Gauge coupling unification in a classically scale invariant model

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ABSTRACT: There are a lot of works within a class of classically scale invariant model, which is motivated by solving the gauge hierarchy problem. In this context, the Higgs mass vanishes at the UV scale due to the classically scale invariance, and is generated via the Coleman-Weinberg mechanism. Since the mass generation should occur not so far from the electroweak scale, we extend the standard model only around the TeV scale. We construct a model which can achieve the gauge coupling unification at the UV scale. In the same way, the model can realize the vacuum stability, smallness of active neutrino masses, baryon asymmetry of the universe, and dark matter relic abundance. The model predicts the existence vector-like fermions charged under $SU(3)_C$ with masses lower than $1\,\mathrm{TeV}$, and the SM singlet Majorana dark matter with mass lower than $2.6\,\mathrm{TeV}$.

Contents

1 Introduction

The Higgs mass parameter m_h^2 is only a dimensionful parameter in the standard model (SM), and its value is estimated by the observed Higgs mass as $\sqrt{-2m_h^2} = M_h = 125.09 \pm 0.21\,\text{(stat.)} \pm 0.11\,\text{(syst.)}\,\text{GeV}$ [1]. Then, a running of the Higgs quartic coupling becomes negative below the Planck scale within the SM. If the SM can be valid up to a high energy scale such as a breaking scale of a gauge symmetry in the grand unification theory (GUT), the electroweak (EW) scale should be stabilized against radiative corrections coming from the high energy physics. To solve the gauge hierarchy problem, there are a lot of works motivated by a classically scale invariance [2]–[29]. The scale invariance prohibits dimensionful parameters at a classical level, while it can be radiatively broken by the Coleman-Weinberg (CW) mechanism [30]. In addition to the classically scale invariance, with an additional U(1)$_X$ gauge symmetry, e.g., U(1)$_{B-L}$ gauge symmetry, it is possible to naturally realize experimentally observed values of the Higgs mass. When the U(1)$_X$ symmetry is broken by the CW mechanism, the EW symmetry could be also broken through the scalar mixing term. If the U(1)$_X$ breaking scale is not far from the EW scale, the Higgs mass corrections would be sufficiently small, and then the hierarchy problem can be solved. Note that these statements are based on the Bardeen's argument [31], and we consider only logarithmic divergences in this paper (see ref. [7] for more detailed discussions).

In this paper, we assume the classically scale invariance at the UV scale, where the SM gauge couplings are unified. We expect that some unknown mechanism, such as a string theory, realizes the classically scale invariance and the gauge coupling unification (GCU). Actually, the GCU can be realized at $3 \times 10^{16}\,\text{GeV}$ in our model, and the scale is near the typical string scale ($\sim 10^{17}\,\text{GeV}$). To realize the GCU, some additional particles with the

SM gauge charges are needed. Conditions of the GCU can be systematically obtained by an analysis of renormalization group equations (RGEs) [32, 33]. When all additional particles are vector-like fermions with the TeV scale masses, the GCU scale can be realized between 10^{16} GeV and 10^{17} GeV, and there are a lot of possibilities to realize the GCU at the scale.[1] For example, vector-like pairs of quark doublet $Q_{L,R}$ and down-type quark singlet $D_{L,R}$ can achieve the GCU [34, 35]. When there are additional fermions charged under the SM gauge symmetries, the gauge couplings and the top Yukawa coupling respectively become larger and smaller compared to the SM case, and then, both changes make the β function of the Higgs quartic coupling become larger. Therefore, the vacuum can become stable when the GCU is realized.

To solve the gauge hierarchy problem, there should be no intermediate scale between the EW and the GCU scales except an energy scale, which is not so far from the EW scale, i.e., the TeV scale. Then, phenomenological and cosmological problems (e.g., smallness of active neutrino masses, baryon asymmetry of the universe, and dark matter (DM)) should be explained with sufficiently small Higgs mass corrections. The first two problems can be explained by the right-handed neutrinos, which are naturally introduced to cancel the anomalies accompanied with the U(1)$_X$ gauge symmetry, via type-I seesaw mechanism [36–40] and resonant leptogenesis [41], respectively. In our model, the DM is identified with the SM singlet Majorana fermions, and its stability can be guaranteed by an additional Z_2 symmetry [42]. In this paper, we will show that our model can explain the above problems as well as realizing the GCU without affecting the hierarchy problem.[2]

In the next section, we will define our model, and explain the U(1)$_X$ gauge symmetry breaking as well as the EW symmetry breaking via the CW mechanism. We also obtain the upper bound on the U(1)$_X$ breaking scale from the naturalness. In section 3, we will discuss the GCU, vacuum stability, smallness of active neutrino masses, baryon asymmetry of the universe, and the DM relic abundance. Our model predicts the existence vector-like fermions charged under SU(3)$_C$ with masses lower than 1 TeV, and the SM singlet Majorana dark matter with mass lower than 2.6 TeV. We summarize our results in section 4.

2 Symmetry breaking mechanism

We consider the U(1)$_X$ gauge extension of the SM with three generations of the right-handed neutrinos ν_{R_i} ($i = 1, 2, 3$), six vector-like fermions (Q_L, Q_R, D_L, D_R, N_L, and N_R), and two SM singlet scalars (Φ and S). Charge assignments of the particles are shown in table 1. The U(1)$_X$ charge are given by $B - L + 2x_H Y$, where x_H, B, L, and Y denote a

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[1]For example, we can consider the origin of the vector-like fermions as the string theory, in which a number of vector-like fermions should appear above the compact scale, which is expected to be the GCU scale in our model. Some of them might have the TeV scale masses due to the fine-tuning of moduli (or Wilson line, extra-dimensional component of anti-symmetric tensor field, and so on).

[2]From theoretical point of view, there are some papers constructing a model which realizes classically scale invariance and gauge coupling unification at the same scale [43]–[45]. Furthermore, asymptotic safety of gravity [46] leads vanishing couplings at the UV scale, which suggests vanishing quartic couplings and gauge coupling unification around the Planck scale [see figure 1 in ref. [47] for example]. In this paper, we simply expect such a situation comes from unknown UV physics.

	$\mathrm{SU(3)}_C \otimes \mathrm{SU(2)}_L \otimes \mathrm{U(1)}_Y$	$\mathrm{U(1)}_X$	Z_2
q_L	(3, 2, 1/6)	$(x_H + 1)/3$	+
u_R	(3, 1, 2/3)	$(4x_H + 1)/3$	+
d_R	(3, 1, −1/3)	$(−2x_H + 1)/3$	+
ℓ_L	(1, 2, −1/2)	$−x_H − 1$	+
e_R	(1, 1, −1)	$−2x_H − 1$	+
ν_R	(1, 1, 0)	−1	+
H	(1, 2, 1/2)	x_H	+
$Q_{L,R}$	(3, 2, 1/6)	$(x_H + 1)/3$	−
$D_{L,R}$	(3, 1, −1/3)	$(−2x_H + 1)/3$	−
$N_{L,R}$	(1, 1, 0)	−1	−
Φ	(1, 1, 0)	2	+
S	(1, 1, 0)	0	+

Table 1. Charge assignment of particles, where x_H is a real number.

real number, the baryon and lepton numbers, and the $\mathrm{U(1)}_Y$ hypercharge, respectively. In particular, $x_H = 0$, $−1$ and $−2/5$ correspond to $\mathrm{U(1)}_{B−L}$, $\mathrm{U(1)}_R$ and $\mathrm{U(1)}_\chi$, respectively. The vector-like fermions $Q_{L,R}$, $D_{L,R}$, and $N_{L,R}$ respectively have the same charges as the SM quark doublet, the SM down-quark singlet, and the right-handed neutrino, while only the vector-like fermions are odd under an additional Z_2 symmetry. Four of the vector-like fermions ($Q_{L,R}$ and $D_{L,R}$) play a role for achieving the GCU, and the others ($N_{L,R}$) are the DM candidates, whose stability is guaranteed by the Z_2 symmetry. These particles are not necessary for the realization of GCU and DM. We choose them for the simplest extension.

