat $\mathbb{R}^4$ parametrized by $x^{3,7,8,9}$ with $\varepsilon_{3789} = +1$.[12] In the the expressions (4.2), (4.3) and (4.8), the function $h(r)$ can be replaced with an arbitrary positive harmonic function on $\mathbb{R}^4$, which corresponds to a supergravity solution describing parallel multiple (p,q) 5-branes distributed in $\mathbb{R}^4$.

The dilaton and R-R 0-from combined into a complex scalar field $\tau \equiv g_s^{-1}(C_0 + ie^{-\phi})$ can be written as

$$\tau = \tau_0 + 4\pi D\, e^{i2\psi} \,, \tag{4.9}$$

where

$$\tau_0 \equiv -\frac{q}{p} + 4\pi D \,, \quad 4\pi D \equiv \frac{\rho}{2p(q + p\chi_0)g_s} \,, \tag{4.10}$$

are real constants and $\psi(r)$ is a real function satisfying

$$\tan\psi(r) = \frac{p\, h(r)^{1/2}}{(q + p\chi_0)g_s} \,. \tag{4.11}$$

Here, we have assumed p, q, χ_0 are all positive and $0 < \psi < \frac{\pi}{2}$. The complex scalar field (4.9) evaluated at $x^i = 0$ corresponds to the complex coupling (2.9). In fact, the expression in (4.9) agrees with the complex coupling obtained in [1]. (See (2.23).) Note here that $\psi|_{x^i=0}$ can be chosen to be a generic real function of x^3, because as mentioned above, $h(r)$ in (4.11) can be replaced with an arbitrary positive harmonic function on $\mathbb{R}^4$ transverse to the (p,q) 5-branes.

[12] One can easily recover the expressions of H_3 and F_3 in (4.4) from (4.8) by using the polar coordinates (4.7) with metric of $\mathbb{R}^4$: $ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi_1^2 + \sin^2\theta\sin^2\phi_1 d\phi_2^2)$ and $\varepsilon_{r\theta\phi_1\phi_2} = r^3\sin^2\theta\sin\phi_1$.

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The metric in the string frame is given by

$$ds^2_{\rm string} = e^{\frac{1}{2}\phi} ds^2_E \tag{4.12}$$
$$= \frac{p}{\sqrt{\rho g_s}\sin\psi}\left[(\eta_{\alpha\beta}dx^\alpha dx^\beta + \delta_{ab}dx^a dx^b) + \left(\frac{\rho}{8\pi Dp^2}\tan\psi\right)^2\left((dx^3)^2 + \delta_{pq}dx^p dx^q\right)\right].$$

It is easy to show that both $V_{\rm DBI}$ and $V_{\rm CS}$ in the potential (3.6) are flat (i.e. Φ^i independent), because $F_5 = 0$, $H_{i\mu\nu} = F_{i\mu\nu} = 0$, $B_{\mu\nu} = 0$ and $L_{\rm DBI}$ defined in (B.31) is a constant:

$$L_{\rm DBI} = e^{-\phi}\sqrt{-\det(g_{\mu\nu})} = 1\ . \tag{4.13}$$

Next, consider N probe D3-branes placed at $x^i = 0$ $(i = 4, \ldots, 9)$ in this background. The metric (4.12) evaluated at $x^i = 0$ is written as

$$ds^2_{\rm string}\big|_{x^i=0} = e^{\xi}\left[\eta_{\mu\nu}dy^\mu dy^\nu + \delta_{ab}dx^a dx^b + e^{2\eta}\delta_{pq}dx^p dx^q\right]\ , \tag{4.14}$$

where we have defined

$$e^{\xi} \equiv \left.\frac{p}{\sqrt{\rho g_s}\sin\psi(r)}\right|_{x^i=0}\ , \quad e^{\eta} \equiv \left. h(r)^{1/2}\right|_{x^i=0} = \left.\frac{\rho}{8\pi Dp^2}\tan\psi(r)\right|_{x^i=0}\ , \tag{4.15}$$

and introduced new coordinates y^μ $(\mu = 0, 1, 2, 3)$ satisfying

$$y^0 = x^0\ , \quad y^1 = x^1\ , \quad y^2 = x^2\ , \quad dy^3 = e^{\eta}dx^3\ . \tag{4.16}$$

Then, using the coordinates y^μ, the bosonic part of the effective action (3.2) becomes

$$\begin{aligned} S^{\rm boson}_{\rm D3} = \frac{T_3\lambda^2}{2}\int d^4y\, e^{-\phi}\,{\rm tr}\Big(&-\frac{1}{2}\eta^{\mu\nu}\eta^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} \pm \frac{e^{\phi}C_0}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \\ &+ e^{2\xi}\eta^{\mu\nu}\delta_{ab}D_\mu\Phi^a D_\nu\Phi^b + e^{2(\xi+\eta)}\eta^{\mu\nu}\delta_{pq}D_\mu\Phi^p D_\nu\Phi^q \\ &+ \frac{1}{2}e^{4\xi}\delta_{ab}\delta_{a'b'}[\Phi^a,\Phi^b][\Phi^{a'},\Phi^{b'}] + \frac{1}{2}e^{4(\xi+\eta)}\delta_{pq}\delta_{p'q'}[\Phi^p,\Phi^q][\Phi^{p'},\Phi^{q'}] \\ &+ e^{4\xi+2\eta}\delta_{ab}\delta_{pq}[\Phi^a,\Phi^b][\Phi^p,\Phi^q] \mp \frac{i}{3}e^{2\xi+\phi}(G^R_\pm)_{ijk}\Phi^i[\Phi^j,\Phi^k]\Big)\ . \end{aligned} \tag{4.17}$$

In order to compare with the action (2.2), it is convenient to rescale the scalar fields as

$$A_a \equiv e^{\xi}\Phi^a\ , \quad A_p \equiv e^{\xi+\eta}\Phi^p\ . \tag{4.18}$$

Then, the kinetic term of the scalar fields can be rewritten as

$$\begin{aligned} &e^{-\phi}{\rm tr}\left(e^{2\xi}\eta^{\mu\nu}\delta_{ab}D_\mu\Phi^a D_\nu\Phi^b + e^{2(\xi+\eta)}\eta^{\mu\nu}\delta_{pq}D_\mu\Phi^p D_\nu\Phi^q\right) \\ &= e^{-\phi}{\rm tr}\left(\eta^{\mu\nu}\delta^{AB}D_\mu A_A D_\nu A_B + \frac{1}{2}m^{AB}A_A A_B\right) + (\text{total derivative})\,, \end{aligned} \tag{4.19}$$

where $A, B = 4, \ldots, 9$ and the non-zero components of m^{AB} are

$$\begin{aligned} m^{ab} &= 2\left(\xi'^2 + \xi'' - \phi'\xi'\right)\delta^{ab} = 2\left(\psi'^2 - (\psi'\cot\psi)'\right)\delta^{ab}\,, \\ m^{pq} &= 2\left((\xi'+\eta')^2 + \xi'' + \eta'' - \phi'(\xi'+\eta')\right)\delta^{pq} = 2\left(\psi'^2 + (\psi'\tan\psi)'\right)\delta^{pq}\ . \end{aligned} \tag{4.20}$$

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Here, the prime denotes the derivative with respect to y^3, e.g., $\xi' = \partial_{y^3}\xi$. These expressions precisely agree with (2.24)

Note that the non-zero components of $(G^R_\pm)_{ijk}$ are

$$(G^R_\pm)_{456} = -e^{-3\eta}(F_{789} + C_0 H_{789}) = 2\sqrt{\frac{g_s}{\rho}}(q + g_s^{-1}C_0 p)\eta' \,, \tag{4.21}$$

$$(G^R_\pm)_{789} = \mp e^{-\phi} H_{789} = \pm e^{-\phi}\frac{2p}{\sqrt{\rho g_s}} e^{3\eta}\eta' \,, \tag{4.22}$$

and one can show the following relations:

$$e^{\phi-\xi}(G^{\rm R}_\pm)_{456} = \frac{2\psi'}{\sin\psi} \,, \quad e^{\phi-\xi-3\eta}(G^{\rm R}_\pm)_{789} = \pm\frac{2\psi'}{\cos\psi} \,. \tag{4.23}$$

Using these, the last term of (4.17) can be written as

$$\begin{aligned} \mp\frac{i}{3}e^{2\xi+\phi}(G^{\rm R}_\pm)_{ijk}\Phi^i[\Phi^j,\Phi^k] &= \mp 2i\left(e^{\phi-\xi}(G^{\rm R}_\pm)_{456}A_4[A_5,A_6] + e^{\phi-\xi-3\eta}(G^{\rm R}_\pm)_{789}A_7[A_8,A_9]\right) \\ &= 4i\left(\mp\frac{\psi'}{\sin\psi}A_4[A_5,A_6] - \frac{\psi'}{\cos\psi}A_7[A_8,A_9]\right) \,. \end{aligned} \tag{4.24}$$

These terms (with the upper sign) agree with (2.25). (The lower sign is obtained, e.g., by a transformation $(x^1, x^4) \to (-x^1, -x^4)$.)

In summary, the bosonic part is written as

$$\begin{aligned} S^{\rm boson}_{\rm D3} = \frac{T_3\lambda^2}{2}\int d^4y\, e^{-\phi}\,{\rm tr}\Big(&-\frac{1}{2}\eta^{II'}\eta^{JJ'}F_{IJ}F_{I'J'} \pm \frac{e^\phi C_0}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \\ &+ 4i\left(\mp\frac{\psi'}{\sin\psi}A_4[A_5,A_6] - \frac{\psi'}{\cos\psi}A_7[A_8,A_9]\right) \\ &- (\psi'^2 - (\psi'\cot\psi)'))\delta^{ab}A_aA_b - (\psi'^2 + (\psi'\tan\psi)'))\delta^{pq}A_pA_q\Big) \end{aligned} \tag{4.25}$$

with ϕ and C_0 given by (4.9).

