

Anomalous dimensions in the WF $O(N)$ model with a monodromy line defect

Alexander Söderberg

*Department of Physics and Astronomy, Uppsala University,
Uppsala, Sweden*

E-mail: alexander.soderberg@physics.uu.se

ABSTRACT: Implications of inserting a conformal, monodromy line defect in three dimensional $O(N)$ models are studied. We consider then the WF $O(N)$ model, and study the two-point Green's function for bulk-local operators found from both the bulk-defect expansion and Feynman diagrams. This yields the anomalous dimensions for bulk- and defect-local primaries as well as one of the OPE coefficients as ϵ -expansions to the first loop order. As a check on our results, we study the $(\phi^k)^2\phi^j$ operator both using the bulk-defect expansion as well as the equations of motion.

KEYWORDS: Conformal Field Theory, Global Symmetries, Spontaneous Symmetry Breaking

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1 Introduction and review

Conformal field theories (CFT) in higher than two dimensions are interesting in several different contexts, e.g. condensed matter physics (three dimensions), particle physics (four dimensions), AdS/CFT correspondence [1] and entanglement [2]. There has been a lot of development in higher dimensional CFTs¹ since the breakthrough in conformal bootstrap [3], where the authors numerically determined an upper bound on the dimensions of leading primaries in the OPE, and after the analytical approaches to the bootstrap program for higher dimensional theories [4, 5], where they studied the large spin behavior of CFTs. The results from [4, 5] are generalized in [6], where a large spin perturbation theory is developed. This method is later used in [7].² Some notable examples of analytical developments in higher dimensional theories are [8–17], as well as numerical developments [18–22].

¹I.e. theories in more than two dimensions.

²We thank Alday for telling us about this development.

More important for this paper, are higher dimensional $O(N)$ models, which also have had a lot of development lately [7, 23–31]. It is interesting to study $O(N)$ models since they are important for the AdS/CFT correspondence, see [26] and references therein.

Lately there has been a lot of development in CFTs with a defect, i.e. defect conformal field theories (DCFT), both analytically [32–36] and numerically [37–40]. Such theories may be used to explain boundary conditions, magnetic-like impurities in spin systems, Rényi entropy and entanglement, see [34, 35, 39, 41, 42] and references therein. A defect is a subspace in the space of a theory, where new operators and interactions between operators may occur. It is therefore important to distinguish between bulk-local operators, which live in the entire space of the theory, and defect-local operators, which only live on the defect. Using the operator product expansion (OPE), it is possible to write bulk-local and defect-local operators in terms of each other when the bulk-local operators are close to the defect [33, 43]. We call these OPEs the bulk-defect as well as defect-bulk expansion. These expansions contain OPE coefficients, that are promoted to tensors (with arbitrary many indices) in theories with a global symmetry, as is the case of $O(N)$ models. The tensors/coefficients in these expansions do not need to be real-valued, unlike the coefficients in the OPE between two bulk-local operators. We expect the global symmetry of the theory to be broken after insertion of a defect, since in general the latter is only left invariant under some subgroups of the global symmetry group. A conformal defect behaves like a CFT on its own. Meaning, conformal transformations parallel to the defect is preserved, i.e. if a conformal defect of codimension m is inserted into a d -dimensional CFT, $SO(d-m+1, 1)$ is left unbroken.³ If the defect is flat or spherical, rotations $SO(m)$ around the defect is preserved as well. This rotation group will act as a global or internal symmetry of the defect-local operators. So a conformal flat defect will break the $SO(d+1, 1)$ conformal group into $SO(m) \times SO(d-m+1, 1)$. In this case, defect-local operators may carry both $SO(m)$ - and $SO(d-m+1, 1)$ -spin, while bulk-local operators may carry $SO(d+1, 1)$ -spin. Bulk-local operators are transformed under an element from the global symmetry group as they are transported around a monodromy defect. We may define several different defects using different group elements from the global symmetry group in the monodromy transformation.

In this paper we study the implications of inserting a monodromy line defect into a conformal, three dimensional $O(N)$ model using the bulk-defect expansion. Inserting this defect will break the conformal $SO(4, 1)$ symmetry into $SO(2) \times SO(2, 1)$. The monodromy action tells us about the $SO(2)$ -spin of the defect-local operators as well as how the global $O(N)$ symmetry is broken after the defect is inserted, while symmetry of the residual subgroups of $O(N)$ tell us what kinds of OPE tensors may exist in the bulk-defect expansion, and thus also restricts what kinds of defect-local operators will live on the defect. We find that the global $O(N)$ symmetry is broken into two or three subgroups, depending on what group element we use in the monodromy action. Operators that transform in different unbroken subgroups do not mix with each other, and defect-local operators in the bulk-defect expansions will transform under the same subgroup as their corresponding bulk-local operator. The $SO(2)$ -spin of the defect-local operators will differ depending on

³In this paper we use Euclidean signature.

what subgroup they transform under. This spin, s_X , can be generic, and does not need to be integer or half-integer

$$s_X \in \mathbb{Z} + v, \quad v \in [0, 1). \tag{1.1}$$

We denote bulk- and defect-local operators that transform in one of the subgroups that are left unbroken, say $O(X)$, as ϕ_X^j and ψ_X^j , where ψ_X^j has $SO(2)$ -spin s_X . By studying this $O(X)$ symmetry we find that only vector operators will appear in the bulk-defect expansion, with OPE tensors of rank zero, i.e. OPE constants (denoted c_X), in the bulk-defect expansion.

The 3D Ising model with a monodromy line defect was studied analytically in [43]. They started from the Wilson-Fisher (WF) fixed point in $4 - \epsilon$ dimensional ϕ^4 theory and let ϵ go to one (the defect is always of co-dimension two). The scaling dimensions of bulk- and defect-local primaries as well as some of the OPE coefficients were found to the first loop order through comparison of the two-point Green's functions for two bulk-local operators on the defect found in two different ways. One being from the bulk-defect expansion, the other from Feynman diagrams. Their results are in agreement with the numerical data from [38]. We will generalize this approach to an $O(N)$ model by promoting the scalar operators in ϕ^4 -theory into vector multiplets of $O(N)$. We call this theory the WF $O(N)$ model. The CFT data we find through this approach are⁴

$$\begin{aligned} |c_X| &= 1 - \frac{\tilde{\psi}(|s_X| + 1) - \tilde{\psi}(1)}{4} \epsilon + \mathcal{O}(\epsilon^2), \\ \Delta_{\psi_X} &= |s_X| + 1 - \left(1 - \frac{v(v-1)(X+2)}{(X+8)|s_X|} \right) \frac{\epsilon}{2} + \mathcal{O}(\epsilon^2), \\ \Delta_{\phi_X} &= 1 - \frac{\epsilon}{2} + \mathcal{O}(\epsilon^2). \end{aligned} \tag{1.2}$$

Another analytical approach is the ϵ -expansion for the 3D Ising model created by Rychkov and Tan in 2015 [44]. This approach (we will call it the Rychkov-Tan analysis) constrains the theory by defining three axioms that contain information about its dynamics. One of these axioms states that every $\phi^n, n \geq 0, n \in \mathbb{Z}$ is a primary, except ϕ^3 which is a descendant of ϕ . This follows from the equations of motion. The Rychkov-Tan analysis has been applied to several different theories, e.g. scalar theories in different dimensions [45–47], the Gross-Neveu model [48, 49], $O(N)$ models [50], theories studied in Mellin space [51, 52], the Lee-Yang model [53], generalized free CFTs [54] and the 3D Ising model with a monodromy line defect [55]. The same scaling dimension of defect-local operators as those from [43] was found using the Rychkov-Tan analysis in [55]. At the end of this paper we generalize the Rychkov-Tan analysis in [55] to the WF $O(N)$ model. We find that the anomalous dimensions for bulk- and defect-local operators are in agreement with the corresponding ones found using the approach in [43], see (1.2), indicating that they are correct.

