

Supersymmetric AdS_6 black holes from $F(4)$ gauged supergravity

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ABSTRACT: In $F(4)$ gauged supergravity in six dimensions, we study supersymmetric AdS_6 black holes with various horizon geometries. We find a new $AdS_2 \times \Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$ horizon solution with $\mathfrak{g}_1 > 1$ and $\mathfrak{g}_2 > 1$, and present the black hole solution numerically. The full black hole is an interpolating geometry between the asymptotically AdS_6 boundary and the $AdS_2 \times \Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$ horizon. We calculate the Bekenstein-Hawking entropy of the black hole and find a match with the recently calculated topologically twisted index of 5d $USp(2N)$ gauge theory on $\Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2} \times S^1$ in the large N limit. We also find black hole horizons of Kähler four-cycles in Calabi-Yau fourfolds and on Cayley four-cycles in $Spin(7)$ manifolds.

KEYWORDS: AdS-CFT Correspondence, Supergravity Models

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1 Introduction

The AdS_6/CFT_5 correspondence remains as one of the less appreciated among its family [1]. In $SU(2) \times U(1)$ -gauged $\mathcal{N} = 4$ supergravity in six dimensions, commonly known as $F(4)$ gauged supergravity named after the $F(4)$ superalgebra in six dimensions, there is a unique supersymmetric fixed point [2]. This fixed point is known to be dual of 5d superconformal $USp(2N)$ gauge theory [3] which is one of the few 5d SCFTs known so far [4, 5]. In [6] it was shown that $F(4)$ gauged supergravity is a consistent truncation of massive type IIA supergravity [7].¹ The fixed point uplifts to $AdS_6 \times S^4$ near-horizon geometry of the D4-D8 brane system [8].

In order to study RG flows from 5d SCFTs to lower dimensional ones via the AdS/CFT correspondence in the spirit of [12], twisted compactifications of $F(4)$ gauged supergravity were studied. The supergravity solutions describe the near-horizon geometries of wrapped D4-branes on various supersymmetric cycles. D4-branes wrapped on two- and three-cycles were studied in [13, 14]. They found AdS_4 and AdS_3 fixed point solutions. See for more recent results on three-cycles in [15].

In this paper, we study supersymmetric AdS_6 black hole solutions by considering D4-branes wrapped on supersymmetric four-cycles. We begin by deriving supersymmetry equations for D4-branes wrapped on two Riemann surfaces and find a new $AdS_2 \times H^2 \times H^2$ solution. When we consider $S^2 \times S^2$ background, there is no AdS_2 fixed point. We also

¹It is also a consistent truncation of type IIB supergravity [9–11].

present the full black hole solution numerically, which is an interpolating geometry between the asymptotically AdS_6 boundary and the $AdS_2 \times H^2 \times H^2$ horizon. We calculate entropy of asymptotically AdS_6 black holes with $AdS_2 \times \Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$ horizon where $\mathfrak{g}_1 > 1$ and $\mathfrak{g}_2 > 1$. Here $\Sigma_{\mathfrak{g}}$ is a Riemann surface of genus \mathfrak{g} with constant curvature. Analogous to the 3d gauge theory examples in [16–18], this entropy would give the topologically twisted index of 5d $USp(2N)$ gauge theory on $\Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2} \times S^1$ in the large N limit. Indeed we find that the entropy nicely matches the recent calculation of topologically twisted index in [19].

In contrast to the black hole solutions which exist only for $\mathfrak{g}_1 > 1$ and $\mathfrak{g}_2 > 1$, topologically twisted index is well defined for arbitrary genus. When a black hole solution exists, the index is counting the entropy of the black hole, however, the inverse is not true, i.e. even if the index is well defined, it does not guarantee that there is a corresponding black hole solution. The index in this case might count some other information then entropy or might not be consistent.²

Then, we study supersymmetric AdS_6 black hole solutions with other horizon geometries by considering D4-branes wrapped on Kähler four-cycles in Calabi-Yau fourfolds and on Cayley four-cycles in Spin(7) manifolds. We believe these are all possible four-cycles on which D4-branes can wrap. A product of two Riemann surfaces falls into a special case of Kähler four-cycles in Calabi-Yau fourfolds. We derive the supersymmetry equations, and obtain new supersymmetric AdS_2 solutions for each case.³

We comment on the comparison with the recent field theory results. The topologically twisted index can be written as the contour integral of meromorphic differential form in variables parametrizing the Cartan subgroup and subalgebra of the gauge group, summed over the lattice of gauge magnetic fluxes, \mathfrak{m} , on the internal manifold [17, 18]. Recently, topologically twisted index of 5d $USp(2N)$ gauge theory on $\Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2} \times S^1$ was calculated at finite N by two independent groups, [19] and [20], and their calculations agree. When considering the large N limit, they both employed a conjecture to extremize the prepotential in order to get a saddle point distribution for dominant eigenvalue distribution. In [20], by considering the non-zero magnetic fluxes as a subleading contribution, the contribution to the twisted index from zero magnetic fluxes, $\mathfrak{m} = 0$, was evaluated. As we explain in detail below, it matches the gravitational entropy of the AdS_2 solution in [14] which is expected to be incorrect, and it counts the half of the full contribution to the twisted index. However, in [19], instead of setting the magnetic fluxes to zero, they extremized the twisted superpotential in order to get a saddle point distribution for the fluxes as well. In this paper, we will show that the full gravitational entropy of asymptotically AdS_6 black holes with $AdS_2 \times \Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$ horizon matches the topologically twisted index in the large N limit, only when the contributions from non-zero gauge theory magnetic fluxes are accounted as it was done in [19].

In section 2, we review $F(4)$ gauged supergravity. In section 3, we study supersymmetric black holes with $AdS_2 \times \Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$ horizon by considering D4-branes wrapped on two

²We would like to thank Alberto Zaffaroni for discussion on this point.

³These cases were previously studied in [14] as D4-branes wrapped on Kähler four-cycles in Calabi-Yau threefolds and on co-associative four-cycles in G_2 manifolds. However, as they have not turned on the two form gauge potential which is needed to have a consistent set of supersymmetry equations and to satisfy the equations of motion, we conclude that their equations and solutions are *not* correct.

