# Connecting $b \rightarrow s \ell \bar{\ell}$ anomalies to enhanced rare nonleptonic $\bar{B}_{s}^{0}$ decays in $Z^{\prime}$ model 

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AbStract: The present data on a number of observables in $b \rightarrow s \mu^{+} \mu^{-}$processes manifest some tensions with the standard model (SM). Assuming that these anomalies have a new physics origin, we consider the possibility that a $Z^{\prime}$ boson is responsible for them. We further assume that its interactions with quarks also affect rare nonleptonic decays of the $\bar{B}_{s}^{0}$ meson which are purely isospin-violating and tend to be dominated by electroweakpenguin contributions, namely $\bar{B}_{s}^{0} \rightarrow\left(\eta, \eta^{\prime}, \phi\right)\left(\pi^{0}, \rho^{0}\right)$. Most of these decays are not yet observed, and their rates are expected to be relatively small in the SM. Taking into account constraints from various measurements, including the evidence for $\bar{B}_{s}^{0} \rightarrow \phi \rho^{0}$ recently seen by LHCb, we find that the $Z^{\prime}$ effects on $\bar{B}_{s}^{0} \rightarrow(\eta, \phi) \pi^{0}$ can make their rates bigger than the SM predictions by up to an order of magnitude. For $\bar{B}_{s}^{0} \rightarrow \eta^{\prime} \pi^{0},\left(\eta, \eta^{\prime}\right) \rho^{0}$, the enhancement factors are at most a few. Since the $Z^{\prime}$ contributions to the different channels depend on different combinations of its couplings, observations of more of these decays in future experiments, along with improved $b \rightarrow s \mu^{+} \mu^{-}$data, will probe this $Z^{\prime}$ scenario more thoroughly.

Keywords: Beyond Standard Model, Heavy Quark Physics, Effective Field Theories, Gauge Symmetry

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## Contents


#### Abstract

1 Introduction


$2 Z^{\prime}$ interactions in $b \rightarrow s \mu \bar{\mu}$ and $B_{s}-\bar{B}_{s}$ mixing 3
$3 Z^{\prime}$ contributions to rare nonleptonic $\bar{B}_{s}$ decays 5
4 Conclusions 12

## 1 Introduction

The latest measurements of various $b \rightarrow s \mu^{+} \mu^{-}$processes have turned up some intriguing discrepancies from the expectations of the standard model (SM) of particle physics. Specifically, the LHCb Collaboration in its angular analysis of the decay $B^{0} \rightarrow K^{* 0} \mu^{+} \mu^{-}$found tensions with the SM at the $3.4 \sigma$ level [1]. This was later confirmed in the Belle experiment on the same process, but with lower statistical confidence [2]. Furthermore, LHCb findings [3, 4] on the ratio $R_{K}$ of the branching fractions of $B^{+} \rightarrow K^{+} \mu^{+} \mu^{-}$and $B^{+} \rightarrow K^{+} e^{+} e^{-}$ decays and on the corresponding ratio $R_{K^{*}}$ for $B^{0} \rightarrow K^{* 0} \mu^{+} \mu^{-}$and $B^{0} \rightarrow K^{* 0} e^{+} e^{-}$decays are all below their SM predictions [5-7] by $2.1 \sigma$ to $2.6 \sigma$. In addition, the current data [8-10] on the branching fractions of $B \rightarrow K^{(*)} \mu^{+} \mu^{-}$and $B_{s} \rightarrow \phi \mu^{+} \mu^{-}$favor values less than their SM estimates.

Although the statistical significance of the aforesaid deviations from SM expectations is still too low for a definite conclusion, they may be early clues about interactions beyond the SM in $b \rightarrow s$ transitions. Recent model-independent theoretical analyses have in fact demonstrated that new physics (NP) could account for these anomalies [11-25]. In view of the possibility that these tentative hints of NP will be confirmed by upcoming experiments, it is of interest to explore the potential implications for other $b \rightarrow s$ processes.

Among them are the nonleptonic decays $\bar{B}_{s} \rightarrow \eta \pi^{0}, \bar{B}_{s} \rightarrow \eta^{\prime} \pi^{0}, \bar{B}_{s} \rightarrow \phi \pi^{0}, \bar{B}_{s} \rightarrow$ $\eta \rho^{0}, \bar{B}_{s} \rightarrow \eta^{\prime} \rho^{0}$, and $\bar{B}_{s} \rightarrow \phi \rho^{0}$. Each of these transitions has a final state with total isospin $I=1$ and thus fully breaks isospin symmetry, implying that their amplitudes receive no contributions from QCD-penguin operators and arise instead from charmless tree and electroweak-penguin (EWP) operators [26, 27]. The product of Cabibbo-KobayashiMaskawa (CKM) matrix elements in the tree contributions is suppressed compared to that in the EWP ones, and the suppression factor is $\left|V_{u s} V_{u b}\right| /\left|V_{t s} V_{t b}\right| \sim 0.02$. Consequently, although the Wilson coefficients of the tree operators are much bigger than those of the EWP operators, the latter turn out to dominate the majority of these channels, and the resulting decay rates are relatively low [26-29]. Most of them are not yet observed, the exception being $\bar{B}_{s} \rightarrow \phi \rho^{0}$. Evidence for it was detected by LHCb last year [30] with
a branching fraction $\mathcal{B}\left(\bar{B}_{s} \rightarrow \phi \rho^{0}\right)=(0.27 \pm 0.08) \times 10^{-6}$ [10], which agrees with some of its estimates in the SM within sizable errors [31-36].

The smallness of the rates of $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}, \phi\right)\left(\pi^{0}, \rho^{0}\right)$ in the SM implies that they may serve as probes of physics beyond it. This has been considered to varying extents in the contexts of different models [34-40]. In this paper, we treat these rare nonleptonic $\bar{B}_{s}$ decays along similar lines and suppose that the NP influencing them also causes the aforementioned $b \rightarrow s \mu^{+} \mu^{-}$anomalies. We adopt in particular a scenario where an electrically neutral and uncolored spin-one particle, the $Z^{\prime}$ boson, is responsible for the new interactions in these two sets of $b \rightarrow s$ transitions. We assume that it couples nonuniversally to SM fermions and does not mix with SM gauge bosons, but it is not necessarily a gauge boson and could even be composite.

Although the possibility of NP effects on $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}, \phi\right)\left(\pi^{0}, \rho^{0}\right)$ in the $Z^{\prime}$ context has been entertained before [36-39], our current paper contains new considerations and results which were not available in the previous literature. Firstly, while the past studies separately examined only subsets of these six modes, ${ }^{1}$ here we treat all of them at the same time, noting that among $B_{s}$ decays into two charmless mesons the six are the only ones which are strangeness changing and purely isospin-violating. This allows us to gain a more complete picture than before concerning the $Z^{\prime}$ contributions, which reveals clearly how they in general modify the different channels in different ways. A second novel aspect of our analysis is that, as stated in the preceding paragraph, we explore a scenario in which the same $Z^{\prime}$ not only modifies these nonleptonic $\bar{B}_{s}$ decays, but also gives rise to the $b \rightarrow s \ell \bar{\ell}$ anomalies. It turns out that assuming this link between the two sets of processes leads to an important consequence for the $Z^{\prime}$ interactions, namely that the left-handed $b s Z^{\prime}$ coupling must be roughly ten times stronger than the right-handed one if both of them exist, as will be detailed later on. This particular finding was absent from the earlier studies [36-39], which did not deal with such a potential link, as most of them appeared before the arrival of the anomalies. A third significant novelty in our analysis is that we take into account the foregoing evidence of $\bar{B}_{s} \rightarrow \phi \rho^{0}$ recently seen by LHCb [30]. As this new measurement, albeit still with a sizable uncertainty, is compatible with its SM expectations, we will show that the implied room for the $Z^{\prime}$ influence on the channels with the $\rho^{0}$, not only $\bar{B}_{s} \rightarrow \phi \rho^{0}$ but also $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}\right) \rho^{0}$, is now limited. In contrast, previously $\bar{B}_{s} \rightarrow \phi \rho^{0}$ and $\bar{B}_{s} \rightarrow \eta \rho^{0}$ were allowed in refs. [36] and [39], respectively, to have rates exceeding their SM predictions by an order of magnitude. Last but not least, we will nevertheless also demonstrate that the viable $Z^{\prime}$ parameter space still accommodates the possibility that the pionic channels $\bar{B}_{s} \rightarrow \phi \pi^{0}$ and $\bar{B}_{s} \rightarrow \eta \pi^{0}$ can have rates which are about a factor of ten higher than their SM values. Needless to say, this should add to the motivation for intensified efforts in upcoming experiments at LHCb and Belle II to investigate all these decays. The acquired data on them would provide especially useful complementary information about the NP responsible for the $b \rightarrow s \ell \bar{\ell}$ anomalies should the latter be established by future measurements to be signals of physics beyond the SM.

[^0]

Figure 1. Allowed $2 \sigma$ (cyan) region of $C_{9^{\prime} \mu}^{\mathrm{NP}}$ versus $C_{9 \mu}^{\mathrm{NP}}$ from the global analysis of $b \rightarrow s \mu^{+} \mu^{-}$ data performed in ref. [18].

The rest of the paper is organized as follows. In section 2 , we address the $Z^{\prime}$ contributions to $b \rightarrow s \mu^{+} \mu^{-}$and apply constraints from the relevant empirical information, including that on $B_{s}-\bar{B}_{s}$ mixing. In section 3 , we examine the impact of the $Z^{\prime}$ interactions with SM quarks on $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}, \phi\right)\left(\pi^{0}, \rho^{0}\right)$. To evaluate their amplitudes, we employ the soft-collinear effective theory [41-48]. We show that in the $Z^{\prime}$ presence the rates of $\bar{B}_{s} \rightarrow \eta^{\prime} \pi^{0},\left(\eta, \eta^{\prime}\right) \rho^{0}$ can increase by as much as factors of a few with respect to the predictions in the SM, while the rates of $\bar{B}_{s} \rightarrow(\eta, \phi) \pi^{0}$ can exceed their SM values by up to an order of magnitude. We make our conclusions in section 4.

