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New A_4 lepton flavor model from S_4 modular symmetry

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Abstract: We study a flavor model with A_4 symmetry which originates from S_4 modular group. In S_4 symmetry, Z_2 subgroup can be anomalous, and then S_4 can be violated to A_4. Starting with a S_4 symmetric Lagrangian at the tree level, the Lagrangian at the quantum level has only A_4 symmetry when Z_2 in S_4 is anomalous. We obtain modular forms of two singlets and a triplet representations of A_4 by decomposing S_4 modular forms into A_4 representations. We propose a new A_4 flavor model of leptons by using those A_4 modular forms. We succeed in constructing a viable neutrino mass matrix through the Weinberg operator for both normal hierarchy (NH) and inverted hierarchy (IH) of neutrino masses. Our predictions of the CP violating Dirac phase $\delta_{\rm CP}$ and the mixing $\sin^2\theta_{23}$ depend on the sum of neutrino masses for NH.

Keywords: Discrete Symmetries, Neutrino Physics, Compactification and String Models, CP violation

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Contents

1 Introduction

The origin of the flavor structure is one of important issues in particle physics. The recent development of the neutrino oscillation experiments provides us important clues to investigate the flavor physics. Indeed, the neutrino oscillation experiments have presented two large flavor mixing angles, which is a contrast to the quark mixing angles. In addition to the precise measurements of the flavor mixing angles of leptons, the T2K and NOνA strongly indicate the CP violation in the neutrino oscillation [1, 2]. We are in the era to develop the flavor theory of leptons with the observation of flavor mixing angles and CP violating phase.

It is interesting to impose non-Abelian discrete symmetries for flavors. In the last twenty years, the studies of discrete symmetries for flavors have been developed through the precise observation of flavor mixing angles of leptons [3–11]. Many models have been proposed by using the non-Abelian discrete groups S_3, A_4, S_4, A_5 and other groups with larger orders to explain the large neutrino mixing angles. Among them, A_4 flavor symmetry is attractive because A_4 group is the minimal one including a triplet irreducible representation. A triplet representation allows us to give a natural explanation of the existence of three families of leptons [12–18]. However, a variety of models is so wide that it is difficult to obtain a clear evidence of the A_4 flavor symmetry.

Superstring theory is a promising candidate for the unified theory of all interactions including gravity and matter fields such as quarks and leptons as well as the Higgs field. Superstring theory predicts six-dimensional compact space in addition to four-dimensional

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space-time. Geometrical aspects, i.e. the size and shape of the compact space, are described by moduli parameters. Gauge couplings and Yukawa couplings as well as higher order couplings in four-dimensional low-energy effective field theory depend on moduli parameters. A geometrical symmetry of the six-dimensional compact space can be the origin of the flavor symmetry.[1]

The torus compactification as well as the orbifold compactification has the modular symmetry Γ.[2] It is interesting that the modular symmetry includes $\Gamma_2 \simeq S_3$, $\Gamma_3 \simeq A_4$, $\Gamma_4 \simeq S_4$, $\Gamma_5 \simeq A_5$ as finite groups [24]. Inspired by these aspects, recently a new type of flavor models was proposed [25]. In ref. [25], the A_4 flavor symmetry is assumed as a finite group of the modular symmetry. Three families of leptons are assigned to certain A_4 representations like conventional flavor models. Furthermore, Yukawa couplings as well as Majorana masses are assumed to be modular forms which are functions of the modular parameter τ and they are non-trivial representations under A_4. We have a modular form of A_4 triplet with weight 2 [25]. The flavor symmetry A_4 is broken when the value of the modular parameter τ is fixed. It is noted that one can construct flavor models without flavon fields.

The modular forms of the weight 2 have been constructed for the S_3 doublet [26], the S_4 triplet and doublet [27], and the A_5 quintet and triplets [28], as well as the $\Delta(96)$ triplet and the $\Delta(384)$ triplet [29]. The modular forms of the weight 1 and higher weights are also given for T' doublet [30]. By use of these modular forms, new flavor models have been constructed [31–44].

Discrete symmetries can be anomalous [45–47]. Anomalies of non-Abelian symmetries were studied in [48]. (See also [4, 5].) The anomaly of the modular symmetry was also discussed [49]. In the S_4 symmetry, the Z_2 subgroup can be anomalous and then S_4 can be violated to A_4. The A_5 symmetry is always anomaly-free. Both S_3 and A_4 can be anomalous, and then they can be violated to Abelian discrete symmetries. Thus, the S_4 is unique among S_3, A_4, S_4, A_5 in the sense that it can be violated by anomalies to another non-Abelian symmetry, A_4. Even starting with a S_4 symmetric Lagrangian at the tree level, the Lagrangian at the quantum level has only the A_4 symmetry when Z_2 subgroup of S_4 is anomalous. Our purpose is to show such a possibility in a phenomenological viewpoint. We decompose S_4 modular forms into A_4 representations. Such modulus functions are different from the modular forms in Γ_3. We propose a new A_4 flavor model with those A_4 modular forms, which is much different from the typical modular A_4 models [25, 31, 32].