This paper is outlined as follows. In section 2 we study the implications of inserting a monodromy, line defect into a three dimensional $O(N)$ model. Here we study constraints on the bulk-defect expansion that arises from the monodromy of the defect and the symmetry of the unbroken subgroups of $O(N)$ that are left preserved after the defect has been inserted.

⁴Here $\tilde{\psi}(x)$ is the digamma function.

Some technical details about the monodromy constraint are gathered in appendix A. We generalize the approach in [43] to the WF $O(N)$ model in section 3. The Green's function for two bulk-local operators are studied using both the bulk-defect expansion and Feynman diagrams (up to one loop level). The results (1.2) are found in this section. We have placed technicalities about the one-loop Feynman integrals in appendix B. Finally in section 4 we generalize the Rychkov-Tan analysis to the WF $O(N)$ model with a monodromy, line defect. This section serves as a check that our results from section 3 are correct.

2 Monodromy line defect in a three dimensional $O(N)$ model

Let us consider a three dimensional CFT with a global $O(N)$ symmetry and a monodromy line defect. We expect a breaking of the $O(N)$ symmetry by inserting this defect. Thus we will consider bulk-local fields that are in a vector representation of the residual symmetry group G . A monodromy defect is defined with the action

$$\Phi^j(r, \theta + 2\pi, y) = g^{j_{j'}} \Phi^{j'}(r, \theta, y), \quad g^{j_{j'}} \in O(N), \quad j \in \{1, \dots, N\}. \quad (2.1)$$

Here r and θ are polar coordinates transverse to the defect, and y is the coordinate parallel to the defect. This condition means that if we transport Φ^j around the defect, we get back a transformed operator. The choice of the group element $g^{j_{j'}}$ from $O(N)$ will define the defect.

Example 1 *In the 3D Ising model, the global symmetry group is Z_2 . Thus the monodromy defect in this theory can be defined with either $g = \pm 1$. In this case, $g = 1$ is the trivial case when there is no defect. See [43] for the implications of $g = -1$.*

If one of these bulk-local operators is close to the defect, we may write it in terms of defect-local operators using the bulk-defect expansion [56]. In a three-dimensional CFT with a codimension two defect, the bulk-defect expansion for the rescaled Φ^j presented in [43] is generalized into

$$\begin{aligned} \Phi^j(r, \theta, y) &= \sum_{\Psi_s^R} C^j_{R,s} \frac{e^{-is\theta}}{r^{\Delta_\Psi - \Delta_\Psi}} B_{\Delta_\Psi}(r^2, \partial_y^2) \Psi_s^R(y), \\ B_\Delta(x, y) &= \sum_{m \geq 0} \frac{(-1)^m (\Delta)_m}{m! (2\Delta)_{2m}} x^m y^m, \\ C^j_{R,s} \Psi_s^R &\equiv (C^j_{R,s})_{k_1 \dots k_l} (\Psi_s^R)^{k_1 \dots k_l}, \quad \Delta_\Psi \equiv \Delta_{\Psi_s^R}(R, s). \end{aligned} \quad (2.2)$$

Here we sum over all tensor primaries, Ψ_s^R , that lives on the defect. These defect-local operators are in irreducible representations R of G , and different defect-local operators may transform in different representations of G . In this expansion $C^j_{R,s}$ is an OPE tensor that transforms as a vector of G when contracted with Ψ_s^R , s is the $SO(2)$ -spin of Ψ_s^R , Δ_Ψ is the scaling dimension of Ψ_s^R and $(x)_m$ is the Pochhammer symbol. Note that both $C^j_{R,s}$ and Δ_Ψ depend on R and s , i.e. they may differ for each Ψ_s^R . We can see that the original $SO(d+1, 1)$ conformal symmetry has been broken into $SO(2) \times SO(d -$

1, 1), where $SO(2)$ describes rotations around the defect, and $SO(2, 1)$ describes conformal transformations parallel to the defect. This expansion is valid only when Φ^j is close to the defect. Since $SO(2)$ is an Abelian group, s will act as a charge under the global $SO(2) \simeq U(1)$ transformations that Ψ_s^R enjoys

$$\Psi_s^R(y) = e^{is\theta} \Psi_s^R(y). \tag{2.3}$$

Note that the continuous parameter θ in this $SO(2)$ -transformation is one of the polar coordinates in the CFT bulk. The factor $e^{-is\theta}$ in (2.2) makes sure that $\Psi_s^R(y)$ can transform globally under $SO(2)$ without affecting $\Phi^j(r, \theta, y)$. Reality of Φ^j implies that [43]

$$\Psi_{-s}^R = \bar{\Psi}_s^R. \tag{2.4}$$

The first thing we need to ask ourselves is what kinds of defect-local operators may appear in the expansion (2.2). We may be able to constrain the theory using the definition of a monodromy action (2.1) as well as the residual symmetry G . Since we expect the global $O(N)$ symmetry to be broken by the monodromy of the defect, we have to study constraints on the dynamics from it first.

2.1 Monodromy action constraint

By conjugation, an $O(N)$ -matrix is given by⁵

$$(g^j_{j'}) (\vartheta) = \begin{bmatrix} R_\vartheta & 0 & 0 \\ 0 & \mathbb{1}_{\chi \times \chi} & 0 \\ 0 & 0 & -\mathbb{1}_{(N-\chi-2) \times (N-\chi-2)} \end{bmatrix}, \quad R_\vartheta = \begin{bmatrix} \pm \cos \vartheta & \mp \sin \vartheta \\ \sin \vartheta & \cos \vartheta \end{bmatrix}. \tag{2.5}$$

Here $\chi \in \{0, 1, \dots, N - 2\}$. Monodromy of the defect (2.1) together with the bulk-defect expansion (2.2) yields

$$\begin{cases} e^{-2\pi is} C^1_{R,s} \Psi_s^R = \pm \cos \vartheta C^1_{R,s} \Psi_s^R \mp \sin \vartheta C^2_{R,s} \Psi_s^R, \\ e^{-2\pi is} C^2_{R,s} \Psi_s^R = \sin \vartheta C^1_{R,s} \Psi_s^R + \cos \vartheta C^2_{R,s} \Psi_s^R, \\ e^{-2\pi is} C^q_{R,s} \Psi_s^R = C^q_{R,s} \Psi_s^R, \quad q \in \{3, \dots, \chi + 2\}, \\ e^{-2\pi is} C^r_{R,s} \Psi_s^R = -C^r_{R,s} \Psi_s^R, \quad r \in \{\chi + 3, \dots, N\}. \end{cases} \tag{2.6}$$

There are two important special cases for the above equation system. These special cases occur when we cannot write $C^1_{R,s} \Psi_s^R$ in terms of $C^2_{R,s} \Psi_s^R$ and vice versa, i.e. when

$$\sin \vartheta = 0 \quad \Leftrightarrow \quad \vartheta = \begin{cases} 0 & \text{mod } 2\pi, \\ \pi & \text{mod } 2\pi. \end{cases} \tag{2.7}$$

We will get two different sets of solutions depending on whether R_ϑ describes a proper ($\det R_\vartheta = 1$) or improper ($\det R_\vartheta = -1$) rotation.

⁵We can think of this as a general $O(N)$ transformation where we have chosen the basis vectors in this $O(N)$ space such that it only rotates the first two vectors.