The relevant Lagrangian is given by

$$
\begin{aligned}
\mathcal{L} = {} & \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{kinetic}} - V(H, \Phi, S) - (Y_\nu \overline{\ell_L} H^c \nu_R + \kappa_1 \overline{Q_L} H D_R + \kappa_2 \overline{D_L} H Q_R \\
& + Y_M \Phi \overline{\nu_R^c} \nu_R + Y_{N_L} \Phi \overline{N_L} N_L^c + Y_{N_R} \Phi \overline{N_R^c} N_R \\
& + f_Q S \overline{Q_L} Q_R + f_D S \overline{D_L} D_R + f_N S \overline{N_L} N_R + \mathrm{h.c.}),
\end{aligned}
\tag{2.1}
$$

where $\mathcal{L}_{\mathrm{SM}}$ is the SM Lagrangian except for the Higgs sector, $\mathcal{L}_{\mathrm{kinetic}}$ includes kinetic terms of the Higgs and new particles, and $V(H, \Phi, S)$ is a scalar potential of the model. Without the Z_2 symmetry, there are also additional Yukawa interactions between the SM particles and the new particles, e.g., $y_1 \overline{Q_L} H^c u_R$, $y_2 \overline{Q_L} H d_R$, and $y_3 \overline{q_L} H D_R$. However, these coupling constants have to be very small due to constraints from the precision electroweak data [48]. To forbid these terms, we have imposed odd parity to only the vector-like fermions under the Z_2 symmetry.

Since there are two $\mathrm{U(1)}$ gauge symmetry, $\mathrm{U(1)}$ kinetic mixing generally arises in the model. We can take covariant derivative as

$$
D_\mu = \partial_\mu + i g_3 T^\alpha G_\mu^\alpha + i g_2 T^a W_\mu^a + i g_Y Y B_\mu + i(g_{\mathrm{mix}} Y + g_X X) Z_\mu', \tag{2.2}
$$

where g's are gauge couplings, T^α and T^a are generators of SU(3)$_C$ and SU(2)$_L$, respectively, and V_μ ($V = G^\alpha, W^a, B, Z'$) are gauge bosons. The coupling constant g_{mix} denotes the kinetic mixing between the U(1)$_Y$ and the U(1)$_X$ gauge symmetries, and we will take $g_{\mathrm{mix}} = 0$ at the GCU scale. This boundary condition naturally arises from breaking a simple unified gauge group into SU(3)$_C \times$ SU(2)$_L \times$ U(1)$_Y \times$ U(1)$_X$.

We impose the classically scale invariance at the GCU scale, and hence, the scalar potential $V(H, \Phi, S)$ is given by

$$V(H, \Phi, S) = \lambda_H |H|^4 + \lambda_\Phi |\Phi|^4 + \lambda_S S^4 + \lambda_{H\Phi} |H|^2 |\Phi|^2 + \lambda_{HS} |H|^2 S^2 + \lambda_{\Phi S} |\Phi|^2 S^2, \quad (2.3)$$

where there is no dimensionful parameter. In the model, a complex scalar singlet Φ spontaneously breaks the U(1)$_X$ gauge symmetry due to radiative corrections, i.e. the CW mechanism. Since the complex scalar field obtains the nonzero vacuum expectation value (VEV), the SM singlet scalar Φ, the U(1)$_X$ gauge boson Z', the right-handed neutrinos and the vector-like fermion $N_{L,R}$ become massive. After the U(1)$_X$ symmetry breaking, negative mass terms of a real scalar singlet S and the SM Higgs doublet H are generated, which induces the EW symmetry breaking. Then, S, the vector-like fermions and the SM particles become massive, and typically their masses are lighter than those obtained by the U(1)$_X$ symmetry breaking.

Let us explain the symmetry breaking mechanism more explicitly. We consider the CW potential for a classical field of the singlet scalar ϕ as

$$V_\Phi(\phi) = \frac{1}{4} \lambda_\Phi(v_\Phi) \phi^4 + \frac{1}{8} \beta_{\lambda_\Phi}(v_\Phi) \phi^4 \left(\ln \frac{\phi^2}{v_\Phi^2} - \frac{25}{6} \right), \quad (2.4)$$

where we have taken $\Phi = \phi/\sqrt{2}$ without loss of generality, and $\langle \phi \rangle = v_\Phi$ is the VEV of ϕ. β functions of Φ, β_{λ_Φ}, almost depends on quartic terms of g_X, Y_M and $Y_{N_{L,R}}$ for $\lambda_\Phi \simeq 0$. (β functions of the model parameters are given in appendix.) The effective potential (2.4) satisfies the following renormalization conditions

$$\left. \frac{\partial^2 V_\Phi}{\partial \phi^2} \right|_{\phi=0} = 0, \qquad \left. \frac{\partial^4 V_\Phi}{\partial \phi^4} \right|_{\phi=v_\Phi} = 6\lambda_\Phi, \quad (2.5)$$

and the minimization condition of V_Φ induces

$$\lambda_\Phi(v_\Phi) \simeq \frac{11}{6\pi^2} \left[6g_X^4(v_\Phi) - \left(\mathrm{tr} Y_M^4(v_\Phi) + Y_{N_L}^4(v_\Phi) + Y_{N_R}^4(v_\Phi) \right) \right], \quad (2.6)$$

where we have assumed that the scalar quartic couplings are negligibly small in the right-hand side. When this relation is satisfied, the U(1)$_X$ symmetry is broken, and Φ and Z' become massive as

$$M_\phi = \sqrt{\frac{6}{11} \lambda_\Phi(v_\Phi)} v_\Phi, \qquad M_{Z'} = 2g_X(v_\Phi) v_\Phi, \quad (2.7)$$

respectively. Since the right-hand side of eq. (2.6) should be positive, $\lambda_\Phi(v_\Phi) \lesssim g_X^4(v_\Phi)$ is required, and hence, $M_\phi < M_{Z'}$ is generally expected. In addition, the quartic terms of Majorana Yukawa couplings (Y_M and $Y_{N_{L,R}}$) are smaller than the quartic terms of g_X

because of $\lambda_\Phi(v_\Phi) > 0$. The masses of right-handed neutrinos and $N_{L,R}$ will be discussed in section 3.3.