Let us next consider to the fermionic part. The action (3.3) in the background (4.2)–(4.4) is

$$S^{\rm fermi}_{\rm D3} = \frac{T_3\lambda^2}{2}\int d^4y\, e^{2\xi}\, e^{-\phi}\,{\rm tr}\left\{i(\overline{\Psi}\Gamma^\mu D_\mu\Psi + \overline{\Psi}\Gamma_k i[\Phi^k,\Psi]) - i\overline{\Psi}M_\pm\Psi\right\} \tag{4.26}$$

with

$$M_\pm \equiv \mp\frac{e^\phi}{4}\left((*_4F_1)_{012}\Gamma^{012} + (G^R_\pm)_{456}\Gamma^{456} + (G^R_\pm)_{789}\Gamma^{789}\right) \,. \tag{4.27}$$

Rescaling Ψ, $M_\pm$ and the gamma matrices as

$$\begin{aligned} &\widehat{\Psi} \equiv e^{\frac{3}{4}\xi}\Psi \,, \quad \widehat{M_\pm} \equiv e^{\xi/2}M_\pm \,, \\ &\widehat{\Gamma}^\mu \equiv e^{\xi/2}\Gamma^\mu \,, \quad \widehat{\Gamma}_a \equiv \widehat{\Gamma}^a \equiv e^{\xi/2+\eta}\Gamma^a = e^{-(\xi/2+\eta)}\Gamma_a \,, \quad \widehat{\Gamma}_p \equiv \widehat{\Gamma}^p \equiv e^{\xi/2}\Gamma^p = e^{-\xi/2}\Gamma_p \,, \end{aligned} \tag{4.28}$$

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we obtain

$$S_{\rm D3}^{\rm fermi} = \frac{T_3\lambda^2}{2}\int d^4y\, e^{-\phi}\,{\rm tr}\left\{i(\overline{\widehat{\Psi}}\widehat{\Gamma}^\mu D_\mu\widehat{\Psi} + \overline{\widehat{\Psi}}\widehat{\Gamma}^A i[A_A,\Psi]) - i\overline{\widehat{\Psi}}\widehat{M}_\pm\widehat{\Psi}\right\}\,. \tag{4.29}$$

Here, the rescaled gamma matrices $\widehat{\Gamma}^I$ $(I=0,1,\ldots,9)$ satisfy the anti-commutation relations with the flat metric $\{\widehat{\Gamma}^I,\widehat{\Gamma}^J\}=\eta^{IJ}$, and we have used $\overline{\Psi}\Gamma^\mu(\partial_\mu\xi)\Psi=0$, which follows because $\Gamma^0\Gamma^\mu$ is a symmetric matrix.

Again, using (4.23), we obtain

$$\begin{aligned}\widehat{M}_\pm &= \mp\frac{1}{4}\left(e^\phi\partial_3 C_0\widehat{\Gamma}^{012} + e^{\phi-\xi}(G_\pm^R)_{456}\widehat{\Gamma}^{456} + e^{\phi-\xi-3\eta}(G_\pm^R)_{789}\widehat{\Gamma}^{789}\right)\\ &= \frac{1}{2}\left(\pm\psi'\,\widehat{\Gamma}^{012} \mp \frac{\psi'}{\sin\psi}\,\widehat{\Gamma}^{456} - \frac{\psi'}{\cos\psi}\,\widehat{\Gamma}^{789}\right)\,,\end{aligned} \tag{4.30}$$

which reproduces (2.26). (Again, the lower sign is obtained by the transformation $(x^1,x^4)\to(-x^1,-x^4)$.)

4.2 Backgrounds with D3-branes

Let us next consider probe D3-branes extended along $x^{0,1,2,3}$ directions in a background corresponding to n D3-branes extended along $x^{0,1,4,5}$ directions:

	0	1	2	3	4	5	6	7	8	9
(probe) D3	o	o	o	o						
D3	o	o			o	o				

(4.31)

In this subsection, we use the letters for the indices as $\alpha,\beta=0,1$; $m,n=2,3$; $a,b=4,5$ and $p,q=6,7,8,9$.

The supergravity solution corresponding to n D3-branes placed at $x^m=0$ and $x^p=x_0^p$, in the string frame, is

$$e^\phi = 1\,, \qquad C_0 = \text{constant}\,, \tag{4.32}$$

$$ds^2_{\rm string} = h(r)^{-\frac{1}{2}}(\eta_{\alpha\beta}dx^\alpha dx^\beta + \delta_{ab}dx^a dx^b) + h(r)^{\frac{1}{2}}(\delta_{mn}dx^m dx^n + \delta_{pq}dx^p dx^q)\,, \tag{4.33}$$

$$F_5 = f_5 + *f_5\,, \quad f_5 \equiv dh(r)^{-1}\wedge dx^0\wedge dx^1\wedge dx^4\wedge dx^5, \tag{4.34}$$

where g_s is a constant and $h(r)$ is given as

$$h(r) \equiv 1 + \frac{Q_3}{r^4}\,, \quad Q_3 \equiv 4\pi g_s n l_s^4\,, \quad r^2 \equiv \sum_{m=2,3}(x^m)^2 + \sum_{p=6}^{9}(x^p-x_0^p)^2\,. \tag{4.35}$$

As in the previous subsection, the function $h(r)$ can be replaced with an arbitrary positive harmonic function on the $\mathbb{R}^6$ parametrized by $x^{2,3,6,7,8,9}$.

The metric evaluated at the position of the probe D3-branes, i.e. $x^i=0$ $(i=4,\ldots,9)$, is

$$ds^2_{\rm string}\big|_{x^i=0} = e^{-\frac{1}{2}\varphi}(\bar{g}_{\mu\nu}dx^\mu dx^\nu + \delta_{ab}dx^a dx^b + e^\varphi\delta_{pq}dx^p dx^q)\,, \tag{4.36}$$

where we have defined

$$e^{\varphi(x^m)} \equiv h(r)|_{x^i=0}\,, \tag{4.37}$$

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and

$$\bar{g}_{\mu\nu}dx^\mu dx^\nu \equiv \eta_{\alpha\beta}dx^\alpha dx^\beta + e^{\varphi}\delta_{mn}dx^m dx^n \ . \tag{4.38}$$

This metric (4.36) has $\mathrm{ISO}(1,1)\times\mathrm{SO}(2)\times\mathrm{SO}(4)$ isometry, where $\mathrm{ISO}(1,1)$ is the Poincaré symmetry acting on $x^{0,1}$, $\mathrm{SO}(2)$ and $\mathrm{SO}(4)$ are rotational symmetry acting on $x^{4,5}$ and $x^{6,7,8,9}$, respectively.[13] The supersymmetry condition for the deformed $\mathcal{N}=4$ SYM (2.2) with this symmetry is analyzed in section 4.1 of [2]. Since our brane configuration (4.31) preserves part of the supersymmetry, the action (3.2) and (3.3) for this background should reproduce one of the general solutions obtained there.

It is again easy to see that both V_{DBI} and V_{CS} in the potential (3.6) are flat, because $F_3 = H_3 = 0$, $F_{\mu\nu\rho\sigma i} = 0$ and L_{DBI} defined in (B.31) is a constant. Then, the bosonic part (3.2) becomes

$$\begin{aligned} S^{\mathrm{boson}}_{\mathrm{D3}} = \frac{T_3\lambda^2}{2}\int d^4x\sqrt{-\bar{g}}\,\mathrm{tr}\Big\{ &-\frac{1}{2}\bar{g}^{\mu\nu}\bar{g}^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} \pm \frac{C_0}{4}\bar{\epsilon}^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \\ &- \bar{g}^{\mu\nu}(e^{-\varphi}\delta_{ab}D_\mu\Phi^a D_\nu\Phi^b + \delta_{pq}D_\mu\Phi^p D_\nu\Phi^q) \\ &+ \frac{1}{2}e^{-2\varphi}\delta_{ab}\delta_{a'b'}[\Phi^a,\Phi^b][\Phi^{a'},\Phi^{b'}] + \frac{1}{2}\delta_{pq}\delta_{p'q'}[\Phi^p,\Phi^q][\Phi^{p'},\Phi^{q'}] \\ &+ e^{-\varphi}\delta_{ab}\delta_{pq}[\Phi^a,\Phi^b][\Phi^p,\Phi^q] \mp e^{-\varphi}(*_4\widetilde{F}_5)^{\mu}{}_{ij}\Phi^i D_\mu\Phi^j\Big\}, \end{aligned} \tag{4.39}$$

where $\bar{\epsilon}^{\mu\nu\rho\sigma}$ is the Levi-Civita symbol with $\bar{\epsilon}^{0123} \equiv 1/\sqrt{-\bar{g}}$. In order to compare with the results in [2], we redefine the scalar fields as

$$A_a \equiv e^{-\varphi/2}\Phi^a\,, \quad A_p \equiv \Phi^p \ . \tag{4.40}$$

Then, the kinetic terms for the scalar fields become

$$\begin{aligned} &\sqrt{-\bar{g}}\,\mathrm{tr}\left(e^{-\varphi}\bar{g}^{\mu\nu}\delta_{ab}D_\mu\Phi^a D_\nu\Phi^b + \bar{g}^{\mu\nu}\delta_{pq}D_\mu\Phi^p D_\nu\Phi^q\right) \\ &= \sqrt{-\bar{g}}\,\mathrm{tr}\left(\bar{g}^{\mu\nu}\delta^{AB}D_\mu A_A D_\nu A_B + \frac{1}{2}m^{AB}A_A A_B\right) + (\text{total derivative})\,, \end{aligned} \tag{4.41}$$

where

$$m^{ab} = \bar{g}^{mn}\left(\frac{1}{2}\partial_m\varphi\partial_n\varphi - \partial_m\partial_n\varphi\right)\delta^{ab}\,, \quad m^{pq} = 0\ . \tag{4.42}$$

The non-zero components of $(*_4\widetilde{F}_5)^{\mu}{}_{ij}$ are

$$(*_4\widetilde{F}_5)^{n}{}_{45} = -\bar{\epsilon}^{mn}\partial_m\varphi\,, \tag{4.43}$$

where $\bar{\epsilon}^{23} = -\bar{\epsilon}^{32} = \sqrt{\bar{g}^{22}\bar{g}^{33}} = e^{-\varphi}$. The last term in (4.39) becomes

$$e^{-\varphi}(*_4\widetilde{F}_5)^{\mu}{}_{ij}\Phi^i D_\mu\Phi^j = -\bar{\epsilon}^{mn}\partial_m\varphi\,\varepsilon^{ab}A_a D_n A_b\,, \tag{4.44}$$

[13] If we use $h(r)$ in (4.35), the metric also has a rotational symmetry on the $x^{2,3}$ plane. However, as mentioned above, $h(r)$ can be replace with an arbitrary positive harmonic function on $\mathbb{R}^6$, in which case the rotational symmetry on $x^{2,3}$ is broken in general.