2.1.1 Proper rotation

We consider first the two special cases (2.7). If ϑ equals zero, (2.6) reduces to

$$\begin{cases} e^{-2\pi is} C^p_{R,s} \Psi_s^R = C^p_{R,s} \Psi_s^R, & p \in \{1, \dots, \chi + 2\}, \\ e^{-2\pi is} C^r_{R,s} \Psi_s^R = -C^r_{R,s} \Psi_s^R, & r \in \{\chi + 3, \dots, N\}. \end{cases} \quad (2.8)$$

This system has two solutions. Either

$$C^r_{R,s} \Psi_s^R = 0 \quad \forall r \in \{\chi + 3, \dots, N\}, \quad s = n, \quad (2.9)$$

where $C^p_{R,s} \Psi_s^R$, $p \in \{1, \dots, \chi + 2\}$, does not receive any constraints, or

$$C^p_{R,s} \Psi_s^R = 0 \quad \forall p \in \{1, \dots, \chi + 2\}, \quad s = n + \frac{1}{2}, \quad (2.10)$$

where $C^r_{R,s} \Psi_s^R$, $r \in \{\chi + 3, \dots, N\}$ does not receive any constraints. In this section n is an integer, i.e. $n \in \mathbb{Z}$. The solutions (2.9) and (2.10) tell us that the global $O(N)$ symmetry group has been broken into

$$G = O(\chi + 2) \times O(N - \chi - 2). \quad (2.11)$$

The branching rule tells us that Φ^j can be separated into bulk-local operators that transform in $O(\chi + 2)$ and bulk-local operators that transform in $O(N - \chi - 2)$

$$\Phi^j = \phi_{\chi+2}^a \oplus \phi_{N-\chi-2}^b, \quad a \in \{1, \dots, \chi + 2\}, \quad b \in \{1, \dots, N - \chi - 2\}. \quad (2.12)$$

Both $\phi_{\chi+2}^a$ and $\phi_{N-\chi-2}^b$ will have bulk-defect expansions similar to (2.2). The defect-local operators in these expansions will transform under the same orthogonal symmetry group as their corresponding bulk-local operator, e.g. the defect-local operators, $\psi_{\chi+2}$, in the bulk-defect expansion of $\phi_{\chi+2}^a$ will transform under $O(\chi + 2)$. The $SO(2)$ -spin of $\psi_{\chi+2}$ will be an integer, while the $SO(2)$ -spin of $\psi_{N-\chi-2}$ will be a half-integer spin, i.e.

$$s_{\chi+2} = n, \quad s_{N-\chi-2} = n + \frac{1}{2}. \quad (2.13)$$

More precisely, we can write the bulk-defect expansion (2.2) for the original bulk-local operator (that transforms in G) as two sums. One that sums over defect-local primaries with integer spins, and one that sums over defect-local primaries with half-integer spins. The first of these sums corresponds to $\phi_{\chi+2}^a$ and contains $\psi_{\chi+2}$, while the second corresponds to $\phi_{N-\chi-2}^b$ and contains $\psi_{N-\chi-2}$. A similar decomposition is possible for all of the other cases studied in this section as well.

$$\begin{aligned} \Phi^j(r, \theta, y) &= \sum_{\psi_{\chi+2}} C_{\chi+2}^j \frac{e^{-is_{\chi+2}\theta}}{r^{\Delta_{\phi_{\chi+2}} - \Delta_{\psi_{\chi+2}}}} B_{\Delta_{\psi_{\chi+2}}} \psi_{\chi+2} + \\ &+ \sum_{\psi_{N-\chi-2}} C_{N-\chi-2}^j \frac{e^{-is_{N-\chi-2}\theta}}{r^{\Delta_{\phi_{N-\chi-2}} - \Delta_{\psi_{N-\chi-2}}}} B_{\Delta_{\psi_{N-\chi-2}}} \psi_{N-\chi-2}, \quad (2.14) \\ C_X^j \psi_X &\equiv (C_X^j)_{k_1 \dots k_l} (\psi_X)^{k_1 \dots k_l}, \\ \Delta_{\psi_X} &\equiv \Delta_{\psi_X}(R_X, s_X), \quad C_X^j \equiv C_X^j(R_X, s_X). \end{aligned}$$

Here $X \in \{\chi + 2, N - \chi - 2\}$ and ψ_X is a defect-local primary that is in the irreducible representation R_X of $O(X)$, and transforms as a vector of $O(X)$ when contracted with the OPE coefficient C_X^j . It has $SO(2)$ -spin s_X and scaling dimension Δ_{ψ_X} . The OPE coefficients, the scaling dimensions (for both the bulk-local and defect-local operators) as well as the irreducible representations of the defect-local operators in these two sums may be different to each other.

It is a similar story when $\vartheta = \pi$. The $O(N)$ symmetry is then broken into $O(\chi) \times O(N - \chi)$, and defect-local operators that transform in $O(\chi)$ have integer $SO(2)$ -spin, while defect-local operators that transform in $O(N - \chi)$ have half-integer $SO(2)$ -spin.

A more interesting case is when we consider ϑ to be generic, i.e. $\sin \vartheta \neq 0$. Then (2.6) yields the following system of equations⁶

$$\begin{cases} C^1_{R,s} \Psi_s^R &= \pm i C^2_{R,s} \Psi_s^R, & s = n + \frac{\vartheta}{2\pi}, & n \in \mathbb{Z}, \\ e^{-2\pi i s} C^q_{R,s} \Psi_s^R &= C^q_{R,s} \Psi_s^R, & q \in \{3, \dots, \chi + 2\}, & s = n', \quad n' \in \mathbb{Z}, \\ e^{-2\pi i s} C^r_{R,s} \Psi_s^R &= -C^r_{R,s} \Psi_s^R, & r \in \{\chi + 3, \dots, N\}, & s = n'' + \frac{1}{2}, \quad n'' \in \mathbb{Z}. \end{cases} \quad (2.15)$$

These constraints are on the dynamics of the theory coming from the monodromy action. We see that the first two components of $C^j_{R,s} \Psi_s^R$ relate to each other, and do not mix with other components of the tensor. The system of equations (2.15) has three solutions.⁷ Either

$$C^1_{R,s} \Psi_s^R = \pm i C^2_{R,s} \Psi_s^R, \quad C^v_{R,s} \Psi_s^R = 0 \quad \forall v \in \{3, \dots, N\}, \quad s = n + \frac{\vartheta}{2\pi}, \quad (2.16)$$

or

$$C^{v'}_{R,s} \Psi_s^R = 0 \quad \forall v' \in \{1, 2, \chi + 3, \dots, N\}, \quad s = n, \quad (2.17)$$

where $C^q_{R,s} \Psi_s^R$, $q \in \{3, \dots, \chi + 2\}$ does not receive any constraints, or

$$C^{v''}_{R,s} \Psi_s^R = 0 \quad \forall v'' \in \{1, \dots, \chi + 2\}, \quad s = n + \frac{1}{2}, \quad (2.18)$$

where $C^r_{R,s} \Psi_s^R$, $r \in \{\chi + 3, \dots, N\}$ does not receive any constraints. Thus the $O(N)$ symmetry has been broken into $O(2) \times O(\chi) \times O(N - \chi - 2)$, where defect-local operators that transform under $O(2)$ have generic $SO(2)$ -spin, defect-local operators that transform under $O(\chi)$ have integer $SO(2)$ -spin and defect-local operators that transform under $O(N - \chi - 2)$ have half-integer $SO(2)$ -spin. Note that the two components of the bulk-defect expansion for the bulk-local operator, $(\phi_2^a) = (\Phi^1, \Phi^2)$, that transforms in $O(2)$ are related through (2.16).

Note 1 If we consider an $O(N)$ -model where the OPE tensors need to be real, the relation (2.16) yields that $C^1_{R,s} \Psi_s^R$ and $C^2_{R,s} \Psi_s^R$ are zero and thus also Φ^1 and Φ^2 are zero. In this case the $O(N)$ symmetry is broken into $O(\chi) \times O(N - \chi - 2)$.

⁶See the ‘‘Proper Rotation’’ section of appendix A for details on this. In this paper we assume that the OPE tensors can be complex-valued.

⁷The solutions can be read off by matching the spin required for the equations to hold.

2.1.2 Improper rotation

The solutions to (2.6) considering the special cases when ϑ equals zero or π will yield similar solutions as those in the proper case. In both of these cases the global $O(N)$ symmetry is broken, leaving a $O(\chi + 1) \times O(N - \chi - 1)$ symmetry. Defect-local operators that transform in $O(\chi + 1)$ will have integer $SO(2)$ -spin, while defect-local operators that transform in $O(N - \chi - 1)$ will have half-integer $SO(2)$ -spin. The procedure of finding this is exactly the same as that discussed in the previous section.