After the $\mathrm{U}(1)_X$ symmetry breaking, the effective potentials for s and h are approximately given by

$$V_S(s) = \frac{1}{4}\lambda_S s^4 + \frac{1}{4}\lambda_{\Phi S} v_\Phi^2 s^2, \qquad V_H(h) = \frac{1}{4}\lambda_H h^4 + \frac{1}{4}\lambda_{H\Phi} v_\Phi^2 h^2, \tag{2.8}$$

where $S = s/\sqrt{2}$ and $H = (0, h/\sqrt{2})^T$. Here, we have assumed that λ_{HS} are negligibly small compared to $\lambda_{\Phi S}$ and $\lambda_{H\Phi}$ for simplicity. For $\kappa_{1,2} \simeq 0$, λ_{HS} is always negligibly small during renormalization group evolution [see eq. (A.16)]. When $\lambda_{\Phi S}$ and $\lambda_{H\Phi}$ are negative, the nonzero VEVs $\langle s \rangle = v_S$ and $\langle h \rangle = v_H$ are obtained as

$$v_S^2 = \frac{-\lambda_{\Phi S}}{2\lambda_S} v_\Phi^2, \qquad v_H^2 = \frac{-\lambda_{H\Phi}}{2\lambda_H} v_\Phi^2. \tag{2.9}$$

Note that v_S and v_H is typically lower than v_Φ, because the ratios of quartic couplings $(\lambda_{\Phi S}/(2\lambda_S)$ and $\lambda_{H\Phi}/(2\lambda_H))$ should be lower than unity to avoid the vacuum instability. The vector-like fermions and the SM particles become massive, while the masses of vector-like fermions ($Q_{L,R}$ and $D_{L,R}$) have to be lower than $1\,\mathrm{TeV}$ to realize the GCU as we will show in section 3.1.

In the end of this section, we mention the $\mathrm{U}(1)_X$ breaking scale, which is described by v_Φ. Since $M_{Z'}/g_X > 6.9\,\mathrm{TeV}$ is required from the LEP-II experiments [49], we obtain the lower bound $v_\Phi \gtrsim 3.5\,\mathrm{TeV}$. On the other hand, the naturalness of the Higgs mass suggests a relatively small v_Φ. A major correction to the Higgs mass is given by Z' intermediating diagrams, and one-loop and two-loop corrections are approximately written as

$$\Delta m_h^2 \sim \frac{4x_H^2 g_X^4 v_\Phi^2}{16\pi^2} \quad \text{for } x_H \neq 0, \tag{2.10}$$

$$\Delta m_h^2 \sim \frac{4(x_H + 1)(4x_H + 1)}{9} \frac{y_t^2 g_X^4 v_\Phi^2}{(16\pi^2)^2}, \tag{2.11}$$

respectively. When one defines requirement of the naturalness as $\Delta m_h^2 < M_h^2$, eqs. (2.10) and (2.11) lead the upper bound on v_Φ as

$$v_\Phi \lesssim \frac{1}{|x_H|} \left(\frac{0.1}{g_X}\right)^2 \times 10^5\,\mathrm{GeV} \text{ for } x_H \neq 0, \tag{2.12}$$

$$v_\Phi \lesssim \left(\frac{0.1}{g_X}\right)^2 \times 10^6\,\mathrm{GeV}, \tag{2.13}$$

where we have taken $y_t \approx 1$. For $|x_H| < 0.1$, the two-loop correction gives stronger bound than one-loop correction. In the following, we will use the stronger bound for fixed x_H. Note that the mass correction from Φ is always negligible because of a small mixing coupling $\lambda_{H\Phi}$.

3 Phenomenological and cosmological aspects

In this section, we will discuss phenomenological and cosmological aspects of the model: the GCU, vacuum stability and triviality, smallness of active neutrino masses, baryon
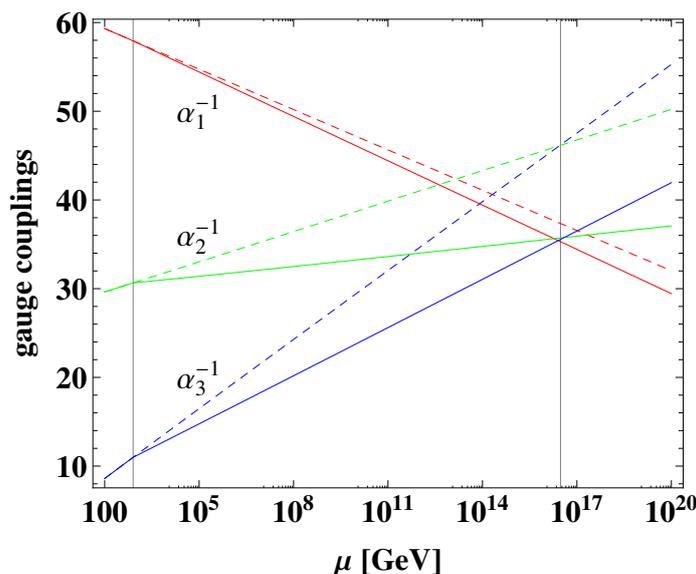
Figure 1. Runnings of gauge couplings α_i^{-1}. The dashed and solid lines correspond to the SM and the U(1)$_X$ extended model cases, respectively. The vertical lines express $M_V = 800\,\text{GeV}$ and $\Lambda_{\text{GCU}} = 3 \times 10^{16}\,\text{GeV}$.

asymmetry of the universe, and dark matter. We will also restrict the model parameters from the naturalness of the Higgs mass.

3.1 Gauge coupling unification

First, we discuss the possibility of the GCU at a high energy scale. Since four additional vector-like fermions ($Q_{L,R}$ and $D_{L,R}$) have gauge charges under the SM gauge groups as shown in table 1, runnings of the SM gauge couplings are modified from the SM. Then, β functions of gauge coupling constants are given by

$$\beta_{g_Y} = \frac{g_Y^3}{16\pi^2}\frac{15}{2}, \qquad \beta_{g_2} = \frac{g_2^3}{16\pi^2}\frac{-7}{6}, \qquad \beta_{g_3} = \frac{g_3^3}{16\pi^2}(-5), \tag{3.1}$$

at 1-loop level. Figure 1 shows runnings of gauge couplings $\alpha_i^{-1} \equiv 4\pi/g_i^2$, where U(1)$_Y$ gauge coupling is normalized as $g_1 \equiv \sqrt{5/3}g_Y$. The calculation has been done for $x_H = 0$ with using 2-loop RGEs. We note that the running of gauge couplings are almost independent of x_H. In the figure, the horizontal axis is the renormalization scale and the vertical axis indicates value of α_i^{-1}. The red, green, and blue lines show α_1^{-1}, α_2^{-1}, and α_3^{-1}, respectively. The dashed and solid lines correspond to the SM and our model, respectively. The left vertical line stands for a typical scale of vector-like fermions, which has been taken as $M_V = 800\,\text{GeV}$ in figure 1. For $\mu < M_V$, the β functions are the SM ones, and we take boundary conditions for the gauge couplings such that experimental values of the Weinberg angle, the fine structure constant, and the strong coupling can be reproduced [50]. The GCU can be achieved at $\Lambda_{\text{GCU}} = (2\text{–}4) \times 10^{16}\,\text{GeV}$, and the unified gauge coupling is $\alpha_{\text{GCU}}^{-1} = (35.4\text{–}35.8)$.[3] This is the same result as in ref. [34], in which only $Q_{L,R}$ and $D_{L,R}$