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where $\varepsilon^{45} = -\varepsilon^{54} = 1$. This gives

$$d^{mab} = \pm\frac{1}{2}\bar{\epsilon}^{mn}\partial_m\varphi\,\varepsilon^{ab}\,, \tag{4.45}$$

in (2.2).

Collecting all these results, (4.39) becomes

$$\begin{aligned} S_{\rm D3}^{\rm boson} = \frac{T_3\lambda^2}{2}\int d^4x\sqrt{-\bar{g}}\,\mathrm{tr}\Big\{ &-\frac{1}{2}\bar{g}^{II'}\bar{g}^{JJ'}F_{IJ}F_{I'J'} \pm \frac{C_0}{4}\bar{\epsilon}^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \\ &-\frac{1}{2}\bar{g}^{mn}\left(\frac{1}{2}\partial_m\varphi\partial_n\varphi - \partial_m\partial_n\varphi\right)\delta^{ab}A_aA_b \pm \bar{\epsilon}^{mn}\partial_m\varphi\,\varepsilon^{ab}A_aD_nA_b\Big\}\,. \end{aligned} \tag{4.46}$$

The fermionic part (3.3) for this configuration is

$$S_{\rm D3}^{\rm fermi} = \frac{T_3\lambda^2}{2}\int d^4x\,\sqrt{-\bar{g}}\,e^{-\varphi}\,\mathrm{tr}\left\{i(\overline{\Psi}\Gamma^\mu D_\mu\Psi + \overline{\Psi}\Gamma_k i[\Phi^k,\Psi]) - i\overline{\Psi}M_\pm\Psi\right\} \tag{4.47}$$

with

$$M_\pm = \pm\frac{1}{4}(*_4\widetilde{F}_5)^n{}_{45}\Gamma^{m45}g_{mn}\,. \tag{4.48}$$

As in the previous subsection, we rescale Ψ, $M_\pm$ and the gamma matrices by

$$\begin{aligned} &\widehat{\Psi} \equiv e^{-\frac{3}{8}\varphi}\Psi\,, \qquad \widehat{M_\pm} \equiv M_\pm e^{-\frac{1}{4}\varphi}\,, \\ &\widehat{\Gamma}^\mu \equiv e^{-\frac{1}{4}\varphi}\Gamma^\mu\,, \qquad \widehat{\Gamma}_a \equiv \widehat{\Gamma}^a \equiv e^{-\frac{1}{4}\varphi}\Gamma^a = e^{\frac{1}{4}\varphi}\Gamma_a\,, \quad \widehat{\Gamma}_p \equiv \widehat{\Gamma}^p \equiv e^{\frac{1}{4}\varphi}\Gamma^p = e^{-\frac{1}{4}\varphi}\Gamma_p\,. \end{aligned} \tag{4.49}$$

The rescaled gamma matrices satisfy

$$\{\widehat{\Gamma}^\mu,\widehat{\Gamma}^\nu\} = 2\bar{g}^{\mu\nu}\,, \quad \{\widehat{\Gamma}^a,\widehat{\Gamma}^b\} = 2\delta^{ab}\,, \quad \{\widehat{\Gamma}^p,\widehat{\Gamma}^q\} = 2\delta^{pq}\,. \tag{4.50}$$

Then, we obtain

$$S_{\rm D3}^{\rm fermi} = \frac{T_3\lambda^2}{2}\int d^4x\,\sqrt{-\bar{g}}\,\mathrm{tr}\left\{i\left(\overline{\widehat{\Psi}}\widehat{\Gamma}^\mu D_\mu\widehat{\Psi} + \overline{\widehat{\Psi}}\widehat{\Gamma}^A i[A_A,\widehat{\Psi}]\right) - i\overline{\widehat{\Psi}}\widehat{M_\pm}\widehat{\Psi}\right\} \tag{4.51}$$

with

$$\widehat{M_\pm} = \pm\frac{1}{4}(*\widetilde{\bar{F}}_5)^n{}_{45}\widehat{\Gamma}^{m45}\bar{g}_{nm} = \mp\frac{1}{4}\bar{\epsilon}^n{}_m\partial_n\varphi\widehat{\Gamma}^{m45}\,, \tag{4.52}$$

where $\bar{\epsilon}^n{}_m \equiv \bar{\epsilon}^{nn'}\bar{g}_{n'm}$. This gives

$$\beta_m = \mp\frac{1}{4}\bar{\epsilon}^n{}_m\partial_n\varphi\,. \tag{4.53}$$

The results (4.42), (4.45) and (4.53) agree with (2.32), (2.31) and (2.30), respectively, for the case with $\tau =$ constant, $\Lambda =$ constant and $s = \mp$.[14]

[14] See the case (C3) with $\beta_m = \mp\frac{1}{4}\bar{\epsilon}^n{}_m\partial_n\varphi$ in section 4.1.2 of [2].

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5 Conclusions and outlook

In this work, we have complemented the study of deformations of $\mathcal{N} = 4$ SYM with varying couplings that we initiated in [2] by showing that some of these gauge theories can be realized on the probe D3-branes in curved backgrounds with fluxes. In particular, we obtained the effective action on the D3-branes for general backgrounds satisfying (3.1) and gave an explicit map between the couplings in the deformed $\mathcal{N} = 4$ SYM and the fluxes of the curved background on which the D3-branes are embedded.

As a check, we explicitly showed that the effective action on the D3-branes in a background with (p, q) 5-branes (see (4.1)) reproduces that of the supersymmetric Janus configuration found in [1]. We also studied D3-branes in a background with another stack of D3-branes intersecting with them (see (4.31)) and found that the action agrees with one of the solutions of SUSY conditions with $\mathrm{ISO}(1,1) \times \mathrm{SO}(2) \times \mathrm{SO}(4)$ symmetry found in [2].

On the other hand, in [2], we found a lot of solutions of SUSY conditions, for which the realization in string theory is not known. Our results in (3.23)–(3.27) suggest that it is possible to extract some information of supergravity fields from the couplings in the deformed $\mathcal{N} = 4$ SYM. Indeed, it is now easy to know which fluxes have non-trivial profiles for the brane configuration that realizes the deformed $\mathcal{N} = 4$ SYM. For example, for the cases with $\mathrm{ISO}(1,1) \times \mathrm{SO}(3) \times \mathrm{SO}(3)$ symmetry, solutions with non-trivial m_{012}, m_{013}, m_{456} and m_{789} are found in [2]. Such configurations, assuming that they can be realized in string theory, should have non-trivial $(*_4 F_1)_{012}$, $(*_4 F_1)_{013}$, $(G^R_\pm)_{456}$ and $(G^R_\pm)_{789}$ fluxes. Despite we have not shown this explicitly, this fact suggests that such a configuration corresponds to D3-branes in a background with (p, q)5- and $[p', q']$7-branes:

	0	1	2	3	4	5	6	7	8	9
(probe) D3	o	o	o	o						
(p, q) 5	o	o	o		o	o	o			
$[p', q']$ 7	o	o			o	o	o	o	o	o

(5.1)

It would be interesting to see this more explicitly.

Finally, we want to stress that we didn't use the equations of motion for the supergravity fields in our analysis in section 3. That is to say, some additional constraints are imposed on the couplings from the supergravity equations of motion. In this respect, some works has been done in [3, 20], where it has been shown that the couplings have to satisfy some algebraic equations obtained from the supergravity equations of motion. Furthermore, if we require SUSY, the background as well as the D3-brane configurations should satisfy BPS conditions. It would be interesting to see whether such conditions agree with the SUSY conditions found in [2]. Actually, there is a logical possibility that the deformed $\mathcal{N} = 4$ SYM action (2.2) could have some additional SUSY solutions which are not necessarily related to backgrounds satisfying the equations of motion in supergravity. It would be important to study the correspondence in more detail and clarify this issue.

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Acknowledgments

We thank Adi Armoni, Ki-Myeong Lee, Takahiro Nishinaka, Yuta Sekiguchi and Shuichi Yokoyama for discussion. We also appreciate useful discussions during the workshops "Geometry, Duality and Strings" Ref. YITP-X-16-11, "Strings and Fields" Ref. YITP-W-17-08 and the KIAS-YITP Joint Workshop 2017 "Strings, Gravity and Cosmology" YITP-W-17-12, held at the Yukawa Institute for Theoretical Physics, Kyoto University. J.J.F.-M. gratefully acknowledges the support of JSPS (Postdoctoral Fellowship) and the Fundación Séneca/Universidad de Murcia (Programa Saavedra Fajardo). The work of S.S. was supported by JSPS KAKENHI (Grant-in-Aid for Scientific Research (C)) Grant Number JP16K05324. The work of J.J.F.-M. and S.S. was also supported by JSPS KAKENHI (Grant-in-Aid for JSPS Fellows) Grant Number JP16F16741.

