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Adiabatic suppression of the axion abundance and isocurvature due to coupling to hidden monopoles

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ABSTRACT: The string theory predicts many light fields called moduli and axions, which cause a cosmological problem due to the overproduction of their coherent oscillation after inflation. One of the prominent solutions is an adiabatic suppression mechanism, which, however, is non-trivial to achieve in the case of axions because it necessitates a large effective mass term which decreases as a function of time. The purpose of this paper is twofold. First, we provide an analytic method to calculate the cosmological abundance of coherent oscillation in a general situation under the adiabatic suppression mechanism. Secondly, we apply our method to some concrete examples, including the one where a string axion acquires a large effective mass due to the Witten effect in the presence of hidden monopoles.

KEYWORDS: Cosmology of Theories beyond the SM, Beyond Standard Model, Solitons Monopoles and Instantons

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1 Introduction

The string theory is a candidate for the theory of everything. While it is hard to derive the Standard Model (SM) of particle physics in a top-down manner, it provides intriguing insight into the low energy physics. One of the important implications is the existence of many light fields called moduli and axions [1, 2]. They acquire a mass from supersymmetry (SUSY) breaking effects and other non-perturbative effects. Since their interactions with the SM particles are typically suppressed by the Planck scale, those light fields are long-lived and play a major role in cosmology. In fact, they are known to cause a catastrophic problem for the evolution of the Universe.

During inflation, those light fields may be displaced from the low-energy minima, since their potentials are very flat and in some case, the potential could be significantly modified

by the SUSY breaking effects of the inflaton. After inflation ends, the energy scale of the Universe decreases and eventually the Hubble parameter becomes as low as the mass scale of the light fields. Then the light fields start to oscillate coherently around the minimum of the potential [3–5]. The amplitude of the oscillation is expected to be of order the Planck scale or the axion decay constant. The energy density of the coherent oscillation behaves like matter and decreases slower than radiation so that they dominate the Universe soon after the onset of oscillation. Since the interactions with the SM particles are suppressed by the Planck scale, their lifetime may be longer than the big bang nucleosynthesis (BBN) epoch and the resulting evolution of the Universe would not be consistent with observations. This is the notorious cosmological moduli problem.¹

A novel solution to the problem was suggested by Linde about two decades ago [22]. If the light fields obtain a time-dependent effective mass much larger than the Hubble parameter, they may follow the slowly-moving potential minimum adiabatically. As a result, the oscillation energy is suppressed when the Hubble parameter becomes comparable to their low-energy mass.² In fact, however, the suppression of the moduli abundance is not so efficient as originally expected [29, 30], since some amount of moduli oscillations is necessarily induced after inflation. This is because the inflaton is lighter than the Hubble parameter for slow-roll inflation and later becomes much heavier than the Hubble parameter and the moduli mass. The oscillation of the inflaton becomes relevant for the moduli dynamics especially when their mass scales become comparable. Also, we note that it is non-trivial to realize the adiabatic suppression mechanism for axions, because the axion potential is protected by a shift symmetry and is induced from non-perturbative effects.

Recently, we found in ref. [31] that the adiabatic suppression mechanism works for axions coupled to a hidden $U(1)_H$ gauge symmetry with a hidden monopole. In the presence of hidden monopoles, the theta parameter in the hidden $U(1)_H$ has a physical effect, known as the Witten effect, where the monopole acquires a hidden electric charge proportional to the theta parameter [32]. Here, when the Peccei-Quinn (PQ) symmetry is also anomalous under the hidden $U(1)_H$ symmetry, the theta parameter of the hidden $U(1)_H$ is promoted to the dynamical axion field [33]. This implies that the hidden monopole acquires an electric charge related to the axion field value. Since the nonzero value of electric charge is not favored to minimize the total energy, the axion starts to cancel the theta parameter of the hidden $U(1)_H$ to make the hidden electric charge of monopole absent. This means that the axion acquires an effective mass via the Witten effect and its abundance can be suppressed by the adiabatic suppression mechanism. Also, the axion isocurvature can be suppressed by the Witten effect [31, 34]. See e.g. refs. [35–49] for other scenarios to suppress the axion isocurvature.

The purpose of the present paper is twofold. First, we provide an analytic method to calculate the abundance of light fields under the adiabatic suppression mechanism. We

¹Another important prediction of the string theory in cosmology is the string landscape. There are an exponentially large number of vacua in the field space of moduli and string axions and the anthropic selection of vacua may explain the fine tuning of the cosmological constant. The slow-roll inflation in the string landscape is also widely studied (see, e.g., ref. [6] for a review and refs. [7–21] for recent works.)