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[3]The GCU can be achieved by adjoint fermions as in ref. [51, 52].
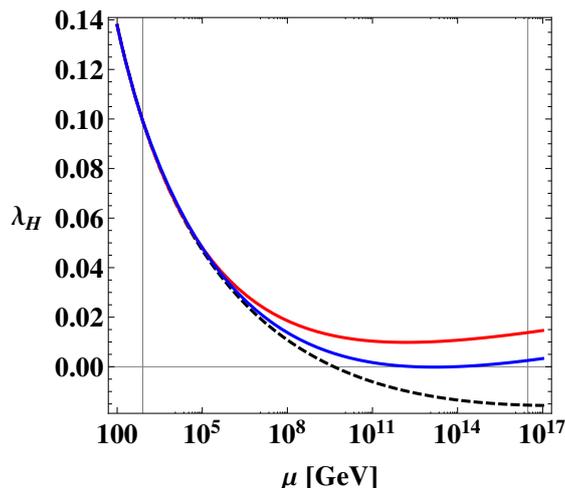
Figure 2. Running of λ_H in the U(1)$_{B-L}$ ($x_H = 0$) case. The red and blue lines correspond to $\kappa = 0$ and $\kappa = 0.33$, respectively. The black dashed line shows running of λ_H in the SM. The vertical lines express $M_V = 800\,\text{GeV}$ and $\Lambda_{\text{GCU}} = 3 \times 10^{16}\,\text{GeV}$.

are added into the SM. As the vector-like fermion masses become larger, the precision of the GCU becomes worse. Thus, the masses of $Q_{L,R}$ and $D_{L,R}$ should be lighter than 1 TeV, while vector-like fermion masses are constrained by the LHC experiments [53–55]. Since the lower bound of vector-like quark lies around 700 GeV, the possibility of the GCU can be testable in the near future.

We note that the proton lifetime in a GUT model. The proton lifetime is roughly derived from a four-fermion approximation for the decay channel $p \to e^+ + \pi^0$, which is given by

$$\tau_{\text{p}} \sim \left(\alpha_{\text{GCU}}^{-1}\right)^2 \frac{\Lambda_{\text{GCU}}^4}{m_{\text{p}}^5}, \tag{3.2}$$

where m_{p} is the proton mass. For $\Lambda_{\text{GCU}} = 3 \times 10^{16}\,\text{GeV}$ and $\alpha_{\text{GCU}}^{-1} = 35.6$, we can estimate $\tau_{\text{p}} \sim 10^{37}$ yrs, which is much longer than the experimental lower bound $\tau_{\text{p}} > 8.2 \times 10^{33}$ yrs [56]. Thus, the model are free from the constraint of the proton decay.

3.2 Vacuum stability and triviality

Next, we discuss the vacuum stability. However, it is difficult to investigate exact vacuum stability conditions, since there are three scalar fields and each of them has nonzero VEVs. Therefore, we simply investigate three necessary conditions: $\lambda_H > 0$, $\lambda_\Phi > 0$ and $\lambda_S > 0$.

The condition $\lambda_H > 0$ depends on additional contributions to β_{λ_H}, i.e., $\kappa_{1,2}$, g_X and scalar mixing couplings.[4] If their contributions to β_{λ_H} are negligible, since the SM gauge couplings are larger compared to the SM case, running of λ_H is raised and always positive. For example, however, the EW vacuum becomes instable for $\kappa \gtrsim 0.33$ in the U(1)$_{B-L}$

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[4]Running of λ_H also depends on mass (or Yukawa coupling) of the top quark. We will use the central value of world average, i.e., $M_t = 173.34\,\text{GeV}$ [57]. If we change this value of top quark mass, the following numerical results can slightly change.

($x_H = 0$) case. We show the running of λ_H for $x_H = 0$ in figure 2, where β_{λ_H} is independent of g_X up to the one-loop level, and contributions of g_X can be negligible. The red and blue lines correspond to $\kappa = 0$ and $\kappa = 0.33$, respectively. The black dashed line shows running of λ_H in the SM. Thus, $\kappa < 0.33$ is required to realize the vacuum stability.

The Higgs mass corrections from $Q_{L,R}$ and $D_{L,R}$ loops are given by

$$\Delta m_h^2 \sim \frac{v_S^2}{16\pi^2} \left[(\kappa_1^2 + \kappa_2^2)(f_Q^2 + f_D^2) + 2\kappa_1\kappa_2 f_Q f_D \right] \sim \frac{12\kappa^2 M_V^2}{16\pi^2}, \tag{3.3}$$

where we have taken $\kappa = \kappa_1 = \kappa_2$, which naturally arises from $L \leftrightarrow R$ symmetry for the vector-like particles, and $M_V = M_Q = M_D$ ($M_Q = f_Q v_S/\sqrt{2}$ and $M_D = f_D v_S/\sqrt{2}$) for simplicity. Then, the naturalness requires $\kappa < 0.1$ for $M_V \sim 1\,\text{TeV}$. Although κv_H is a contribution to the vector-like fermion masses from the Higgs, it can be ignored because of $\kappa v_H \ll M_V$. Since the contribution of κ to β_{λ_H}, i.e., $24\lambda_H\kappa^2 - 12\kappa^4$, is always positive for $\kappa < 0.1$, the naturalness condition also guarantees the vacuum stability. Note that $\kappa \simeq 0$ guarantees $\lambda_{HS} \simeq 0$ at any energy scale, which is required to justify our potential analysis for eq. (2.8).

Here, we check contributions of vector-like fermions to the S and T parameters, which are approximately given by [58, 59]

$$\delta S \approx \frac{43}{30\pi} \left(\frac{\kappa v_H}{M_V} \right)^2, \qquad \delta T \approx \frac{3(\kappa v_H)^2}{10\pi \sin^2\theta_W M_W^2} \left(\frac{\kappa v_H}{M_V} \right)^2, \tag{3.4}$$

where θ_W and M_W are the Weinberg angle and the W boson mass, respectively. For $\kappa < 0.1$, the parameters are estimated as $\delta S < 3 \times 10^{-4}$ and $\delta T < 2 \times 10^{-5}$, which are consistent with the precision EW data $S = 0.00 \pm 0.08$ and $T = 0.05 \pm 0.07$ [56].

The condition $\lambda_\Phi > 0$ is almost always satisfied when g_X is dominant in the right-hand side of eq. (2.6), i.e., $\lambda_\Phi(v_\Phi) \sim g_X^4(v_\Phi)$. In this case, β_{λ_Φ} is positive up to the GCU scale, and then λ_Φ is also positive up to the GCU scale. It is also possible to realize the critical condition $\lambda_\Phi(\Lambda_{\text{GCU}}) = 0$ as well as $\lambda_\Phi > 0$, where the running of λ_Φ is curved upward as in the so-called flatland scenario [9, 14, 16, 21, 24]. Then, both g_X and Majorana Yukawa couplings are dominant in β_{λ_Φ}, while λ_Φ is much smaller than them. This means that there is a fine-tuning to satisfy eq. (2.6).

When λ_S is negligible in its β function, a solution of its RGE is approximately given by

$$\lambda_S(\mu) \approx \lambda_S(v_S) - \frac{1}{16\pi^2} \left(12 f_Q^4(v_S) + 6 f_D^4(v_S) + 2 f_N^4(v_S) \right) \ln \frac{\mu}{v_S}, \tag{3.5}$$

where μ is a renormalization scale. Once v_S is fixed, f_Q and f_D are determined to realize the GCU, while f_N remains a free parameter. To estimate the condition of $\lambda_S > 0$, we assume $f_N = f_Q = f_D$ at $\mu = v_S$ for simplicity. Then, we can find that λ_S is positive up to the GCU scale for $\lambda_S(v_S) \gtrsim 0.01$. This lower bound of $\lambda_S(v_S)$ is almost unchanged for different values of v_S, because v_S dependence is logarithmic.