If we consider a generic angle, i.e. $\sin \vartheta \neq 0$, the results will differ from the proper case. The system of equations (2.6) yields⁸

$$\begin{cases} C^1_{R,s} \Psi_s^R &= \frac{\sin(\vartheta)}{e^{-2\pi i s} + \cos(\vartheta)} C^2_{R,s} \Psi_s^R, & s = \frac{n}{2}, & n \in \mathbb{Z}, \\ e^{-2\pi i s} C^q_{R,s} \Psi_s^R &= C^q_{R,s} \Psi_s^R, & q \in \{3, \dots, \chi + 2\}, & s = n', & n' \in \mathbb{Z}, \\ e^{-2\pi i s} C^r_{R,s} \Psi_s^R &= -C^r_{R,s} \Psi_s^R, & r \in \{\chi + 3, \dots, N\}, & s = n'' + \frac{1}{2}, & n'' \in \mathbb{Z}. \end{cases} \quad (2.19)$$

As in the proper case, these are constraints on the OPE tensors coming from the monodromy action. Here it is convenient to define two linear combinations of $C^1_{R,s} \Psi_s^R$ and $C^2_{R,s} \Psi_s^R$

$$\begin{aligned} \tilde{C}^1_{R,s} \Psi_s^R &\equiv C^1_{R,s} \Psi_s^R + \frac{\sin \vartheta}{1 - \cos \vartheta} C^2_{R,s} \Psi_s^R, \\ \tilde{C}^2_{R,s} \Psi_s^R &\equiv C^1_{R,s} \Psi_s^R - \frac{\sin \vartheta}{1 + \cos \vartheta} C^2_{R,s} \Psi_s^R. \end{aligned} \quad (2.20)$$

The system of equations (2.19) have two solutions. Either

$$\tilde{C}^2_{R,s} \Psi_s^R = 0, \quad C^r_{R,s} \Psi_s^R = 0 \quad \forall r \in \{\chi + 3, \dots, N\}, \quad s = n, \quad (2.21)$$

where $\tilde{C}^1_{R,s} \Psi_s^R$ and $C^q_{R,s} \Psi_s^R$, $q \in \{3, \dots, \chi + 2\}$ does not receive any constraints, or

$$\tilde{C}^1_{R,s} \Psi_s^R = 0, \quad C^q_{R,s} \Psi_s^R = 0 \quad \forall q \in \{3, \dots, \chi + 2\}, \quad s = n + \frac{1}{2}. \quad (2.22)$$

where $\tilde{C}^2_{R,s} \Psi_s^R$ and $C^r_{R,s} \Psi_s^R$, $r \in \{\chi + 3, \dots, N\}$ does not receive any constraints. These solutions tells us that the symmetry group has again been broken into $O(\chi + 1) \times O(N - \chi - 1)$, where defect-local operators that transform in $O(\chi + 1)$ have integer $SO(2)$ -spin, while defect-local operators that transform in $O(N - \chi - 1)$ have half-integer $SO(2)$ -spin. The linear combination $\tilde{C}^1_{R,s} \Psi_s^R$ will appear in the bulk-defect expansion of $\phi_{\chi+1}^a$, and $\tilde{C}^2_{R,s} \Psi_s^R$ will appear in the bulk-defect expansion of $\phi_{N-\chi-1}^b$. We can check that this result is correct by representing the $C^j_{R,s} \Psi_s^R$ -terms in the bulk-defect expansions of bulk-local operators that transform in $O(\chi + 1)$ and $O(N - \chi - 1)$ as vectors, $\sigma_{\chi+1}$ and $\sigma_{N-\chi-1}$, both containing N elements. These elements are the coefficients in front of $C^1_{R,s} \Psi_s^R, \dots, C^N_{R,s} \Psi_s^R$, i.e.

$$\begin{aligned} \sigma_{\chi+1} &\equiv (C^1_{R,s}, \dots, C^N_{R,s}) = (1, (1 - \cos \vartheta)^{-1} \sin \vartheta, \underbrace{1, \dots, 1}_{\chi}, \underbrace{0, \dots, 0}_{N-\chi-2}), \\ \sigma_{N-\chi-1} &\equiv (C^1_{R,s}, \dots, C^N_{R,s}) = (1, -(1 + \cos \vartheta)^{-1} \sin \vartheta, \underbrace{0, \dots, 0}_{\chi}, \underbrace{1, \dots, 1}_{N-\chi-2}). \end{aligned} \quad (2.23)$$

⁸See the ‘‘Improper Rotation’’ section of appendix A for details on this.

Since operators that transform in $O(\chi + 1)$ should not mix with operators that transform in $O(N - \chi - 1)$, the two vectors $\sigma_{\chi+1}$ and $\sigma_{N-\chi-1}$ should be orthogonal to each other. Indeed, using the trigonometric identity we see that this is the case. Moreover, these two vectors should be eigenvectors to improper $O(N)$ matrix (2.5). One can check that this is the case, where $\sigma_{\chi+1}$ has eigenvalue $+1$, and $\sigma_{N-\chi-1}$ has eigenvalue -1 . Actually, eigenvectors of improper $g^j_{j'}(\vartheta)$ can only obtain the two different eigenvalues ± 1 , indicating that our results are correct.

Putting it all together, inserting a monodromy defect using a proper $O(2)$ rotation, i.e. $\det R_\vartheta = 1$, possibly (depending on the angle ϑ) breaks the global $O(N)$ symmetry into three parts $O(2) \times O(\chi) \times O(N - \chi - 2)$, where operators that transform in one of these subgroups does not mix with operators from the other subgroups. Each of these bulk-local operators will have a bulk-defect expansion with defect-local operators that transform under the same unbroken subgroup as their corresponding bulk-local operator. The defect-local operators will have different $SO(2)$ -spins depending on what subgroup they transform under. The situation is very similar when considering an improper $O(2)$ rotation, i.e. $\det R_\vartheta = -1$, when defining the defect. In this case however, the global $O(N)$ symmetry (independently of the angle ϑ) breaks into $O(\chi+1) \times O(N-\chi-1)$, meaning that in general, using $\det R_\vartheta = -1$ does not break the symmetry as much as when using $\det R_\vartheta = 1$.

Note 2 *Similar to [43], the monodromy action constrains the spin of defect-local operators.*

The theory is consistent with flipping the defect, i.e. the discussion in this section is the same when we use the following monodromy action

$$\Phi^j(r, \theta - 2\pi, y) = (g^j_{j'})^{-1} \Phi^{j'}(r, \theta, y), \quad g^j_{j'} \in \mathcal{G}. \quad (2.24)$$

2.2 Symmetry constraints

In this section we study constraints from the broken $O(N)$ symmetry. The transformed bulk-local operator, ϕ^j_X , is to be the same as when we transform the defect-local operators, $\psi^{k_1 \dots k_l}_X$, inside the bulk-defect expansion (2.2). Let $\Omega^j_k \in O(X)$ be a transformation matrix from one of the subgroups that is preserved after the global $O(N)$ symmetry has been broken. Then the transformation of ϕ^j_X under Ω^j_k must be compatible with the transformation of $\psi^{k_1 \dots k_l}_X$ under the same Ω^j_k

$$\Omega^j_{j'} \phi^j_X = \sum_{\psi^{k_1 \dots k_l}_X} (C^j_X)_{k'_1 \dots k'_l} \frac{e^{-is_X \theta}}{r^{\Delta_{\phi_X} - \Delta_{\psi_X}}} B_{\Delta_{\psi_X}}(r, \partial_y) \prod_{n=1}^l \Omega^{k'_n}_{k_n} (\psi_X)^{k_1 \dots k_l}. \quad (2.25)$$

Comparing the two sides of this constrains the OPE tensors. It tells us that $(C^j_X)_{k_1 \dots k_l}$ is an isotropic tensor (or tensor invariant) of $O(X)$ ⁹

$$(C^j_X)_{k_1 \dots k_l} = \Omega^{j'}_j (C^{j'}_X)_{k'_1 \dots k'_l} \prod_{n=1}^l \Omega^{k'_n}_{k_n}. \quad (2.26)$$

⁹Remember that the inverse of an $O(X)$ matrix is its own transpose.