This paper is organized as follows. In section 2, we give a brief review on the modular symmetry and the S_4 anomaly. In section 3, we present our model for lepton mass matrices. In section 4, we show our numerical results for lepton mixing angles, the CP violating Dirac phase and neutrino masses. Section 5 is devoted to a summary. Relevant representations of S_4 and A_4 groups are presented in appendix A. We list the input data of neutrinos in appendix B.

[1] It was shown that stringy selection rules in addition to geometrical symmetries lead to certain non-Abelian flavor symmetries [19–22].

[2] For example, zero-modes in the torus compactification with magnetic fluxes transform non-trivially under the modular symmetry [23].

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2 Modular symmetry and S_4 anomaly

2.1 Modular forms

We give a brief review on the modular symmetry and modular forms. The torus compactification is the simplest compactification. We consider a two-dimensional torus which can be constructed as a division of the two-dimensional real space $\mathbb{R}^2$ by a lattice Λ, i.e. $T^2 = \mathbb{R}^2/\Lambda$. We use the complex coordinate on $\mathbb{R}^2$. The lattice Λ is spanned by two vectors, $\alpha_1 = 2\pi R$ and $\alpha_2 = 2\pi R\tau$, where R is a real and τ is a complex modulus parameter. The same lattice is spanned by the following lattice vectors,

$$\begin{pmatrix} \alpha_2' \\ \alpha_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix}, \tag{2.1}$$

where a, b, c, d are integer with satisfying $ad - bc = 1$. That is, the $\mathrm{SL}(2,\mathbb{Z})$ symmetry. Under $\mathrm{SL}(2,\mathbb{Z})$, the modulus parameter transforms

$$\tau \longrightarrow \tau' = \gamma\tau = \frac{a\tau + b}{c\tau + d}. \tag{2.2}$$

This modular symmetry is generated by two elements, S and T, which transform τ as

$$S : \tau \longrightarrow -\frac{1}{\tau}, \qquad T : \tau \longrightarrow \tau + 1. \tag{2.3}$$

They satisfy the following algebraic relations,

$$S^2 = (ST)^3 = \mathbb{I}. \tag{2.4}$$

If we impose the algebraic relation $T^N = \mathbb{I}$, we obtain the finite groups Γ_N for $N = 2, 3, 4, 5$, and these are isomorphic to S_3, A_4, S_4, A_5, respectively. We define the congruence subgroups of level N as

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}), \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}. \tag{2.5}$$

For $N = 2$, we define $\bar{\Gamma}(2) \equiv \Gamma(2)/\{\mathbb{I}, -\mathbb{I}\}$. Since the element $-\mathbb{I}$ does not belong to $\Gamma(N)$ for $N > 2$, we have $\bar{\Gamma}(N) = \Gamma(N)$. The quotient groups defined as $\Gamma_N \equiv \bar{\Gamma}/\bar{\Gamma}(N)$ are finite modular groups.

Modular forms of weight k are the holomorphic functions of τ and transform as

$$f_i(\tau) \longrightarrow (c\tau + d)^k \rho(\gamma)_{ij} f_j(\gamma\tau), \tag{2.6}$$

where $\rho(\gamma)_{ij}$ is a unitary matrix. Also, matter fields $\phi^{(I)}$ with the modular weight k_I transform

$$(\phi^{(I)})_i(x) \longrightarrow (c\tau + d)^{k_I} \rho(\gamma)_{ij} (\phi^{(I)})_j(x), \tag{2.7}$$

under the modular symmetry.

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In ref. [27], the modular form of the level $N = 4$ for $\Gamma_4 \simeq S_4$ have been constructed with the Dedekind eta function, $\eta(\tau)$,

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n)\,, \tag{2.8}$$

where $q = e^{2\pi i\tau}$. The modular forms of the weight 2 are written by

$$\begin{aligned}
Y_1(\tau) &= Y(1, 1, \omega, \omega^2, \omega, \omega^2|\tau),\\
Y_2(\tau) &= Y(1, 1, \omega^2, \omega, \omega^2, \omega|\tau),\\
Y_3(\tau) &= Y(1, -1, -1, -1, 1, 1|\tau),\\
Y_4(\tau) &= Y(1, -1, -\omega^2, -\omega, \omega^2, \omega|\tau),\\
Y_5(\tau) &= Y(1, -1, -\omega, -\omega^2, \omega, \omega^2|\tau),
\end{aligned} \tag{2.9}$$

where $\omega = e^{2\pi i/3}$ and

$$\begin{aligned}
Y(a_1, a_2, a_3, a_4, a_5, a_6|\tau) \,=\, & a_1 \frac{\eta'(\tau + 1/2)}{\eta(\tau + 1/2)} + 4a_2 \frac{\eta'(4\tau)}{\eta(4\tau)}\\
& + \frac{1}{4} \sum_{m=0}^{3} a_{m+3} \frac{\eta'((\tau + m)/4)}{\eta((\tau + m)/4)}.
\end{aligned} \tag{2.10}$$

These five modular forms are decomposed into the $\mathbf{3'}$ and $\mathbf{2}$ representations under S_4,

$$Y_{S_4\mathbf{2}}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}, \qquad Y_{S_4\mathbf{3'}}(\tau) = \begin{pmatrix} Y_3(\tau) \\ Y_4(\tau) \\ Y_5(\tau) \end{pmatrix}. \tag{2.11}$$

The generators, S and T, are represented on the above modular forms,

$$\rho(S) = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{2.12}$$

for $\mathbf{2}$, and

$$\rho(S) = -\frac{1}{3} \begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega & 2 & -\omega^2 \\ 2\omega^2 & -\omega & 2 \end{pmatrix}, \qquad \rho(T) = -\frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}, \tag{2.13}$$

for $\mathbf{3'}$. The modular form of larger weights are obtained as products of $Y_{S_4\mathbf{2}}(\tau)$ and $Y_{S_4\mathbf{3'}}(\tau)$. Other representations are shown in appendix A.