## $2 \quad Z^{\prime}$ interactions in $b \rightarrow s \mu \bar{\mu}$ and $B_{s}-\bar{B}_{s}$ mixing

Global analyses $[18,19]$ have found that some of the best fits to the most recent anomalous $b \rightarrow s \mu^{+} \mu^{-}$measurements result from effective interactions given by

$$
\begin{align*}
\mathcal{L}_{\mathrm{eff}} & \supset \frac{\alpha_{\mathrm{e}} \lambda_{t} G_{\mathrm{F}}}{\sqrt{2} \pi}\left(C_{9 \mu} \bar{s} \gamma^{\kappa} P_{L} b+C_{9^{\prime} \mu} \bar{s} \gamma^{\kappa} P_{R} b\right) \bar{\mu} \gamma_{\kappa} \mu+\text { H.c. } \\
\lambda_{t} & =V_{t s}^{*} V_{t b}, \quad P_{L, R}=\frac{1}{2}\left(1 \mp \gamma_{5}\right) \tag{2.1}
\end{align*}
$$

where $C_{9 \mu}=C_{9 \ell}^{\mathrm{SM}}+C_{9 \mu}^{\mathrm{NP}}$ and $C_{9^{\prime} \mu}=C_{9^{\prime} \mu}^{\mathrm{NP}}$ are the Wilson coefficients, $\alpha_{\mathrm{e}}=1 / 133$ is the fine structure constant at the $b$-quark mass $\left(m_{b}\right)$ scale, and $G_{F}$ is the Fermi constant. The same SM part $C_{9 \ell}^{\mathrm{SM}}$ occurs in the electron and tau channels $b \rightarrow s\left(e^{+} e^{-}, \tau^{+} \tau^{-}\right)$, but they are not affected by the NP. In figure 1 , for later use, we display the $2 \sigma$ (cyan) region of $C_{9^{\prime} \mu}^{\mathrm{NP}}$ versus $C_{9 \mu}^{\mathrm{NP}}$ permitted by the data, from the global fit carried out in ref. [18].

In the literature, many models possessing some kind of $Z^{\prime}$ particle with different sets of fermionic couplings have been studied in relation to the $b \rightarrow s \mu^{+} \mu^{-}$anomalies [49-88]. In the $Z^{\prime}$ scenario considered here, the interactions responsible for $C_{9 \mu, 9^{\prime} \mu}^{\mathrm{NP}}$ in eq. (2.1) are described by

$$
\begin{equation*}
\mathcal{L}_{Z^{\prime}} \supset-\left[\bar{s} \gamma^{\kappa}\left(\Delta_{L}^{s b} P_{L}+\Delta_{R}^{s b} P_{R}\right) b Z_{\kappa}^{\prime}+\text { H.c. }\right]-\Delta_{V}^{\mu \mu} \bar{\mu} \gamma^{\kappa} \mu Z_{\kappa}^{\prime} \tag{2.2}
\end{equation*}
$$

where the constants $\Delta_{L, R}^{s b}$ are generally complex and $\Delta_{V}^{\mu \mu}$ is real due to the Hermiticity of $\mathcal{L}_{Z^{\prime}}$. Any other possible $Z^{\prime}$ couplings to leptons are taken to be negligible. To simplify the


Figure 2. Regions of $\rho_{R}$ versus $\rho_{L}$ for $m_{Z^{\prime}}=1 \mathrm{TeV}$ which are consistent with the $C_{9^{\prime} \mu^{-}}^{\mathrm{NP}} C_{9 \mu}^{\mathrm{NP}}$ constraint depicted in figure 1 for $\Delta_{V}^{\mu \mu}= \pm 0.03$ (red), $\pm 0.05$ (orange), $\pm 0.1$ (yellow), and $\pm 0.3$ (green). The blue area fulfills the condition in eq. (2.7) from $B_{s}-\bar{B}_{s}$ mixing data.
analysis, hereafter we focus on the special case in which

$$
\begin{equation*}
\Delta_{L}^{s b}=\rho_{L} V_{t s}^{*} V_{t b}, \quad \Delta_{R}^{s b}=\rho_{R} V_{t s}^{*} V_{t b} \tag{2.3}
\end{equation*}
$$

where $\rho_{L, R}$ are real numbers, and so they do not supply any new $C P$-violation phase. Accordingly, for a heavy $Z^{\prime}$ with mass $m_{Z^{\prime}}$ we obtain

$$
\begin{equation*}
C_{9 \mu}^{\mathrm{NP}}=\frac{-\sqrt{2} \pi \rho_{L} \Delta_{V}^{\mu \mu}}{\alpha_{\mathrm{e}} G_{\mathrm{F}} m_{Z^{\prime}}^{2}}, \quad C_{9^{\prime} \mu}^{\mathrm{NP}}=\frac{-\sqrt{2} \pi \rho_{R} \Delta_{V}^{\mu \mu}}{\alpha_{\mathrm{e}} G_{\mathrm{F}} m_{Z^{\prime}}^{2}} . \tag{2.4}
\end{equation*}
$$

In figure 2, we illustrate the ranges of $\rho_{L}$ and $\rho_{R}$ corresponding to the allowed $C_{9^{\prime} \mu}^{\mathrm{NP}}-C_{9 \mu}^{\mathrm{NP}}$ (cyan) region in figure 1 for $m_{Z^{\prime}}=1 \mathrm{TeV}$ and some sample choices of $\Delta_{V}^{\mu \mu}$, namely $\pm 0.03$ (red), $\pm 0.05$ (orange), $\pm 0.1$ (yellow), and $\pm 0.3$ (green). We note that these $\Delta_{V}^{\mu \mu}$ values contribute positively to the SM muon anomalous magnetic moment, but with $m_{Z^{\prime}}=1 \mathrm{TeV}$ are too small to explain the disagreement with its measurement [89].

The $Z^{\prime}$ couplings in eq. (2.2) also induce tree-level effects on $\Delta M_{s}=2\left|M_{12}^{s}\right|$, which pertains to $B_{s}-\bar{B}_{s}$ mixing and has been measured to be $\Delta M_{s}^{\exp }=(17.757 \pm 0.021) / \mathrm{ps}[10]$. We can express the sum of the SM and $Z^{\prime}$ contributions as [90]

$$
\begin{equation*}
M_{12}^{s}=M_{12}^{s, \mathrm{SM}}\left(1+4 \tilde{r} \frac{\rho_{L}^{2}+\rho_{R}^{2}+\kappa_{L R} \rho_{L} \rho_{R}}{g_{\mathrm{SM}}^{2} S_{0} m_{Z^{\prime}}^{2}}\right) \tag{2.5}
\end{equation*}
$$

where [90] $\tilde{r}=0.985$ for $m_{Z^{\prime}}=1 \mathrm{TeV}$ is a QCD factor, $g_{\mathrm{SM}}^{2}=1.7814 \times 10^{-7} \mathrm{GeV}^{-2}$, the SM loop function $S_{0} \simeq 2.35$ for a top-quark mass $m_{t} \simeq 165 \mathrm{GeV}$, and

$$
\begin{equation*}
\kappa_{L R}=\frac{6\left(C_{1}^{L R}\left\langle Q_{1}^{L R}\right\rangle+C_{2}^{L R}\left\langle Q_{2}^{L R}\right\rangle\right)}{\eta_{B} \hat{B}_{B_{s}} f_{B_{s}}^{2} m_{B_{s}} \tilde{r}}, \tag{2.6}
\end{equation*}
$$

with $[90] C_{1}^{L R}=1-\alpha_{\mathrm{s}}\left[1 / 6+2 \log \left(m_{Z^{\prime}} / \mu^{\prime}\right)\right] /(4 \pi)$ and $C_{2}^{L R}=\alpha_{\mathrm{s}}\left[-1-12 \log \left(m_{Z^{\prime}} / \mu^{\prime}\right)\right] /(4 \pi)$ containing the strong coupling constant $\alpha_{\mathrm{s}}$, all evaluated at a scale $\mu^{\prime} \sim m_{Z^{\prime}},\left\langle Q_{1}^{L R}\right\rangle=$
$-0.37 \mathrm{GeV}^{3},\left\langle Q_{2}^{L R}\right\rangle=0.51 \mathrm{GeV}^{3}, \eta_{B}=0.55 \pm 0.01$, and [91, 92] $f_{B_{s}} \hat{B}_{B_{s}}^{1 / 2}=(262.2 \pm$ 9.7) MeV . With the central values of these parameters and $m_{B_{s}}$ from ref. [10], we get $\kappa_{L R}=-11.2$ for $m_{Z^{\prime}}=1 \mathrm{TeV}$.

To apply restrictions on $\rho_{L, R}$ from the $B_{s}-\bar{B}_{s}$ mixing data, we impose

$$
\begin{equation*}
0.899 \leq \frac{\Delta M_{s}}{\Delta M_{s}^{\text {sM }}}=\left|\frac{M_{12}^{s}}{M_{12}^{S, S M}}\right| \leq 1.252 \tag{2.7}
\end{equation*}
$$

which is the $95 \%$ confidence level (CL) range from the latest UTfit global analysis [92]. Since some of the numbers quoted in the last paragraph have uncertainties up to a few percent, we let $\kappa_{L R}$ vary by up to $10 \%$ from its central value when scanning the parameter space for $\rho_{L, R}$ values which conform to eq. (2.7). For $m_{Z^{\prime}}=1 \mathrm{TeV}$, we incorporate the scan result into figure 2 , represented by the blue area. Thus, in this figure each overlap of the blue area with one of the other colored ones of a particular $\Delta_{V}^{\mu \mu}$ value corresponds to the parameter space that can explain the $b \rightarrow s \mu^{+} \mu^{-}$anomalies and simultaneously satisfies eq. (2.7). With smaller choices of $\left|\Delta_{V}^{\mu \mu}\right|$, such overlaps could be found at larger $\left|\rho_{L, R}\right|$ values. This graph also reveals that in the absence of the right-handed coupling, $\rho_{R}=0$, the allowed range of $\rho_{L}$ would be rather narrow, indicating the importance of nonvanishing $\rho_{R}$ for gaining bigger viable parameter space [51].

## $3 \quad Z^{\prime}$ contributions to rare nonleptonic $\bar{B}_{s}$ decays

Given that $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}, \phi\right)\left(\pi^{0}, \rho^{0}\right)$ change both strangeness and isospin, in the SM their amplitudes proceed from $b \rightarrow s$ four-quark operators $O_{1,2}^{u}$ and $O_{7,8,9,10}$ which are derived from charmless tree and electroweak-penguin diagrams, respectively. In contrast, the QCDpenguin operators $O_{3,4,5,6}$, which preserve isospin symmetry, do not affect these processes. ${ }^{2}$ In many models beyond the SM , new interactions may modify the Wilson coefficients $C_{i}$ of $O_{i}$ and/or give rise to extra operators $\tilde{O}_{i}$ which are the chirality-flipped counterparts of $O_{i}$. A flavor-violating $Z^{\prime}$ boson may contribute to some of them, depending on the details of its properties.