On the other hand, when λ_S is dominant in β_{λ_S}, the Landau pole might exist, at which the theory is not valid from the point of view of perturbativity (triviality). The energy scale where the Landau pole appears is approximately estimated as

$$\Lambda_{\text{LP}} = v_S \exp\left[\frac{4\pi^2 v_S^2}{9 M_s^2} \right], \tag{3.6}$$
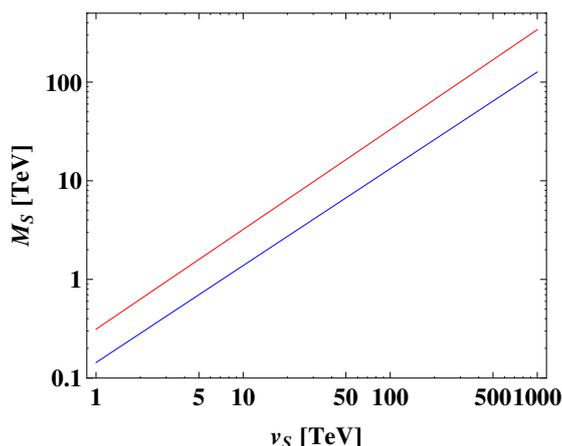
Figure 3. v_S dependence on the upper (red) and lower (blue) bounds of M_s, which correspond to the Landau pole and vacuum stability conditions, respectively. For the Landau pole bound, we take $\Lambda_{\mathrm{LP}} = \Lambda_{\mathrm{GCU}} = 3 \times 10^{16}$ GeV in eq. (3.6).
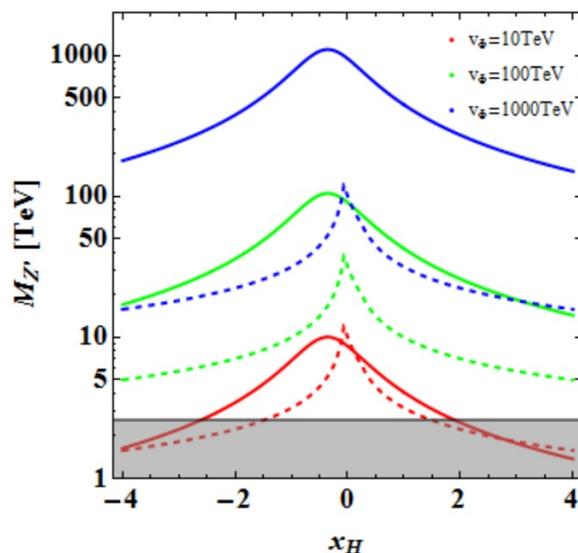
Figure 4. The upper bound of $M_{Z'}$ for fixed v_Φ, which depends on x_H. The solid and dashed lines show the Landau pole (3.7) and the naturalness (eqs. (2.12) and (2.13)) bounds, respectively. For the Landau pole bound, we take $\Lambda_{\mathrm{LP}} = \Lambda_{\mathrm{GCU}} = 3 \times 10^{16}$ GeV in eq. (3.7). The shaded region ($M_{Z'} < 2.6$ TeV) is excluded by the LHC experiments.

where $M_s = \sqrt{2\lambda_S(v_S)}\, v_S$ is a mass of the real singlet scalar field. Figure 3 shows v_S dependence on the upper (red) and lower (blue) bonds of M_s, which correspond to the Landau pole and vacuum stability conditions, respectively. Since the both bounds are almost proportional to v_S, allowed values of $\lambda_S(v_S)$ are almost unchanged for different v_S. We can find a strong constraint for λ_S as $0.01 \lesssim \lambda_S(v_S) \lesssim 0.05$.

In the same way, the Landau pole also exists when $g_X(v_\Phi)$ is sufficiently large. The energy scale where the Landau pole appears is approximately estimated by the one-loop

RGE of g_X as

$$\Lambda_{\mathrm{LP}} = v_\Phi \exp\left[\frac{32\pi^2 v_\Phi^2}{(44/3 + 64/3 x_H + 30 x_H^2) M_{Z'}^2}\right],\tag{3.7}$$

where $M_{Z'}$ is given in eq. (2.7). Figure 4 shows the upper bound of $M_{Z'}$ for fixed v_Φ, which depends on x_H. The solid lines show the maximal value of $M_{Z'}$ allowed in the model, which are calculated by eq. (3.7) for $\Lambda_{\mathrm{LP}} = \Lambda_{\mathrm{GCU}} = 3 \times 10^{16}\,\mathrm{GeV}$. Note that the peak of solid lines at $x_H = -16/45$ corresponds to the orthogonal basis of two U(1) gauges. The dashed lines show the naturalness bound estimated by eqs. (2.12) and (2.13). The red, green, and blue colors correspond to $v_\Phi = 10$, 100, and 1000 TeV, respectively. The shaded region ($M_{Z'} < 2.6\,\mathrm{TeV}$) is excluded by the LHC experiments [60, 61]. When we define the triviality bound as $\Lambda_{\mathrm{GCU}} < \Lambda_{\mathrm{LP}}$, it prohibits the regions above the solid lines. One can see that the bound leads $g_X(v_\Phi) \lesssim 0.5$ from eq. (2.7), which is almost independent of v_Φ. Since the naturalness requires the stronger constraints than the triviality bound in almost all parameter space, we can say that the naturalness guarantees no Landau pole below the GCU scale. Note that the both bounds are almost the same for $v_\Phi = 10\,\mathrm{TeV}$, and they exclude $M_{Z'} > 10\,\mathrm{TeV}$.

3.3 Neutrino masses and baryon asymmetry of the universe

From the Lagrangian (2.1), the neutrino mass terms are given by

$$(\overline{\nu_L}, \overline{\nu_R^c}, \overline{N_L}, \overline{N_R^c})\begin{pmatrix} 0 & m_D & 0 & 0 \\ m_D^T & M_M & 0 & 0 \\ 0 & 0 & M_{N_L} & m_N \\ 0 & 0 & m_N & M_{N_R} \end{pmatrix}\begin{pmatrix} \nu_L^c \\ \nu_R \\ N_L^c \\ N_R \end{pmatrix},\tag{3.8}$$

where $m_D = Y_\nu v_H/\sqrt{2}$, $M_M = Y_M v_\Phi/\sqrt{2}$, $M_{N_{L,R}} = Y_{N_{L,R}} v_\Phi/\sqrt{2}$, and $m_N = f_N v_S/\sqrt{2}$. There is no mixing term between $\nu_{L,R}$ and $N_{L,R}$ due to the Z_2 symmetry. The active neutrino masses can be obtained by the usual type-I seesaw mechanism [36–40], i.e., $m_\nu \approx m_D M_M^{-1} m_D^T$. The heavier mass eigenvalue is nearly equal to M_M, whose upper bound is given by the naturalness of the Higgs mass. Neutrino one-loop diagram contributes the Higgs mass as

$$\Delta m_h^2 \sim \frac{Y_\nu^2 Y_M^2 v_\Phi^2}{16\pi^2} \sim \frac{m_\nu M_M^3}{16\pi^2 v_H^2},\tag{3.9}$$

where we have used the seesaw relation. For $m_\nu \sim 0.1\,\mathrm{eV}$, the naturalness requires $M_M \lesssim 10^7\,\mathrm{GeV}$.