2.2 Anomaly

A discrete symmetry can be anomalous. Each element g in a non-Abelian symmetry satisfies $g^N = 1$, that is, the Abelian Z_N symmetry. If all of such Abelian symmetries in a non-Abelian symmetry are anomaly-free, the whole non-Abelian symmetry is anomaly-free. Otherwise, the non-Abelian symmetry is anomalous, and anomalous sub-group is violated.

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Furthermore, each element g is represented by a matrix $\rho(g)$. If $\det\rho(g) = 1$, the corresponding Z_N is always anomaly-free. On the other hand, if $\det\rho(g) \neq 1$, the corresponding Z_N symmetry can be anomalous. See anomalies of non-Abelian symmetries [4, 5, 48].

In particular, in refs. [4, 5], it shows which sub-groups can be anomalous in non-Abelian discrete symmetries. The S_4 group is isomorphic to $(Z_2 \times Z_2) \rtimes S_3$, and then the Z_2 symmetry of S_3 can be anomalous in S_4. In general, the $\mathbf{2}$ and $\mathbf{3}$ representations as well as $\mathbf{1}'$ have $\det\rho(g) = -1$ while the $\mathbf{1}$ and $\mathbf{3}'$ representations have $\det\rho(g) = 1$. Indeed $\rho(S)$ and $\rho(T)$ for $\mathbf{2}$ as well as $\mathbf{3}$ and $\mathbf{1}'$ have $\det(\rho(S)) = \det(\rho(T)) = -1$. Thus, the odd number of $\mathbf{2}$'s as well as $\mathbf{3}$ and $\mathbf{1}'$ can lead to anomalies.

If the above Z_2 symmetry in S_4 is anomalous, S_4 is violated to A_4. In this case, S and T themselves are anomalous, but $\tilde{S} = T^2$ and $\tilde{T} = ST$ are anomaly-free. These anomaly-free elements satisfy

$$(\tilde{S})^2 = (\tilde{S}\tilde{T})^3 = (\tilde{T})^3 = \mathbb{I}, \tag{2.14}$$

if we impose $T^4 = \mathbb{I}$. That is, the A_4 algebra is realized. The explicit representations of generators $\tilde{S}$ and $\tilde{T}$ for the A_4 triplet and singlets are presented in appendix A. The modular forms for S_4 act under the A_4 symmetry as follows:

$$Y_{S_4\mathbf{2}}(\tau) \to (\ Y_{A_4\mathbf{1}''}(\tau),\ Y_{A_4\mathbf{1}'}(\tau)\ )\,, \qquad Y_{S_4\mathbf{3}'}(\tau) \to Y_{A_4\mathbf{3}}(\tau)\ . \tag{2.15}$$

That is, we have

$$Y_{A_4\mathbf{1}'}(\tau) = Y_2(\tau), \qquad Y_{A_4\mathbf{1}''}(\tau) = Y_1(\tau), \qquad Y_{A_4\mathbf{3}}(\tau) = \begin{pmatrix} Y_3(\tau) \\ Y_4(\tau) \\ Y_5(\tau) \end{pmatrix}. \tag{2.16}$$

Note that these are different from modular forms of the level $N = 3$ for Γ_3 because they do not transform as A_4 multiplets under S and T.

Anomalies of the S_4 symmetry, in particular its Z_2 sub-symmetry, depend on models, that is, the numbers of $\mathbf{2}$, $\mathbf{3}$ and $\mathbf{1}'$. If the S_4 symmetry is anomaly-free and exact, the model building follows the study in ref. [27] and its extension. If the S_4 is anomalous and violated to A_4, that leads to a new type of model building. In the next section, we study such a new possibility for lepton mass matrices.

3 A_4 lepton model from S_4 modular symmetry

We present a viable A_4 model of leptons originated from the subgroup of S_4 group. The charge assignment of the fields and modular forms is summarized in table 1. We assign the modular weight -1 to the left- and right-handed leptons. If the S_4 is exact, $\mu^c_{\mathbf{1}''}$ and $\tau^c_{\mathbf{1}'}$ are combined to the S_4 doublet. The odd number of doublets can lead to anomalies.

The modular forms of weight 2 that transform non-trivially under the A_4 symmetry are given in S_4 modular group as discussed in section 2. The A_4 triplet $Y_{A_4\mathbf{3}}$ and non-trivial A_4 singlets $Y_{A_4\mathbf{1}'}$, $Y_{A_4\mathbf{1}''}$ are constructed by five modular forms in eq. (2.9), which is a difference from the $\Gamma_3 \simeq A_4$ modular symmetry with three modular forms.