Since there are no vector invariants of $O(X)$, there cannot be any scalars on the defect.

A general isotropic tensor of $O(X)$ is given by a sum over all possible permutations of Kronecker deltas [57].¹⁰ Since the defect-local operators are in irreducible representations of $O(X)$, and since tensors with two or more indices in such representations are traceless,¹¹ we end up with only vector-operators on the defect

$$\phi_X^j(r, \theta, y) = \sum_{\psi_X^j} c_X(s_X) \frac{e^{-is_X\theta}}{r^{\Delta_{\phi_X} - \Delta_{\psi_X}}} B_{\Delta_{\psi_X}}(r, \partial_y) \psi_X^j(y). \quad (2.27)$$

Here c_X is an OPE coefficient and ψ_X^j is an $O(X)$ vector primary on the defect. Note that it only exist one vector representation of $O(X)$.

In this section we inserted a monodromy line defect into a three dimensional CFT with a global symmetry. From the monodromy action we found how the global symmetry is broken as well as what kinds of $SO(2)$ -spin the defect-local operators will carry, while from symmetry arguments we found what kinds of defect-local operators can appear in the bulk-defect expansion. All we needed in order to perform this procedure was essentially the bulk-defect expansion (2.2), which can be used for any three-dimensional CFT with bulk-local vector operators and a codimension two defect. Thus we should be able to apply this procedure to other three-dimensional CFTs with other global symmetries as well. It would be interesting to study bulk-defect expansions in d -dimensional CFTs with a monodromy defect of codimension other than two, such that we could perform this procedure to those kinds of theories as well. Note that equation (2.26) should hold for any symmetry preserved by the defect. Thus OPE tensors in bulk-defect expansions will always be isotropic tensors of the global symmetry group their respective bulk-local operators transform under.

3 Green's function

In this section we generalize the steps in [43] to the case with $O(X)$ symmetry. Our starting point for this discussion is Green's function, i.e. the correlator, for two bulk-local operators close to the defect that transform under the same unbroken symmetry group, say $O(X)$. If the bulk-local operators in this correlator would not be close to the defect, this Green's function would be the usual one we encounter in a CFT without a defect. We proceed to find this Green's function from both the bulk-defect expansion and Feynman diagrams.¹²

The WF $O(N)$ model is governed by the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^j)^2 + \frac{\lambda}{4!}[(\Phi^j)^2]^2, \quad j \in \{1, \dots, N\}. \quad (3.1)$$

¹⁰These tensors can also be written as a sum over products of Kronecker deltas, even number of Levi-Civitas and combinations of those two. However, even numbers of Levi-Civitas can be written as a product of several Kronecker deltas. Terms with uneven numbers of Levi-Civitas are not invariants of $O(X)$, but of the smaller group $SO(X)$. We thank Jian Qiu for telling us about this.

¹¹Meaning that if we contract any two of one of these tensors' indices with eachother, it is zero.

¹²Our results from section 2.1 tell us that if the bulk-local operators in the two-point correlators transform in same unbroken symmetry group, the $SO(2)$ -spin in their bulk-defect expansions will be of the same kind, i.e. integer, half-integer or neither.

We renormalize it using dimensional regularization, i.e. we consider $4 - \epsilon$ dimensions. The β -function is given by [58]

$$\beta(\lambda) = \frac{\lambda}{3!} \left(-\epsilon + \frac{N+8}{3!8\pi^2} \lambda \right) + \mathcal{O}(\epsilon^3), \quad (3.2)$$

which have fixed points at

$$\lambda = 0 \quad \text{and} \quad \lambda = \frac{3!8\pi^2\epsilon}{N+8} + \mathcal{O}(\epsilon^2). \quad (3.3)$$

We consider the CFT at the fixed point where the coupling constant is non-zero.

3.1 Green's function from the bulk-defect expansion

Since we expand (in ϵ) around the free theory, the CFT data will not be degenerate, i.e. there will only be one defect-local operator with $\text{SO}(2)$ -spin s_X . Thus we can sum over s_X instead of the defect-local primaries in the bulk-defect expansion (2.27). This yields the full two-point correlator for two bulk-local operators that transforms in $\text{O}(X)$ ¹³

$$\begin{aligned} G^{jj'} &\equiv \langle 0 | \phi_X^j(r_1, \theta_1, y_1) \phi_X^{j'}(r_2, \theta_2, y_2) | 0 \rangle \\ &= \sum_{s_X, s'_X} c_X^\dagger c_X \frac{e^{i(s_X \theta_1 - s'_X \theta_2)}}{r_1^{\Delta_{\phi_X} - \Delta_{\psi_X}} r_2^{\Delta_{\phi_X} - \Delta_{\psi'_X}}} \times \\ &\quad \times [1 + \mathcal{O}(r_1^2 \partial_{y_1}^2) + \mathcal{O}(r_2^2 \partial_{y_2}^2)] \langle 0 | \psi_X^j(y_1) \psi_X^{j'}(y_2) | 0 \rangle, \end{aligned} \quad (3.4)$$

where the $\text{SO}(2)$ -spins, s_X and s'_X , will be of the same kind

$$s_X, s'_X \in \mathbb{Z} + \nu, \quad \nu \in [0, 1). \quad (3.5)$$

Here ν is fixed and the same for both s_X and s'_X since operators with different kinds of $\text{SO}(2)$ -spin do not mix with each other (see section 2.1). The defect-local operators are normalized through its two-point correlator

$$\langle 0 | \psi_X^j(y_1) \psi_X^{j'}(y_2) | 0 \rangle = \frac{\delta^{\psi_X^j \psi_X^{j'}}}{|y_{12}|^{2\Delta_{\psi_X}}}, \quad \delta^{\psi_X^j \psi_X^{j'}} = \delta_{s_X s'_X} \delta^{jj'}, \quad y_{12} \equiv y_1 - y_2. \quad (3.6)$$

We place the bulk-local operators on the same distance from the defect, i.e. $r \equiv r_1 = r_2$

$$G_{s_X}^{jj'} = |c_X|^2 \frac{e^{is_X \theta_{12}}}{r^{2\Delta_{\phi_X}}} \rho^{2\Delta_{\psi_X}} \delta^{jj'} [1 + \mathcal{O}(\rho^2)], \quad \theta_{12} \equiv \theta_1 - \theta_2, \quad \rho \equiv \frac{r}{|y_{12}|}. \quad (3.7)$$

Here $G_{s_X}^{jj'}$ is the summand of (3.4). By comparing this OPE with the result that we will calculate from diagrams at tree-level, we find the zeroth loop order correction to Δ_{ϕ_X} , Δ_{ψ_X} and $|c_X|$. The logarithm of $G_{s_X}^{jj'}$ will be useful when finding correction from one-loop diagrams

$$\log G_{s_X}^{jj'} = (2 \log |c_X| + is_X \theta_{12} - 2\Delta_{\phi_X} \log r + 2\Delta_{\psi_X} \log \rho) \delta^{jj'} + \mathcal{O}(\rho^2). \quad (3.8)$$

Note 3 *Since bulk-local operators will not be affected by the defect if they are far away from it, we expect the CFT data (in our case Δ_{ϕ_X}) for those kind of operators to be the same as the theory without a defect.*

¹³We do not make any assumptions on whether the OPE coefficients, c_X , are real-valued.