We mention the baryon asymmetry of the universe. In the normal thermal leptogenesis [62], there is a lower bound on the right-handed neutrino mass as $M_M \gtrsim 10^9\,\mathrm{GeV}$ [63]. However, the resonant leptogenesis can work even at the TeV scale, where two right-handed neutrino masses are well-degenerated [41]. In our model, additional U(1)$_X$ gauge interactions make the right-handed neutrinos be in thermal equilibrium with the SM particles [64]. A large efficiency factor can be easily obtained, and the sufficient baryon asymmetry of the
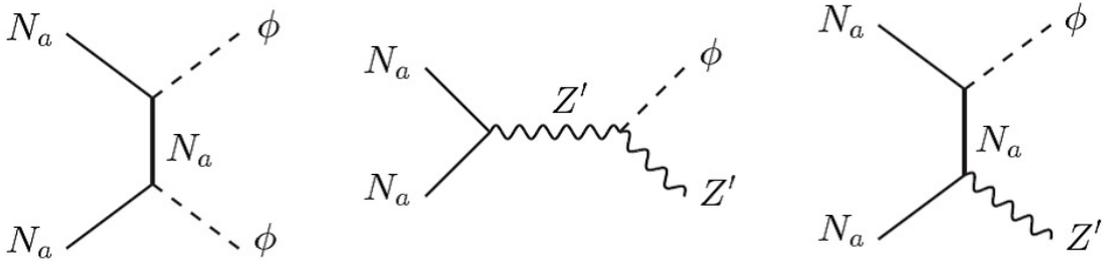
Figure 5. Annihilation processes of the dark matter N_a ($a = 1, 2$).

universe can be generated by the right-handed neutrinos with a few TeV masses. Since the neutrino Yukawa coupling Y_N and Y_M almost do not depend on the other phenomenological problems, we can do the same analysis as in ref. [64], and hence, the result is also the same as in ref. [64].

For the vector-like neutrinos ($N_{L,R}$), we consider $M_N = M_{N_L} = M_{N_R}$, which naturally arises from $L \leftrightarrow R$ symmetry for the vector-like fermions. Then, the mass eigenvalues are respectively $M_{N_1} = |M_N - m_N|$ and $M_{N_2} = |M_N + m_N|$ for $N_1 = (N_L^c - N_R)/\sqrt{2}$ and $N_2 = (N_L^c + N_R)/\sqrt{2}$. The lighter mass eigenstate N_1 is a DM candidate, because its stability is guaranteed by the Z_2 symmetry. In the limit of $m_N \to 0$ ($M_{N_1} = M_{N_2}$), N_1 and N_2 are degenerate, and N_2 is also effective for a calculation of the DM relic abundance. In the next subsection, we will investigate the degenerate $N_{1,2}$ case.

In our model, the U(1)$_X$ gauge symmetry is successfully achieved via the CW mechanism. It requires $\lambda_\Phi(v_\Phi) > 0$ in eq. (2.6), that is,

$$n_\nu M_M^4 + 2M_N^4 < \frac{3}{32} M_{Z'}^4, \tag{3.10}$$

where n_ν is a relevant number of right-handed neutrinos, which is defined as $\mathrm{tr} Y_M^4 (v_\Phi/\sqrt{2})^4 = n_\nu M_M^4$. Thus, the Majorana masses must be lighter than the Z' boson mass. We have made sure that this constraint is always satisfied when $N_{1,2}$ explain the DM relic abundance.

3.4 Dark matter

To calculate the DM relic abundance, we use the same formula for the DM annihilation cross sections as in ref. [19], where a new vector-like fermion is only $N_{L,R}$ (or $N_{1,2}$), and the SM fermions do not have U(1)$_X$ charges. The annihilation processes are t-channel $NN \to \phi\phi$, t-channel $NN \to Z'\phi$, and Z' mediated s-channel $NN \to Z'\phi$. The corresponding diagrams are shown in figure 5. Although our model has other contributions to the annihilation cross sections, they are all negligible in the following setup. We consider the degenerate case for simplicity, in which there is no vector-like mass term of N. Thus, t-channel $NN \to ss$ process and s mediated s-channel $NN \to \nu_R \nu_R$ process does not occur at tree level. From eq. (3.10), $(2M_N)^2 < M_{Z'}^2$ is always required. Then, the annihilation cross section $\sigma(NN \to Z'^* \to f\bar{f})$, where f is some U(1)$_X$ charged fermion, is suppressed by $1/M_{Z'}^2$. As a result, we can use the same formula for the DM annihilation cross sections as in ref. [19].
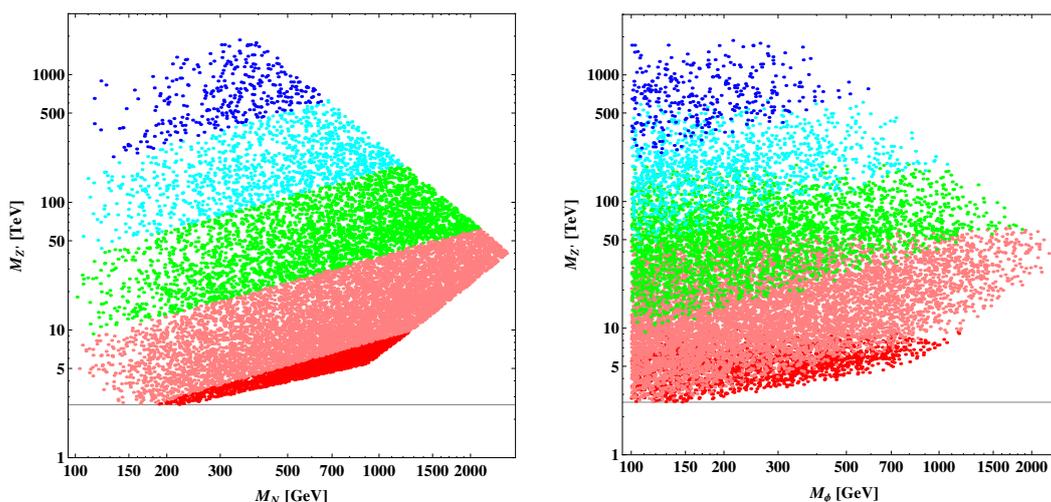
Figure 6. Scatter plots in $(M_N, M_{Z'})$ plane (left) and $(M_\phi, M_{Z'})$ plane (right), which realize the DM relic abundance $\Omega_{\mathrm{DM}}h^2 = 0.1187$, and satisfy all constraints as discussed in this paper as well as the LUX bound. The horizontal line shows the lower bound on $M_{Z'}$ by the LHC experiments. The red, pink green, cyan, and blue dots correspond to $6\,\mathrm{TeV} \leq v_\Phi < 10\,\mathrm{TeV}$, $10\,\mathrm{TeV} \leq v_\Phi < 100\,\mathrm{TeV}$, $100\,\mathrm{TeV} \leq v_\Phi < 10^3\,\mathrm{TeV}$, $10^3\,\mathrm{TeV} \leq v_\Phi < 10^4\,\mathrm{TeV}$, $10^4\,\mathrm{TeV} \leq v_\Phi < 10^5\,\mathrm{TeV}$, respectively.

The spin independent cross section for the direct detection is almost dominated by t-channel exchange of scalars h and ϕ, which has been considered in ref. [19]. However, our model has an additional contribution due to Z' exchange diagrams, which is given by [65]

$$\sigma_{SI} = \frac{m_n^2 M_N^2}{\pi (m_n + M_N)^2} \frac{g_X^4}{M_{Z'}^4} = 7.75 \times 10^{-42} \left(\frac{\mu_n}{1\,\mathrm{GeV}}\right)^2 \left(\frac{1\,\mathrm{TeV}}{v_\Phi}\right)^4 \mathrm{cm}^2, \quad (3.11)$$

where m_n is the nucleon mass, and $\mu_n = m_n M_N/(m_n + M_N)$ is the reduced nucleon mass. For the DM with the masses of $100\,\mathrm{GeV}$ and $1\,\mathrm{TeV}$, the small v_Φ regions such as $v_\Phi < 11\,\mathrm{TeV}$ and $v_\Phi < 6\,\mathrm{TeV}$ are excluded by the LUX experiment, respectively [67]. These bound are stronger than the LEP bound, where $v_\Phi < 3.5\,\mathrm{TeV}$ is excluded.