Riemann surfaces. We derive the supersymmetry equations, find a new AdS_2 solution, and calculate entropy of the black holes. We also present the full black hole solution numerically. In section 4, we consider more horizon geometries by studying D4-branes wrapped on Kähler four-cycles in Calabi-Yau fourfolds and on Cayley four-cycles in Spin(7) manifolds. In section 5, we conclude. The equations of motion of $F(4)$ gauged supergravity are presented in appendix A.

2 $F(4)$ gauged supergravity in six dimensions

We review $SU(2) \times U(1)$ -gauged $\mathcal{N} = 4$ supergravity in six dimensions [2]. The bosonic field content consists of the metric, $g_{\mu\nu}$, a real scalar, ϕ , an $SU(2)$ gauge field, A_μ^I , $I = 1, 2, 3$, a $U(1)$ gauge field, \mathcal{A}_μ , and a two-form gauge potential, $B_{\mu\nu}$. The fermionic field content is gravitinos, $\psi_{\mu i}$, and dilatinos, χ_i , $i = 1, 2$. The field strengths are defined by

$$\begin{aligned}
 \mathcal{F}_{\mu\nu} &= \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu, \\
 F_{\mu\nu}^I &= \partial_\mu A_\nu^I - \partial_\nu A_\mu^I + g\epsilon^{IJK} A_\mu^J A_\nu^K, \\
 G_{\mu\nu\rho} &= 3\partial_{[\mu} B_{\nu\rho]}, \\
 \mathcal{H}_{\mu\nu} &= \mathcal{F}_{\mu\nu} + mB_{\mu\nu}.
 \end{aligned} \tag{2.1}$$

The bosonic Lagrangian is given by

$$\begin{aligned}
 e^{-1}\mathcal{L} &= -\frac{1}{4}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{8}\left(g^2e^{\sqrt{2}\phi} + 4gme^{-\sqrt{2}\phi} - m^2e^{-3\sqrt{2}\phi}\right) \\
 &\quad - \frac{1}{4}e^{-\sqrt{2}\phi}\left(\mathcal{H}_{\mu\nu}\mathcal{H}^{\mu\nu} + F_{\mu\nu}^IF^{I\mu\nu}\right) + \frac{1}{12}e^{2\sqrt{2}\phi}G_{\mu\nu\rho}G^{\mu\nu\rho} \\
 &\quad - \frac{1}{8}\epsilon^{\mu\nu\rho\sigma\tau\kappa}B_{\mu\nu}\left(\mathcal{F}_{\rho\sigma}\mathcal{F}_{\tau\kappa} + mB_{\rho\sigma}\mathcal{F}_{\tau\kappa} + \frac{1}{3}m^2B_{\rho\sigma}B_{\tau\kappa} + F_{\rho\sigma}^IF_{\tau\kappa}^I\right),
 \end{aligned} \tag{2.2}$$

where g is the $SU(2)$ gauge coupling constant and m is the mass of the two-form gauge potential. The supersymmetry transformations of the fermionic fields are

$$\begin{aligned}
 \delta\psi_{\mu i} &= \nabla_\mu\epsilon_i + gA_\mu^I(T^I)_i{}^j\epsilon_j - \frac{1}{8\sqrt{2}}\left(ge^{-\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}}\right)\gamma_\mu\gamma_7\epsilon_i \\
 &\quad - \frac{1}{8\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}\left(\mathcal{F}_{\nu\lambda} + mB_{\nu\lambda}\right)\left(\gamma_\mu{}^{\nu\lambda} - 6\delta_\mu{}^\nu\gamma^\lambda\right)\epsilon_i \\
 &\quad - \frac{1}{4\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}F_{\nu\lambda}^I\left(\gamma_\mu{}^{\nu\lambda} - 6\delta_\mu{}^\nu\gamma^\lambda\right)\gamma_7(T^I)_i{}^j\epsilon_j \\
 &\quad - \frac{1}{24}e^{\sqrt{2}\phi}G_{\nu\lambda\rho}\gamma_7\gamma^{\nu\lambda\rho}\gamma_\mu\epsilon_i,
 \end{aligned} \tag{2.3}$$

$$\begin{aligned}
\delta\chi_i &= \frac{1}{\sqrt{2}}\gamma^\mu\partial_\mu\phi\epsilon_i + \frac{1}{4\sqrt{2}}\left(ge^{-\frac{\phi}{\sqrt{2}}} - 3me^{-\frac{3\phi}{\sqrt{2}}}\right)\gamma_7\epsilon_i \\
&+ \frac{1}{4\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}(\mathcal{F}_{\mu\nu} + mB_{\mu\nu})\gamma^{\mu\nu}\epsilon_i \\
&+ \frac{1}{2\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}F_{\mu\nu}^I\gamma^{\mu\nu}\gamma_7(T^I)_i{}^j\epsilon_j \\
&- \frac{1}{12}e^{\sqrt{2}\phi}G_{\mu\nu\lambda}\gamma_7\gamma^{\mu\nu\lambda}\epsilon_i,
\end{aligned} \tag{2.4}$$

where T^I , $I = 1, 2, 3$, are the $SU(2)$ left-invariant one-forms,

$$T^I = -\frac{i}{2}\sigma^I. \tag{2.5}$$

Described by the above Lagrangian, there are five inequivalent theories: $\mathcal{N} = 4^+$ ($g > 0$, $m > 0$), $\mathcal{N} = 4^-$ ($g < 0$, $m > 0$), $\mathcal{N} = 4^g$ ($g > 0$, $m = 0$), $\mathcal{N} = 4^m$ ($g = 0$, $m > 0$), $\mathcal{N} = 4^0$ ($g = 0$, $m = 0$). The $\mathcal{N} = 4^+$ theory admits a supersymmetric AdS_6 fixed point when $g = 3m$. At the supersymmetric AdS_6 fixed point, all the fields are vanishing except the AdS_6 metric.