A Conventions for supergravity fields

We follow the conventions for the supergravity fields used in [21]. The bosonic part of the type IIB supergravity action in the string frame is

$$S_{\rm IIB} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\Big\{e^{-2\phi}\left(R+4|d\phi|^2-\frac{1}{2}|H_3|^2\right)-\frac{1}{2}\left(|F_1|^2+|\widetilde{F}_3|^2+\frac{1}{2}|\widetilde{F}_5|^2\right)\Big\} + \frac{1}{4\kappa^2}\int\left(C_4+\frac{1}{2}B_2\wedge C_2\right)\wedge F_3\wedge H_3\,, \tag{A.1}$$

where

$$H_3 = dB_2\,,\quad F_n = dC_{n-1}\,,\quad \widetilde{F}_n = F_n + H_3\wedge C_{n-3}\,. \tag{A.2}$$

and $|\omega_n|^2$ for an n-form ω_n is defined as

$$|\omega_n|^2 \equiv \frac{1}{n!}\omega_{I_1\cdots I_n}\omega_{J_1\cdots J_n}g^{I_1J_1}\cdots g^{I_nJ_n}\,. \tag{A.3}$$

In our convention, the dilaton ϕ vanishes asymptotically and κ is related to the Newton's constant G_N, string length l_s and string coupling g_s as

$$2\kappa^2 = 16\pi G_N = (2\pi)^7 l_s^8 g_s^2\,. \tag{A.4}$$

In addition, we have to impose the self-duality condition

$$\widetilde{F}_5 = *\widetilde{F}_5\,. \tag{A.5}$$

Here, the Hodge star $*$ is defined by

$$*(dx^{I_1}\wedge\cdots\wedge dx^{I_n}) = \frac{1}{(10-n)!}\epsilon^{I_1\cdots I_{10}}g_{I_{n+1}J_{n+1}}\cdots g_{I_{10}J_{10}}dx^{J_{n+1}}\wedge\cdots\wedge dx^{J_{10}}\,, \tag{A.6}$$

where $\epsilon^{M_1\cdots M_{10}}$ is the 10-dimensional Levi-Civita symbol with $\epsilon^{01\cdots9} = 1/\sqrt{-g}$.

It is useful to define $\widetilde{F}_n$ with $n>5$ by

$$\widetilde{F}_n \equiv (-1)^{\frac{1}{2}n(n+1)+1} * \widetilde{F}_{10-n}\,. \tag{A.7}$$

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Then, the equations of motion and the Bianchi identities for the R-R fields are written as

$$d\widetilde{F}_n + H_3 \wedge \widetilde{F}_{n-2} = 0\,, \quad (n = 1, 3, 5, 7, 9)\,, \tag{A.8}$$

which allows us to introduce C_{n-1} satisfying (A.2) for $n = 1, 3, 5, 7, 9$.

ϕ, B_2, C_0, C_2 and C_4 are related to those used in [22], denoted with superscript "P", as

$$e^{\phi^{\rm P}} = g_s e^{\phi}\,, \quad B_2^{\rm P} = -B_2\,, \quad C_0^{\rm P} = g_s^{-1} C_0\,, \quad C_2^{\rm P} = g_s^{-1} C_2\,, \quad C_4^{\rm P} = g_s^{-1}\left(C_4 + \frac{1}{2} B_2 \wedge C_2\right)\,. \tag{A.9}$$

The metric in the Einstein frame is defined as

$$g^{\rm E}_{IJ} = e^{-\frac{1}{2}\phi} g_{IJ}\,. \tag{A.10}$$

The action can be written as

$$\begin{aligned} S_{\rm IIB} &= \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g^{\rm E}}\left\{R^{\rm E} - \frac{1}{2}\left(|d\phi|^2_{\rm E} + e^{-\phi}|H_3|^2_{\rm E} + e^{2\phi}|F_1|^2_{\rm E} + e^{\phi}|\widetilde{F}_3|^2_{\rm E} + \frac{1}{2}|\widetilde{F}_5|^2_{\rm E}\right)\right\} \\ &\quad + \frac{1}{4\kappa^2}\int \left(C_4 + \frac{1}{2} B_2 \wedge C_2\right) \wedge F_3 \wedge H_3 \\ &= \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g^{\rm E}}\left\{R^{\rm E} - \frac{1}{2}\left(g^{MN}_{\rm E}\frac{\partial_M \overline{\tau}\partial_N \tau}{(\mathrm{Im}\,\tau)^2} + \mathcal{M}_{ij} F_3^i \cdot F_3^j + \frac{1}{2}|\widetilde{F}_5|^2_{\rm E}\right)\right\} \\ &\quad + \frac{\varepsilon_{ij}}{8\kappa^2}\int \left(C_4 + \frac{1}{2} B_2 \wedge C_2\right) \wedge F_3^i \wedge F_3^j\,, \end{aligned} \tag{A.11}$$

where $|\omega_n|^2_{\rm E}$ is defined as in (A.3) with the metric in the Einstein frame,

$$\tau \equiv g_s^{-1}(C_0 + i e^{-\phi})\,, \qquad F_3^1 \equiv -g_s^{1/2} H_3\,, \qquad F_3^2 \equiv g_s^{-1/2} F_3\,, \tag{A.12}$$

$$(\mathcal{M}_{ij}) = \frac{1}{\mathrm{Im}\,\tau}\begin{pmatrix} |\tau|^2 & -\mathrm{Re}\,\tau \\ -\mathrm{Re}\,\tau & 1 \end{pmatrix}\,, \qquad (\varepsilon_{ij}) = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \tag{A.13}$$

and

$$F_3^i \cdot F_3^j \equiv \frac{1}{3!} F^i_{I_1 I_2 I_3} F^j_{J_1 J_2 J_3} g^{I_1 J_1}_{\rm E} g^{I_2 J_2}_{\rm E} g^{I_3 J_3}_{\rm E}\,. \tag{A.14}$$

This action is invariant under the $\mathrm{SL}(2,\mathbb{R})$ transformation:

$$\tau \to \frac{a\tau + b}{c\tau + d}\,, \quad \begin{pmatrix} F_3^2 \\ F_3^1 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} F_3^2 \\ F_3^1 \end{pmatrix}\,, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{R})\,, \tag{A.15}$$

with κ, $g^{\rm E}_{MN}$ and $C_4 + \frac{1}{2} B_2 \wedge C_2$ kept fixed.

B Derivation of the D3-brane effective action

In this appendix, we show the detailed derivation of the action (3.2) and (3.3).

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B.1 Dp-branes in curved backgrounds (review)

For convenience, we first review the effective action of Dp-branes embedded in general backgrounds in appendix B.1, following [21] and [16] for bosonic and fermionic parts, respectively.

B.1.1 Bosonic part

In this subsection, we review the bosonic part of the effective action on Dp-branes embedded in a curved background with fluxes following [21].

The 10-dimensional space-time coordinates are denoted as x^I $(I = 0, 1, \ldots, 9)$. We choose the static gauge in which x^μ $(\mu = 0, 1, 2, 3)$ are identified as the coordinates on the Dp-brane world-volume and x^i $(i = 4, \ldots, 9)$ parametrize the transverse directions. The bosonic sector of the effective theory contains a U(N) gauge field A_μ $(\mu = 0, \ldots, p)$, $(9-p)$ scalar fields Φ^i $(i = p+1, \ldots, 9)$, which belong to the adjoint representation of the gauge group U(N). The reference position of the Dp-brane is chosen to be $x^i = 0$ and small deviations from it is described by the values of the scalar fields.

The effective action that describes the light open-string bosonic fluctuations of a set of N coincident Dp-branes in type II string theory consists of

$$S_{\mathrm{D}p}^{\mathrm{boson}} = S_{\mathrm{D}p}^{\mathrm{DBI}} + S_{\mathrm{D}p}^{\mathrm{CS}}\,, \tag{B.1}$$

where the Dirac-Born-Infeld (DBI) and Chern-Simons (CS) terms are given by

$$S_{\mathrm{D}p}^{\mathrm{DBI}} = -T_p \int d^{p+1}x\, \mathrm{Str}\left\{e^{-\widehat{\phi}}\sqrt{-\det(M_{\mu\nu})\det(Q^i{}_j)}\right\}\,, \tag{B.2}$$

$$S_{\mathrm{D}p}^{\mathrm{CS}} = \mu_p \int \mathrm{Str}\left\{\mathrm{P}\left[e^{i\lambda\iota_\Phi\iota_\Phi}\left(\sum_n \widehat{C}_n \wedge e^{\widehat{B}_2}\right)\right]\wedge e^{\lambda F}\right\}\,. \tag{B.3}$$