3.2 Green's function from Feynman rules

When calculating diagrams using Feynman rules, we calculate one loop order at a time, hence we write Green's function as a sum over loop order corrections, where G_n represents the correction from the n^{th} loop order

$$G^{jj'} = \sum_{n \geq 0} G_n^{jj'} . \tag{3.9}$$

The logarithm of $G^{jj'}$ will be useful when finding first loop order corrections to the CFT data. We Taylor expand the logarithm of the above sum so it later can be compared with the result from the OPE (3.8)

$$\log G^{jj'} = \log G_0^{jj'} + (G_0^{-1})^j_{j''} G_1^{j''j'} + \mathcal{O}(\epsilon^2) . \tag{3.10}$$

3.2.1 Tree-level diagram

The calculation of the tree-level diagram is the same as in [43], but with an overall factor of $\delta^{jj'}$ as well as different spin in the spectrum of defect-local operators. These calculations will be expressed in terms of the dimension of the defect

$$D = 2 - \epsilon . \tag{3.11}$$

Our starting point is the Laplace equation for the two-point correlator

$$-\nabla^2 G_0^{jj'}(x_1, x_2) = \frac{4\pi^{D/2+1}}{\Gamma(D/2)} \delta^{jj'} \delta^D(x_1 - x_2) . \tag{3.12}$$

Note 4 *This Green's function is normalized such that it has the asymptotic*

$$G_0^{jj'}(x_1, x_2) \equiv \langle 0 | \phi_X^j(x_1) \phi_X^{j'}(x_2) | 0 \rangle = \frac{\delta^{jj'}}{|x_1 - x_2|^D} . \tag{3.13}$$

In momentum space, the Laplace equation (3.12) has the solution

$$G_0^{jj'}(x_1, x_2) = \frac{2\pi^{D/2}}{\Gamma(D/2)} \delta^{jj'} \sum_{s_X} \int \frac{d^D k}{(2\pi)^D} e^{is_X \theta_{12}} e^{iky_{12}} I_{|s_X|}(kr_-) K_{|s_X|}(kr_+) , \tag{3.14}$$

$$r_- = \min(r_1, r_2) , \quad r_+ = \max(r_1, r_2) .$$

Here $I_{|s_X|}$ and $K_{|s_X|}$ are modified Bessel functions. Using some relations that the modified Bessel functions satisfy, we can rewrite the summand, $G_{0s_X}^{jj'}$, of $G_0^{jj'}$ as

$$G_{0s_X}^{jj'}(x_1, x_2) = \frac{\Gamma(|s_X| + D/2)}{\Gamma(D/2)\Gamma(|s_X| + 1)} \frac{e^{is_X \theta_{12}}}{(r_1 r_2)^{D/2}} (4\xi)^{-(|s_X| + D/2)} \delta^{jj'} \times$$

$$\times {}_2F_1(|s_X| + D/2, |s_X| + 1/2, 2|s_X| + 1, -\xi^{-1}) , \tag{3.15}$$

$$\xi = \frac{y_{12}^2 + r_{12}^2}{4r_1 r_2} , \quad r_{12} = r_1 - r_2 .$$

Here ${}_2F_1$ is a hyper geometric function, and ξ is one of the two conformally invariant cross-ratios. The other one being the relative angle, θ_{12} , between the two bulk-local operators with respect to the defect [32]. We place the bulk-local operators on the same distance from the defect, i.e. $r \equiv r_1 = r_2$, so we can compare it with the result from the OPE

$$G_{0s_X}^{jj'}(x_1, x_2) = \frac{\Gamma(|s_X| + D/2)}{\Gamma(D/2)\Gamma(|s_X| + 1)} \frac{e^{is_X\theta_{12}}}{r^D} \rho^{2|s_X|+D} \delta^{jj'} [1 + \mathcal{O}(\rho^2)] . \quad (3.16)$$

Comparing this with (3.7) yields¹⁴

$$\begin{aligned} |c_X|_0 &= 1 - \frac{\tilde{\psi}(|s_X| + 1) - \tilde{\psi}(1)}{4} \epsilon + \mathcal{O}(\epsilon^2), \\ \Delta_{\phi_X}^0 &= 1 - \frac{\epsilon}{2}, \quad \Delta_{\psi_X}^0 = |s_X| + 1 - \frac{\epsilon}{2}. \end{aligned} \quad (3.17)$$

Here $\tilde{\psi}(x)$ is the digamma function, $|c_X|_m$ is the m -loop correction to $|c_X|$, and $\Delta_{\phi}^m/\Delta_{\psi}^m$ is the m -loop correction to $\Delta_{\phi_X}/\Delta_{\psi_X}$. We do not have any constraints on whether OPE coefficients in bulk-defect expansions are real.

Note 5 *It is important to remember that in all of the ϵ -expansions in this section, ϵ is not small, but one. Taking ϵ to one is not strictly speaking justified, but it is common practice, and it is known to be a good approximation, e.g. the results in the ϕ^4 case seems to be true by comparing it to numerical data [38, 43].*

3.2.2 One-loop diagram

The two-point, one-loop diagram (not in momentum space) for bulk-local operators on the defect is given by

$$\begin{aligned} G_1^{jj'}(x_1, x_2) &= \sum_{s_X} G_{1s_X}^{jj'}(x_1, x_2), \\ G_{1s_X}^{jj'}(x_1, x_2) &= -\frac{2\lambda}{(2\pi)^4 S} \int_{\mathbb{R}^4} d^4x_0 \left(G_{0s_X}^{jk}(x_1, x_0) G_{0kl}(x_0, x_0) G_{0s_X}^{lj'}(x_0, x_2) + \right. \\ &\quad + G_{0s_X}^{jk}(x_1, x_0) G_{0lk}(x_0, x_0) G_{0s_X}^{lj'}(x_0, x_2) + \\ &\quad \left. + G_{0s_X}^{jk}(x_1, x_0) G_{0l}^l(x_0, x_0) G_{0s_X k}^{j'}(x_0, x_2) \right), \end{aligned} \quad (3.18)$$

$$S = 3!2.$$

Here λ is the coupling constant at the WF fixed point, see (3.3), and S is the symmetry factor. Please note that in each of the terms in $G_{1s_X}^{jj'}$, one of the Green's functions, $G_{0s_X}^{jj''}$, is the whole sum and not only the summand, $G_{0s_X}^{jj''}$, of (3.15). In appendix section B.1 we rewrite $G_{0s_X}^{jj''}$ using hypergeometric function relations

$$\begin{aligned} G_{0s_X}^{jj''}(x_k, x_l) &= e^{is_X\theta_{kl}} \frac{(4r_k r_l)^{|s_X|}}{d_{kl}^- d_{kl}^+ (d_{kl}^- + d_{kl}^+)^{2|s_X|}} \delta^{jj''}, \\ d_{kl}^\pm &= \sqrt{y_{kl}^2 + (r_{kl}^\pm)^2 + z_{kl}^2}, \quad r_{kl}^\pm = r_k \pm r_l. \end{aligned} \quad (3.19)$$

¹⁴Here we have Taylor expanded $|c_X|_0$ around $\epsilon = 0$.

The sum $G_0^{jj''}$ is the propagator for the theory. Renormalization yields that we only need to care about the finite piece of this propagator when we perform the resummation¹⁵

$$G_0^{jj'}(x_0, x_0) = \frac{v(v-1)}{2r_0^2} \delta^{jj'}. \quad (3.20)$$

Inserting $G_{0s_X}^{jj'}$ and $G_0^{jj'}$ back into (3.18) yields

$$G_{1s_X}^{jj'}(x_1, x_2) = -\frac{v(v-1)\lambda}{(2\pi)^4 S} e^{is_X \theta_{12}} \left(2 + \delta^l_l\right) \delta^{jj'} \times \\ \times \int_{\kappa} \frac{dy_0 dz_0 r_0 dr_0 d\theta_0}{r_0^2} \frac{(4rr_0)^{2|s_X|}}{d_{10}^- d_{10}^+ (d_{10}^- + d_{10}^+)^{2|s_X|} d_{02}^- d_{02}^+ (d_{02}^- + d_{02}^+)^{2|s_X|}}, \quad (3.21) \\ \kappa = \{y_0, z_0 \in \mathbb{R}, \quad r_0 \in \{0, \infty\}, \quad \theta_0 \in \{0, 2\pi\}\}.$$