3 Black holes with $AdS_2 \times \Sigma_{g_1} \times \Sigma_{g_2}$ horizon

3.1 The supersymmetry equations

In this section, we obtain supersymmetric AdS_6 black holes with a horizon which is a product of two Riemann surfaces. We consider the metric,

$$ds^2 = e^{2f(r)}(dt^2 - dr^2) - e^{2g_1(r)}(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) - e^{2g_2(r)}(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2), \tag{3.1}$$

for the $S^2 \times S^2$ background, and

$$ds^2 = e^{2f(r)}(dt^2 - dr^2) - e^{2g_1(r)}(d\theta_1^2 + \sinh^2\theta_1 d\phi_1^2) - e^{2g_2(r)}(d\theta_2^2 + \sinh^2\theta_2 d\phi_2^2), \tag{3.2}$$

for the $H^2 \times H^2$ background. The only non-vanishing component of the non-Abelian $SU(2)$ gauge field, A_μ^I , $I = 1, 2, 3$, is given by

$$A^3 = -a_1 \cos\theta_1 d\phi_1 - a_2 \cos\theta_2 d\phi_2, \tag{3.3}$$

for the $S^2 \times S^2$ background, and

$$A^3 = a_1 \cosh\theta_1 d\phi_1 + a_2 \cosh\theta_2 d\phi_2, \tag{3.4}$$

for the $H^2 \times H^2$ background, where the magnetic charges, a_1 and a_2 , are constant. In order to have equal signs for field strengths, we set opposite signs of the gauge fields for $S^2 \times S^2$ and $H^2 \times H^2$ backgrounds. We also have a non-trivial two-form gauge potential, $B_{\mu\nu}$, and we will determine it later. We turn off the Abelian $U(1)$ gauge field, \mathcal{A}_μ .

The supersymmetry equations are obtained by setting the supersymmetry variations of the fermionic fields to zero. From the supersymmetry variations, we obtain

$$f'e^{-f}\gamma^{\hat{r}}\epsilon_i - \frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}}\right)\gamma_7\epsilon_i - \frac{1}{2\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}\left(a_1e^{-2g_1}\gamma^{\hat{\theta}_1\hat{\phi}_1} + a_2e^{-2g_2}\gamma^{\hat{\theta}_2\hat{\phi}_2}\right)\gamma_7 2(T^3)_i{}^j\epsilon_j - \frac{3}{\sqrt{2}m}a_1a_2e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}\gamma^{\hat{r}}\epsilon_i = 0, \quad (3.5)$$

$$g'_1e^{-f}\gamma^{\hat{r}}\epsilon_i - \frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}}\right)\gamma_7\epsilon_i + \frac{1}{2\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}\left(3a_1e^{-2g_1}\gamma^{\hat{\theta}_1\hat{\phi}_1} - a_2e^{-2g_2}\gamma^{\hat{\theta}_2\hat{\phi}_2}\right)\gamma_7 2(T^3)_i{}^j\epsilon_j + \frac{1}{\sqrt{2}m}a_1a_2e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}\gamma^{\hat{r}}\epsilon_i = 0, \quad (3.6)$$

$$g'_2e^{-f}\gamma^{\hat{r}}\epsilon_i - \frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}}\right)\gamma_7\epsilon_i + \frac{1}{2\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}\left(3a_2e^{-2g_2}\gamma^{\hat{\theta}_2\hat{\phi}_2} - a_1e^{-2g_1}\gamma^{\hat{\theta}_1\hat{\phi}_1}\right)\gamma_7 2(T^3)_i{}^j\epsilon_j + \frac{1}{\sqrt{2}m}a_1a_2e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}\gamma^{\hat{r}}\epsilon_i = 0, \quad (3.7)$$

$$\frac{1}{\sqrt{2}}\phi'e^{-f}\gamma^{\hat{r}}\epsilon_i + \frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{\sqrt{2}}} - 3me^{-\frac{3\phi}{\sqrt{2}}}\right)\gamma_7\epsilon_i + \frac{1}{2\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}\left(a_1e^{-2g_1}\gamma^{\hat{\theta}_1\hat{\phi}_1} + a_2e^{-2g_2}\gamma^{\hat{\theta}_2\hat{\phi}_2}\right)\gamma_7 2(T^3)_i{}^j\epsilon_j - \frac{1}{\sqrt{2}m}a_1a_2e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}\gamma^{\hat{r}}\epsilon_i = 0, \quad (3.8)$$

where the hatted indices are the flat indices. The t -, θ_1 -, and θ_2 -components of the gravitino variations give (3.5), (3.6), (3.7), and the dilatino variation gives (3.8). The ϕ_1 -, ϕ_2 -components of the variations are identical to the θ_1 -, and θ_2 -components beside few more terms,

$$\epsilon_i = -2ga_1\gamma^{\hat{\theta}_1\hat{\phi}_1}(T^3)_i{}^j\epsilon_j, \quad \epsilon_i = -2ga_2\gamma^{\hat{\theta}_2\hat{\phi}_2}(T^3)_i{}^j\epsilon_j. \quad (3.9)$$

We employ the projection conditions,

$$\gamma^{\hat{r}}\gamma^7\epsilon_i = \epsilon_i, \quad \gamma^{\hat{\theta}_1\hat{\phi}_1}(T^3)_i{}^j\epsilon_j = \frac{\lambda}{2}\epsilon_i, \quad \gamma^{\hat{\theta}_2\hat{\phi}_2}(T^3)_i{}^j\epsilon_j = \frac{\lambda}{2}\epsilon_i, \quad (3.10)$$

where $\lambda = \pm 1$. Solutions with the projection conditions preserve 1/8 of the supersymmetries. By employing the projection conditions, we obtain the complete supersymmetry equations,

$$\begin{aligned} f'e^{-f} &= -\frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}}\right) - \frac{\lambda}{2\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}\left(a_1e^{-2g_1} + a_2e^{-2g_2}\right) - \frac{3}{\sqrt{2}m}a_1a_2e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}, \\ g'_1e^{-f} &= -\frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}}\right) + \frac{\lambda}{2\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}\left(3a_1e^{-2g_1} - a_2e^{-2g_2}\right) + \frac{1}{\sqrt{2}m}a_1a_2e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}, \\ g'_2e^{-f} &= -\frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}}\right) + \frac{\lambda}{2\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}\left(3a_2e^{-2g_2} - a_1e^{-2g_1}\right) + \frac{1}{\sqrt{2}m}a_1a_2e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}, \\ \frac{1}{\sqrt{2}}\phi'e^{-f} &= \frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{\sqrt{2}}} - 3me^{-\frac{3\phi}{\sqrt{2}}}\right) + \frac{\lambda}{2\sqrt{2}}e^{-\frac{\phi}{\sqrt{2}}}\left(a_1e^{-2g_1} + a_2e^{-2g_2}\right) - \frac{1}{\sqrt{2}m}a_1a_2e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}. \end{aligned} \quad (3.11)$$