In the following, we consider $x_H = 0$ ($\mathrm{U}(1)_{B-L}$) case. There are six new parameters in the model: the $\mathrm{U}(1)_{B-L}$ gauge coupling g_X, the two Majorana Yukawa coupling Y_{N_L}, Y_{N_R}, the two quartic couplings λ_Φ, $\lambda_{H\Phi}$, and the VEV of the complex scalar field v_Φ. On the other hand, there are two conditions $Y_{N_L} = Y_{N_R}$ and eq. (2.9), and we require that N explains the DM relic abundance $\Omega_{\mathrm{DM}}h^2 = 0.1187$ [66]. Thus, we have three free parameters for the DM analysis.

Figure 6 shows scatter plots in $(M_N, M_{Z'})$ plane (left) and $(M_\phi, M_{Z'})$ plane (right), which realize the DM relic abundance $\Omega_{\mathrm{DM}}h^2 = 0.1187$, and satisfy all constraints as discussed above as well as the LUX bound. The parameter space starts from the initial values $M_\phi = 100\,\mathrm{GeV}$, $M_N = 100\,\mathrm{GeV}$, and $M_{Z'} = 2.6\,\mathrm{TeV}$. Although the two figures in figure 6 are very similar, $M_N > M_\phi$ is always satisfied. The region of $M_{Z'} < 2.6\,\mathrm{TeV}$ is excluded by the current LHC bound [60, 61]. Since $g_X \lesssim 0.5$ is required to avoid the Landau pole, the upper bound on $M_{Z'}$ is given by $M_{Z'} \lesssim v_\Phi$, while the upper bound in the $M_{N,\phi} \gtrsim 500\,\mathrm{GeV}$ region is given by the naturalness (2.13). In the $200\,\mathrm{GeV} \lesssim M_{N,\phi} \lesssim$

900 GeV region, the lower bound on $M_{Z'}$ is given by the LUX bound. To realize the DM relic abundance, sufficiently large annihilation cross sections are required, which induce the lower bound on $M_{Z'}$ in the $M_N \gtrsim 900$ GeV region. From figure 6, we can see the upper bound on the DM mass as $M_N \lesssim 2.6$ TeV, and the bound of M_ϕ is almost the same as M_N.

4   Conclusion

To solve the gauge hierarchy problem, we have constructed a classically scale invariant model with a U(1)$_X$ gauge extension. We have assumed the classical scale invariance at the GCU scale, where the Higgs mass completely vanishes even with some quantum corrections. The scale invariance is violated around the TeV scale by the CW mechanism, and the Higgs mass can be naturally generated through the scalar mixing term. The GCU is realized by vector-like fermions $Q_{L,R}$ and $D_{L,R}$, which respectively have the same quantum number as the SM quark doublet and down-type quark singlet but distinguished by the additional Z_2 symmetry, and their masses lie in 800 GeV $\lesssim M_V \lesssim 1$ TeV. The GCU scale is $\Lambda_{\rm GCU} = 3 \times 10^{16}$ GeV with $\alpha_{\rm GCU}^{-1} = 35.6$, and the proton life time is estimated as $\tau_{\rm p} \sim 10^{37}$ yrs, which is much longer than the experimental lower bound $\tau_{\rm p} > 8.2 \times 10^{33}$ yrs.

In addition, we have shown that the model can explain the vacuum stability, smallness of active neutrino masses, baryon asymmetry of the universe, and dark matter relic abundance without inducing large Higgs mass corrections. Since there are additional fermions with the SM gauge charges, the SM gauge couplings become larger than the SM case, which leads smaller top Yukawa couplings. Then, the β function of the Higgs quartic coupling becomes larger, and hence the EW vacuum becomes stable. The smallness of active neutrino masses and the baryon asymmetry of the universe can be explained by the right-handed neutrinos via the type-I seesaw mechanism and resonant leptogenesis, respectively. The DM candidate is the SM singlet Majorana fermions $N_{1,2}$, and stability of the DM is guaranteed by the additional Z_2 symmetry. We have analyzed the DM relic abundance in the degenerate case ($M_{N_1} = M_{N_2}$), and found the upper bound on the DM mass as $M_N \lesssim 2.6$ TeV.

A β functions in the U(1)$_X$ extended SM

We give one-loop β-functions in our model:

$$\beta_{g_Y} = \frac{g_Y^3}{16\pi^2}\frac{15}{2}, \qquad \beta_{g_2} = \frac{g_2^3}{16\pi^2}\frac{-7}{6}, \qquad \beta_{g_3} = \frac{g_3^3}{16\pi^2}(-5), \tag{A.1}$$

$$\beta_{g_X} = \frac{g_X}{16\pi^2}\left[\left(\frac{44}{3} + \frac{64}{3}x_H + 30x_H^2\right)g_X^2 + \frac{15}{2}g_{\rm mix}^2 + \left(\frac{32}{3} + 30x_H\right)g_{\rm mix}g_X\right], \tag{A.2}$$

$$
\beta_{g_{\rm mix}} = \frac{1}{16\pi^2}\left[g_{\rm mix}\left(\frac{15}{2}(g_{\rm mix}^2 + 2g_Y^2) + \left(\frac{44}{3} + \frac{64}{3}x_H + 30x_H^2 \right) g_X^2 \right) \right.
$$
$$
\left. + \left(\frac{32}{3} + 30x_H \right) g_X(g_Y^2 + g_{\rm mix}^2) \right], \tag{A.3}
$$

$$
\beta_{y_t} = \frac{y_t}{16\pi^2}\left[\frac{9}{2}y_t^2 - \left(8g_3^2 + \frac{9}{4}g_2^2 + \frac{17}{12}(g_Y^2 + g_{\rm mix}^2) + \left(\frac{2}{3} + \frac{10}{3}x_H + \frac{17}{3}x_H^2 \right) g_X^2 \right. \right.
$$
$$
\left. \left. + \left(\frac{5}{3} + \frac{17}{3}x_H \right) g_{\rm mix}g_X \right) + 3(\kappa_1^2 + \kappa_2^2) \right], \tag{A.4}
$$

$$
\beta_{Y_M} = \frac{Y_M}{16\pi^2}\left[4Y_M^2 + 2{\rm tr}Y_M^2 + 2(Y_{N_L}^2 + Y_{N_R}^2) - 6g_X^2 \right], \tag{A.5}
$$

$$
\beta_{Y_{N_L}} = \frac{1}{16\pi^2}\left[Y_{N_L}(6Y_{N_L}^2 + f_N^2 + 2({\rm tr}Y_M^2 + Y_{N_R}^2) - 6g_X^2) + 2f_N^2 Y_{N_R} \right], \tag{A.6}
$$

$$
\beta_{Y_{N_R}} = \frac{1}{16\pi^2}\left[Y_{N_R}(6Y_{N_R}^2 + f_N^2 + 2({\rm tr}Y_M^2 + Y_{N_L}^2) - 6g_X^2) + 2f_N^2 Y_{N_L} \right], \tag{A.7}
$$