In our scenario of interest, besides its couplings in eq. (2.2), the $Z^{\prime}$ has flavor-conserving interactions with the $u$ and $d$ quarks via

$$
\begin{equation*}
\mathcal{L}_{Z^{\prime}} \supset-\left[\bar{u} \gamma^{\kappa}\left(\Delta_{L}^{u u} P_{L}+\Delta_{R}^{u u} P_{R}\right) u+\bar{d} \gamma^{\kappa}\left(\Delta_{L}^{d d} P_{L}+\Delta_{R}^{d d} P_{R}\right) d\right] Z_{\kappa}^{\prime}, \tag{3.1}
\end{equation*}
$$

the constants $\Delta_{L, R}^{u u, d d}$ being real, but does not couple flavor-diagonally to other quarks. From eqs. (2.2) and (3.1), we can derive tree-level $Z^{\prime}$-mediated diagrams contributing to nonleptonic $b \rightarrow s$ reactions. For a heavy $Z^{\prime}$, these diagrams yield

$$
\begin{equation*}
\mathcal{L}_{4-\text { quark }}^{Z^{\prime}} \supset \frac{-\lambda_{t}}{m_{Z^{\prime}}^{2}} \bar{s} \gamma^{\kappa}\left(\rho_{L} P_{L}+\rho_{R} P_{R}\right) b \sum_{q=u, d} \bar{q} \gamma_{\kappa}\left(\Delta_{L}^{q q} P_{L}+\Delta_{R}^{q q} P_{R}\right) q \tag{3.2}
\end{equation*}
$$

after applying eq. (2.3). It is straightforward to realize that these additional terms bring about modifications to the coefficients of the QCD- and electroweak-penguin operators

[^1]$O_{3,5,7,9}$ in the SM and also generate their chirality-flipped partners $\tilde{O}_{3,5,7,9}$ [93]. We can express them in the effective Lagrangian for $b \rightarrow s$ transitions as
\[

$$
\begin{align*}
& \mathcal{L}_{\text {eff }} \supset \sqrt{8} \lambda_{t} G_{\mathrm{F}} \sum_{q=u, d}\left\{\bar{s} \gamma^{\kappa} P_{L} b\left[\left(C_{3}+\frac{3}{2} C_{9} e_{q}\right) \bar{q} \gamma_{\kappa} P_{L} q+\left(C_{5}+\frac{3}{2} C_{7} e_{q}\right) \bar{q} \gamma_{\kappa} P_{R} q\right]\right. \\
&\left.+\bar{s} \gamma^{\kappa} P_{R} b\left[\left(\tilde{C}_{3}+\frac{3}{2} \tilde{C}_{9} e_{q}\right) \bar{q} \gamma_{\kappa} P_{R} q+\left(\tilde{C}_{5}+\frac{3}{2} \tilde{C}_{7} e_{q}\right) \bar{q} \gamma_{\kappa} P_{L} q\right]\right\}, \tag{3.3}
\end{align*}
$$
\]

where $C_{j}=C_{j}^{\mathrm{SM}}+C_{j}^{Z^{\prime}}$ and $\tilde{C}_{j}=\tilde{C}_{j}^{Z^{\prime}}$ for $j=3,5,7,9$ are the Wilson coefficients. Thus, from eq. (3.2) we have [36, 38, 93]

$$
\begin{align*}
& C_{3,5}^{Z^{\prime}}=\frac{\rho_{L}\left(-\Delta_{L, R}^{u u}-2 \Delta_{L, R}^{d d}\right)}{6 \sqrt{2} G_{\mathrm{F}} m_{Z^{\prime}}^{2}}=\frac{\rho_{L}\left(-\delta_{L, R}-3 \Delta_{L, R}^{d d}\right)}{6 \sqrt{2} G_{\mathrm{F}} m_{Z^{\prime}}^{2}}, \\
& \tilde{C}_{3,5}^{Z^{\prime}}=\frac{\rho_{R}\left(-\Delta_{R, L}^{u u}-2 \Delta_{R, L}^{d d}\right)}{6 \sqrt{2} G_{\mathrm{F}} m_{Z^{\prime}}^{2}}=\frac{\rho_{R}\left(-\delta_{R, L}-3 \Delta_{R, L}^{d d}\right)}{6 \sqrt{2} G_{\mathrm{F}} m_{Z^{\prime}}^{2}}, \\
& C_{7,9}^{Z^{\prime}}=\frac{\rho_{L}\left(-\Delta_{R, L}^{u u}+\Delta_{R, L}^{d d}\right)}{3 \sqrt{2} G_{\mathrm{F}} m_{Z^{\prime}}^{2}}=\frac{-\rho_{L} \delta_{R, L}}{3 \sqrt{2} G_{\mathrm{F}} m_{Z^{\prime}}^{2}}, \\
& \tilde{C}_{7,9}^{Z^{\prime}}=\frac{\rho_{R}\left(-\Delta_{L, R}^{u u}+\Delta_{L, R}^{d d}\right)}{3 \sqrt{2} G_{\mathrm{F}} m_{Z^{\prime}}^{2}}=\frac{-\rho_{R} \delta_{L, R}}{3 \sqrt{2} G_{\mathrm{F}} m_{Z^{\prime}}^{2}}, \tag{3.4}
\end{align*}
$$

where

$$
\begin{equation*}
\delta_{L}=\Delta_{L}^{u u}-\Delta_{L}^{d d}, \quad \delta_{R}=\Delta_{R}^{u u}-\Delta_{R}^{d d} . \tag{3.5}
\end{equation*}
$$

As $O_{3,5}$ and $\tilde{O}_{3,5}$ do not break isospin, only $C_{7,9}^{Z^{\prime}}$ and $\tilde{C}_{7,9}^{Z^{\prime}}$ contribute to $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}, \phi\right)\left(\pi^{0}, \rho^{0}\right)$.
To estimate the $Z^{\prime}$ impact on these decays, we make use of the soft-collinear effective theory (SCET) [41-48], similarly to what was done in ref. [39] in the case of a leptophobic$Z^{\prime}$ model. For any one of them, the SCET amplitude at leading order in $\alpha_{s}\left(m_{b}\right)$ can be written as [48]

$$
\begin{align*}
\mathcal{A}_{\bar{B}_{s} \rightarrow M_{1} M_{2}}= & \frac{f_{M_{1}} G_{\mathrm{F}} m_{B_{s}}^{2}}{\sqrt{2}}\left[\int_{0}^{1} d \nu\left(\zeta_{J}^{B M_{2}} T_{1 J}(\nu)+\zeta_{J g}^{B M_{2}} T_{1 J g}(\nu)\right) \phi_{M_{1}}(\nu)+\zeta^{B M_{2}} T_{1}+\zeta_{g}^{B M_{2}} T_{1 g}\right] \\
& +(1 \leftrightarrow 2), \tag{3.6}
\end{align*}
$$

where $f_{M}$ is the decay constant of meson $M$, the $\zeta$ 's are nonperturbative hadronic parameters which can be fixed from experiment, the $T$ 's are hard kernels which are functions of the Wilson coefficients $C_{i}$ and $\tilde{C}_{i}$, and $\phi_{M}(\nu)$ is the light-cone distribution amplitude of $M$ which is normalized as $\int_{0}^{1} d \nu \phi_{M}(\nu)=1$. The so-called charming-penguin term, which in this case conserves isospin, is absent from $\mathcal{A}_{\bar{B}_{s} \rightarrow M_{1} M_{2}}$. The hard kernels for the decays of concern are available from the literature $[33,47,48]$ and have been listed in table 1 , where the flavor states $\eta_{q} \sim(u \bar{u}+d \bar{d}) / \sqrt{2}$ and $\eta_{s} \sim s \bar{s}$ are related to the physical meson states $\eta$ and $\eta^{\prime}$ by $\eta=\eta_{q} \cos \theta-\eta_{s} \sin \theta$ and $\eta^{\prime}=\eta_{q} \sin \theta+\eta_{s} \cos \theta$ with mixing angle $\theta=39.3^{\circ}[47,48,95,96]$.

In the presence of NP which also generates the extra operators $\tilde{O}_{i}$, the quantities $c_{2,3}$ and $b_{2,3}$ in table 1 depend not only on $C_{i}$ and $\tilde{C}_{i}$, but also on the final mesons $M_{1}$ and

| Decay mode | $T_{1}$ | $T_{2}$ | $T_{1 g}$ | $T_{2 g}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{B}_{s} \rightarrow \eta_{s} \pi^{0}$ | 0 | $\frac{1}{\sqrt{2}}\left(c_{2}-c_{3}\right)$ | 0 | $\frac{1}{\sqrt{2}}\left(c_{2}-c_{3}\right)$ |
| $\bar{B}_{s} \rightarrow \eta_{s} \rho^{0}$ | 0 | $\frac{1}{\sqrt{2}}\left(c_{2}+c_{3}\right)$ | 0 | $\frac{1}{\sqrt{2}}\left(c_{2}+c_{3}\right)$ |
| $\bar{B}_{s} \rightarrow \eta_{q} \pi^{0}$ | 0 | 0 | 0 | $c_{2}-c_{3}$ |
| $\bar{B}_{s} \rightarrow \eta_{q} \rho^{0}$ | 0 | 0 | 0 | $c_{2}+c_{3}$ |
| $\bar{B}_{s} \rightarrow \phi \pi^{0}$ | 0 | $\frac{1}{\sqrt{2}}\left(c_{2}-c_{3}\right)$ | 0 | 0 |
| $\bar{B}_{s} \rightarrow \phi \rho^{0}$ | 0 | $\frac{1}{\sqrt{2}}\left(c_{2}+c_{3}\right)$ | 0 | 0 |