²Thermal inflation is another possible solution to the cosmological moduli problem, where the moduli density is diluted by a mini-inflation around the TeV scale [23–26]. See refs. [27, 28] for recent works.

calculate an approximately conserved adiabatic invariant, which represents the comoving number density of light fields, and show that the adiabatic invariant is nonzero but exponentially suppressed if we neglect the initial abundance which may be induced by the effect of the inflaton oscillation. Our result is consistent with ref. [22], but our method can be applied to a more generic situation. Secondly, we apply our calculation to some concrete models, including the one with the Witten effect on the string axion.

This paper is organized as follows. In the next section, we briefly review the moduli problem and the axion overproduction problem. Then in section 3, we calculate the abundance of coherent oscillation of scalar field under the adiabatic suppression mechanism. Then in the subsequent sections, we apply the calculation to scenarios where an axion acquires an effective mass due to the Witten effect in the presence of monopoles in a hidden sector. In section 4, we focus on the QCD axion and discuss a possibility that the dark matter (DM) is a hidden monopole. In section 5, we show that the overproduction problem of string axion can be ameliorated by the Witten effect when abundant monopoles disappear before the BBN epoch. Finally, we conclude in section 6.

2 Overproduction problem of light scalar fields

The string theory predicts many moduli and axions that have extremely flat potentials compared with the fundamental scale, such as the Planck scale [1, 2]. Such light fields start to oscillate around the low energy vacuum after inflation. Since the amplitude of the coherent oscillation is expected to be of order the Planck or GUT scale, they soon dominate the Universe, and the subsequent evolution of the Universe is inconsistent with that of our Universe. In this section, we briefly explain the overproduction problem of moduli and axions.

2.1 Moduli problem

The string theory predicts many light singlet scalar fields in low energy effective theory. Its potential is generically written as

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2, \quad (2.1)$$

around the low-energy vacuum, where m_ϕ is the mass of moduli that is related to the SUSY breaking scale. Here we take $\phi = 0$ at the low energy vacuum by shifting its field value.

The mass of moduli is expected to be in the range between TeV scale and PeV scale in most SUSY models motivated by addressing the hierarchy problem and the gauge coupling unification. In this paper, we consider the case where the Hubble parameter during inflation is much larger than the mass of moduli. In this case, the potential of the moduli is so flat that it may have a nonzero vacuum expectation value (VEV) during inflation because of the Hubble friction effect. Since the coherent length is stretched by the exponential expansion of the Universe during inflation, the moduli VEV becomes spatially homogeneous over the observable Universe. After inflation ends, the Hubble parameter $H(t)$ decreases and the moduli start to oscillate around the low-energy vacuum at $H(t) \simeq m_\phi$. The amplitude of

its oscillation is expected to be of order the Planck scale, so that the resulting abundance is given by

$$\frac{\rho_\phi}{s} \simeq \frac{1}{8} T_{\text{RH}}, \quad (2.2)$$

where T_{RH} is reheat temperature after inflation. This energy density is much larger than the DM abundance. If the moduli is stable, the Universe would be in matter domination before the BBN starts. Even if the moduli is unstable, its decay products may spoil the success of the BBN [50]. This is the notorious cosmological moduli problem. Also, if the moduli is stabilized by SUSY breaking effects in the Kähler potential, their axionic partners remain much lighter than the moduli. In this case, the moduli decay into a pair of axions with a large branching ratio, leading to the overproduction of axions [51–53]. Similarly, the moduli generically decay into a pair of gravitinos with a sizable branching fraction if kinematically accessible [54–57]. Thus produced gravitinos may spoil the BBN or produce too many lightest SUSY particles.

2.2 Axion overproduction problem

The potential of axion can be similarly written as follows around the low-energy vacuum:

$$V(a) \approx \frac{1}{2} m_a^2 a^2, \quad (2.3)$$

where a is the axion field and m_a is the axion mass at the potential minimum.