Here, the parameters T_p, μ_p and λ are given by

$$T_p \equiv \frac{1}{(2\pi)^p l_s^{p+1} g_s}\,, \quad \mu_p \equiv \pm T_p\,, \quad \lambda \equiv 2\pi l_s^2\,, \tag{B.4}$$

where l_s is the string length, g_s is the string coupling, and the upper (lower) sign appearing in μ_p, which is proportional to the R-R charge of the Dp-brane, corresponds to the case of Dp-branes ($\overline{\mathrm{D}p}$-branes). The quantities $M_{\mu\nu}$ and $Q^i{}_j$ are given by

$$M_{\mu\nu} \equiv \mathrm{P}\left[\widehat{E}_{\mu\nu} + \widehat{E}_{\mu i}(Q^{-1} - \delta)^{ij}\widehat{E}_{j\nu}\right] + \lambda F_{\mu\nu}\,, \tag{B.5}$$

$$Q^i{}_j \equiv \delta^i{}_j + i\lambda[\Phi^i, \Phi^k]\widehat{E}_{kj}\,, \tag{B.6}$$

where F is the field strength of the gauge field A living on the brane,

$$F = dA + iA \wedge A = \frac{1}{2}F_{\mu\nu}dx^\mu \wedge dx^\nu\,, \tag{B.7}$$

and

$$\widehat{E}_{IJ} \equiv \widehat{g}_{IJ} + \widehat{B}_{IJ}\,. \tag{B.8}$$

ϕ is the dilaton field, g_{IJ} is the background metric, $B_2 = \frac{1}{2}B_{IJ}dx^I \wedge dx^J$ is the Kalb-Ramond 2-form field and C_n ($n = 0, 2, 4, 6, 8$) are the Ramond-Ramond (R-R) n-form potential. The hat "$\widehat{\ }$" on the background fields indicates that they are evaluated at the position of the Dp-branes placed at $x^i = \lambda\Phi^i$, which is defined via a Taylor expansion as, e.g.,

$$\widehat{\phi}(x^\mu, \lambda\Phi^i) \equiv \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \Phi^{i_1} \dots \Phi^{i_n}\, \partial_{i_1} \dots \partial_{i_n} \phi(x^\mu, x^i)\big|_{x^i=0} \ . \tag{B.9}$$

The symbol $\mathrm{P}[\cdots]$ in (B.3) and (B.5) denotes the pull-back of the bulk fields over the Dp-brane world-volume, in which the ordinary derivative $\partial_\mu\Phi^i$ is replaced by the covariant derivative $D_\mu\Phi^i$:

$$D_\mu\Phi^i \equiv \partial_\mu\Phi^i + i[A_\mu, \Phi^i] \ . \tag{B.10}$$

For example, the pull-back of $E_{\mu\nu}$ is given by

$$\mathrm{P}[E_{\mu\nu}] = \widehat{E}_{\mu\nu} + \lambda\widehat{E}_{\mu i}D_\nu\Phi^i + \lambda\widehat{E}_{i\nu}D_\mu\Phi^i + \lambda^2\widehat{E}_{ij}D_\mu\Phi^i D_\nu\Phi^j \ . \tag{B.11}$$

$\imath_\Phi$ in (B.3) denotes the interior product by a vector (Φ^i), e.g.,

$$\imath_\Phi\imath_\Phi\left(\frac{1}{2}C_{ij}dx^i \wedge dx^j\right) = -\frac{1}{2}C_{ij}[\Phi^i, \Phi^j] \ . \tag{B.12}$$

The symbol $\mathrm{Str}\{\cdots\}$ in (B.2) and (B.3) denotes the symmetrized trace, which means Φ^i in the expansion (B.9), $F_{\mu\nu}$, $D_\mu\Phi^i$ and $[\Phi^i, \Phi^j]$ are symmetrized before taking the trace.

B.1.2 Fermionic part

In this subsection, we write down the fermionic part (quadratic terms with respect to the fermion fields) of the effective action on a Dp-brane embedded in any supergravity background following [16]. Here, we consider the cases with a single Dp-brane in type IIB string theory.

The action, after fixing the κ-symmetry, is given by

$$\begin{aligned} S_{\mathrm{D}p}^{\mathrm{fermi}} = \frac{T_p}{2}\int d^{p+1}x\, e^{-\phi}\sqrt{-\det(M_{\mu\nu})}\Big\{ & i\overline{\psi}\left[(M^{-1})^{\mu\nu}\Gamma_\mu\nabla_\nu^{(H)} - \Delta^{(1)}\right]\psi \\ & - i\overline{\psi}\,\breve{\Gamma}_{Dp}^{-1}\left[(M^{-1})^{\mu\nu}\Gamma_\nu W_\mu - \Delta^{(2)}\right]\psi\Big\}, \end{aligned} \tag{B.13}$$

where ψ is the fermion field (dimensional reduction of the 10-dimensional positive chirality Majorana-Weyl spinor field), $\Gamma_\mu \equiv \Gamma_{\hat{I}}e_I^{\hat{I}}\partial_\mu x^I$ is the pull-back of the 10-dimensional gamma matrices, $M_{\mu\nu}$ is the Abelian version of (B.5):

$$M_{\mu\nu} = \mathrm{P}[\widehat{g}_{\mu\nu} + \widehat{B}_{\mu\nu}] + \lambda F_{\mu\nu} \, , \tag{B.14}$$

and other quantities are defined as follows.

The covariant derivative $\nabla_\nu^{(H)}$ is the pull-back of the 10-dimensional covariant derivative including the H-flux:

$$\nabla_I^{(H)} \equiv \partial_I + \frac{1}{4}\omega_I{}^{\hat{J}\hat{K}}\Gamma_{\hat{J}\hat{K}} + \frac{1}{4\cdot 2!}H_{IJK}\Gamma^{JK} \, , \tag{B.15}$$

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where $\omega_I{}^{\hat{J}\hat{K}}$ is the spin connection and H_{IJK} is the field strength of the Kalb-Ramond 2-form field.

W_μ, $\Delta^{(1)}$ and $\Delta^{(2)}$ are defined as

$$W_\mu \equiv \frac{1}{8}e^{\phi}\left(-F_J\Gamma^J + \frac{1}{3!}\widetilde{F}_{JKL}\Gamma^{JKL} - \frac{1}{2\cdot 5!}\widetilde{F}_{JKLMN}\Gamma^{JKLMN}\right)\Gamma_\mu\,, \tag{B.16}$$

$$\Delta^{(1)} \equiv \frac{1}{2}\left(\Gamma^I\partial_I\phi + \frac{1}{2\cdot 3!}H_{IJK}\Gamma^{IJK}\right)\,, \tag{B.17}$$

$$\Delta^{(2)} \equiv -\frac{1}{2}e^{\phi}\left(-F_I\Gamma^I + \frac{1}{2\cdot 3!}\widetilde{F}_{IJK}\Gamma^{IJK}\right)\,, \tag{B.18}$$

where F_I, $\widetilde{F}_{IJK}$ and $\widetilde{F}_{IJKLM}$ are the field strength of the R-R fields defined as

$$\begin{aligned} &F_1 = F_I dx^I \equiv dC_0\,, \qquad \widetilde{F}_3 = \frac{1}{3!}\widetilde{F}_{IJK}dx^I\wedge dx^J\wedge dx^K \equiv dC_2 + C_0H_3\,,\\ &\widetilde{F}_5 = \frac{1}{5!}\widetilde{F}_{IJKLM}dx^I\wedge dx^J\wedge dx^K\wedge dx^L\wedge dx^M \equiv dC_4 + H_3\wedge C_2\,. \end{aligned} \tag{B.19}$$

(See appendix A for our conventions.)

Finally, $\breve{\Gamma}^{-1}_{\mathrm{D}p}$ is defined by

$$\breve{\Gamma}^{-1}_{\mathrm{D}p} = (-1)^{p-2}\Gamma^{(0)}_{\mathrm{D}p}\frac{\sqrt{-g}}{\sqrt{-\det(M_{\mu\nu})}}\sum_{q\geq 0}\frac{(-1)^q}{q!2^q}\Gamma^{\mu_1\cdots\mu_{2q}}\mathcal{F}_{\mu_1\mu_2}\cdots\mathcal{F}_{\mu_{2q-1}\mu_{2q}}\,, \tag{B.20}$$

where $\sqrt{-g} \equiv \sqrt{-\det \mathrm{P}[\hat{g}_{\mu\nu}]}$,

$$\mathcal{F}_{\mu\nu} \equiv \mathrm{P}[\widehat{B}_{\mu\nu}] + \lambda F_{\mu\nu}\,, \tag{B.21}$$

and

$$\Gamma^{(0)}_{\mathrm{D}p} \equiv \frac{1}{(p+1)!}\epsilon^{\mu_1\cdots\mu_{p+1}}\Gamma_{\mu_1\cdots\mu_{p+1}} \tag{B.22}$$

with the Levi-Civita symbol $\epsilon^{\mu_1\cdots\mu_{p+1}}$ with $\epsilon^{01\cdots p} = 1/\sqrt{-g}$.

B.2 D3-brane effective action

In this appendix, we consider the particular case of D3-branes under some simple and relatively general assumptions (3.1). We will study the expansion of the full action to leading and sub-leading orders that survive in the field theory limit and establish a relation between the backgrounds fields and the couplings in the action (2.2) of the deformed $\mathcal{N}=4$ SYM. The first two subsections B.2.1 and B.2.2 correspond to the analyses for the DBI action and the CS term, respectively, and appendix B.2.3 is the summary of the total bosonic sector. In appendix B.2.4, we carry out the calculations for the fermionic sector.

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B.2.1 DBI action

In this section, we present the extended calculations of the expansion of the DBI term (B.2) with respect to λ. Let us first consider the quantity $M_{\mu\nu}$ defined in (B.5). The pull-back of the first term of (B.5) is given in (B.11) and it is expanded as

$$\begin{aligned}\mathrm{P}[\widehat{E}_{\mu\nu}] &= \widehat{E}_{\mu\nu} + \lambda\widehat{E}_{\mu i}D_\nu\Phi^i + \lambda\widehat{E}_{i\nu}D_\mu\Phi^i + \lambda^2\widehat{E}_{ij}D_\mu\Phi^iD_\nu\Phi^j \\ &= \widehat{E}_{\mu\nu} + \lambda^2(\partial_jB_{\mu i}\Phi^jD_\nu\Phi^i + \partial_jB_{i\nu}\Phi^jD_\mu\Phi^i + g_{ij}D_\mu\Phi^iD_\nu\Phi^j) + \mathcal{O}(\lambda^3)\,, \end{aligned} \tag{B.23}$$

where we have used the assumptions in (3.1). The expansion of $Q^i{}_j$ in (B.6) is

$$Q^i{}_j = \delta^i{}_j + i\lambda[\Phi^i,\Phi^k]g_{kj} + i\lambda^2[\Phi^i,\Phi^k]\Phi^l\partial_lE_{kj} + \mathcal{O}(\lambda^3)\ . \tag{B.24}$$

Because $(Q^{-1}-\delta)^i_j = \mathcal{O}(\lambda)$, it is easy to see that

$$\mathrm{P}\left[E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}\right] = \mathcal{O}(\lambda^3)\,, \tag{B.25}$$

under the assumptions (3.1) and we can discard such higher-order contributions.