Table 1. Hard kernels $T_{1,2,1 g, 2 g}$ for $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}, \phi\right)\left(\pi^{0}, \rho^{0}\right)$ decays. The hard kernels $T_{r J, r J g}(\nu)$ for $r=1,2$ are obtainable from $T_{r, r g}$, respectively, through the replacement $c_{k} \rightarrow b_{k}$, where $b_{k}$ has dependence on $\nu$.
$M_{2}$, as well as on the CKM factors $\lambda_{t}$ and $\lambda_{u}=V_{u s}^{*} V_{u b}$. The dependence on $M_{1}$ and $M_{2}$ arises from the fact that, with regard to the nonzero kernels in this table, for each 4-quark operator the $\bar{B}_{s} \rightarrow M_{1}$ and vacuum $\rightarrow M_{2}$ matrix elements and their contraction in the amplitude can lead to an overall negative or positive sign for the contribution of the operator, the sign being determined by the chirality combination of the operator and by whether the final mesons are pseudoscalars $(P P)$, vectors $(V V), P V$, or $V P$. Thus, for $\bar{B}_{s} \rightarrow\left(\eta_{q}, \eta_{s}\right) \pi^{0}$ and $\bar{B}_{s} \rightarrow \phi \rho^{0}$ we have ${ }^{3}$

$$
\begin{align*}
& c_{2}=\lambda_{u}\left(C_{2}-\tilde{C}_{2}+\frac{C_{1}-\tilde{C}_{1}}{N_{\mathrm{c}}}\right)-\frac{3 \lambda_{t}}{2}\left(C_{9}-\tilde{C}_{9}+\frac{C_{10}-\tilde{C}_{10}}{N_{\mathrm{c}}}\right), \\
& c_{3}=-\frac{3 \lambda_{t}}{2}\left(C_{7}-\tilde{C}_{7}+\frac{C_{8}-\tilde{C}_{8}}{N_{\mathrm{c}}}\right), \\
& b_{2}=\lambda_{u}\left[C_{2}-\tilde{C}_{2}+\left(1-\frac{m_{b}}{\omega_{3}}\right) \frac{C_{1}-\tilde{C}_{1}}{N_{\mathrm{c}}}\right]-\frac{3 \lambda_{t}}{2}\left[C_{9}-\tilde{C}_{9}+\left(1-\frac{m_{b}}{\omega_{3}}\right) \frac{C_{10}-\tilde{C}_{10}}{N_{\mathrm{c}}}\right], \\
& b_{3}=-\frac{3 \lambda_{t}}{2}\left[C_{7}-\tilde{C}_{7}+\left(1-\frac{m_{b}}{\omega_{2}}\right) \frac{C_{8}-\tilde{C}_{8}}{N_{\mathrm{c}}}\right], \tag{3.7}
\end{align*}
$$

where $N_{\mathrm{c}}=3$ is the color number and $b_{2,3}$, which are contained in $T_{2 J}(\nu)$ and $T_{2 J g}(\nu)$, are also functions of $\nu$ via [48] $\omega_{2}=\nu m_{B_{s}}$ and $\omega_{3}=(\nu-1) m_{B_{s}}$. However, for $\bar{B}_{s} \rightarrow\left(\eta_{q}, \eta_{s}\right) \rho^{0}$ and $\bar{B}_{s} \rightarrow \phi \pi^{0}$ we need to make the sign change $-\tilde{C}_{i} \rightarrow+\tilde{C}_{i}$ in $c_{2,3}$ and $b_{2,3}$.

The expressions in eq. (3.7) generalize the SM ones provided previously in refs. [47, 48]. They also supplied the values of the SM coefficients $C_{i}^{\mathrm{SM}}$ at the $m_{b}$ scale, $C_{1,2}^{\mathrm{SM}}=$ $(1.11,-0.253)$ and $C_{7,8,9,10}^{\mathrm{SM}}=(0.09,0.24,-10.3,2.2) \times 10^{-3}[94]$, which we will use in $c_{2,3}$ and $b_{2,3}$. Our $Z^{\prime}$ contributions of interest, in eq. (3.4), enter eq. (3.7) only via $C_{7,9}=$ $C_{7,9}^{\mathrm{SM}}+C_{7,9}^{Z \prime}$ and $\tilde{C}_{7,9}=\tilde{C}_{7,9}^{Z \prime}$.

For numerical computation of $\mathcal{A}_{\bar{B}_{s} \rightarrow M_{1} M_{2}}$, in view of table 1, the meson decay constants which we need are only $f_{\pi}=131 \mathrm{MeV}$ and $f_{\rho}=209 \mathrm{MeV}$, and the integral in eq. (3.6) can

[^2]| Decay mode | QCDF | PQCD | SCET |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | Solution 1 | Solution 2 |
| $\bar{B}_{s} \rightarrow \eta \pi^{0}$ | $0.05_{-0.01}^{+0.03+0.02}$ | $0.05_{-0.02}^{+0.02+0.01+0.00}$ | $0.032 \pm 0.013 \pm 0.008$ | $0.025 \pm 0.010 \pm 0.003$ |
| $\bar{B}_{s} \rightarrow \eta^{\prime} \pi^{0}$ | $0.04{ }_{-0.00}^{+0.01+0.01}$ | $0.11_{-0.03-0.01-0.00}^{+0.05+0.02+0.00}$ | $0.001 \pm 0.000 \pm 0.005$ | $0.052 \pm 0.021 \pm 0.015$ |
| $\bar{B}_{s} \rightarrow \phi \pi^{0}$ | $0.12_{-0.01-0.02}^{+0.02+0.04}$ | $0.16_{-0.05}^{+0.06+0.02+0}$ | $0.074 \pm 0.030 \pm 0.009$ | $0.091 \pm 0.036 \pm 0.016$ |
| $\bar{B}_{s} \rightarrow \eta \rho^{0}$ | $0.10_{-0.01}^{+0.02+0.02}$ | $0.06_{-0.02-0.01+0.00}^{+0.03+0.01+0.00}$ | $0.078 \pm 0.031 \pm 0.022$ | $0.059 \pm 0.023 \pm 0.006$ |
| $\bar{B}_{s} \rightarrow \eta^{\prime} \rho^{0}$ | $0.16_{-0.02-0.03}^{+0.06+0.03}$ | $0.13_{-0.04{ }_{-0.02}^{+0.06+0.01}}$ | $0.003 \pm 0.001 \pm 0.013$ | $0.141 \pm 0.056 \pm 0.042$ |
| $\bar{B}_{s} \rightarrow \phi \rho^{0}$ | $0.18{ }_{-0.01}^{+0.01+0.09}$ | $0.23_{-0.07}^{+0.09+0.03+01}{ }_{-0.01}^{+0.00}$ | $0.36 \pm 0.14 \pm 0.04$ |  |

Table 2. Branching fractions, in units of $10^{-6}$, of $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}, \phi\right)\left(\pi^{0}, \rho^{0}\right)$ decays in the SM. For the first five modes, the last two columns correspond to the two solutions of SCET parameters in eq. (3.8). The errors of the SCET predictions are due to assumed $20 \%$ flavor-SU(3)-breaking effects and the errors in the $\zeta$ s from fits to data, respectively. For comparison, the second and third columns contain results calculated in the frameworks of QCDF [31] and PQCD [32].
be treated with the aid of the relations $\int_{0}^{1} d \nu \phi_{M}(\nu) / \nu=\int_{0}^{1} d \nu \phi_{M}(\nu) /(1-\nu) \equiv\left\langle\chi^{-1}\right\rangle_{M}$ for $M=\pi, \rho$, in which cases $\left\langle\chi^{-1}\right\rangle_{\pi}=3.3$ and $\left\langle\chi^{-1}\right\rangle_{\rho}=3.45[47,48]$. Moreover, for the $\zeta$ 's we adopt the two solutions derived from the fit to data done in ref. [48]:

$$
\begin{align*}
& \left(\zeta^{P}, \zeta_{J}^{P}, \zeta^{V}, \zeta_{J}^{V}, \zeta_{g}, \zeta_{J g}\right)_{1}=(0.137,0.069,0.117,0.116,-0.049,-0.027) \\
& \left(\zeta^{P}, \zeta_{J}^{P}, \zeta^{V}, \zeta_{J}^{V}, \zeta_{g}, \zeta_{J g}\right)_{2}=(0.141,0.056,0.227,0.065,-0.100,0.051) \tag{3.8}
\end{align*}
$$

From these, we can obtain $\zeta_{(J)}^{B \eta_{q}}=\zeta_{(J)}^{B \eta_{s}}=\zeta_{(J)}^{P}, \zeta_{(J)}^{B \phi}=\zeta_{(J)}^{V}$, and $\zeta_{(J) g}^{B \eta_{q}}=\zeta_{(J) g}^{B \eta_{s}}=\zeta_{(J) g}$ under the assumption of flavor-SU(3) symmetry [48]. In eq. (3.8), we have not displayed the errors of the $\zeta$ s from the fit to data, which are available from ref. [48]. Other input parameters that we will employ are the meson masses $m_{\pi^{0}}=134.977, m_{\eta}=547.862$, $m_{\eta^{\prime}}=957.78, m_{\rho^{0}}=769, m_{\phi}=1019.46$, and $m_{B_{s}}=5366.89$, all in units of MeV , and the $B_{s}$ lifetime $\tau_{B_{s}}=1.505 \times 10^{-12} \mathrm{~s}$, which are their central values from ref. [10].

Before addressing the $Z^{\prime}$ influence on $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}, \phi\right)\left(\pi^{0}, \rho^{0}\right)$, we provide the SM predictions for their branching fractions, which are collected in table 2. For the first five modes, the SCET numbers have been evaluated with the preceding formulas and parameter values, and the last two columns correspond to the two solutions of SCET parameters in eq. (3.8). For the sixth $\left(\phi \rho^{0}\right)$ mode, the SCET entry has been computed with the CKM and SCET parameters supplied very recently in ref. [33]. The two errors in each of the SCET predictions are due to flavor-SU(3)-breaking effects which we have assumed to be $20 \%$ and due to the errors in the $\zeta$ s from the fits to data, respectively, the latter errors being given in refs. [33, 48]. The SCET numbers for $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}\right) \rho^{0}, \phi \pi^{0}\left(\bar{B}_{s} \rightarrow\right.$ $\phi \rho^{0}$ ) are close to the corresponding ones determined in ref. [48] ([33]). ${ }^{4}$ For comparison, in the second and third columns we quote numbers calculated with QCD factorization (QCDF) [31] and perturbative QCD (PQCD) [32]. Evidently, these two methods produce results comparable to those of SCET, especially with its Solution 2 in the case of the first five modes, considering the errors in the predictions. The entries for $\bar{B}_{s} \rightarrow \phi \rho^{0}$ are

[^3]also compatible with the new measurement $\mathcal{B}\left(\bar{B}_{s} \rightarrow \phi \rho^{0}\right)_{\exp }=(0.27 \pm 0.08) \times 10^{-6}$ [10] mentioned earlier. An important implication of what we see in this table is that NP would not be easily noticeable in the rates of these decays unless it could enhance them by more than a factor of 2 . This possibility may be unlikely to be realized in the case of $\bar{B}_{s} \rightarrow \phi \rho^{0}$ which has been detected having a rate consistent with SM expectations. Nevertheless, as we demonstrate below, substantial enhancement can still occur in some of the other channels.