$$
\beta_{\kappa_1} = \frac{1}{16\pi^2}\left[\kappa_1\left(-8g_3^2 - \frac{9}{4}g_2^2 - \frac{5}{12}(g_Y^2 + g_{\rm mix}^2) + \frac{1}{3}(1 - 5x_H)g_{\rm mix}g_X \right. \right.
$$
$$
\left. \left. + \frac{1}{3}(-2 + 2x_H - 5x_H^2)g_X^2 + \frac{1}{2}(f_Q^2 + f_D^2) + 3y_t^2 + \frac{9}{2}\kappa_1^2 + 3\kappa_2^2 \right) + 2f_Q f_D \kappa_2 \right], \tag{A.8}
$$

$$
\beta_{\kappa_2} = \frac{1}{16\pi^2}\left[\kappa_2\left(-8g_3^2 - \frac{9}{4}g_2^2 - \frac{5}{12}(g_Y^2 + g_{\rm mix}^2) + \frac{1}{3}(1 - 5x_H)g_{\rm mix}g_X \right. \right.
$$
$$
\left. \left. + \frac{1}{3}(-2 + 2x_H - 5x_H^2)g_X^2 + \frac{1}{2}(f_Q^2 + f_D^2) + 3y_t^2 + \frac{9}{2}\kappa_2^2 + 3\kappa_1^2 \right) + 2f_Q f_D \kappa_1 \right], \tag{A.9}
$$

$$
\beta_{f_Q} = \frac{1}{16\pi^2}\left[f_Q\left(-8g_3^2 - \frac{9}{2}g_2^2 - \frac{1}{6}(g_Y^2 + g_{\rm mix}^2) - \frac{2}{3}(1 + x_H)g_{\rm mix}g_X \right. \right.
$$
$$
\left. \left. - \frac{2}{3}(1 + x_H)^2 g_X^2 + \frac{1}{2}(\kappa_1^2 + \kappa_2^2) + 15f_Q^2 + 6f_D^2 + 2f_N^2 \right) + 2f_D\kappa_1\kappa_2 \right], \tag{A.9}
$$

$$
\beta_{f_D} = \frac{1}{16\pi^2}\left[f_D\left(-8g_3^2 - \frac{2}{3}(g_Y^2 + g_{\rm mix}^2) + \frac{4}{3}(1 - 2x_H)g_{\rm mix}g_X - \frac{2}{3}(1 - 2x_H)^2 g_X^2 \right. \right.
$$
$$
\left. \left. + (\kappa_1^2 + \kappa_2^2) + 12f_Q^2 + 9f_D^2 + 2f_N^2 \right) + 4f_Q\kappa_1\kappa_2 \right], \tag{A.10}
$$

$$
\beta_{f_N} = \frac{f_N}{16\pi^2}\left[-6g_X^2 + 2Y_{N_L}^2 + 8Y_{N_L}Y_{N_R} + 2Y_{N_R}^2 + 12f_Q^2 + 6f_D^2 + 5f_N^2 \right], \tag{A.11}
$$

$$
\beta_{\lambda_H} = \frac{1}{16\pi^2}\left[24\lambda_H^2 + \lambda_{H\Phi}^2 + 2\lambda_{HS}^2 + \lambda_H(12y_t^2 - 9g_2^2 - 3(g_Y^2 + g_{\rm mix}^2) - 12x_H^2 g_X^2 \right.
$$
$$
- 12x_H g_{\rm mix}g_X + 12(\kappa_1^2 + \kappa_2^2)) - 6y_t^4 - 6(\kappa_1^4 + \kappa_2^4) + \frac{3}{8}\left\{ 2g_2^4 + (g_2^2 + (g_Y^2 + g_{\rm mix}^2))^2 \right\}
$$
$$
\left. + 3x_H g_{\rm mix}g_X(g_2^2 + g_Y^2 + g_{\rm mix}^2 + 4x_H^2 g_X^2) + 3x_H^2 g_X^2(g_2^2 + g_Y^2 + 3g_{\rm mix}^2 + 2x_H^2 g_X^2) \right], \tag{A.12}
$$

$$
\beta_{\lambda_\Phi} = \frac{1}{16\pi^2}\left[20\lambda_\Phi^2 + 2\lambda_{H\Phi}^2 + 2\lambda_{\Phi S}^2 + \lambda_\Phi(8({\rm tr}Y_M^2 + Y_{N_L}^2 + Y_{N_R}^2) - 48g_X^2) + 96g_X^4 \right.
$$
$$
\left. - 16({\rm tr}Y_M^4 + Y_{N_L}^4 + Y_{N_R}^4) \right], \tag{A.13}
$$

$$
\beta_{\lambda_S} = \frac{1}{16\pi^2}\left[72\lambda_S^2 + 2\lambda_{HS}^2 + \lambda_{\Phi S}^2 + \lambda_S(48f_Q^2 + 24f_D^2 + 8f_N^2) - 12f_Q^4 - 6f_D^4 - 2f_N^4 \right], \tag{A.14}
$$

$$\beta_{\lambda_{H\Phi}} = \frac{1}{16\pi^2}\left[4\lambda_{HS}\lambda_{\Phi S} + \lambda_{H\Phi}\left(12\lambda_H + 8\lambda_\Phi + 4\lambda_{H\Phi} - \frac{9}{2}g_2^2 - \frac{3}{2}(g_Y^2 + g_{\mathrm{mix}}^2)\right.\right.$$

$$\left.-6(4+x_H^2)g_X^2 - 6x_H g_{\mathrm{mix}}g_X + 6y_t^2 + 6(\kappa_1^2 + \kappa_2^2) + 4(\mathrm{tr}Y_M^2 + Y_{N_L}^2 + Y_{N_R}^2)\right)$$

$$\left.+12g_X^2(g_{\mathrm{mix}} + 2x_H g_X)^2\right], \tag{A.15}$$

$$\beta_{\lambda_{HS}} = \frac{1}{16\pi^2}\left[2\lambda_{H\Phi}\lambda_{\Phi S} + \lambda_{HS}\left(12\lambda_H + 24\lambda_S + 8\lambda_{HS} - \frac{9}{2}g_2^2 - \frac{3}{2}(g_Y^2 + g_{\mathrm{mix}}^2)\right.\right.$$

$$\left.-6x_H^2 g_X^2 - 6x_H g_{\mathrm{mix}}g_X + 6y_t^2 + 6(\kappa_1^2 + \kappa_2^2) + 24f_Q^2 + 12f_D^2 + 4f_N^2\right)$$

$$\left.-12(f_Q^2 + f_D^2)(\kappa_1^2 + \kappa_2^2) - 24f_Q f_D \kappa_1 \kappa_2\right], \tag{A.16}$$

$$\beta_{\lambda_{\Phi S}} = \frac{1}{16\pi^2}\left[4\lambda_{HS}\lambda_{H\Phi} + \lambda_{\Phi S}\left(24\lambda_S + 8\lambda_\Phi + 8\lambda_{\Phi S} - 24g_X^2 + 24f_Q^2 + 12f_D^2 + 4f_N^2\right.\right.$$

$$\left.+4(\mathrm{tr}Y_M^2 + Y_{N_L}^2 + Y_{N_R}^2)\right) - 16f_N^2(Y_{N_L}^2 + Y_{N_R}^2 + Y_{N_L}Y_{N_R})\right]. \tag{A.17}$$

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