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	$L_{\mathbf{3}}$	$e^c_{\mathbf{1}}, \mu^c_{\mathbf{1}''}, \tau^c_{\mathbf{1}'}$	$H_{u,d}$	$Y_{A_4\mathbf{3}}$	$Y_{A_4\mathbf{1}'}$	$Y_{A_4\mathbf{1}''}$
SU(2)	2	1	2	1	1	1
A_4	3	$1\,,1''\,,1'$	1	3	$1'$	$1''$
$-k_I$	-1	-1	0	$k=2$	$k=2$	$k=2$

Table 1. The charge assignment of SU(2), A_4, and the modular weight ($-k_I$ for fields and k for coupling Y).

Suppose neutrinos to be Majorana particles. The superpotential of the neutrino mass term is given by the Weinberg operator:

$$w_\nu = \frac{1}{\Lambda}\left[Y_{A_4\mathbf{3}} + aY_{A_4\mathbf{1}''} + bY_{A_4\mathbf{1}'}\right] L_{\mathbf{3}} L_{\mathbf{3}} H_u H_u, \tag{3.1}$$

where $L_{\mathbf{3}}$ denote the A_4 triplet of the left-handed lepton doublet, $(L_e, L_\mu, L_\tau)^T$, and H_u stands for the Higgs doublet which couples to the neutrino sector. Parameters a and b are complex constants in general. If the S_4 symmetry is exact, $Y_{A_4\mathbf{1}'}$ and $Y_{A_4\mathbf{1}''}$ are combined to the S_4 doublet $Y_{S_4\mathbf{2}}$. That is, the second and third terms are originated from $aY_{S_4\mathbf{2}} LLH_uH_u$, where L is taken to be $\mathbf{3}'$ of S_4, and we have $a = b$. Breaking of S_4 to A_4 leads to the above terms with $a \neq b$. One naively expects to be $a \sim b$, although their difference depends on breaking effects. At any rate, we treat them as independent parameters from the phenomenological viewpoint. We also discuss the situation with $a \sim b$.

The superpotential of the mass term of charged leptons is described as

$$w_e = \left[\alpha e^c_{\mathbf{1}} + \beta \mu^c_{\mathbf{1}''} + \gamma \tau^c_{\mathbf{1}'}\right] Y_{A_4\mathbf{3}} L_{\mathbf{3}} H_d, \tag{3.2}$$

where charged leptons $e^c_{\mathbf{1}}, \mu^c_{\mathbf{1}''}, \tau^c_{\mathbf{1}'}$ are assigned to the A_4 singlets of $\mathbf{1}, \mathbf{1}'', \mathbf{1}'$ respectively. The H_d is a Higgs doublet which couples to the charged lepton sector. Coefficients α, β and γ can be taken to be real. Then, charged lepton masses are given in terms of τ, $\langle H_d\rangle$, α, β and γ. Similar to eq. (3.1), if the S_4 is exact, $\mu^c_{\mathbf{1}''}$ and $\tau^c_{\mathbf{1}'}$ are combined to the S_4 doublet. That is, we have to require $\beta = \gamma$. Here, we also treat these parameters as independent parameters from the phenomenological viewpoint.

The relevant mass matrices are given by using the multiplication rules based on $\tilde{S}$ and $\tilde{T}$ in appendix A. The Majorana neutrino mass matrix is:

$$M_\nu = \frac{\langle H_u\rangle^2}{\Lambda}\left[\begin{pmatrix} 2Y_3 & -Y_5 & -Y_4 \\ -Y_5 & 2Y_4 & -Y_3 \\ -Y_4 & -Y_3 & 2Y_5 \end{pmatrix} + aY_1 \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + bY_2 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\right], \tag{3.3}$$

while the charged lepton matrix is given as:

$$M_e = \langle H_d\rangle \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} Y_3 & Y_5 & Y_4 \\ Y_4 & Y_3 & Y_5 \\ Y_5 & Y_4 & Y_3 \end{pmatrix}_{RL}, \tag{3.4}$$

where α, β and γ are taken to be real positive without loss of generality.

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4 Numerical result

We discuss numerical results for the lepton flavor mixing by using eqs. (3.3) and (3.4). Parameters of the model are α, β, and γ of the charge lepton mass matrix; and a and b of the neutrino mass matrix in addition to modulus τ. Parameters α, β, and γ are real while a and b are complex in general. However, we take a and b to be real in order to present a simple viable model, that is to say, the CP violation comes from modular forms in section 2. Parameters α, β, and γ are given in terms of τ after inputting three charged lepton masses. Therefore, we scan the parameters in the following ranges as:

$$\tau = [-2.0, 2.0] + i[0.1, 2.8], \quad a = [-15, 15], \quad b = [-15, 15], \tag{4.1}$$

where the fundamental domain of $\Gamma(4)$ is taken into account. The fundamental region is shown in figure 5. The lower-cut 0.1 of $\mathrm{Im}[\tau]$ is artificial to keep the accurate numerical calculation. The upper-cut 2.8 is enough large to estimate the modular forms.