Here we are using cylindrical coordinates and the positions x_1 and x_2 are at the same distance from the defect, i.e. $r \equiv r_1 = r_2$, as well as $z_1 = z_2 = 0$. We rewrite this integral using the variable change

$$y'_0 = y_0 + \frac{y}{2}, \quad y \equiv y_{12}, \quad (3.22)$$

which yields

$$d_{10}^{\pm} \stackrel{(3.22)}{=} \sqrt{\left(y'_0 - \frac{y}{2}\right)^2 + (r_0 \pm r)^2 + z_0^2} \equiv e_{-}^{\pm}, \quad (3.23) \\ d_{02}^{\pm} \stackrel{(3.22)}{=} \sqrt{\left(y'_0 + \frac{y}{2}\right)^2 + (r_0 \pm r)^2 + z_0^2} \equiv e_{+}^{\pm}.$$

Thus

$$G_{1s_X}^{jj'}(x_1, x_2) \stackrel{(3.22)}{=} -\frac{v(v-1)(X+2)\lambda}{(2\pi)^3 S} e^{is_X \theta_{12}} \delta^{jj'} H_{s_X}(r, y), \quad (3.24) \\ H_{s_X}(r, y) = \int_{\mathbb{R}^2} dy'_0 dz_0 \int_0^{\infty} dr_0 \frac{1}{r_0} \frac{(4rr_0)^{2|s_X|}}{e_{-}^{-} e_{-}^{+} e_{+}^{-} e_{+}^{+} (e_{-}^{-} + e_{-}^{+})^{2|s_X|} (e_{+}^{-} + e_{+}^{+})^{2|s_X|}}.$$

The asymptotic of the integral $H_{s_X}(r, y)$ is carefully studied in [43].

$$G_{1s_X}^{jj'}(x_1, x_2) = \frac{v(v-1)(X+2)\epsilon}{(X+8)|s_X|} e^{is_X \theta_{12}} \delta^{jj'} \frac{\rho^{2(|s_X|+1)}}{r^2} \log \rho + \mathcal{O}(\rho^0). \quad (3.25)$$

From (3.10) we know that we can find the first loop order correction to some of the CFT data from $(G_{0s_X}^{-1})^j j'' G_{1s_X}^{j''j}$, with $G_{0s_X}^{jj'}$ from (3.16). Taylor expanding $(G_{0s_X}^{-1})^j j''$ around $\epsilon = 0$

$$(G_{0s_X}^{-1})^j j'' G_{1s_X}^{j''j} = \frac{v(v-1)(X+2)\epsilon}{2(X+8)|s_X|} \delta^{jj'} \log \rho + \mathcal{O}(\rho^0) + \mathcal{O}(\epsilon^2). \quad (3.26)$$

Comparing this with the result from the OPE (3.8) and we find that only Δ_{ψ_X} receives corrections from the one-loop diagram. This correction is given by

$$\Delta_{\psi_X}^1 = \frac{v(v-1)(X+2)\epsilon}{2(X+8)|s_X|}. \quad (3.27)$$

¹⁵Details about this resummation is in appendix section B.2.

Putting it all together, up to one-loop corrections (or up to order ϵ), we have

$$\begin{aligned}
 |c_X| &= \delta^{jj'} - \frac{\tilde{\psi}(|s_X|+1) - \tilde{\psi}(1)}{4} \delta^{jj'} \epsilon + \mathcal{O}(\epsilon^2), \\
 \Delta_{\psi_X} &= |s_X| + 1 - \left(1 - \frac{v(v-1)(X+2)}{(X+8)|s_X|}\right) \frac{\epsilon}{2} + \mathcal{O}(\epsilon^2), \\
 \Delta_{\phi_X} &= 1 - \frac{\epsilon}{2} + \mathcal{O}(\epsilon^2).
 \end{aligned}
 \tag{3.28}$$

Note 6 *As a consistency check, one can see that this reduces to the results in [43] when $X = 1$ and $v = 2^{-1}$*

$$X = 1, \quad v = \frac{1}{2} \quad \Rightarrow \quad \Delta_{\psi} = |s| + 1 - \left(\frac{1}{12|s|} + 1\right) \frac{\epsilon}{2} + \mathcal{O}(\epsilon^2).
 \tag{3.29}$$

4 Rychkov-Tan analysis

In this section we generalize the $O(N)$ framework created in [44] to the WF $O(N)$ model with a co-dimension two, monodromy defect. This approach is very similar to that in [55]. We define three axioms for the theory that contains information about its dynamics. This section will serve as another consistency check on the results (3.28). In this section we need to rescale the bulk-local fields so that they matches the normalization (3.13)

$$\Phi^j \rightarrow \frac{1}{2\pi} \Phi^j.
 \tag{4.1}$$

Axiom 1 *The WF fixed point in the WF $O(N)$ model, see (3.3), is conformally invariant, hence the theory at this point is a CFT.*

Axiom 2 *Correlators in the WF fixed point approach free theory correlators (when the coupling constant is zero) in the limit*

$$\epsilon \rightarrow 0.
 \tag{4.2}$$

This is because the coupling constant at this fixed point is proportional to ϵ . It yields that every operator in the $4 - \epsilon$ dimensional theory tends to operators in the free theory in the above limit.

Axiom 3 *The operators*

$$T_{2p} = \left(\phi_X^k \phi_X^k\right)^p, \quad T_{2p+1}^j = \phi_X^j \left(\phi_X^k \phi_X^k\right)^p, \quad j, k \in \{1, \dots, X\},
 \tag{4.3}$$

are all primary except T_3^j . The equations of motion from (3.1), with the rescaling of bulk-local operators (4.1), tells us that it is a descendant of T_1

$$\alpha T_3^j = \partial_\mu^2 T_1^j, \quad \alpha = \frac{\lambda}{3!(2\pi)^2} = \frac{2\epsilon}{X+8} + \mathcal{O}(\epsilon^2).
 \tag{4.4}$$

We will find T_3^j first from (4.3) using Wick's theorem, and then compare it with the T_3^j that we find from (4.4). The Wick contraction between two bulk-local primaries close to the defect is the propagator (3.20). From (4.3) we find

$$T_3^j = \frac{v(v-1)(X+2)}{2r^2} \phi_X^j + \mathcal{O}(r^0). \quad (4.5)$$

Using the bulk-defect expansion (2.27) of ϕ_X^j

$$T_3^j = \frac{v(v-1)(X+2)}{2} \sum_{s_X} \left(c_X \frac{e^{-is_X\theta}}{r^{\Delta_{\phi_X} - \Delta_{\psi_X} + 2}} \psi_X^j + \mathcal{O}(r^{\Delta_{\phi_X} - \Delta_{\psi_X}}) \right). \quad (4.6)$$

We move on to find T_3^j using (4.4). With cylindrical coordinates

$$T_3^j = \alpha^{-1} \sum_{s_X} \left(c_X \left[(\Delta_{\phi_X} - \Delta_{\psi_X})^2 - s_X^2 \right] \frac{e^{-is_X\theta}}{r^{\Delta_{\phi_X} - \Delta_{\psi_X} + 2}} \psi_X^j + \mathcal{O}(r^{\Delta_{\phi_X} - \Delta_{\psi_X}}) \right).$$

Compare the $r^{-\Delta_{\phi_X} + \Delta_{\psi_X} - 2}$ -terms above with those in (4.6) to get the relation

$$\frac{v(v-1)(X+2)}{2} = \frac{(\Delta_{\phi_X} - \Delta_{\psi_X})^2 - s_X^2}{\alpha}. \quad (4.7)$$

The scaling dimension, Δ_{ϕ_X} , for bulk-local operators is found using the framework for $O(N)$ models from [44]. It is the same as in section 3, see (3.28). If we write Δ_{ψ_X} as a power series in ϵ , we find it to be the same as in section 3 as well.