For the QCD axion, its mass is given as

$$m_a|_{T=0} \simeq \frac{z}{(1+z)^2} \frac{m_\pi f_\pi}{f_a}, \quad (2.4)$$

at the zero temperature, where f_a is the axion decay constant, z ($\simeq 0.56$) is the ratio of u - and d -quark masses, m_π ($\simeq 140$ MeV) is the pion mass, and f_π ($\simeq 130$ MeV) is the pion decay constant. In a finite temperature plasma with $T \gg \Lambda_{\text{QCD}}$, the axion mass depends on the temperature as

$$m_a^2(T) \simeq c_T \frac{\Lambda_{\text{QCD}}^4}{f_a^2} \left(\frac{T}{\Lambda_{\text{QCD}}} \right)^{-n}, \quad (2.5)$$

where $c_T \simeq 1.68 \times 10^{-7}$, $n = 6.68$, and $\Lambda_{\text{QCD}} = 400$ MeV [58, 59].

Let us consider a case in which the PQ symmetry is broken before inflation. In this case, the axion stays at a certain phase during inflation because it is massless and is affected by the Hubble friction effect. Since the coherent length is stretched by the exponential expansion of the Universe during inflation, the axion VEV is spatially homogeneous. After inflation ends and before the QCD phase transition, the axion mass is much smaller than the Hubble parameter, so that it continues to stay at a certain VEV due to the Hubble friction effect. Then at a time around the QCD phase transition, the axion mass becomes larger than the Hubble parameter and starts to oscillate around the low energy vacuum, at which the axion VEV cancels the undesirable strong CP phase [3–5]. The temperature

of the Universe at the onset of oscillation is given as

$$T_{\text{osc},0} \simeq \Lambda_{\text{QCD}} \left(\frac{90c_T M_{\text{Pl}}^2}{\pi^2 g_*(T_{\text{osc},0}) f_a^2} \right)^{1/(4+n)} \quad (2.6)$$

$$\simeq 1.2 \text{ GeV} \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{-0.187}, \quad (2.7)$$

where M_{Pl} ($\simeq 2.4 \times 10^{18}$ GeV) is the reduced Planck mass. The parameter $g_*(T)$ is the effective number of relativistic particles in the plasma and we use $g_*(T_{\text{osc},0}) \approx 85$. The energy density of the axion oscillation decreases with time just like matter after the axion mass becomes constant, and hence it is a good candidate for cold DM. Neglecting the anharmonic effect [60], one obtains the axion abundance as [61]

$$\Omega_a h^2 \simeq 0.2 \theta_{\text{ini}}^2 \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{1.19}, \quad (2.8)$$

where h is the Hubble parameter in units of 100 km/s/Mpc and θ_{ini} ($-\pi < \theta_{\text{ini}} \leq \pi$) is the initial misalignment angle. The observed DM abundance, $\Omega_{\text{DM}} h^2 \simeq 0.12$, can be explained when the axion decay constant is given by

$$f_a \simeq 7.4 \times 10^{11} \text{ GeV} \times |\theta_{\text{ini}}|^{-1.68}. \quad (2.9)$$

Note that the string theory predicts axions with the axion decay constants of order the grand-unified theory (GUT) scale. With such a large decay constant, the axion abundance easily overcloses the Universe unless the initial misalignment angle θ_{ini} is fine-tuned to be much smaller than unity [see eq. (2.8)]. Although the string axion is well motivated, it confronts the overproduction problem of the axion energy density.

3 Adiabatic suppression mechanism

The moduli and axion overproduction problems can be avoided when the VEV of these fields changes adiabatically at the onset of oscillation. This can be achieved when they have a large time-dependent mass term in addition to the low-energy mass term [22]. In this section, we explain the adiabatic suppression mechanism and show that it results in the exponential suppression of an adiabatic invariant, which describes the comoving number density of particles.

Suppose that a light field (moduli or axion) has a time-dependent mass $\tilde{m}(t)$. The potential of the light field ϕ is then given by

$$V(\phi) = \frac{1}{2} m_\phi^2(t) \phi^2 + \frac{1}{2} \tilde{m}^2(t) (\phi - \phi_0)^2, \quad (3.1)$$

where ϕ_0 is of order the Planck scale for the case of moduli or of order the axion decay constant for the case of axion. We denote the mass of light field as $m_\phi(t)$, which may depend on time in some cases. Note that the model considered in ref. [22] corresponds to

the case with $m_\phi(t) = \text{const.}$ ($\equiv m_\phi$) and $\tilde{m}^2(t) = C^2 H^2(t)$ (C : constant). The equation of motion is then given by

$$\ddot{\phi} + 3H\dot{\phi} = -m_\phi^2(t)\phi - \tilde{m}^2(t)(\phi - \phi_0), \quad (3.2)$$

where $H = p/t$ with $p = 1/2$ in the radiation dominated era.