From (3.9) we also obtain the twist conditions on the magnetic charges,

$$a_1 = -\frac{k}{\lambda g}, \quad a_2 = -\frac{k}{\lambda g}, \quad (3.12)$$

where $k = +1$ for the $S^2 \times S^2$ background and $k = -1$ for the $H^2 \times H^2$ background.⁴

In the derivation of the supersymmetry equations, we determined the non-zero components of the two-form gauge potential, $B_{\mu\nu}$. We determined the normalization by solving the equations of motion,

$$B_{\text{tr}} = -\frac{2}{m^2} a_1 a_2 e^{\sqrt{2}\phi + 2f - 2g_1 - 2g_2}. \quad (3.13)$$

The three-form field strength of the two-form gauge potential, $G_{\mu\nu\lambda}$, vanishes identically. The supersymmetry equations satisfy the equations of motion. We present the equations of motion in appendix A.

When we plug the twist conditions, (3.12), in the supersymmetry equations, we obtain

$$\begin{aligned} f' e^{-f} &= -\frac{1}{4\sqrt{2}} \left(g e^{\frac{\phi}{\sqrt{2}}} + m e^{-\frac{3\phi}{\sqrt{2}}} \right) + \frac{k}{2\sqrt{2}g} e^{-\frac{\phi}{\sqrt{2}}} (e^{-2g_1} + e^{-2g_2}) - \frac{3}{\sqrt{2}g^2 m} e^{\frac{\phi}{\sqrt{2}} - 2g_1 - 2g_2}, \\ g'_1 e^{-f} &= -\frac{1}{4\sqrt{2}} \left(g e^{\frac{\phi}{\sqrt{2}}} + m e^{-\frac{3\phi}{\sqrt{2}}} \right) - \frac{k}{2\sqrt{2}g} e^{-\frac{\phi}{\sqrt{2}}} (3e^{-2g_1} - e^{-2g_2}) + \frac{1}{\sqrt{2}g^2 m} e^{\frac{\phi}{\sqrt{2}} - 2g_1 - 2g_2}, \\ g'_2 e^{-f} &= -\frac{1}{4\sqrt{2}} \left(g e^{\frac{\phi}{\sqrt{2}}} + m e^{-\frac{3\phi}{\sqrt{2}}} \right) - \frac{k}{2\sqrt{2}g} e^{-\frac{\phi}{\sqrt{2}}} (3e^{-2g_2} - e^{-2g_1}) + \frac{1}{\sqrt{2}g^2 m} e^{\frac{\phi}{\sqrt{2}} - 2g_1 - 2g_2}, \\ \frac{1}{\sqrt{2}} \phi' e^{-f} &= \frac{1}{4\sqrt{2}} \left(g e^{\frac{\phi}{\sqrt{2}}} - 3m e^{-\frac{3\phi}{\sqrt{2}}} \right) - \frac{k}{2\sqrt{2}g} e^{-\frac{\phi}{\sqrt{2}}} (e^{-2g_1} + e^{-2g_2}) - \frac{1}{\sqrt{2}g^2 m} e^{\frac{\phi}{\sqrt{2}} - 2g_1 - 2g_2}, \end{aligned} \quad (3.14)$$

where $k = +1$ for the $S^2 \times S^2$ background and $k = -1$ for the $H^2 \times H^2$ background. The supersymmetry equations in (3.14) are analogous to the equations for M5-branes wrapped on two Riemann surfaces in [21], and more recently generalized in [22].

3.2 The AdS_2 solution and entropy of black holes

Now we will consider the $\mathcal{N} = 4^+$ theory, $g > 0$, $m > 0$. We find a new AdS_2 fixed point solution for the $H^2 \times H^2$ background with $k = -1$,⁵

$$e^f = \frac{2^{1/4}}{g^{3/4} m^{1/4}} \frac{1}{r}, \quad e^{g_1} = e^{g_2} = \frac{2^{3/4}}{g^{3/4} m^{1/4}}, \quad e^{\frac{\phi}{\sqrt{2}}} = \frac{2^{1/4} m^{1/4}}{g^{1/4}}. \quad (3.16)$$

When we consider the $S^2 \times S^2$ background with $k = +1$, AdS_2 fixed point does not exist.

⁴It is possible to have geometries like $S^2 \times H^2$ for $k_1 = +1$ and $k_2 = -1$, or vice versa. One can easily generalize our supersymmetry equations and the twist conditions to that case.

⁵The solution can also be presented as

$$e^f = \frac{\sqrt{2}}{g} e^{-\frac{\phi}{\sqrt{2}}} \frac{1}{r}, \quad e^{g_1} = e^{g_2} = \frac{2}{g} e^{-\frac{\phi}{\sqrt{2}}}, \quad e^{-2\sqrt{2}\phi} = \frac{g}{2m}. \quad (3.15)$$

By employing the uplift formulae in [6], the AdS_2 solution can be uplifted to a solution of massive type IIA supergravity. We only present the uplift formulae for the metric,

$$ds_{10}^2 = \sin^{1/12} \xi X^{1/8} \left[\Delta^{3/8} ds_6^2 + \frac{2\Delta^{3/8} X^2}{g^2} d\xi^2 + \frac{\cos^2 \xi}{2g^2 \Delta^{5/8} X} \Sigma_{I=1}^3 (\sigma^I - gA^I)^2 \right], \quad (3.17)$$

and the dilaton field, Φ ,

$$e^\Phi = \frac{\Delta^{1/4}}{\sin^{5/6} \xi X^{5/4}}, \quad (3.18)$$

where we define

$$\Delta = X \cos^2 \xi + \frac{1}{X^3} \sin^2 \xi, \quad X = e^{-\frac{\phi}{\sqrt{2}}}. \quad (3.19)$$

The solution in (3.16) describes the $AdS_2 \times H^2 \times H^2$ horizon of six-dimensional black holes. Moreover, when we consider the $\mathcal{N} = 4^+$ theory with $g = 3m$, there is a supersymmetric AdS_6 fixed point which is known to be dual of 5d $USp(2N)$ gauge theory in the large N limit. Recently, there has been development in calculating topologically twisted index of 5d $USp(2N)$ gauge theory on $M_4 \times S^1$ by two independent groups, [19] and [20]. See the introduction for more details on their calculations. Analogous to the 3d gauge theory examples in [16–18], the entropy of the black holes could match the topologically twisted index of 5d $USp(2N)$ gauge theory on $\Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2} \times S^1$ in the large N limit.