On the other hand, using the formula

$$\begin{aligned}\sqrt{\det(X+\delta X)} =& \sqrt{\det X}\bigg(1+\frac{1}{2}\,\mathrm{tr}(X^{-1}\delta X) + \frac{1}{8}(\mathrm{tr}(X^{-1}\delta X))^2 \\ &- \frac{1}{4}\,\mathrm{tr}(X^{-1}\delta X X^{-1}\delta X) + \mathcal{O}(\delta X^3)\bigg)\end{aligned} \tag{B.26}$$

for general matrices X and δX, we get

$$\sqrt{\det(Q^i{}_j)} = 1 + \frac{i\lambda^2}{2}[\Phi^i,\Phi^k]\Phi^l\partial_lB_{ki} - \frac{\lambda^2}{4}g_{ii'}g_{jj'}[\Phi^i,\Phi^j][\Phi^{i'},\Phi^{j'}] + \mathcal{O}(\lambda^3)\ . \tag{B.27}$$

Similarly, $\sqrt{-\det(M_{\mu\nu})}$ is expanded as

$$\begin{aligned}\sqrt{-\det(M_{\mu\nu})} =& \sqrt{-\det(\widehat{E}_{\mu\nu})} \\ &+\sqrt{-g}\,\frac{\lambda^2}{2}\bigg(g^{\mu\nu}g_{ij}D_\mu\Phi^iD_\nu\Phi^j + \frac{1}{2}g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} + (\partial_iB^{\mu\nu})\Phi^iF_{\mu\nu}\bigg) + \mathcal{O}(\lambda^3)\,.\end{aligned} \tag{B.28}$$

Now, making use of the partial results of the expansions (B.27) and (B.28), we calculate the full integrand of the DBI term (B.2):

$$\begin{aligned}&\mathrm{Str}\left\{e^{-\widehat{\phi}}\sqrt{-\det(M_{\mu\nu})\det(Q^i{}_j)}\right\} \\ &= \mathrm{Str}\left\{e^{-\widehat{\phi}}\sqrt{-\det(\widehat{E}_{\mu\nu})}\right\} \\ &\quad + \frac{\lambda^2}{2}e^{-\phi}\sqrt{-g}\,\mathrm{tr}\bigg(\frac{1}{2}g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} + g^{\mu\nu}g_{ij}D_\mu\Phi^iD_\nu\Phi^j - \frac{1}{2}g_{ii'}g_{jj'}[\Phi^i,\Phi^j][\Phi^{i'},\Phi^{j'}] \\ &\quad + (\partial_iB^{\mu\nu})\Phi^iF_{\mu\nu} - i(\partial_iB_{jk})\Phi^i[\Phi^j,\Phi^k]\bigg) + \mathcal{O}(\lambda^3)\end{aligned}$$

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$$= \mathrm{Str}\left\{e^{-\widehat{\phi}}\sqrt{-\det(\widehat{E}_{\mu\nu})}\right\}$$
$$+ \frac{\lambda^2}{2}e^{-\phi}\sqrt{-g}\,\mathrm{tr}\left(\frac{1}{2}g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} + g^{\mu\nu}g_{ij}D_\mu\Phi^i D_\nu\Phi^j - \frac{1}{2}g_{ii'}g_{jj'}[\Phi^i,\Phi^j][\Phi^{i'},\Phi^{j'}]\right)$$
$$+ H_i{}^{\mu\nu}\Phi^i F_{\mu\nu} - \frac{i}{3}H_{ijk}\Phi^i[\Phi^j,\Phi^k]\Big) + \mathcal{O}(\lambda^3)\ . \tag{B.29}$$

The first term in (B.29) gives the DBI part of the scalar potential. Let us define

$$V_{\mathrm{DBI}}(\Phi) \equiv \frac{1}{\sqrt{-g}}e^{-\widehat{\phi}}\sqrt{-\det(\widehat{E}_{\mu\nu})} = \frac{1}{\sqrt{-g}}\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\Phi^{i_1}\cdots\Phi^{i_n}\partial_{i_1}\cdots\partial_{i_n}L_{\mathrm{DBI}}|_{x^i=0} \tag{B.30}$$

with

$$L_{\mathrm{DBI}} \equiv e^{-\phi}\sqrt{-\det(E_{\mu\nu})}\ . \tag{B.31}$$

The derivatives are

$$\partial_i L_{\mathrm{DBI}} = e^{-\phi}\sqrt{-\det(E_{\mu\nu})}\left(-\partial_i\phi + \frac{1}{2}(E^{-1})^{\mu\nu}\partial_i E_{\nu\mu}\right)\ , \tag{B.32}$$

$$\partial_i\partial_j L_{\mathrm{DBI}} = e^{-\phi}\sqrt{-\det(E_{\mu\nu})}\left[\left(-\partial_i\phi + \frac{1}{2}(E^{-1})^{\mu\nu}\partial_i E_{\nu\mu}\right)\left(-\partial_j\phi + \frac{1}{2}(E^{-1})^{\mu'\nu'}\partial_j E_{\nu'\mu'}\right)\right.$$
$$\left.-\partial_i\partial_j\phi + \frac{1}{2}\left(-(E^{-1})^{\mu\mu'}\partial_i E_{\mu'\nu'}(E^{-1})^{\nu'\nu}\partial_j E_{\nu\mu} + (E^{-1})^{\mu\nu}\partial_i\partial_j E_{\nu\mu}\right)\right]\ . \tag{B.33}$$

Evaluating these quantities at $x^i = 0$, we obtain

$$V_{\mathrm{DBI}}(\Phi) = e^{-\phi}\left(1 + \lambda v_i^{\mathrm{DBI}}\Phi^i + \frac{\lambda^2}{2}m_{ij}^{\mathrm{DBI}}\Phi^i\Phi^j + \mathcal{O}(\lambda^3)\right)\ , \tag{B.34}$$

where the coefficients are

$$v_i^{\mathrm{DBI}} \equiv\ -\partial_i\phi + \frac{1}{2}g^{\mu\nu}\partial_i g_{\nu\mu} = \partial_i\log(\sqrt{-g}\,e^{-\phi})\,, \tag{B.35}$$

$$m_{ij}^{\mathrm{DBI}} \equiv v_i^{\mathrm{DBI}}v_j^{\mathrm{DBI}} - \partial_i\partial_j\phi + \frac{1}{2}\left(g^{\mu\nu}\partial_i\partial_j g_{\nu\mu} - g^{\mu\mu'}\partial_i g_{\mu'\nu'}g^{\nu'\nu}\partial_j g_{\nu\mu} - g^{\mu\mu'}H_{i\mu'\nu'}g^{\nu'\nu}H_{j\nu\mu}\right)$$
$$= \frac{1}{\sqrt{-g}\,e^{-\phi}}\partial_i\partial_j\left(\sqrt{-g}\,e^{-\phi}\right) + \frac{1}{2}H_i{}^{\mu\nu}H_{j\mu\nu}\ . \tag{B.36}$$

Then, the final expression for the DBI action is

$$S_{\mathrm{D3}}^{\mathrm{DBI}} = T_3\lambda^2\int d^4x\sqrt{-g}\,e^{-\phi}\,\mathrm{tr}\Big(-\frac{1}{4}g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} - \frac{1}{2}g^{\mu\nu}g_{ij}D_\mu\Phi^i D_\nu\Phi^j \tag{B.37}$$
$$+\frac{1}{4}g_{ii'}g_{jj'}[\Phi^i,\Phi^j][\Phi^{i'},\Phi^{j'}] - \frac{1}{2}H_i{}^{\mu\nu}\Phi^i F_{\mu\nu} + \frac{i}{3!}H_{ijk}\Phi^i[\Phi^j,\Phi^k] - e^{\phi}\lambda^{-2}V_{\mathrm{DBI}}(\Phi)\Big).$$

B.2.2 CS term

Let us study now the expansion of the CS-term of the D3-brane action, (B.3).