We input the experimental data within $3\,\sigma$ C.L. [51] of three mixing angles in the lepton mixing matrix [52] in order to constrain magnitudes of parameters. We also put the two observed neutrino mass square differences ($\Delta m^2_{\rm sol}$, $\Delta m^2_{\rm atm}$) and the cosmological bound for the neutrino masses $\sum m_i < 0.12\,[\mathrm{eV}]$ [53, 54]. Since parameters are severely restricted due to experimental data, the Dirac phase $\delta_{\rm CP}$ is predicted. Furthermore, we also discuss the effective mass of the $0\nu\beta\beta$ decay $\langle m_{ee}\rangle$:

$$\langle m_{ee}\rangle = \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31} - 2\delta_{\rm CP})} \right| , \tag{4.2}$$

where α_{21} and α_{31} are Majorana phases defined in ref. [52].

There are two possible spectra of neutrinos masses m_i, which are the normal hierarchy (NH), $m_3 > m_2 > m_1$, and the inverted hierarchy (IH), $m_2 > m_1 > m_3$. At first, we show the predicted region of $\sin^2\theta_{23}$–$\delta_{\rm CP}$ in figure 1, where cyan-points and red-points denote cases of NH and IH, respectively. For NH of neutrino masses, the predicted $\delta_{\rm CP}$ is $|\delta_{\rm CP}| < 70^\circ$ and $|\delta_{\rm CP}| = 135^\circ$–$160^\circ$. It is noticed that $|\delta_{\rm CP}| \simeq 90^\circ$ is excluded. The prediction of $\delta_{\rm CP}$ becomes clear if $\sin^2\theta_{23}$ is precisely measured. Indeed, $\delta_{\rm CP}$ is predicted around $\pm 40^\circ$ and $\pm 140^\circ$ at the observed best fit point of $\sin^2\theta_{23} = 0.582$ [51].

For IH of neutrino masses, the predicted $\delta_{\rm CP}$ is $|\delta_{\rm CP}| = 40^\circ$–$70^\circ$ and $|\delta_{\rm CP}| = 110^\circ$–$180^\circ$. It is found that $|\delta_{\rm CP}| \simeq 90^\circ$ is also excluded for IH.

We present the prediction of the effective mass of the $0\nu\beta\beta$ decay, $\langle m_{ee}\rangle$ versus the lightest neutrino mass for both NH and IH of neutrino masses in figure 2. The upper-bound of the lightest neutrino mass is given by the cosmological upper-bound of the sum of neutrino masses. For NH, the lower-bound of the lightest neutrino mass is 12 [meV]. The predicted range of $\langle m_{ee}\rangle$ is 5–22 [meV] depending on the lightest neutrino mass. For IH, $\langle m_{ee}\rangle$ is predicted in 15–30 [meV]. Hence, the $0\nu\beta\beta$ decay will be possibly observed in the future [55].

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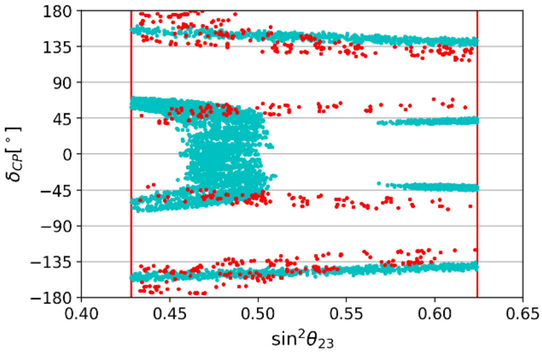

Figure 1. Predicted $\delta_{\rm CP}$ versus $\sin^2\theta_{23}$, where cyan-points and red-points denote cases of NH and IH, respectively. The vertical red lines denote 3σ interval of data.

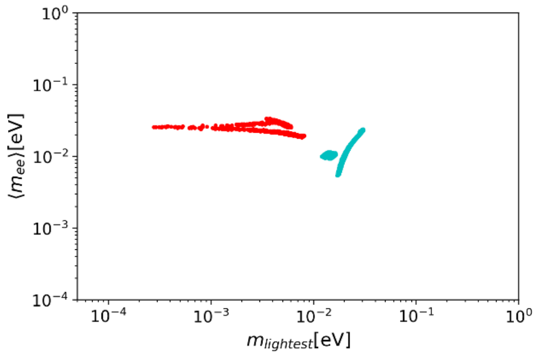

Figure 2. Predicted $\langle m_{ee}\rangle$ versus the lightest neutrino mass, where cyan-points and red-points denote cases of NH and IH, respectively. The cosmological bound of $\sum m_i$ is imposed.

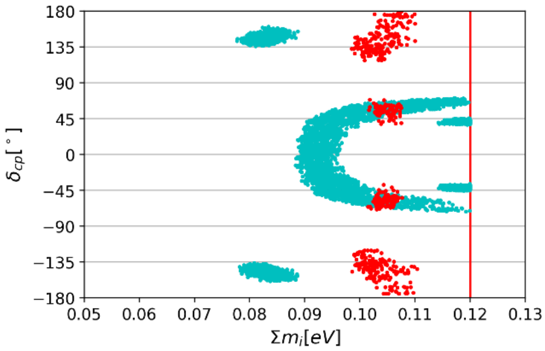

Figure 3. Predicted $\delta_{\rm CP}$ versus $\sum m_i$, where cyan-points and red-points denote cases of NH and IH, respectively. The vertical red line denotes the cosmological upper-bound.