Now we include the $Z^{\prime}$ contributions from eq. (3.4) in order to examine their impact on these decays. As table 2 indicates that the predictions of the SCET Solution 1 for $\bar{B}_{s} \rightarrow \eta^{\prime}\left(\pi^{0}, \rho^{0}\right)$ are comparatively quite suppressed, from this point on we employ only Solution 2 parameters in our treatment of $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}\right)\left(\pi^{0}, \rho^{0}\right), \phi \pi^{0}$. Thus, summing the SM and $Z^{\prime}$ terms for $m_{Z^{\prime}}=1 \mathrm{TeV}$, with the central values of the input parameters, we find the amplitudes (in units of GeV ) for the $\pi^{0}$ channels to be

$$
\begin{align*}
& 10^{9} \mathcal{A}_{\bar{B}_{s} \rightarrow \eta \pi^{0}} \simeq 1.67+0.47 i+(3.96-0.08 i)\left(\rho_{L}+\rho_{R}\right)\left(\delta_{L}-\delta_{R}\right), \\
& 10^{9} \mathcal{A}_{\bar{B}_{s} \rightarrow \eta^{\prime} \pi^{0}} \simeq 0.48-2.48 i-(1.90-0.04 i)\left(\rho_{L}+\rho_{R}\right)\left(\delta_{L}-\delta_{R}\right), \\
& 10^{9} \mathcal{A}_{\bar{B}_{s} \rightarrow \phi \pi^{0}} \simeq-2.88-1.69 i-(7.85-0.15 i)\left(\rho_{L}-\rho_{R}\right)\left(\delta_{L}-\delta_{R}\right) \tag{3.9}
\end{align*}
$$

and for the $\rho^{0}$ channels

$$
\begin{align*}
& 10^{9} \mathcal{A}_{\bar{B}_{s} \rightarrow \eta \rho^{0}} \simeq 2.56+0.77 i+(6.32-0.12 i)\left(\rho_{L}+\rho_{R}\right)\left(\delta_{L}+\delta_{R}\right), \\
& 10^{9} \mathcal{A}_{\bar{B}_{s} \rightarrow \eta^{\prime} \rho^{0}} \simeq 0.78-4.12 i-(3.03-0.06 i)\left(\rho_{L}+\rho_{R}\right)\left(\delta_{L}+\delta_{R}\right), \\
& 10^{9} \mathcal{A}_{\bar{B}_{s} \rightarrow \phi \rho^{0}} \simeq-6.53-1.47 i-(15.3-0.3 i)\left(\rho_{L}-\rho_{R}\right)\left(\delta_{L}+\delta_{R}\right), \tag{3.10}
\end{align*}
$$

where $\delta_{L, R}$ are defined in eq. (3.5).
We notice that the amplitudes in eqs. (3.9) and (3.10) do not all have the same dependence on $\rho_{L, R}$ and $\delta_{L, R}$. Therefore, although $\mathcal{B}\left(\bar{B}_{s} \rightarrow \phi \rho^{0}\right)_{\exp }$ implies a restraint on the values of $\left(\rho_{L}-\rho_{R}\right)\left(\delta_{L}+\delta_{R}\right)$ in $\mathcal{A}_{\bar{B}_{s} \rightarrow \phi \rho^{0}}$, the amplitudes for the other channels, which have different combinations of $\rho_{L, R}$ and $\delta_{L, R}$, may generally still be altered considerably with respect to their SM parts. However, in our particular $Z^{\prime}$ case $\rho_{L, R}$ must satisfy $\rho_{R} \sim 0.1 \rho_{L}$, as can be inferred from figure 2. Hence, based on eq. (3.10), we may expect that the amplitudes for $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}\right) \rho^{0}$ do not deviate hugely from their SM values. To look into this more concretely, for definiteness we take $\rho_{R}=0.1 \rho_{L}$ and impose

$$
\begin{equation*}
0.11 \leq 10^{6} \mathcal{B}\left(\bar{B}_{s} \rightarrow \phi \rho^{0}\right) \leq 0.43, \tag{3.11}
\end{equation*}
$$

which is the $2 \sigma$ range of $\mathcal{B}\left(\bar{B}_{s} \rightarrow \phi \rho^{0}\right)_{\text {exp }}$. From the allowed values of the product $\rho_{L}\left(\delta_{L}+\right.$ $\left.\delta_{R}\right)$ we can assess how much the branching fractions of $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}\right) \rho^{0}$ are modified compared to the central values of their respective SM predictions in table 2 under SCET Solution 2. We show the results in figure 3, which also depicts eq. (3.11) relative to the SM prediction. We further find that the ranges $\rho_{L}\left(\delta_{L}+\delta_{R}\right) \in[-0.99,-0.71]$ and [ $-0.23,0.048$ ] fulfill eq. (3.11). Within these ranges, represented by the horizontal portions of the unshaded areas in this figure, we learn that $\mathcal{B}\left(\bar{B}_{s} \rightarrow \eta \rho^{0}\right)$ (red solid curve) can reach


Figure 3. The calculated branching fractions of $\bar{B}_{s} \rightarrow \eta \rho^{0}$ (red solid curve), $\bar{B}_{s} \rightarrow \eta^{\prime} \rho^{0}$ (blue solid curve), and $\bar{B}_{s} \rightarrow \phi \rho^{0}$ (black curve), normalized by their respective SM predictions listed in table 2 , versus the product $\rho_{L}\left(\delta_{L}+\delta_{R}\right)$ in the case where $\rho_{R}=0.1 \rho_{L}$ and $m_{Z^{\prime}}=1 \mathrm{TeV}$. The vertical length of the unshaded areas and the $\rho_{L}\left(\delta_{L}+\delta_{R}\right)$ values within them satisfy the restriction in eq. (3.11).
up to $\sim 2.7$ times its SM value, whereas for $\bar{B}_{s} \rightarrow \eta^{\prime} \rho^{0}$ (blue solid curve) the enhancement is at most about 1.9 times. ${ }^{5}$

For $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}, \phi\right) \pi^{0}$, the $Z^{\prime}$-induced terms in eq. (3.9) are proportional to $\delta_{L}-\delta_{R}$. Therefore, these channels are not subject to the condition in eq. (3.11), and their amplitudes may be affected by the $Z^{\prime}$ contributions more than their $\rho^{0}$ counterparts. To examine this more quantitatively, we set $\rho_{R}=0.1 \rho_{L}$ as in the previous paragraph and subsequently compute the branching fractions of these $\pi^{0}$ modes for $-1 \leq \rho_{L}\left(\delta_{L}-\delta_{R}\right) \leq 1$. In figure 4 we present the results divided by the central values of their respective SM predictions in table 2 under SCET Solution 2. We observe that over most of the $\rho_{L}\left(\delta_{L}-\delta_{R}\right)>0$ region covered in this plot the $Z^{\prime}$ effects can cause the branching fractions of $\bar{B}_{s} \rightarrow(\eta, \phi) \pi^{0}$ to exceed their SM counterparts by at least a factor of 2 and up to about an order of magnitude. Moreover, the $\bar{B}_{s} \rightarrow(\eta, \phi) \pi^{0}$ rates tend to be enhanced together with roughly similar enlargement factors. One also notices that the $Z^{\prime}$ impact could instead bring about substantial reduction of their rates. In table 3, we provide examples of the enhancement factors for some representative values of $\rho_{L}\left(\delta_{L}-\delta_{R}\right)$.

It is worth remarking that the $Z^{\prime}$-generated coefficients $C_{3,5,7,9}^{Z^{\prime}}$ and $\tilde{C}_{3,5,7,9}^{Z^{\prime}}$ in eq. (3.4) enter the amplitudes for nonleptonic $b \rightarrow s$ decays which are not dominated by the contributions of the electroweak-penguin operators, such as $B_{s} \rightarrow K^{(*)} \bar{K}^{(*)}$ and $B \rightarrow \pi K^{(*)}$. Since these transitions have been observed, their data imply restrictions on the size of $\rho_{L}\left(\delta_{L} \pm \delta_{R}\right)$, as the requirement $\rho_{R} \sim 0.1 \rho_{L}$ in our $Z^{\prime}$ scenario implies that the role of $\tilde{C}_{3,5,7,9}^{Z^{\prime}}$ is minor. Our choices $\left|\rho_{L}\left(\delta_{L} \pm \delta_{R}\right)\right| \leq 1$ above correspond to $\left|C_{7}^{Z^{\prime}} \pm C_{9}^{Z^{\prime}}\right| \leq 0.0202 \simeq$ $2\left|C_{9}^{\mathrm{SM}}\right|$, where $C_{9}^{\mathrm{SM}}=-0.0103$ as quoted before. We have checked that the changes to the rates of those decays due to $\left|C_{7,9}^{Z^{\prime}}\right| \lesssim\left|C_{9}^{\mathrm{SM}}\right|$ are less than the uncertainties of the SCET es-

[^4]

Figure 4. The calculated branching fractions of $\bar{B}_{s} \rightarrow \eta \pi^{0}$ (red curve), $\bar{B}_{s} \rightarrow \eta^{\prime} \pi^{0}$ (blue curve), and $\bar{B}_{s} \rightarrow \phi \pi^{0}$ (black curve), normalized by their respective SM values listed in table 2 under SCET Solution 2, versus the product $\rho_{L}\left(\delta_{L}-\delta_{R}\right)$ in the case where $\rho_{R}=0.1 \rho_{L}$ and $m_{Z^{\prime}}=1 \mathrm{TeV}$.

| $\rho_{L}\left(\delta_{L}-\delta_{R}\right)$ | $\bar{B}_{s} \rightarrow \eta \pi^{0}$ | $\bar{B}_{s} \rightarrow \eta^{\prime} \pi^{0}$ | $\bar{B}_{s} \rightarrow \phi \pi^{0}$ |
| :---: | :---: | :---: | :---: |
| -1 | 2.5 | 2.0 | 1.9 |
| -0.5 | 0.17 | 1.3 | 0.32 |
| 0.5 | 5.0 | 1.0 | 3.9 |
| 1 | 12 | 1.3 | 9.1 |

Table 3. Enhancement factors of the branching fractions of $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}, \phi\right) \pi^{0}$ with respect to their SM predictions at a few representative values of $\rho_{L}\left(\delta_{L}-\delta_{R}\right)$ in the $\rho_{R}=0.1 \rho_{L}$ and $m_{Z^{\prime}}=1 \mathrm{TeV}$ case.
timates in the SM, which are typically around $20 \%$ to $40 \%$ [ $33,47,48$ ]. As for the influence of $C_{3,5}^{Z \prime}$, it can be minimized by adjusting the extra free parameters $\Delta_{L, R}^{d d}$ in eq. (3.4). ${ }^{6}$ For comparison, earlier studies [36, 38, 40] concerning potential NP in $\bar{B}_{s} \rightarrow \phi\left(\pi^{0}, \rho^{0}\right)$ pointed out that rate enhancement factors of a few to an order of magnitude could still occur in the $\phi \pi^{0}$ mode and that $\left|C_{j}^{\mathrm{NP}} / C_{9}^{\mathrm{SM}}\right| \lesssim 2$ was not yet disfavored.