Now we calculate entropy of asymptotically AdS_6 black holes with $AdS_2 \times H^2 \times H^2$ horizon. It can be easily generalized to $AdS_2 \times \Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$ horizon where $\mathfrak{g}_1 > 1$ and $\mathfrak{g}_2 > 1$. Here $\Sigma_{\mathfrak{g}}$ is a Riemann surface of genus \mathfrak{g} with constant curvature. We would like to consider the AdS_6 fixed point in the $\mathcal{N} = 4^+$ theory by taking $g = 3m$. In order to have unit radius, $L_{AdS_6} = 1$, we set $m = \sqrt{2}$.⁶ The Bekenstein-Hawking entropy of the black hole is given by

$$S_{\text{BH}} = \frac{L_{AdS_6}^{p-2}}{4G_N^{(p)}} = \frac{1}{4G_N^{(2)}} = \frac{\text{vol}(M_4)}{4G_N^{(6)}} = \frac{e^{2g_1} \text{vol}(\Sigma_{\mathfrak{g}_1}) e^{2g_2} \text{vol}(\Sigma_{\mathfrak{g}_2})}{4G_N^{(6)}} = \frac{8\pi^2(\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)}{27G_N^{(6)}}, \quad (3.20)$$

where L , vol , and G_N are radius, volume, and the Newton's gravitational constant for the corresponding spaces, respectively. We used that the volume of Riemann surfaces with genus, $\mathfrak{g} > 1$, is⁷

$$\text{vol}(\Sigma_{\mathfrak{g}}) = 4\pi(\mathfrak{g} - 1), \quad (3.21)$$

and, in the last equality, we used the value of the warp factor at our AdS_2 solution, $e^{4g} = 2/3^3$. We can relate the gravitational entropy to the free energy of 5d SCFTs on S^5 by using the universal formula,⁸

$$F_{S^5} = -\frac{\pi^2 L_{AdS_6}^4}{3G_N^{(6)}}, \quad (3.22)$$

⁶See discussions around (4.6) in [24].

⁷See, for example, (73) in [12].

⁸See, for example, (4.5) in [24].

and then entropy of the black hole is

$$S_{\text{BH}} = -\frac{8}{9}(\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)F_{S^5}. \quad (3.23)$$

The free energy of 5d USp(2N) gauge theory with N_f flavors on S^5 in the large N limit is given by [25],

$$F_{S^5} = -\frac{9\sqrt{2}\pi N^{5/2}}{5\sqrt{8 - N_f}}. \quad (3.24)$$

Therefore, entropy of the black hole can be written by

$$S_{\text{BH}} = \frac{8\sqrt{2}\pi(\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)N^{5/2}}{5\sqrt{8 - N_f}}, \quad (3.25)$$

where $\mathfrak{g}_1 > 1$ and $\mathfrak{g}_2 > 1$. This nicely matches the topologically twisted index of 5d USp(2N) gauge theory on $\Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2} \times S^1$ in the large N limit calculated in [19]. As it was explained in the introduction, the contribution to the topologically twisted index with zero magnetic fluxes, $\mathfrak{m} = 0$, calculated in [20], counts only the half of the gravitational entropy.

3.3 Numerical black hole solutions

Now we present the full black hole solution numerically. The full black hole solution is an interpolating geometry between the asymptotically AdS_6 boundary and the $AdS_2 \times \Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$ horizon. We introduce a new radial coordinate,

$$\rho = f + \frac{\phi}{\sqrt{2}}. \quad (3.26)$$

This kind of coordinate was introduced in [22]. Employing the supersymmetry equations, we obtain

$$\frac{\partial \rho}{\partial r} = f' + \frac{\phi'}{\sqrt{2}} = -e^f D, \quad (3.27)$$

where we define

$$D = \frac{m}{\sqrt{2}}e^{-\frac{3\phi}{\sqrt{2}}} + \frac{4}{\sqrt{2}m}a_1a_2e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}. \quad (3.28)$$

Then, the supersymmetry equations are

$$\begin{aligned} -D \frac{\partial f}{\partial \rho} &= -\frac{1}{4\sqrt{2}} \left(ge^{\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}} \right) - \frac{\lambda}{2\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} (a_1 e^{-2g_1} + a_2 e^{-2g_2}) - \frac{3}{\sqrt{2}m} a_1 a_2 e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}, \\ -D \frac{\partial g_1}{\partial \rho} &= -\frac{1}{4\sqrt{2}} \left(ge^{\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}} \right) + \frac{\lambda}{2\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} (3a_1 e^{-2g_1} - a_2 e^{-2g_2}) + \frac{1}{\sqrt{2}m} a_1 a_2 e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}, \\ -D \frac{\partial g_2}{\partial \rho} &= -\frac{1}{4\sqrt{2}} \left(ge^{\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}} \right) + \frac{\lambda}{2\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} (3a_2 e^{-2g_2} - a_1 e^{-2g_1}) + \frac{1}{\sqrt{2}m} a_1 a_2 e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}, \\ -\frac{1}{\sqrt{2}} D \frac{\partial \phi}{\partial \rho} &= \frac{1}{4\sqrt{2}} \left(ge^{\frac{\phi}{\sqrt{2}}} - 3me^{-\frac{3\phi}{\sqrt{2}}} \right) + \frac{\lambda}{2\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} (a_1 e^{-2g_1} + a_2 e^{-2g_2}) - \frac{1}{\sqrt{2}m} a_1 a_2 e^{\frac{\phi}{\sqrt{2}}-2g_1-2g_2}. \end{aligned} \quad (3.29)$$