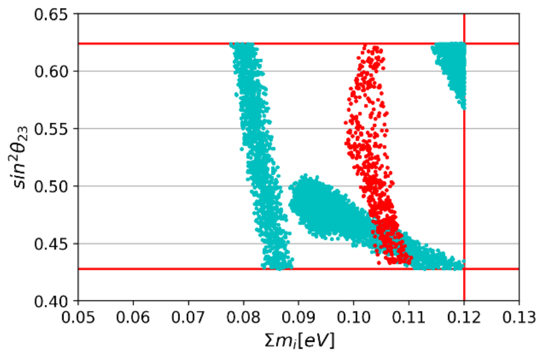

Figure 4. Predicted $\sin^2\theta_{23}$ versus $\sum m_i$. The notation is the same as in figure 3. Horizontal red lines denote 3σ interval of the experimental data.

Let us discuss the neutrino mass dependence of $\delta_{\rm CP}$ and $\sin^2\theta_{23}$. We present the predicted $\delta_{\rm CP}$ versus the sum of neutrino masses $\sum m_i$ in figure 3, where the cosmological bound $\sum m_i < 120\,[\text{meV}]$ is imposed. The predicted $\delta_{\rm CP}$ depends on the sum of neutrino masses, where $\sum m_i > 78\,[\text{meV}]$ for NH of neutrino masses. In the range of $78 < \sum m_i < 88\,[\text{meV}]$, $\delta_{\rm CP} \simeq \pm(135^\circ\text{–}160^\circ)$ is predicted. In the range of $\sum m_i > 88\,[\text{meV}]$, we obtain $|\delta_{\rm CP}| < 70^\circ$). For IH, the sum of neutrino mass is predicted for $98\,[\text{meV}] < \sum m_i < 110\,[\text{meV}]$ with $|\delta_{\rm CP}| > 110^\circ$ or $40^\circ < |\delta_{\rm CP}| < 70^\circ$.

The predicted $\sin^2\theta_{23}$ is also presented versus $\sum m_i$ in figure 4. In the case of NH, the observed best fit point of $\sin^2\theta_{23} = 0.582$ [51] is realized at $\sum m_i = 80\text{–}85\,[\text{meV}]$. For IH, we get $\sum m_i = 100\text{–}105\,[\text{meV}]$ for the best fit point of $\sin^2\theta_{23} = 0.582$. Hence, the observation of the sum of neutrino masses in the cosmology will provide a severe constraint to the flavor model.

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We present the set of best-fit parameters and observables. For NH, we obtain:

$$\begin{aligned}
&\tau = -1.717 + 0.5852i, \qquad a = 0.2178, \qquad b = -1.141, \\
&\alpha v_d = 1.73\times 10^5\,\mathrm{eV}, \qquad \beta v_d = 4.64\times 10^8\,\mathrm{eV}, \qquad \gamma v_d = 3.34\times 10^7\,\mathrm{eV}, \\
&\sin^2\theta_{12} = 0.299, \qquad \sin^2\theta_{23} = 0.587, \qquad \sin^2\theta_{13} = 0.0228, \qquad \delta_{\rm CP} = -142.9^\circ, \\
&\Delta m_{21}^2 = 7.38\times 10^{-5}\,\mathrm{eV}^2, \qquad \Delta m_{31}^2 = 2.54\times 10^{-3}\,\mathrm{eV}^2\,, \\
&\langle m_{ee}\rangle = 13.4\,\mathrm{meV}, \qquad \sum m_i = 81.5\,\mathrm{meV},
\end{aligned} \tag{4.3}$$

where $\chi^2 = 0.31$. For IH, we have:

$$\begin{aligned}
&\tau = -1.508 + 1.288i, \qquad a = -1.230, \qquad b = -3.616, \\
&\alpha v_d = 7.01 \times 10^7\,\mathrm{eV}, \qquad \beta v_d = 1.04 \times 10^9\,\mathrm{eV}, \qquad \gamma v_d = 3.57 \times 10^5\,\mathrm{eV}, \\
&\sin^2\theta_{12} = 0.292, \qquad \sin^2\theta_{23} = 0.586, \qquad \sin^2\theta_{13} = 0.0227, \qquad \delta_{\rm CP} = -133.4^\circ, \\
&\Delta m_{21}^2 = 7.16 \times 10^{-5}, \qquad \Delta m_{31}^2 = -2.51 \times 10^{-3}\,, \\
&\langle m_{ee}\rangle = 27.7\,\mathrm{meV}, \qquad \sum m_i = 101.5\,\mathrm{meV},
\end{aligned} \tag{4.4}$$

where $\chi^2 = 0.70$.

We show the allowed region of Re[τ]–Im[τ] in figure 5, where cyan-points and red-points denote the NH and IH cases, respectively. The fundamental domain of $\Gamma(4)$ is also presented by olive-green in this figure, where the real part of τ is $[-2, 2]$ and the imaginary part of τ is expanded downward. Some points are outside of the fundamental domain of $\Gamma(4)$. Those points are transformed into the inside of the fundamental domain by the S_4 transformations. In this figure, the $\tau \to \tau + 1$ shift symmetry of eq. (2.3) is clearly seen. In order to show $\tau \to -1/\tau$ symmetry of eq. (2.3), we plot one pair by small white triangles. It is seen that the plotted points (red) on Re[τ] = 0.5 inside the fundamental region of $\mathrm{SL}(2, \mathbb{Z})$ are converted to the points on the circles.