Finally, we would like to mention that the $Z^{\prime}$ coupling parameters of interest are separately consistent with constraints which may be pertinent from collider measurements. We illustrate this with the specific examples in table 4 for different sets of $\rho_{L}\left(\delta_{L} \pm \delta_{R}\right)$ and $\Delta_{V}^{\mu \mu}$ values in the aforesaid case where $\rho_{R}=0.1 \rho_{L}$ and $m_{Z^{\prime}}=1 \mathrm{TeV}$. The choice $\left(\rho_{L}, \Delta_{V}^{\mu \mu}\right)=$ $(0.8,0.03)$ is evidently within the region covered in figure 2 , while the points $\left(\rho_{L}, \Delta_{V}^{\mu \mu}\right)=$ $((1.0,1.2), 0.02)$ lie in the extension thereof. In this table, the displayed numbers for $\delta_{L, R}$ can comfortably comply with the condition $\left|\delta_{L, R}\right| \leq 1.0\left[1+(1.3 \mathrm{TeV})^{2} / m_{Z^{\prime}}^{2}\right] m_{Z^{\prime}} /(3 \mathrm{TeV})$

[^5]| $\rho_{L}\left(\delta_{L}+\delta_{R}\right)$ | $\rho_{L}\left(\delta_{L}-\delta_{R}\right)$ | $\rho_{L}$ | $\delta_{L}$ | $\delta_{R}$ | $\Delta_{V}^{\mu \mu}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -0.85 | 0.5 | 0.8 | -0.219 | -0.844 | 0.03 |
| -0.90 | 0.7 | 1.0 | -0.1 | -0.8 | 0.02 |
| -0.95 | 0.9 | 1.2 | -0.021 | -0.771 | 0.02 |
| -0.99 | 1.0 | 1.2 | 0.004 | -0.829 | 0.02 |

Table 4. The quark- $Z^{\prime}$ coupling parameters $\rho_{L}$ and $\delta_{L, R}$ corresponding to a few sample sets of $\rho_{L}\left(\delta_{L} \pm \delta_{R}\right)$ and $\Delta_{V}^{\mu \mu}$ in the $\rho_{R}=0.1 \rho_{L}$ and $m_{Z^{\prime}}=1 \mathrm{TeV}$ case.
inferred in ref. [97] from the study on LHC bounds in ref. [98]. For the lepton sector, the results of refs. [80, 100] imply that the selected $\Delta_{V}^{\mu \mu}$ values are compatible with LEP data on $Z$-boson decays into lepton pairs [10]. Furthermore, as the $\bar{e} e Z^{\prime}$ interaction is supposed to be vanishing, restraints from LEP II measurements on $e^{+} e^{-} \rightarrow f \bar{f}$ can be evaded. Lastly, LHC searches for new high-mass phenomena in the dilepton final states have the potential for significantly probing $\delta_{L, R}$ and $\Delta_{V}^{\mu \mu}$ at the same time. Nevertheless, their samples values in table 4 can be checked to be consistent with the most recent $p p \rightarrow \ell^{+} \ell^{-} X$ results from the ATLAS experiment [99]. ${ }^{7}$

## 4 Conclusions

We have explored the possibility that the recently observed anomalies in several $b \rightarrow$ $s \mu^{+} \mu^{-}$processes are attributable to the interactions of a $Z^{\prime}$ boson which also contribute to rare nonleptonic decays of the $\bar{B}_{s}$ meson, namely $\bar{B}_{s} \rightarrow\left(\eta, \eta^{\prime}, \phi\right)\left(\pi^{0}, \rho^{0}\right)$. Given that the amplitudes for these purely isospin-violating decays have CKM-suppressed tree components and tend to be controlled mainly by the electroweak-penguin operators, their decay rates are expected to be relatively small in the SM, making these modes potentially sensitive to signals beyond the SM. The $Z^{\prime}$ couplings are subject to various restrictions, particularly from the data on $B_{s}-\bar{B}_{s}$ mixing and the new experimental finding on $\bar{B}_{s}^{0} \rightarrow \phi \rho^{0}$, besides the measurements of $b \rightarrow s \mu^{+} \mu^{-}$transitions. We showed that, within the allowed parameter space, the $Z^{\prime}$ impact on $\bar{B}_{s}^{0} \rightarrow(\eta, \phi) \pi^{0}$ can cause their rates to grow up to an order of magnitude greater than their expectations in the SM. On the other hand, the enhancement factors for $\bar{B}_{s}^{0} \rightarrow \eta^{\prime} \pi^{0},\left(\eta, \eta^{\prime}\right) \rho^{0}$ are at most a few. The different enlargement factors of these different channels depend not only on the combinations of the $Z^{\prime}$ couplings occurring in their amplitudes, but also on how the SM and $Z^{\prime}$ terms in the amplitudes interfere with each other. Therefore, the observations of more of these decays in future experiments, together with improved upcoming data on $b \rightarrow s \mu^{+} \mu^{-}$, will test our $Z^{\prime}$ model more comprehensively.