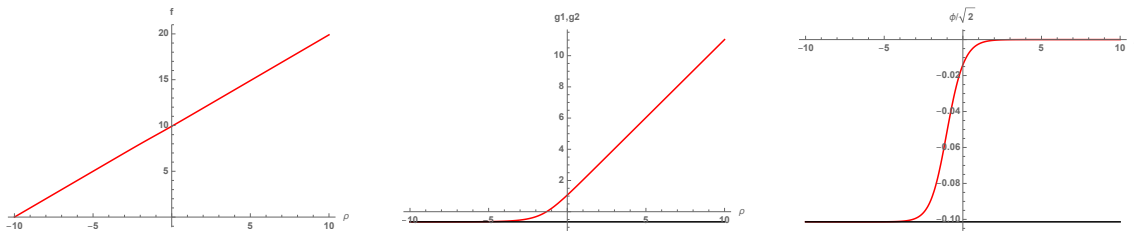


Figure 1. Numerical black hole solution with $m = \sqrt{2}$ and $g = 3m$. The black straight lines are the values at the AdS_2 horizon given in (3.16).

In the r -coordinate, the UV or asymptotically AdS_6 boundary is at $r = 0$, and the IR or $AdS_2 \times \Sigma_{g_1} \times \Sigma_{g_2}$ horizon is at $r = \infty$. In this ρ -coordinate, the UV is at $\rho = +\infty$, and the IR is at $\rho = -\infty$. We present the plot of the full black hole solution in figure 1. We have set $m = \sqrt{2}$ and $g = 3m$, and there is no free parameter left. Therefore, there is only one plot.

4 Black holes with other horizons

In this section, we obtain more black hole solutions with other horizon geometries by considering D4-branes wrapped on Kähler four-cycles in Calabi-Yau fourfolds and on Cayley four-cycles in Spin(7) manifolds. We believe these are all possible four-cycles on which D4-branes can wrap in $F(4)$ gauged supergravity. D4-branes on two Riemann surfaces in the previous section fall into a special case of D4-branes on Kähler four-cycles in Calabi-Yau fourfolds. The analogous results of M5-branes wrapped on supersymmetric four-cycles were studied in [21, 23].

4.1 Kähler four-cycles in Calabi-Yau fourfolds

We consider the metric,

$$ds^2 = e^{2f(r)} (dt^2 - dr^2) - e^{2g(r)} ds_{M_4}^2, \tag{4.1}$$

where M_4 is a Kähler four-cycle in Calabi-Yau fourfolds. The curved coordinates on the Kähler four-cycles will be denoted by $\{x_1, x_2, x_3, x_4\}$, and the hatted ones are the flat coordinates. For Kähler four-cycles in Calabi-Yau fourfolds, there are four directions transverse to D4-branes in the fourfolds. The normal bundle of the four-cycle has $U(2) \subset SO(4)$ structure group. We identify $U(1)$ part of the structure group with $U(1)$ gauge field from the non-Abelian $SU(2)$ gauge group, [23, 26]. The only non-vanishing component of the non-Abelian $SU(2)$ gauge field, A_μ^I , $I = 1, 2, 3$, is given by

$$F_{\hat{x}_1 \hat{x}_2}^3 = a_1 e^{-2g}, \quad F_{\hat{x}_3 \hat{x}_4}^3 = a_2 e^{-2g}, \tag{4.2}$$

where the magnetic charges, a_1 and a_2 , are constant. The only non-vanishing component of the two-form gauge potential is

$$B_{tr} = -\frac{2}{m^2} a_1 a_2 e^{\sqrt{2}\phi + 2f - 4g}. \tag{4.3}$$

We employ the projection conditions,

$$\gamma^{\hat{r}}\gamma^7\epsilon_i = \epsilon_i, \quad \gamma^{\hat{x}_1\hat{x}_2}(T^3)_i{}^j\epsilon_j = \frac{\lambda}{2}\epsilon_i, \quad \gamma^{\hat{x}_3\hat{x}_4}(T^3)_i{}^j\epsilon_j = \frac{\lambda}{2}\epsilon_i, \quad (4.4)$$

where $\lambda = \pm 1$. Solutions with the projection conditions preserve 1/8 of the supersymmetries. By employing the projection conditions, we obtain the complete supersymmetry equations,

$$\begin{aligned} f'e^{-f} &= -\frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}}\right) + \frac{k}{\sqrt{2}g}e^{-\frac{\phi}{\sqrt{2}}-2g} - \frac{3}{\sqrt{2}g^2m}e^{\frac{\phi}{\sqrt{2}}-4g}, \\ g'e^{-f} &= -\frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}}\right) - \frac{k}{\sqrt{2}g}e^{-\frac{\phi}{\sqrt{2}}-2g} + \frac{1}{\sqrt{2}g^2m}e^{\frac{\phi}{\sqrt{2}}-4g}, \\ \frac{1}{\sqrt{2}}\phi'e^{-f} &= \frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{\sqrt{2}}} - 3me^{-\frac{3\phi}{\sqrt{2}}}\right) - \frac{k}{\sqrt{2}g}e^{-\frac{\phi}{\sqrt{2}}-2g} - \frac{1}{\sqrt{2}g^2m}e^{\frac{\phi}{\sqrt{2}}-4g}, \end{aligned} \quad (4.5)$$

with the twist conditions,

$$a_1 = -\frac{k}{\lambda g}, \quad a_2 = -\frac{k}{\lambda g}, \quad (4.6)$$

where k determines the curvature of the Kähler four-cycles in Calabi-Yau fourfolds.

A product of two Riemann surfaces considered in the previous section is a special case of Kähler four-cycles in Calabi-Yau fourfolds. When we identify $g \equiv g_1 = g_2$ in the supersymmetry equations for D4-branes wrapped on two Riemann surfaces, (3.14), we obtain the supersymmetry equations here, (4.5). By solving the supersymmetry equations, we find an AdS_2 fixed point solution which is identical to the one obtained in the previous section, (3.16).