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First, we define

$$K \equiv \sum_{n:\text{even}} C_n \wedge e^{B_2} \equiv K_0 + K_2 + K_4 + K_6 + \cdots , \tag{B.38}$$

where[15]

$$\begin{aligned} &K_0 \equiv C_0 , \quad K_2 \equiv C_2 + C_0 B_2 , \quad K_4 \equiv C_4 + C_2 B_2 + \frac{1}{2} C_0 B_2^2 , \\ &K_6 \equiv C_6 + C_4 B_2 + \frac{1}{2} C_2 B_2^2 + \frac{1}{3!} C_0 B_2^3 . \end{aligned} \tag{B.39}$$

Note that it satisfies

$$dK = \sum_{n:\text{odd}} \widetilde{F}_n \wedge e^{B_2} . \tag{B.40}$$

For an n-form ω_n, we define an m-form with $(n-m)$ indices in the transverse directions $(\omega_n)_{m,i_1\cdots i_{n-m}}$ as

$$(\omega_n)_{m,i_1\cdots i_{n-m}} \equiv \frac{1}{m!} (\omega_n)_{\mu_1\cdots\mu_m i_1\cdots i_{n-m}} dx^{\mu_1}\cdots dx^{\mu_m} . \tag{B.41}$$

For example,

$$(\widetilde{F}_5)_{3,ij} \equiv \frac{1}{3!}(\widetilde{F}_5)_{\mu\nu\rho ij} dx^\mu dx^\nu dx^\rho , \quad (\widetilde{F}_5)_{4,j} \equiv \frac{1}{4!}\widetilde{F}_{\mu\nu\rho\sigma j} dx^\mu dx^\nu dx^\rho dx^\sigma , \quad \text{etc.} \tag{B.42}$$

Under the assumptions (3.1), the CS-term (B.3) is expanded as

$$\begin{aligned} S_{\text{D3}}^{\text{CS}} &= \mu_3 \int \text{Str}\left\{ \text{P}\left[e^{i\lambda \imath_\Phi \imath_\Phi} \widehat{K} \right] \wedge e^{\lambda F} \right\} \\ &= \mu_3 \int \text{Str}\left\{ \text{P}[\widehat{K}_4] + \lambda^2 \left(i\text{P}[\imath_\Phi \imath_\Phi \partial_i K_6]\Phi^i + \text{P}[\partial_i K_2]\Phi^i F + \frac{1}{2} C_0 F^2 \right) + \mathcal{O}(\lambda^3) \right\} . \end{aligned} \tag{B.43}$$

Expanding the first term, we get

$$\text{P}[\widehat{K}_4] = \left[\lambda \partial_i K_4 \Phi^i + \lambda^2 \partial_i (K_4)_{3,j} \Phi^i D\Phi^j + \frac{\lambda^2}{2} \partial_i \partial_j K_4 \Phi^i \Phi^j \right]_0 + \mathcal{O}(\lambda^3) , \tag{B.44}$$

where $[\cdots]_0$ denotes the pull-back on the world-volume at $x^i = 0$ (obtained by setting $x^i = 0$ and $dx^i = 0$), $D\Phi^j$ is a 1-form defined as

$$D\Phi^j \equiv D_\mu \Phi^j dx^\mu = d\Phi^j + i[A, \Phi^j] , \tag{B.45}$$

and we have used the notation (B.41).

The trace of the second term in (B.44) can be rewritten as

$$\begin{aligned} &\left[\lambda^2 \partial_i (K_4)_{3,j} \,\text{tr}(\Phi^i D\Phi^j) \right]_0 \\ &= \frac{\lambda^2}{2} \left[\frac{1}{2} (\partial_i (K_4)_{3,j} + \partial_j (K_4)_{3,i}) d \,\text{tr}(\Phi^i \Phi^j) + (\partial_i (K_4)_{3,j} - \partial_j (K_4)_{3,i}) \,\text{tr}(\Phi^i D\Phi^j) \right]_0 \\ &= \frac{\lambda^2}{2} \left[(\partial_i d (K_4)_{3,j} \,\text{tr}(\Phi^i \Phi^j) - (\widetilde{F}_5)_{3,ij} \,\text{tr}(\Phi^i D\Phi^j) + (\text{total derivative}) \right]_0 . \end{aligned} \tag{B.46}$$

[15] In this section, we often omit the symbol "$\wedge$" in the products of differential forms.

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Using this equation and the identities

$$
\begin{aligned}
[\partial_i K_4]_0 &= [(\widetilde{F}_5)_{4,i}]_0\,, \\
[\partial_i d(K_4)_{3,j} + \partial_i\partial_j K_4]_0 &= [\partial_i (dK_4)_{4,j}]_0 = [\partial_i(\widetilde{F}_5)_{4,j} + (\widetilde{F}_3)_{2,j}(H_3)_{2,i}]_0\,,
\end{aligned}
\tag{B.47}
$$

which are valid under the assumptions (3.1), we obtain

$$
\int \mathrm{Str}\,\mathrm{P}[\widehat{K}_4] = \int \mathrm{tr}\left\{\lambda(\widetilde{F}_5)_{4,i}\Phi^i + \frac{\lambda^2}{2}\left(\partial_i(\widetilde{F}_5)_{4,j} + (\widetilde{F}_3)_{2,j}(H_3)_{2,i}\right)\Phi^i\Phi^j - \frac{\lambda^2}{2}(\widetilde{F}_5)_{3,ij}\Phi^i D\Phi^j\right\}.
\tag{B.48}
$$

The second and third terms in (B.43) are rewritten by using

$$
\mathrm{tr}\{i\mathrm{P}[\imath_\Phi\imath_\Phi\partial_i K_6]\Phi^i\} = -\frac{i}{2}[\partial_i(K_6)_{4,jk}]_0\,\mathrm{tr}\{[\Phi^j,\Phi^k]\Phi^i\} = -\frac{i}{3!}[(\widetilde{F}_7)_{4,ijk}]_0\,\mathrm{tr}\{[\Phi^j,\Phi^k]\Phi^i\}\,,
\tag{B.49}
$$

$$
\mathrm{P}[\partial_i K_2]\Phi^i F_2 = [(\widetilde{F}_3)_{2,i}]_0\,\Phi^i F\,,
\tag{B.50}
$$

respectively.

Plugging these results in (B.43), the CS-term becomes

$$
\begin{aligned}
S^{\mathrm{CS}}_{\mathrm{D3}} = \mu_3\lambda^2\int \mathrm{tr}\,\Big\{&\lambda^{-1}(\widetilde{F}_5)_{4,i}\Phi^i + \frac{1}{2}\left(\partial_i(\widetilde{F}_5)_{4,j} + (\widetilde{F}_3)_{2,j}(H_3)_{2,i}\right)\Phi^i\Phi^j - \frac{1}{2}(\widetilde{F}_5)_{3,ij}\Phi^i D\Phi^j \\
&- \frac{i}{3!}(\widetilde{F}_7)_{4,ijk}\Phi^i[\Phi^j,\Phi^k] + (\widetilde{F}_3)_{2,i}\Phi^i F + \frac{1}{2}C_0F^2\Big\}\,.
\end{aligned}
\tag{B.51}
$$

It can also be written as

$$
\begin{aligned}
S^{\mathrm{CS}}_{\mathrm{D3}} = \mu_3\lambda^2\int d^4x\sqrt{-g}\,\mathrm{tr}\,\Big\{&\frac{1}{2}(*_4\widetilde{F}_3)_i{}^{\mu\nu}\Phi^i F_{\mu\nu} - \frac{i}{3!}(*_6\widetilde{F}_3)_{ijk}\Phi^i[\Phi^j,\Phi^k] \\
&- \frac{1}{2}(*_4\widetilde{F}_5)^{\mu}{}_{ij}\Phi^i D_\mu\Phi^j + \frac{1}{8}C_0F_{\mu\nu}F_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} - \lambda^{-2}V_{\mathrm{CS}}(\Phi)\Big\}\,,
\end{aligned}
\tag{B.52}
$$

where the potential $V_{\mathrm{CS}}(\Phi)$ is

$$
V_{\mathrm{CS}}(\Phi) \equiv -\lambda(*_4\widetilde{F}_5)_i\Phi^i - \frac{\lambda^2}{2}\left(\partial_i(*_4\widetilde{F}_5)_j + \frac{1}{2}(*_4\widetilde{F}_3)_j{}^{\mu\nu}(H_3)_{\mu\nu i}\right)\Phi^i\Phi^j
\tag{B.53}
$$

and we have defined

$$
\begin{aligned}
(*_4\widetilde{F}_3)_i{}^{\mu\nu} &\equiv \frac{1}{2}\epsilon^{\rho\sigma\mu\nu}\widetilde{F}_{i\rho\sigma}\,, & (*_6\widetilde{F}_3)_{ijk} &\equiv \frac{1}{3!}\epsilon_{lmnijk}\widetilde{F}^{lmn}\,, \\
(*_4\widetilde{F}_5)^{\mu}{}_{ij} &\equiv \frac{1}{3!}\epsilon^{\nu\rho\sigma\mu}\widetilde{F}_{\nu\rho\sigma ij}\,, & (*_4\widetilde{F}_5)_i &\equiv \frac{1}{4!}\epsilon^{\mu\nu\rho\sigma}\widetilde{F}_{\mu\nu\rho\sigma i}\,,
\end{aligned}
\tag{B.54}
$$

and used the relation

$$
\widetilde{F}_7 = -*\widetilde{F}_3\,.
\tag{B.55}
$$

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B.2.3 Bosonic part

Summing (B.37) and (B.52), we obtain

$$\begin{aligned} S_{\rm D3}^{\rm boson} = \frac{T_3\lambda^2}{2}\int d^4x\sqrt{-g}\,{\rm tr}\,\Big(&-\frac{e^{-\phi}}{2}g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} \pm \frac{C_0}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \\ &- e^{-\phi}g^{\mu\nu}g_{ij}D_\mu\Phi^i D_\nu\Phi^j + \frac{e^{-\phi}}{2}g_{ii'}g_{jj'}[\Phi^i,\Phi^j][\Phi^{i'},\Phi^{j'}] - 2V(\Phi) \\ &\pm (G^R_\pm)_i{}^{\mu\nu}\Phi^i F_{\mu\nu} \mp \frac{i}{3}(G^R_\pm)_{ijk}\Phi^i[\Phi^j,\Phi^k] \mp (*_4\widetilde{F}_5)^\mu{}_{ij}\Phi^i D_\mu\Phi^j\Big)\,, \end{aligned} \tag{B.56}$$

where we have set

$$\mu_3 = \pm T_3 \tag{B.57}$$

for D3-branes and $\overline{\rm D3}$-branes, respectively, and defined

$$G_3 \equiv F_3 + (C_0 + ie^{-\phi})H_3 = \widetilde{F}_3 + ie^{-\phi}H_3\,, \tag{B.58}$$

$$(G^R_\pm)_i{}^{\mu\nu} \equiv {\rm Re}((*_4G_3)_i{}^{\mu\nu} \pm iG_i{}^{\mu\nu}) = (*_4\widetilde{F}_3)_i{}^{\mu\nu} \mp e^{-\phi}H_i{}^{\mu\nu}\,, \tag{B.59}$$

$$(G^R_\pm)_{ijk} \equiv {\rm Re}((*_6G_3)_{ijk} \pm iG_{ijk}) = (*_6\widetilde{F}_3)_{ijk} \mp e^{-\phi}H_{ijk}\,, \tag{B.60}$$

and

$$V(\Phi) \equiv \lambda^{-2}(V_{\rm DBI}(\Phi) \pm V_{\rm CS}(\Phi))\,. \tag{B.61}$$