We show the allowed region of a–b in figure 6. The magnitudes of a and b are found to be of order one for both NH and IH, which is consistent with the conventional A_4 flavor model [16]. It is noticed that the $(a, b) = (0, 0)$ point is excluded. That is to say, we need either singlet modular forms of $1'$ or $1''$ in order to reproduce the experimental data of leptons in appendix B. One naively expects to be $a \simeq b$ since $\Gamma_4 \simeq S_4$ is broken to A_4 due to quantum effects (anomaly) as discussed in section 3. Obtained values of a and b deviate from $a \simeq b$ as seen in eq. (4.3) for NH while the desirable region of $a \simeq b$ exists as seen in eq. (4.4) for IH. Thus, we should discuss the magnitude of the S_4 breaking beyond the naive expectation. However, it is out of scope in this paper.

In our work, we take a and b to be real in a simple viable model. Our predicted regions of $\delta_{\rm CP}$ and $\langle m_{ee}\rangle$ are possibly enlarged if a and b are complex. Whereas, it is worthwhile to discuss the case of real a and b because the case is attractive in the context of the generalized CP violation of modular-invariant flavor model [39].

Finally, we also comment on numerical values on α, β and γ of the charged lepton mass matrix. These ratios are typically $\gamma/\beta = \mathcal{O}(0.1)$ and $\alpha/\beta = \mathcal{O}(10^{-4})$ for the case of NH as seen in eq. (4.3). The value of α is much smaller than β and γ, on the other hand, we need

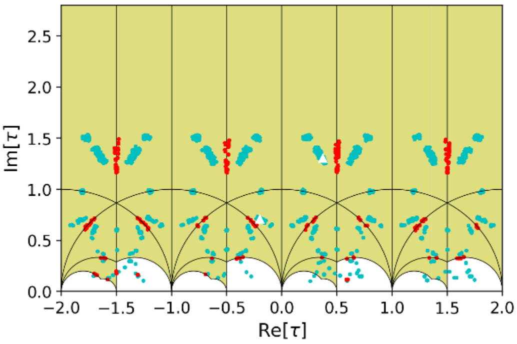

Figure 5. Allowed region on the Re[τ]–Im[τ] plane. The fundamental domain of $\Gamma(4)$ are shown by olive-green. Cyan-points and red-points denote cases of NH and IH, respectively. Two small white triangles denote a pair connecting by the S symmetry.

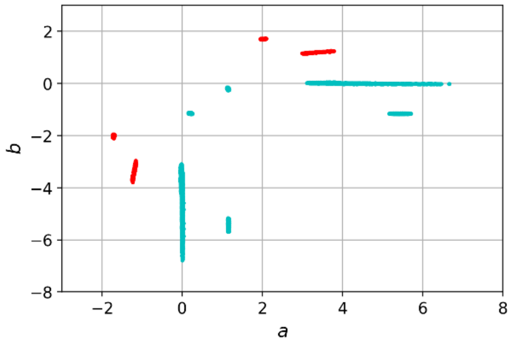

Figure 6. Allowed region on the a–b plane, where a and b are taken to be real. Cyan-points and red-points denote cases of NH and IH, respectively.

a mild hierarchy of $\mathcal{O}(0.1)$ between β and γ although one may naively expect $\beta \sim \gamma$ as discussed in section 3. Thus, the magnitude of the S_4 breaking is somewhat large beyond the naive expectation. For the case of IH, we need a strong hierarchy between β and γ as seen in eq. (4.4). Therefore, the IH case is not favored in our model.

In our calculations, we take Yukawa couplings of charged leptons at the GUT scale 2×10^{16} GeV, where $v_u/v_d = 2.5$ is taken as discussed in appendix B. However, we input the data of NuFIT 4.0 [51] for three lepton mixing angles and neutrino mass parameters. The renormalization group equation (RGE) effects of mixing angles and the mass ratio $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ are negligibly small in the case of $\tan\beta = 2.5$ even if IH of neutrino masses is considered (see appendix B).

5 Summary

In the S_4 symmetry, the Z_2 subgroup can be anomalous, and then S_4 can be violated to A_4. The S_4 symmetry is unique among S_3, A_4, S_4, A_5 in the sense that it can be violated by anomalies to another non-Abelian symmetry, A_4. Starting with a S_4 symmetric Lagrangian at the tree level, the Lagrangian at the quantum level has only A_4 symmetry when Z_2 in S_4 is anomalous. We have studied such a possibility that the A_4 flavor symmetry is originated from the S_4 modular group. Decomposing S_4 modular forms into A_4 representations, we have obtained the modular forms of two singlets, $\mathbf{1}'$ and $\mathbf{1}''$, in addition to triplet, $\mathbf{3}$ for A_4. Using those modular forms, we have succeeded in constructing the viable neutrino mass matrix through the Weinberg operator for both NH and IH of neutrino masses. Our model presents a new possibility of flavor model with the modular symmetry.

Indeed, we have obtained an interesting prediction of $\delta_{\rm CP}$ for both NH and IH, and their predictions also depend on the sum of neutrino masses. Hence, the observation of the sum of neutrino masses in the cosmology will provide a severe constraint to the flavor model.