4.2 Cayley four-cycles in Spin(7) manifolds

We consider the metric,

$$ds^2 = e^{2f(r)}(dt^2 - dr^2) - e^{2g(r)}ds_{M_4}^2, \quad (4.7)$$

where M_4 is a Cayley four-cycle in manifolds with Spin(7) holonomy. The curved coordinates on the Cayley four-cycles will be denoted by $\{x_1, x_2, x_3, x_4\}$, and the hatted ones are the flat coordinates. In order to preserve supersymmetry for D4-branes wrapped on Cayley four-cycles in Spin(7) manifolds, we identify self-dual $SU(2)_+$ subgroup of the SO(4) isometry of the four-cycle,

$$SO(4) \rightarrow SU(2)_+ \times SU(2)_-, \quad (4.8)$$

with the non-Abelian SU(2) gauge group, [23, 26]. The self-duality is defined by

$$\gamma_{\mu\nu} = \pm \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\gamma^{\rho\sigma}, \quad (4.9)$$

and we denoted the self-duality and anti-self-duality by + and -, respectively. For the self-dual part, components are identified by

$$\gamma^{\hat{x}_1\hat{x}_2} = \gamma^{\hat{x}_3\hat{x}_4}, \quad \gamma^{\hat{x}_1\hat{x}_3} = \gamma^{\hat{x}_4\hat{x}_2}, \quad \gamma^{\hat{x}_1\hat{x}_4} = \gamma^{\hat{x}_2\hat{x}_3}. \quad (4.10)$$

The only non-vanishing components of the non-Abelian SU(2) gauge field, A_μ^I , $I = 1, 2, 3$, are given by

$$\begin{aligned} F_{\hat{x}_1\hat{x}_2}^1 &= F_{\hat{x}_3\hat{x}_4}^1 = a_1 e^{-2g}, \\ F_{\hat{x}_1\hat{x}_3}^2 &= F_{\hat{x}_4\hat{x}_2}^2 = a_2 e^{-2g}, \\ F_{\hat{x}_1\hat{x}_4}^3 &= F_{\hat{x}_2\hat{x}_3}^3 = a_3 e^{-2g}, \end{aligned} \quad (4.11)$$

where the magnetic charges, a_1 , a_2 and a_3 , are constant. The only non-vanishing component of the two-form gauge potential is

$$B_{\text{tr}} = -\frac{2}{m^2} (a_1^2 + a_2^2 + a_3^2) e^{\sqrt{2}\phi + 2f - 4g}. \quad (4.12)$$

We employ the projection conditions,

$$\begin{aligned} \gamma^{\hat{r}} \gamma^7 \epsilon_i &= \epsilon_i, \\ \gamma^{\hat{x}_1\hat{x}_2} (T^1)_i{}^j \epsilon_j &= \gamma^{\hat{x}_3\hat{x}_4} (T^1)_i{}^j \epsilon_j = \frac{\lambda}{2} \epsilon_i, \\ \gamma^{\hat{x}_1\hat{x}_3} (T^2)_i{}^j \epsilon_j &= \gamma^{\hat{x}_4\hat{x}_2} (T^2)_i{}^j \epsilon_j = \frac{\lambda}{2} \epsilon_i, \\ \gamma^{\hat{x}_1\hat{x}_4} (T^3)_i{}^j \epsilon_j &= \gamma^{\hat{x}_2\hat{x}_3} (T^3)_i{}^j \epsilon_j = \frac{\lambda}{2} \epsilon_i, \end{aligned} \quad (4.13)$$

where $\lambda = \pm 1$. Solutions with the projection conditions preserve 1/16 of the supersymmetries. By employing the projection conditions, we obtain the complete supersymmetry equations,

$$\begin{aligned} f' e^{-f} &= -\frac{1}{4\sqrt{2}} \left(g e^{\frac{\phi}{\sqrt{2}}} + m e^{-\frac{3\phi}{\sqrt{2}}} \right) + \frac{k}{\sqrt{2}g} e^{-\frac{\phi}{\sqrt{2}} - 2g} - \frac{1}{\sqrt{2}g^2 m} e^{\frac{\phi}{\sqrt{2}} - 4g}, \\ g' e^{-f} &= -\frac{1}{4\sqrt{2}} \left(g e^{\frac{\phi}{\sqrt{2}}} + m e^{-\frac{3\phi}{\sqrt{2}}} \right) - \frac{k}{\sqrt{2}g} e^{-\frac{\phi}{\sqrt{2}} - 2g} + \frac{1}{3\sqrt{2}g^2 m} e^{\frac{\phi}{\sqrt{2}} - 4g}, \\ \frac{1}{\sqrt{2}} \phi' e^{-f} &= \frac{1}{4\sqrt{2}} \left(g e^{\frac{\phi}{\sqrt{2}}} - 3m e^{-\frac{3\phi}{\sqrt{2}}} \right) - \frac{k}{\sqrt{2}g} e^{-\frac{\phi}{\sqrt{2}} - 2g} - \frac{1}{3\sqrt{2}g^2 m} e^{\frac{\phi}{\sqrt{2}} - 4g}, \end{aligned} \quad (4.14)$$

with the twist conditions,

$$a_1 = -\frac{k}{3\lambda g}, \quad a_2 = -\frac{k}{3\lambda g}, \quad a_3 = -\frac{k}{3\lambda g}, \quad (4.15)$$

where k determines the curvature of the Cayley four-cycles in Spin(7) manifolds.

Now we will consider the $\mathcal{N} = 4^+$ theory, $g > 0$, $m > 0$. By solving the supersymmetry equations, we find a new AdS_2 fixed point solution for the negatively curved Cayley four-cycles with $k = -1$,

$$e^f = \frac{3^{1/4}}{g^{3/4} m^{1/4}} \frac{1}{r}, \quad e^g = \frac{2}{3^{1/4} g^{3/4} m^{1/4}}, \quad e^{\frac{\phi}{\sqrt{2}}} = \frac{2^{1/2} m^{1/4}}{3^{1/4} g^{1/4}}. \quad (4.16)$$

When we consider $k = +1$, AdS_2 fixed point does not exist. It will be interesting to have a field theory interpretation of this AdS_2 fixed point solution.