B.2.4 Fermionic part

Let us consider the fermionic action (B.13) for a D3-brane. To make the kinetic term of the fermions $\mathcal{O}(\lambda^0)$, we rescale the fermion as

$$\psi = \lambda\Psi\,. \tag{B.62}$$

Since we are interested in the terms that survive in the $l_s \to 0$ limit, we can set $M_{\mu\nu} = g_{\mu\nu}$ and $\breve{\Gamma}^{-1}_{\rm D3} = -\Gamma^{(0)}_{\rm D3} = \Gamma^{(4)}$ where

$$\Gamma^{(4)} \equiv \Gamma^{\hat{0}}\Gamma^{\hat{1}}\Gamma^{\hat{2}}\Gamma^{\hat{3}}\,. \tag{B.63}$$

Inserting (B.15)–(B.18) into the action (B.13) we obtain

$$\begin{aligned} S_{\rm D3}^{\rm fermi} = \frac{T_3\lambda^2}{2}\int d^4x\,\sqrt{-g}\,e^{-\phi}\,i\overline{\Psi}\Big[&\Gamma^\mu\nabla_\mu + \frac{1}{4\cdot 2!}H_{\mu IJ}\Gamma^{\mu IJ} - \frac{1}{4\cdot 3!}H_{IJK}\Gamma^{IJK} \\ &\mp e^{\phi}\Gamma^{(4)}\left(\frac{1}{8}\Gamma^\mu\left(-F_I\Gamma^I + \frac{1}{3!}\widetilde{F}_{IJK}\Gamma^{IJK} - \frac{1}{2\cdot 5!}\widetilde{F}_{IJKL}\Gamma^{IJKL}\right)\Gamma_\mu\right. \\ &\left. - \frac{1}{2}F_I\Gamma^I + \frac{1}{4\cdot 3!}\widetilde{F}_{IJK}\Gamma^{IJK}\right)\Big]\Psi\,, \end{aligned} \tag{B.64}$$

where ∇_μ is

$$\nabla_\mu\Psi \equiv \partial_\mu\Psi + \frac{1}{4}\omega_\mu{}^{\hat{I}\hat{J}}\Gamma_{\hat{I}\hat{J}}\Psi\,. \tag{B.65}$$

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Again, the upper (lower) signs correspond to the case of D3- ($\overline{\text{D3}}$-) branes. Note that the first term in (B.17) does not contribute, because $\Gamma^0\Gamma^I$ is a symmetric matrix. In general, one can show

$$\overline{\Psi}\,\Gamma^{I_1\ldots I_n}\Psi = 0 \quad \text{for} \quad n \neq 3 \ (\text{mod}\,4)\ . \tag{B.66}$$

Using this fact and the identities:

$$\Gamma^\mu(F_I\Gamma^I)\Gamma_\mu = -\,2F_\mu\Gamma^\mu - 4F_i\Gamma^i\,, \tag{B.67}$$

$$\Gamma^\mu(\widetilde{F}_{IJK}\Gamma^{IJK})\Gamma_\mu = 2\widetilde{F}_{\mu\nu\rho}\Gamma^{\mu\nu\rho} - 6\widetilde{F}_{\mu jk}\Gamma^{\mu jk} - 4\widetilde{F}_{ijk}\Gamma^{ijk}\,, \tag{B.68}$$

$$\begin{aligned}\Gamma^\mu(\widetilde{F}_{IJKLM}\Gamma^{IJKLM})\Gamma_\mu = {}& 20\widetilde{F}_{\mu\nu\rho\sigma m}\Gamma^{\mu\nu\rho\sigma m} + 20\widetilde{F}_{\mu\nu\rho lm}\Gamma^{\mu\nu\rho lm} \\ & - 10\widetilde{F}_{\mu jklm}\Gamma^{\mu jklm} - 4\widetilde{F}_{ijklm}\Gamma^{ijklm}\,,\end{aligned} \tag{B.69}$$

we obtain

$$\begin{aligned}S^{\text{fermi}}_{\text{D3}} = \frac{T_3\lambda^2}{2}\int d^4x\,\sqrt{-g}\,e^{-\phi}\,i\overline{\Psi}\Big[&\Gamma^\mu\nabla_\mu + \frac{1}{4\cdot 3!}(3H_{i\mu\nu}\Gamma^{i\mu\nu} - H_{ijk}\Gamma^{ijk}) \\ & \mp e^{\phi}\Gamma^{(4)}\Big(-\frac{1}{4}F_\mu\Gamma^\mu + \frac{1}{4\cdot 3!}(3\widetilde{F}_{i\mu\nu}\Gamma^{i\mu\nu} - \widetilde{F}_{ijk}\Gamma^{ijk}) \\ & - \frac{1}{3!\cdot 2^5}(2\widetilde{F}_{\mu\nu\rho ij}\Gamma^{\mu\nu\rho ij} - \widetilde{F}_{\mu ijkl}\Gamma^{\mu ijkl})\Big)\Big]\Psi\ .\end{aligned} \tag{B.70}$$

Furthermore, using the following identities

$$\begin{aligned}&\Gamma^{(4)}\Gamma^\mu = \frac{1}{3!}\epsilon^{\mu\nu\rho\sigma}\Gamma_{\nu\rho\sigma}\,, \qquad \Gamma^{(4)}\Gamma^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\Gamma_{\rho\sigma}\,, \qquad \Gamma^{(4)}\Gamma^{\mu\nu\rho} = -\epsilon^{\mu\nu\rho\sigma}\Gamma_\sigma\,, \\ &\Gamma^{(4)}\Gamma^{ijk} = \frac{1}{3!}\epsilon^{ijklmn}\Gamma_{lmn}\Gamma^{(10)}\,, \qquad \Gamma^{(4)}\Gamma^{ijkl} = -\frac{1}{2}\epsilon^{ijklmn}\Gamma_{mn}\Gamma^{(10)}\,,\end{aligned} \tag{B.71}$$

together with the chirality condition (2.1) and the relation

$$(*_4\widetilde{F}_5)_{\mu ij} = (*_6\widetilde{F}_5)_{\mu ij}\,, \tag{B.72}$$

that follows from the self-duality condition (A.5), the action can be rewritten as

$$\begin{aligned}S^{\text{fermi}}_{\text{D3}} = \frac{T_3\lambda^2}{2}\int d^4x\,\sqrt{-g}\,e^{-\phi}\,i\overline{\Psi}\Big[&\Gamma^\mu\nabla_\mu + \frac{1}{4\cdot 3!}(3H_{i\mu\nu}\Gamma^{i\mu\nu} - H_{ijk}\Gamma^{ijk}) \\ & \mp e^{\phi}\Big(-\frac{1}{4\cdot 3!}(*_4F_1)_{\mu\nu\rho}\Gamma^{\mu\nu\rho} + \frac{1}{4\cdot 3!}\Big(3(*_4\widetilde{F}_3)_{i\mu\nu}\Gamma^{i\mu\nu} - (*_6\widetilde{F}_3)_{ijk}\Gamma^{ijk}\Big) \\ & + \frac{1}{8}(*_4\widetilde{F}_5)_{\mu ij}\Gamma^{\mu ij}\Big)\Big]\Psi\,,\end{aligned} \tag{B.73}$$

where we have used the notation (B.54) and defined

$$(*_4F_1)_{\nu\rho\sigma} \equiv \epsilon_{\mu\nu\rho\sigma}F^\mu = \epsilon^\mu{}_{\nu\rho\sigma}\partial_\mu C_0\ . \tag{B.74}$$

For the non-Abelian case, the covariant derivative ∇_μ should be replaced with

$$\Gamma^\mu\nabla_\mu\Psi \quad \to \quad \Gamma^\mu D_\mu\Psi + i\Gamma_k[\Phi^k,\Psi] + \frac{1}{4}\omega_{\mu\hat{i}\hat{j}}\Gamma^{\mu\hat{i}\hat{j}}\,, \tag{B.75}$$

where $D_\mu\Psi$ is defined in (2.6) and we have assumed $g_{\mu i} = 0$ and $\omega_{\mu\hat{\nu}\hat{i}} = -\omega_{\mu\hat{i}\hat{\nu}} = 0$.

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Then, our final expression for the fermionic part of the action is

$$S_{\rm D3}^{\rm fermi} = \frac{T_3\lambda^2}{2}\int d^4x\,\sqrt{-g}\,e^{-\phi}\,\mathrm{tr}\left\{i(\overline{\Psi}\Gamma^\mu D_\mu\Psi + \overline{\Psi}\Gamma_k i[\Phi^k,\Psi]) - i\overline{\Psi}\left(M_\pm - \frac{1}{4}\omega_{\mu\hat{i}\hat{j}}\Gamma^{\mu\hat{i}\hat{j}}\right)\Psi\right\} \tag{B.76}$$

with

$$M_\pm \equiv \mp\frac{e^\phi}{8}\left(\frac{1}{3}(*_4F_1)_{\mu\nu\rho}\Gamma^{\mu\nu\rho} - (G^R_\pm)_{i\mu\nu}\Gamma^{i\mu\nu} + \frac{1}{3}(G^R_\pm)_{ijk}\Gamma^{ijk} - (*_4\widetilde{F}_5)_{\mu ij}\Gamma^{\mu ij}\right), \tag{B.77}$$

where $(G^R_\pm)_{i\mu\nu}$ and $(G^R_\pm)_{ijk}$ are defined in (B.59) and (B.60), respectively.

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