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A Proper and improper $O(2)$ solutions

In this appendix we solve the first two equations from (2.6) when $\sin \vartheta \neq 0$

$$\begin{cases} e^{-2\pi is} C^1_{R,s} \Psi_s^R = \pm \cos \vartheta C^1_{R,s} \Psi_s^R \mp \sin \vartheta C^2_{R,s} \Psi_s^R, \\ e^{-2\pi is} C^2_{R,s} \Psi_s^R = \sin \vartheta C^1_{R,s} \Psi_s^R + \cos \vartheta C^2_{R,s} \Psi_s^R. \end{cases} \quad (A.1)$$

The first of these equations yields

$$C^1_{R,s} \Psi_s^R = \mp \frac{\sin \vartheta}{e^{-2\pi is} \mp \cos \vartheta} C^2_{R,s} \Psi_s^R. \quad (A.2)$$

Inserting this into the second equation in (A.1) gives us

$$(e^{-2\pi is} - \cos \vartheta) (e^{-2\pi is} \mp \cos \vartheta) = \mp \sin^2 \vartheta. \quad (A.3)$$

This will yield different results depending on whether R_ϑ in (2.5) has determinant one or minus one.

A.1 Proper rotation

A proper R_ϑ , i.e. $\det R_\vartheta = 1$, yields

$$(e^{-2\pi is} - \cos \vartheta)^2 = -\sin^2 \vartheta. \tag{A.4}$$

Solving for s

$$e^{-2\pi is} = \cos \vartheta \pm i \sin \vartheta = e^{\pm i(\vartheta + 2\pi n)}, \quad n \in \mathbb{Z} \quad \Leftrightarrow \quad s = n + \frac{\vartheta}{2\pi}. \tag{A.5}$$

Insert this back into (A.2) and we find the relation

$$C^1_{R,s} \Psi_s^R = \pm i C^2_{R,s} \Psi_s^R. \tag{A.6}$$

A.2 Improper rotation

An improper R_ϑ , i.e. $\det R_\vartheta = -1$, yields

$$(e^{-2\pi is} - \cos \vartheta) (e^{-2\pi is} + \cos \vartheta) = \sin^2 \vartheta. \tag{A.7}$$

Solving for s

$$e^{-4\pi is} = 1 \quad \Leftrightarrow \quad s = \frac{n}{2}, \quad n \in \mathbb{Z}. \tag{A.8}$$

B One-loop diagram integral

If we study the components of the integral (3.18), we can solve it by carefully study its asymptotic expansion. First though, we need to massage the expression for the summand, $G_{0sX}^{jj'}$, and then resum this expression to find the propagator $G_0^{jj'}$. The asymptotic behavior of (3.18) will not be studied here. The interested reader may find details on its asymptotics in [43].

B.1 Summand

We start with the summand $G_{0s}^{jj'}$. We cannot consider $r \equiv r_1 = r_2$, which corresponds to (3.16), since we are integrating over one of the coordinates. Thus we need to massage (3.15) using hypergeometric function relations

$$\begin{aligned} G_{0s}^{jj'}(x_k, x_l) &= \frac{\Gamma(|s| + 1)}{\Gamma(1)\Gamma(|s| + 1)} \frac{e^{is\theta_{kl}}}{r_k r_l} \alpha^{-(|s|+1)} \delta^{jj'} \times \\ &\quad \times {}_2F_1(|s| + 1, |s| + 1/2, 2|s| + 1, -4/\alpha) + \mathcal{O}(\epsilon) \\ &= \frac{e^{is\theta_{kl}}}{r_k r_l} \frac{4^s}{\sqrt{\alpha}\sqrt{4+\alpha}(\sqrt{\alpha} + \sqrt{4+\alpha})^{2|s|}} \delta^{jj'} + \mathcal{O}(\epsilon), \quad \alpha = 4\xi. \end{aligned} \tag{B.1}$$

Taylor expand this expression around $\epsilon = 0$

$$\begin{aligned}
 G_{0s}^{jj'}(x_k, x_l) &= \frac{e^{is\theta_{kl}}}{r_k r_l} \frac{4^s}{(r_k r_l)^{-1} \sqrt{y_{kl}^2 + r_{kl}^2 + z_{kl}^2} \sqrt{4r_k r_l + y_{kl}^2 + r_{kl}^2 + z_{kl}^2}} \times \\
 &\times \frac{1}{(r_k r_l)^{-s} \left(\sqrt{y_{kl}^2 + r_{kl}^2 + z_{kl}^2} + \sqrt{4r_k r_l + y_{kl}^2 + r_{kl}^2 + z_{kl}^2} \right)^{2|s|}} \delta^{jj'} + \\
 &+ \mathcal{O}(\epsilon) \\
 &= e^{is\theta_{kl}} \frac{(4r_k r_l)^{|s|}}{d_{kl}^- d_{kl}^+ (d_{kl}^- + d_{kl}^+)^{2|s|}} \delta^{jj'} + \mathcal{O}(\epsilon), \\
 d_{kl}^\pm &= \sqrt{y_{kl}^2 + (r_{kl}^\pm)^2 + z_{kl}^2}, \quad r_{kl}^\pm = r_k \pm r_l.
 \end{aligned} \tag{B.2}$$

Note 7 The z -components are zero unless it is one of the integration variables in (3.18)

$$z_k = 0 \quad \text{if } k \neq 0. \tag{B.3}$$

B.2 Resummation

The next component in (3.18) that we need to study is the sum $G_0^{jj'}(x_0, x_0)$. This component will be divergent, but we renormalize the theory such that we only care about its finite part. Let us denote

$$x \equiv \sqrt{y_{00}^2 + z_{00}^2} \quad \Rightarrow \quad d_{00}^- = \lim_{x \rightarrow 0} x, \quad d_{00}^+ = \lim_{x \rightarrow 0} \sqrt{(2r_0)^2 + x^2}. \tag{B.4}$$

We consider the defect-local operators in the bulk-defect expansion to have generic spin (3.5) with v fixed (since the operators we study in our Green's function transform in the same unbroken subgroup, $O(X)$, of $O(N)$). Using (B.2)

$$G_0^{jj'}(x_0, x_0) = \lim_{x \rightarrow 0} \frac{\delta^{jj'}}{x \sqrt{(2r_0)^2 + x^2}} \sum_{s \in \mathbb{Z}+v} \left(\frac{2r_0}{\left(x + \sqrt{(2r_0)^2 + x^2} \right)} \right)^{2|s|}. \tag{B.5}$$

Resumming a geometric sum on the form

$$\sum_{s \in \mathbb{Z}+v} \eta^{|s|} = 2 \sum_{s \geq v} \eta^s - \delta_{v0} = [s' = s - v] = 2 \sum_{s' \geq 0} \eta^{s'+v} - \delta_{v0} = \frac{2\eta^v}{1 - \eta} - \delta_{v0}, \tag{B.6}$$

and using the following Taylor expansions

$$\frac{1}{\sqrt{(2r_0)^2 + x^2}} = \frac{1}{2r_0} + \mathcal{O}(x^2), \tag{B.7}$$

$$\begin{aligned}
 \frac{1}{\sqrt{(2r_0)^2 + x^2}} \frac{\left(x + \sqrt{(2r_0)^2 + x^2} \right)^{-2v}}{1 - (2r_0)^2 \left(x + \sqrt{(2r_0)^2 + x^2} \right)^{-2}} &= \\
 &= \frac{1}{(2r_0)^{2v}} \left(\frac{1}{2x} + \frac{1 - 2v}{2(2r_0)} + \frac{v(v-1)}{(2r_0)^2} x \right) + \mathcal{O}(x^2),
 \end{aligned} \tag{B.8}$$

yields

$$G_0^{jj'}(x_0, x_0) = \delta^{jj'} \left(\lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{x} + \frac{1 - 2v - \delta_{v0}}{2r_0} \right) + \frac{v(v-1)}{2r_0^2} \right). \quad (\text{B.9})$$

We renormalize the theory such that we can ignore the divergent part (x^{-2} - and x^{-1} -terms) in the above propagator. This propagator is correct since we reproduce the result from [43], with an overall factor of $\delta^{jj'}$, in the half-integer case ($v = 1/2$), i.e.

$$G_0^{jj'}(x_0, x_0) = \delta^{jj'} \left(\lim_{x \rightarrow 0} \frac{1}{x^2} - \frac{1}{8r_0^2} \right). \quad (\text{B.10})$$

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