[^6]
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## References

[1] LHCb collaboration, Angular analysis of the $B^{0} \rightarrow K^{* 0} \mu^{+} \mu^{-}$decay using $3 \mathrm{fb}^{-1}$ of integrated luminosity, JHEP 02 (2016) 104 [arXiv:1512.04442] [INSPIRE].
[2] Belle collaboration, S. Wehle et al., Lepton-flavor-dependent angular analysis of $B \rightarrow K^{*} \ell^{+} \ell^{-}$, Phys. Rev. Lett. 118 (2017) 111801 [arXiv:1612.05014] [inSPIRE].
[3] LHCb collaboration, Test of lepton universality using $B^{+} \rightarrow K^{+} \ell^{+} \ell^{-}$decays, Phys. Rev. Lett. 113 (2014) 151601 [arXiv: 1406.6482 [ [inSPIRE].
[4] LHCb collaboration, Test of lepton universality with $B^{0} \rightarrow K^{* 0} \ell^{+} \ell^{-}$decays, JHEP 08 (2017) 055 [arXiv:1705.05802] [inSPIRE].
[5] G. Hiller and F. Krüger, More model-independent analysis of $b \rightarrow s$ processes, Phys. Rev. D 69 (2004) 074020 [hep-ph/0310219] [inSPIRE].
[6] HPQCD collaboration, C. Bouchard, G.P. Lepage, C. Monahan, H. Na and J. Shigemitsu, Standard Model predictions for $B \rightarrow K \ell^{+} \ell^{-}$with form factors from lattice QCD, Phys. Rev. Lett. 111 (2013) 162002 [Erratum ibid. 112 (2014) 149902] [arXiv:1306.0434] [NSPIRE].
[7] M. Bordone, G. Isidori and A. Pattori, On the Standard Model predictions for $R_{K}$ and $R_{K^{*}}$, Eur. Phys. J. C 76 (2016) 440 [arXiv:1605.07633] [inSPIRE].
[8] LHCb collaboration, Differential branching fractions and isospin asymmetries of $B \rightarrow K^{(*)} \mu^{+} \mu^{-}$decays, JHEP 06 (2014) 133 [arXiv: 1403.8044] [inSPIRE].
[9] LHCb collaboration, Angular analysis and differential branching fraction of the decay $B_{s}^{0} \rightarrow \phi \mu^{+} \mu^{-}$, JHEP 09 (2015) 179 [arXiv:1506.08777] [inSPIRE].
[10] Particle Data Group collaboration, C. Patrignani et al., Review of particle physics, Chin. Phys. C 40 (2016) 100001 [inSPIRE].
[11] S. Descotes-Genon, J. Matias and J. Virto, Understanding the $B \rightarrow K^{*} \mu^{+} \mu^{-}$anomaly, Phys. Rev. D 88 (2013) 074002 [arXiv:1307.5683] [inSPIRE].
[12] F. Beaujean, C. Bobeth and D. van Dyk, Comprehensive Bayesian analysis of rare (semi)leptonic and radiative B decays, Eur. Phys. J. C 74 (2014) 2897 [Erratum ibid. C 74 (2014) 3179] [arXiv:1310.2478] [inSPIRE].
[13] T. Hurth and F. Mahmoudi, On the LHCb anomaly in $B \rightarrow K^{*} \ell^{+} \ell^{-}$, JHEP 04 (2014) 097 [arXiv:1312.5267] [inSPIRE].
[14] R. Alonso, B. Grinstein and J. Martin Camalich, $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge invariance and the shape of new physics in rare B decays, Phys. Rev. Lett. 113 (2014) 241802 [arXiv:1407.7044] [INSPIRE].
[15] G. Hiller and M. Schmaltz, $R_{K}$ and future $b \rightarrow$ sll physics beyond the Standard Model opportunities, Phys. Rev. D 90 (2014) 054014 [arXiv:1408.1627] [inSPIRE].
[16] D. Ghosh, M. Nardecchia and S.A. Renner, Hint of lepton flavour non-universality in B meson decays, JHEP 12 (2014) 131 [arXiv:1408.4097] [INSPIRE].
[17] T. Hurth, F. Mahmoudi and S. Neshatpour, Global fits to $b \rightarrow$ slौ data and signs for lepton non-universality, JHEP 12 (2014) 053 [arXiv:1410.4545] [INSPIRE].
[18] B. Capdevila, A. Crivellin, S. Descotes-Genon, J. Matias and J. Virto, Patterns of new physics in $b \rightarrow s \ell^{+} \ell^{-}$transitions in the light of recent data, JHEP 01 (2018) 093 [arXiv:1704.05340] [INSPIRE].
[19] W. Altmannshofer, P. Stangl and D.M. Straub, Interpreting hints for lepton flavor universality violation, Phys. Rev. D 96 (2017) 055008 [arXiv:1704.05435] [INSPIRE].
[20] G. D'Amico et al., Flavour anomalies after the $R_{K^{*}}$ measurement, JHEP 09 (2017) 010 [arXiv:1704.05438] [INSPIRE].
[21] G. Hiller and I. Nisandzic, $R_{K}$ and $R_{K^{*}}$ beyond the Standard Model, Phys. Rev. D 96 (2017) 035003 [arXiv:1704.05444] [INSPIRE].
[22] L.-S. Geng, B. Grinstein, S. Jäger, J. Martin Camalich, X.-L. Ren and R.-X. Shi, Towards the discovery of new physics with lepton-universality ratios of $b \rightarrow$ sll decays, Phys. Rev. $\mathbf{D}$ 96 (2017) 093006 [arXiv:1704.05446] [InSPIRE].
[23] M. Ciuchini et al., On flavourful easter eggs for new physics hunger and lepton flavour universality violation, Eur. Phys. J. C 77 (2017) 688 [arXiv:1704.05447] [InSPIRE].
[24] A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, Gauge-invariant implications of the LHCb measurements on lepton-flavor nonuniversality, Phys. Rev. D 96 (2017) 035026 [arXiv:1704.05672] [INSPIRE].
[25] T. Hurth, F. Mahmoudi, D. Martinez Santos and S. Neshatpour, Lepton nonuniversality in exclusive $b \rightarrow$ slौ decays, Phys. Rev. D 96 (2017) 095034 [arXiv:1705.06274] [InSPIRE].
[26] R. Fleischer, Search for the angle $\gamma$ in the electroweak penguin dominated decay $B_{s} \rightarrow \pi^{0} \phi$, Phys. Lett. B 332 (1994) 419 [inSPIRE].
[27] N.G. Deshpande, X.-G. He and J. Trampetic, Unique signature of electroweak penguin in pure hadronic $B$ decays, Phys. Lett. B 345 (1995) 547 [hep-ph/9410388] [inSPIRE].
[28] B. Tseng, Exclusive charmless $B_{s}$ hadronic decays into $\eta^{\prime}$ and $\eta$, Phys. Lett. B 446 (1999) 125 [hep-ph/9807393] [inSPIRE].
[29] Y.-H. Chen, H.-Y. Cheng and B. Tseng, Charmless hadronic two-body decays of $B_{s}$ mesons, Phys. Rev. D 59 (1999) 074003 [hep-ph/9809364] [inSPIRE].
[30] LHCb collaboration, Observation of the decay $B_{s}^{0} \rightarrow \phi \pi^{+} \pi^{-}$and evidence for $B^{0} \rightarrow \phi \pi^{+} \pi^{-}$, Phys. Rev. D 95 (2017) 012006 [arXiv:1610.05187] [InSPIRE].
[31] H.-Y. Cheng and C.-K. Chua, $Q C D$ factorization for charmless hadronic $B_{s}$ decays revisited, Phys. Rev. D 80 (2009) 114026 [arXiv:0910.5237] [inSPIRE].
[32] A. Ali et al., Charmless non-leptonic $B_{s}$ decays to $P P, P V$ and $V V$ final states in the pQCD approach, Phys. Rev. D 76 (2007) 074018 [hep-ph/0703162] [INSPIRE].
[33] C. Wang, S.-H. Zhou, Y. Li and C.-D. Lu, Global analysis of charmless B decays into two vector mesons in soft-collinear effective theory, Phys. Rev. D 96 (2017) 073004 [arXiv:1708.04861] [INSPIRE].
[34] G. Faisel, Supersymmetric contributions to $\bar{B}_{s} \rightarrow \phi \pi^{0}$ and $\bar{B}_{s} \rightarrow \phi \rho^{0}$ decays in SCET, JHEP 08 (2012) 031 [arXiv:1106.4651] [inSPIRE].
[35] G. Faisel, Charged Higgs contribution to $\bar{B}_{s} \rightarrow \phi \pi^{0}$ and $\bar{B}_{s} \rightarrow \phi \rho^{0}$, Phys. Lett. B 731 (2014) 279 [arXiv:1311.0740] [InSPIRE].
[36] L. Hofer, D. Scherer and L. Vernazza, $B_{s} \rightarrow \phi \rho^{0}$ and $B_{s} \rightarrow \phi \pi^{0}$ as a handle on isospin-violating new physics, JHEP 02 (2011) 080 [arXiv:1011.6319] [INSPIRE].
[37] J. Hua, C.S. Kim and Y. Li, Testing the non-universal $Z^{\prime}$ model in $B_{s} \rightarrow \phi \pi^{0}$ decay, Phys. Lett. B 690 (2010) 508 [arXiv:1002.2532] [inSPIRE].
[38] Q. Chang, X.-Q. Li and Y.-D. Yang, A comprehensive analysis of hadronic $b \rightarrow s$ transitions in a family non-universal $Z^{\prime}$ model, J. Phys. G 41 (2014) 105002 [arXiv:1312.1302] [INSPIRE].
[39] G. Faisel, New physics contributions to $\bar{B}_{s} \rightarrow \pi^{0}\left(\rho^{0}\right) \eta^{\left({ }^{\prime}\right)}$ decays, Eur. Phys. J. C 77 (2017) 380 [arXiv:1412.3011] [inSPIRE].
[40] C. Bobeth, M. Gorbahn and S. Vickers, Weak annihilation and new physics in charmless $B \rightarrow M M$ decays, Eur. Phys. J. C 75 (2015) 340 [arXiv:1409.3252] [INSPIRE].
[41] C.W. Bauer, S. Fleming and M.E. Luke, Summing Sudakov logarithms in $B \rightarrow X_{s} \gamma$ in effective field theory, Phys. Rev. D 63 (2000) 014006 [hep-ph/0005275] [INSPIRE].
[42] C.W. Bauer, S. Fleming, D. Pirjol and I.W. Stewart, An effective field theory for collinear and soft gluons: heavy to light decays, Phys. Rev. D 63 (2001) 114020 [hep-ph/0011336] [INSPIRE].
[43] J.-G. Chay and C. Kim, Factorization of $B$ decays into two light mesons in soft collinear effective theory, Phys. Rev. D 68 (2003) 071502 [hep-ph/0301055] [INSPIRE].
[44] J. Chay and C. Kim, Nonleptonic B decays into two light mesons in soft collinear effective theory, Nucl. Phys. B 680 (2004) 302 [hep-ph/0301262] [inSPIRE].
[45] C.W. Bauer, D. Pirjol, I.Z. Rothstein and I.W. Stewart, $B \rightarrow M_{1} M_{2}$ : factorization, charming penguins, strong phases and polarization, Phys. Rev. D 70 (2004) 054015 [hep-ph/0401188] [INSPIRE].
[46] C.W. Bauer, I.Z. Rothstein and I.W. Stewart, SCET analysis of $B \rightarrow K \pi, B \rightarrow K \bar{K}$ and $B \rightarrow \pi \pi$ decays, Phys. Rev. D 74 (2006) 034010 [hep-ph/0510241] [inSPIRE].
[47] A.R. Williamson and J. Zupan, Two body B decays with isosinglet final states in SCET, Phys. Rev. D 74 (2006) 014003 [Erratum ibid. D 74 (2006) 03901] [hep-ph/0601214] [inSPIRE].
[48] W. Wang, Y.-M. Wang, D.-S. Yang and C.-D. Lu, Charmless two-body $B_{(s)} \rightarrow V P$ decays in soft-collinear-effective-theory, Phys. Rev. D 78 (2008) 034011 [arXiv:0801.3123] [InSPIRE].
[49] A. Crivellin, G. D'Ambrosio and J. Heeck, Explaining $h \rightarrow \mu^{ \pm} \tau^{\mp}, B \rightarrow K^{*} \mu^{+} \mu^{-}$and $B \rightarrow K \mu^{+} \mu^{-} / B \rightarrow K e^{+} e^{-}$in a two-Higgs-doublet model with gauged $L_{\mu}-L_{\tau}$, Phys. Rev. Lett. 114 (2015) 151801 [arXiv:1501.00993] [INSPIRE].
[50] A. Crivellin, G. D'Ambrosio and J. Heeck, Addressing the LHC flavor anomalies with horizontal gauge symmetries, Phys. Rev. D 91 (2015) 075006 [arXiv:1503.03477] [inSPIRE].
[51] A. Crivellin, L. Hofer, J. Matias, U. Nierste, S. Pokorski and J. Rosiek, Lepton-flavour violating $B$ decays in generic $Z^{\prime}$ models, Phys. Rev. D 92 (2015) 054013 [arXiv:1504.07928] [INSPIRE].
[52] G. Bélanger, C. Delaunay and S. Westhoff, A dark matter relic from muon anomalies, Phys. Rev. D 92 (2015) 055021 [arXiv:1507.06660] [InSPIRE].
[53] B. Allanach, F.S. Queiroz, A. Strumia and S. Sun, $Z^{\prime}$ models for the LHCb and $g-2$ muon anomalies, Phys. Rev. D 93 (2016) 055045 [arXiv:1511.07447] [INSPIRE].
[54] K. Fuyuto, W.-S. Hou and M. Kohda, $Z^{\prime}$-induced FCNC decays of top, beauty and strange quarks, Phys. Rev. D 93 (2016) 054021 [arXiv:1512.09026] [inSPIRE].
[55] C.-W. Chiang, X.-G. He and G. Valencia, $Z^{\prime}$ model for $b \rightarrow s \ell \bar{\ell}$ flavor anomalies, Phys. Rev. D 93 (2016) 074003 [arXiv:1601.07328] [INSPIRE].
[56] D. Bečirević, O. Sumensari and R. Zukanovich Funchal, Lepton flavor violation in exclusive $b \rightarrow s$ decays, Eur. Phys. J. C 76 (2016) 134 [arXiv:1602.00881] [InSPIRE].
[57] C.S. Kim, X.-B. Yuan and Y.-J. Zheng, Constraints on a $Z^{\prime}$ boson within minimal flavor violation, Phys. Rev. D 93 (2016) 095009 [arXiv:1602.08107] [inSPIRE].
[58] W. Altmannshofer, S. Gori, S. Profumo and F.S. Queiroz, Explaining dark matter and B decay anomalies with an $L_{\mu}-L_{\tau}$ model, JHEP 12 (2016) 106 [arXiv:1609.04026] [INSPIRE].
[59] B. Bhattacharya, A. Datta, J.-P. Guévin, D. London and R. Watanabe, Simultaneous explanation of the $R_{K}$ and $R_{D^{(*)}}$ puzzles: a model analysis, JHEP 01 (2017) 015 [arXiv:1609.09078] [inSPIRE].
[60] A. Crivellin, J. Fuentes-Martin, A. Greljo and G. Isidori, Lepton flavor non-universality in B decays from dynamical Yukawas, Phys. Lett. B 766 (2017) 77 [arXiv:1611.02703] [InSPIRE].
[61] P. Ko, T. Nomura and H. Okada, A flavor dependent gauge symmetry, predictive radiative seesaw and LHCb anomalies, Phys. Lett. B 772 (2017) 547 [arXiv:1701.05788] [inSPIRE].
[62] P. Ko, T. Nomura and H. Okada, Explaining $B \rightarrow K^{(*)} \ell^{+} \ell^{-}$anomaly by radiatively induced coupling in $\mathrm{U}(1)_{\mu-\tau}$ gauge symmetry, Phys. Rev. D 95 (2017) 111701 [arXiv:1702.02699] [InSPIRE].
[63] P. Ko, Y. Omura, Y. Shigekami and C. Yu, LHCb anomaly and B physics in flavored $Z^{\prime}$ models with flavored Higgs doublets, Phys. Rev. D 95 (2017) 115040 [arXiv:1702.08666] [INSPIRE].
[64] J.F. Kamenik, Y. Soreq and J. Zupan, Lepton flavor universality violation without new sources of quark flavor violation, Phys. Rev. D 97 (2018) 035002 [arXiv:1704.06005] [INSPIRE].
[65] S. Di Chiara et al., Minimal flavor-changing $Z^{\prime}$ models and muon $g-2$ after the $R_{K^{*}}$ measurement, Nucl. Phys. B 923 (2017) 245 [arXiv:1704.06200] [INSPIRE].
[66] D. Ghosh, Explaining the $R_{K}$ and $R_{K^{*}}$ anomalies, Eur. Phys. J. C 77 (2017) 694 [arXiv:1704.06240] [inSPIRE].
[67] A.K. Alok, D. Kumar, J. Kumar and R. Sharma, Lepton flavor non-universality in the $B$-sector: a global analyses of various new physics models, arXiv:1704.07347 [INSPIRE].
[68] A.K. Alok, B. Bhattacharya, A. Datta, D. Kumar, J. Kumar and D. London, New physics in $b \rightarrow s \mu^{+} \mu^{-}$after the measurement of $R_{K^{*}}$, Phys. Rev. D 96 (2017) 095009 [arXiv:1704.07397] [INSPIRE].
[69] W. Wang and S. Zhao, Implications of the $R_{K}$ and $R_{K^{*}}$ anomalies, Chin. Phys. C 42 (2018) 013105 [arXiv:1704.08168] [inSPIRE].
[70] A. Greljo and D. Marzocca, High-p $p_{T}$ dilepton tails and flavor physics, Eur. Phys. J. C 77 (2017) 548 [arXiv:1704.09015] [INSPIRE].
[71] R. Alonso, P. Cox, C. Han and T.T. Yanagida, Anomaly-free local horizontal symmetry and anomaly-full rare B-decays, Phys. Rev. D 96 (2017) 071701 [arXiv:1704.08158] [INSPIRE].
[72] R. Alonso, P. Cox, C. Han and T.T. Yanagida, Flavoured B-L local symmetry and anomalous rare B decays, Phys. Lett. B 774 (2017) 643 [arXiv:1705.03858] [inSPIRE].
[73] C. Bonilla, T. Modak, R. Srivastava and J.W.F. Valle, $\mathrm{U}(1)_{B_{3}-3 L_{\mu}}$ gauge symmetry as the simplest description of $b \rightarrow s$ anomalies, arXiv:1705.00915 [INSPIRE].
[74] J. Ellis, M. Fairbairn and P. Tunney, Anomaly-free models for flavour anomalies, arXiv:1705.03447 [INSPIRE].
[75] F. Bishara, U. Haisch and P.F. Monni, Regarding light resonance interpretations of the $B$ decay anomalies, Phys. Rev. D 96 (2017) 055002 [arXiv:1705.03465] [inSPIRE].
[76] Y. Tang and Y.-L. Wu, Flavor non-universality gauge interactions and anomalies in $B$-meson decays, arXiv:1705.05643 [INSPIRE].
[77] A. Datta, J. Kumar, J. Liao and D. Marfatia, New light mediators for the $R_{K}$ and $R_{K^{*}}$ puzzles, arXiv:1705.08423 [INSPIRE].
[78] S. Matsuzaki, K. Nishiwaki and R. Watanabe, Phenomenology of flavorful composite vector bosons in light of $B$ anomalies, JHEP 08 (2017) 145 [arXiv:1706.01463] [inSPIRE].
[79] L. Di Luzio and M. Nardecchia, What is the scale of new physics behind the B-flavour anomalies?, Eur. Phys. J. C 77 (2017) 536 [arXiv:1706.01868] [INSPIRE].
[80] C.-W. Chiang, X.-G. He, J. Tandean and X.-B. Yuan, $R_{K^{(*)}}$ and related $b \rightarrow$ sl $\bar{\ell}$ anomalies in minimal flavor violation framework with $Z^{\prime}$ boson, Phys. Rev. D 96 (2017) 115022 [arXiv:1706.02696] [inSPIRE].
[81] S.F. King, Flavourful $Z^{\prime}$ models for $R_{K^{(*)}}$, JHEP 08 (2017) 019 [arXiv:1706.06100] [inSPIRE].
[82] R.S. Chivukula, J. Isaacson, K.A. Mohan, D. Sengupta and E.H. Simmons, $R_{K}$ anomalies and simplified limits on $Z^{\prime}$ models at the LHC, Phys. Rev. D 96 (2017) 075012 [arXiv:1706.06575] [INSPIRE].
[83] J.M. Cline and J. Martin Camalich, B decay anomalies from non-Abelian local horizontal symmetry, Phys. Rev. D 96 (2017) 055036 [arXiv:1706.08510] [inSPIRE].
[84] C.-H. Chen and T. Nomura, Penguin $b \rightarrow s \ell^{\prime+} \ell^{\prime-}$ and $B$-meson anomalies in a gauged $L_{\mu}-L_{\tau}$, Phys. Lett. B 777 (2018) 420 [arXiv:1707.03249] [inSPIRE].
[85] S. Baek, Dark matter contribution to $b \rightarrow s \mu^{+} \mu^{-}$anomaly in local $\mathrm{U}(1)_{L_{\mu}-L_{\tau}}$ model, arXiv:1707. 04573 [inSPIRE].
[86] L. Bian, S.-M. Choi, Y.-J. Kang and H.M. Lee, A minimal flavored $\mathrm{U}(1)^{\prime}$ for $B$-meson anomalies, Phys. Rev. D 96 (2017) 075038 [arXiv:1707.04811] [INSPIRE].
[87] M. Dalchenko, B. Dutta, R. Eusebi, P. Huang, T. Kamon and D. Rathjens, Bottom-quark fusion processes at the LHC for probing $Z^{\prime}$ models and $B$-meson decay anomalies, arXiv:1707. 07016 [inSPIRE].
[88] N.B. Beaudry, A. Datta, D. London, A. Rashed and J.-S. Roux, The $B \rightarrow \pi K$ puzzle revisited, JHEP 01 (2018) 074 [arXiv:1709.07142] [INSPIRE].
[89] C.-W. Chiang, Y.-F. Lin and J. Tandean, Probing leptonic interactions of a family-nonuniversal $Z^{\prime}$ boson, JHEP 11 (2011) 083 [arXiv:1108.3969] [INSPIRE].
[90] A.J. Buras, F. De Fazio and J. Girrbach, The anatomy of $Z^{\prime}$ and $Z$ with flavour changing neutral currents in the flavour precision era, JHEP 02 (2013) 116 [arXiv:1211.1896] [INSPIRE].
[91] UTFIT collaboration, M. Bona et al., Model-independent constraints on $\Delta F=2$ operators and the scale of new physics, JHEP 03 (2008) 049 [arXiv:0707.0636] [inSPIRE].
[92] UTfit collaboration webpage, http://www.utfit.org.
[93] V. Barger, L.L. Everett, J. Jiang, P. Langacker, T. Liu and C.E.M. Wagner, $b \rightarrow s$ transitions in family-dependent $\mathrm{U}(1)^{\prime}$ models, JHEP 12 (2009) 048 [arXiv:0906.3745] [INSPIRE].
[94] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Weak decays beyond leading logarithms, Rev. Mod. Phys. 68 (1996) 1125 [hep-ph/9512380] [INSPIRE].
[95] T. Feldmann, P. Kroll and B. Stech, Mixing and decay constants of pseudoscalar mesons, Phys. Rev. D 58 (1998) 114006 [hep-ph/9802409] [inSPIRE].
[96] T. Feldmann, P. Kroll and B. Stech, Mixing and decay constants of pseudoscalar mesons: the sequel, Phys. Lett. B 449 (1999) 339 [hep-ph/9812269] [INSPIRE].
[97] A.J. Buras, New physics patterns in $\varepsilon^{\prime} / \varepsilon$ and $\varepsilon_{K}$ with implications for rare kaon decays and $\Delta M_{K}, J H E P 04$ (2016) 071 [arXiv:1601.00005] [INSPIRE].
[98] M. de Vries, Four-quark effective operators at hadron colliders, JHEP 03 (2015) 095 [arXiv:1409.4657] [INSPIRE].
[99] ATLAS collaboration, Search for new high-mass phenomena in the dilepton final state using $36 \mathrm{fb}^{-1}$ of proton-proton collision data at $\sqrt{s}=13 \mathrm{Te} V$ with the ATLAS detector, JHEP 10 (2017) 182 [arXiv:1707.02424] [inSPIRE].
[100] C.-W. Chiang, T. Nomura and J. Tandean, Effects of family nonuniversal Z' boson on leptonic decays of Higgs and weak bosons, Phys. Rev. D 87 (2013) 075020 [arXiv:1302.2894] [INSPIRE].