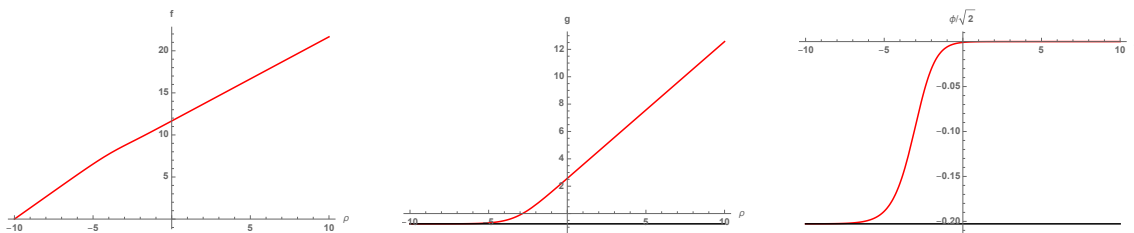


Figure 2. Numerical black hole solution with $m = \sqrt{2}$ and $g = 3m$. The black straight lines are the values at the AdS_2 horizon given in (4.16).

Now we present the full black hole solution numerically. The full black hole solution is an interpolating geometry between the asymptotically AdS_6 boundary and the $AdS_2 \times Cayley_4$ horizon with negative curvature, $k = -1$. We introduce the radial coordinate, ρ , in (3.26), and solve the supersymmetry equations in ρ coordinate, as we did in section 3.3. We again set $m = \sqrt{2}$ and $g = 3m$. The full black hole solution is presented in figure 2.

5 Conclusions

In this paper, we studied supersymmetric AdS_6 black holes with various horizon geometries. We found asymptotically AdS_6 black holes with $AdS_2 \times \Sigma_{g_1} \times \Sigma_{g_2}$ horizon with $g_1 > 1$ and $g_2 > 1$ and presented the full black hole solution numerically. We calculated the Bekenstein-Hawking entropy of the black holes, and found that it nicely matches the topologically twisted index of 5d $USp(2N)$ gauge theory on $\Sigma_{g_1} \times \Sigma_{g_2} \times S^1$ in the large N limit obtained in [19]. The contribution to the topologically twisted index with zero magnetic fluxes, $\mathbf{m} = 0$, calculated in [20], counts only the half of the gravitational entropy. We also found black hole solutions by considering D4-branes wrapped on Kähler four-cycles in Calabi-Yau fourfolds and Cayley four-cycles in Spin(7) manifolds.

For the near-horizon geometries we considered in this paper, we have shown that there should be non-zero two-form gauge potential, $B_{\mu\nu}$, in order to have a consistent set of supersymmetry equations, and satisfy the equations of motion. For this reason, as explained in the introduction, we concluded that the equations and solutions for the corresponding cases in the previous study, [14], are *not* correct.

It would be interesting to study supersymmetric four-cycles in matter coupled $F(4)$ gauged supergravity [27] along the line of [28], to obtain more examples of supersymmetric black holes in AdS_6 [29, 30].

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A The equations of motion

In this appendix, we present the equations of motion of $F(4)$ gauged supergravity,

$$\begin{aligned}
 R_{\mu\nu} = & 2\partial_\mu\phi\partial_\nu\phi + \frac{1}{8}g_{\mu\nu} \left(g^2e^{\sqrt{2}\phi} + 4gme^{-\sqrt{2}\phi} - m^2e^{-3\sqrt{2}\phi} \right) \\
 & - 2e^{-\sqrt{2}\phi} \left(\mathcal{H}_\mu{}^\rho\mathcal{H}_{\nu\rho} - \frac{1}{8}g_{\mu\nu}\mathcal{H}_{\rho\sigma}\mathcal{H}^{\rho\sigma} \right) \\
 & - 2e^{-\sqrt{2}\phi} \left(F_\mu{}^I{}^\rho F_{\nu\rho}^I - \frac{1}{8}g_{\mu\nu}F_{\rho\sigma}^IF^{I\rho\sigma} \right) \\
 & + e^{2\sqrt{2}\phi} \left(G_\mu{}^{\rho\sigma}G_{\nu\rho\sigma} - \frac{1}{6}g_{\mu\nu}G_{\rho\sigma\tau}G^{\rho\sigma\tau} \right), \tag{A.1}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) = & \frac{1}{4\sqrt{2}} \left(g^2e^{\sqrt{2}\phi} - 4gme^{-\sqrt{2}\phi} + 3m^2e^{-3\sqrt{2}\phi} \right) \\
 & + \frac{1}{2\sqrt{2}}e^{-\sqrt{2}\phi} (\mathcal{H}_{\mu\nu}\mathcal{H}^{\mu\nu} + F_{\mu\nu}^IF^{I\mu\nu}) + \frac{1}{3\sqrt{2}}e^{2\sqrt{2}\phi}G_{\mu\nu\rho}G^{\mu\nu\rho}, \tag{A.2}
 \end{aligned}$$

$$\mathcal{D}_\nu \left(e^{-\sqrt{2}\phi}\mathcal{H}^{\nu\mu} \right) = \frac{1}{6}e\epsilon^{\mu\nu\rho\sigma\tau\kappa}\mathcal{H}_{\nu\rho}G_{\sigma\tau\kappa}, \tag{A.3}$$

$$\mathcal{D}_\nu \left(e^{-\sqrt{2}\phi}F^{I\nu\mu} \right) = \frac{1}{6}e\epsilon^{\mu\nu\rho\sigma\tau\kappa}F_{\nu\rho}^IG_{\sigma\tau\kappa}, \tag{A.4}$$

$$\mathcal{D}_\rho \left(e^{2\sqrt{2}\phi}G^{\rho\mu\nu} \right) = -\frac{1}{4}e\epsilon^{\mu\nu\rho\sigma\tau\kappa} (\mathcal{H}_{\rho\sigma}\mathcal{H}_{\tau\kappa} + F_{\rho\sigma}^IF_{\tau\kappa}^I) - me^{-\sqrt{2}\phi}\mathcal{H}^{\mu\nu}. \tag{A.5}$$

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