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Realistic mass matrices are realized in the parameter region with small $\mathrm{Im}[\tau]$ as well as large $\mathrm{Im}[\tau]$. If our four-dimensional field theory is originated from extra dimensional theory or superstring theory on a compact space, the volume of compact space is proportional to $\mathrm{Im}[\tau]$. Such volume of the compact space must be larger than the string scale. For example, the volume of torus compactification is obtained by $(2\pi R)^2\mathrm{Im}[\tau]$. Thus, larger $2\pi R$ will be required for smaller $\mathrm{Im}[\tau]$.

Furthermore, it is important how to derive the preferred values of τ in such compactified theory. That is the so-called moduli stabilization problem. However, that is beyond our scope. We can study this problem elsewhere.[3]

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A S_4 and A_4 representations

The representations S and T of $\Gamma_4 \simeq S_4$ are given for the representations $\mathbf{2}$ and $\mathbf{3}'$ in section 2. Here, we give other representations. The generators S and T are represented by

$$\rho(S) = \frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega & 2 & -\omega^2 \\ 2\omega^2 & -\omega & 2 \end{pmatrix}, \qquad \rho(T) = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}, \tag{A.1}$$

on the S_4 $\mathbf{3}$ representation, where $\omega = e^{i\frac{2}{3}\pi}$, and

$$\rho(S) = \rho(T) = -1, \tag{A.2}$$

for $\mathbf{1}'$, while $\rho(S) = \rho(T) = 1$ for $\mathbf{1}$.

On the other hand, we take the generators of A_4 group $\tilde{S}$ and $\tilde{T}$ for $\mathbf{3}$ by using the S and T of the S_4 group as follows:

$$\rho(\tilde{S}) = \rho(T^2) = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \qquad \rho(\tilde{T}) = \rho(ST) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}. \tag{A.3}$$

Since the doublet $\mathbf{2}$ of S_4 group is transformed by $\tilde{S}$ and $\tilde{T}$ as

$$\rho(\tilde{S}) = \rho(T^2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \rho(\tilde{T}) = \rho(ST) = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \tag{A.4}$$

[3] Realistic results are obtained at some points of τ near edges of the $\mathrm{SL}(2,\mathbb{Z})$ fundamental domain and domains transformed by S, T and their products. The edges of the fundamental domain can be candidates for the minimum of the modulus potential. (See e.g. ref. [59] and its references therein.)

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observable &3 σ range for NH	3 σ range for IH	
$\Delta m^2_{\rm atm}$	$(2.431$–$2.622) \times 10^{-3}\,\mathrm{eV}^2$	$-(2.413$–$2.606) \times 10^{-3}\,\mathrm{eV}^2$
$\Delta m^2_{\rm sol}$	$(6.79$–$8.01) \times 10^{-5}\,\mathrm{eV}^2$	$(6.79$–$8.01) \times 10^{-5}\,\mathrm{eV}^2$
$\sin^2\theta_{23}$	0.428–0.624	0.433–0.623
$\sin^2\theta_{12}$	0.275–0.350	0.275–0.350
$\sin^2\theta_{13}$	0.02044–0.02437	0.02067–0.02461

Table 2. The 3 σ ranges of neutrino parameters from NuFIT 4.0 for NH and IH [51].

the doublet of S_4 can be decomposed into singlets of A_4 transformed as

$$\rho(\tilde{S})_{\mathbf{1}'} = \rho(\tilde{S})_{\mathbf{1}''} = 1, \quad \rho(\tilde{T})_{\mathbf{1}'} = \omega^2, \quad \rho(\tilde{T})_{\mathbf{1}''} = \omega. \tag{A.5}$$

In this base, the multiplication rule of the A_4 triplet is

$$\begin{aligned}
\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} &= (a_1 b_1 + a_2 b_3 + a_3 b_2)_{\mathbf{1}} \oplus (a_3 b_3 + a_1 b_2 + a_2 b_1)_{\mathbf{1}'} \\
&\quad \oplus (a_2 b_2 + a_1 b_3 + a_3 b_1)_{\mathbf{1}''} \\
&\quad \oplus \frac{1}{3} \begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_1 b_3 - a_3 b_1 \end{pmatrix}_{\mathbf{3}} \oplus \frac{1}{2} \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{pmatrix}_{\mathbf{3}}, \\
\mathbf{1} \otimes \mathbf{1} = \mathbf{1}, \quad \mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}'', \quad & \mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}', \quad \mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}\,.
\end{aligned} \tag{A.6}$$

More details are shown in the review [4, 5].

B Input data

We input charged lepton masses in order to constrain the model parameters. We take Yukawa couplings of charged leptons at the GUT scale 2×10^{16} GeV, where $\tan\beta = 2.5$ is taken [31, 56–58]:

$$y_e = (1.97 \pm 0.02) \times 10^{-6}, \quad y_\mu = (4.16 \pm 0.05) \times 10^{-4}, \quad y_\tau = (7.07 \pm 0.07) \times 10^{-3}, \tag{B.1}$$

where lepton masses are given by $m_\ell = \sqrt{2} y_\ell v_H$ with $v_H = 174\,\mathrm{GeV}$. We also use the following lepton mixing angles and neutrino mass parameters in table 2 given by NuFIT 4.0 [51]. The RGE effects of mixing angles and the mass ratio $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ are negligibly small in the case of $\tan\beta = 2.5$ for both NH and IH as seen in appendix E of ref. [31].

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