[^0]:    ${ }^{1}$ Of the six modes, only $\bar{B}_{s} \rightarrow \phi \pi^{0}$ was discussed in [37, 38], $\bar{B}_{s} \rightarrow \phi\left(\pi^{0}, \rho^{0}\right)$ in [36], and $\bar{B}_{s} \rightarrow$ $\left(\eta, \eta^{\prime}\right)\left(\pi^{0}, \rho^{0}\right)$ in [39].

[^1]:    ${ }^{2}$ The expressions for $O_{i}, i=1,2, \cdots, 10$, can be found in, e.g., [47].

[^2]:    ${ }^{3}$ The formula for $b_{2}$ given in [39] contains typos which we have corrected here in eq. (3.7).

[^3]:    ${ }^{4}$ The SCET predictions in table 2 differ from those obtained in [39] because some of the input parameters used in our two papers are not the same.

[^4]:    ${ }^{5}$ Before the LHCb detection of the $\bar{B}_{s} \rightarrow \phi \rho^{0}$ evidence [30], the possibilities of the $\bar{B}_{s} \rightarrow(\phi, \eta) \rho^{0}$ rates exceeding their SM predictions by an order of magnitude were entertained in [36, 39], respectively, for the $\delta_{L}=0$ case.

[^5]:    ${ }^{6}$ For instance, selecting $2 \Delta_{L, R}^{d d}=-\Delta_{L, R}^{u u}$ would lead to $C_{3,5}^{Z^{\prime}}=0$, which was considered in [36, 39].

[^6]:    ${ }^{7}$ We may test our $Z^{\prime}$ coupling choices with the latest ATLAS [99] constraint on a nonstandard quarkmuon contact interaction of the form $\mathcal{L}=\left(4 \pi / \Lambda^{2}\right) \eta_{\chi \chi^{\prime}} \bar{q}_{\chi} \gamma^{\beta} q_{\chi} \bar{\mu}_{\chi^{\prime}} \gamma_{\beta} \mu_{\chi^{\prime}}$, where $\Lambda$ is a heavy mass scale, $\eta_{\chi \chi^{\prime}}=-1(1)$ if the new and SM contributions to $q \bar{q} \rightarrow \mu^{+} \mu^{-}$interfere constructively (destructively), and $\chi, \chi^{\prime}=L, R$. It turns out that the strongest restriction applies to the $\chi \chi^{\prime}=R L$ or $R R$ case and arises from the $95 \%$-CL limit $\Lambda>28 \mathrm{TeV}$ [99] corresponding to $4 \pi / \Lambda^{2}<0.016 \mathrm{TeV}^{-2}$. This can be fulfilled by $\left|\Delta_{R}^{q q} \Delta_{V}^{\mu \mu}\right|$ for $q=u, d$ and the entries in the last two columns of table 4 with selections such as $\Delta_{R}^{u u}=-\Delta_{R}^{d d}=\delta_{R} / 2$.