We assume that $\tilde{m}(t)$ is larger than $H(t)$ at least until $\tilde{m}(t) \simeq m_\phi(t)$. In this case ϕ stays at or oscillates around $\phi = \phi_0$. Then, at the time around $\tilde{m}(t) \simeq m_\phi(t)$, the minimum of the potential decreases as $v(t) \equiv \phi_0 \tilde{m}^2(t) / (m_\phi(t)^2 + \tilde{m}^2(t))$. Here, the time scale of $v(t)$ is of order the Hubble parameter ($\dot{v}/v \sim H(t)$) while that of the oscillation is of order $\tilde{m}(t)$ ($\gg H(t)$). This means that the minimum of the potential changes adiabatically and almost no oscillation is induced through this dynamics.

Note that we should take a particular care of the origin of the effective mass term. Suppose that, for example, the effective mass term comes from a coupling between the moduli field and the inflaton. During inflation, the inflaton mass is lighter than the Hubble parameter for successful slow-roll inflation. After inflation, the inflaton oscillates about its potential minimum and its mass becomes larger than the Hubble parameter. This implies that the effective modulus mass becomes necessarily comparable to the inflaton mass sometime after inflation. Then, the modulus dynamics can be significantly affected by the inflaton oscillations, leading to a production of the modulus field after inflation [29]. The model used in ref. [22] confronts this issue. On the other hand, the examples we use in section 4 and 5 are free from this issue, since the effective axion mass appears after inflation when the inflaton mass is already much heavier than the effective axion mass.

Below, we evaluate the abundance of the light field by calculating the time evolution of an adiabatic invariant. Our calculation not only reproduces the result of ref. [22] but also is applicable to more generic cases.

3.1 Calculation of adiabatic invariant

We rewrite the equation of motion as

$$\ddot{\varphi} + m^2(t)(\varphi - v(t)) = 0, \quad (3.3)$$

where

$$\varphi \equiv \left(\frac{t}{t_0}\right)^{3p/2} \phi, \quad (3.4)$$

$$m^2(t) \equiv m_\phi^2(t) + \frac{3}{2} \left(\frac{1}{p} - \frac{3}{2}\right) H^2(t) + \tilde{m}^2(t) \quad (3.5)$$

$$v(t) \equiv \frac{\tilde{m}^2(t)}{m^2(t)} \phi_0 \left(\frac{t}{t_0}\right)^{3p/2}. \quad (3.6)$$

The dynamics of such a field is analogous to a motion of a particle with the Hamiltonian,

$$\mathcal{H} = \frac{1}{2} \Pi^2 + \frac{1}{2} m^2(t) (\varphi - v(t))^2. \quad (3.7)$$

where Π is a canonical momentum of the particle φ . This is the Hamiltonian of a harmonic oscillator with two time-dependent parameters.

We can define an adiabatic invariant for a one-particle system with a compact trajectory. We assume that the typical time scale of its oscillation $m(t)$ is much larger than $H(t)$ and those of the two parameters $(\dot{m}/m)^{-1}$ and $(\dot{v}/v)^{-1}$ are of order $H(t)$. In this case, the adiabatic invariant approximately conserves. It is explicitly written as

$$I = \frac{1}{2\pi} \int_T \Pi d\varphi, \tag{3.8}$$

where the integral is taken over the interval of one periodic motion assuming constant m and v . In an oscillating homogeneous scalar field, the adiabatic invariant can be interpreted as the comoving number density of particles, and its approximate conservation law means that almost no particles are produced due to slowly changing parameters. However, exponentially suppressed but nonzero amount of particles are produced throughout the dynamics and we can calculate it in the following way. We follow and generalize a method explained in ref. [62].

3.1.1 Case with constant parameters

First, let us consider a trivial case where the parameters m and v are constant in time. The result is of course given by

$$\varphi(t) = \sqrt{\frac{2E}{m^2}} \sin \theta(t) + v, \tag{3.9}$$

$$\Pi(t) = \sqrt{2E} \cos \theta(t), \tag{3.10}$$

where E is the energy, $\Pi = \dot{\varphi}$ is the canonical momentum, and $\theta(t)$ is given by mt . The adiabatic invariant, which is exactly conserved in this case, is calculated as

$$I = E/m. \tag{3.11}$$

Note that since we define φ as eq. (3.4), the adiabatic invariant is proportional to the comoving number density of the field ϕ .

Let us take the action as a function of φ and t such as

$$S(\varphi, t) = \int^{\varphi, t} \mathcal{L} dt'. \tag{3.12}$$

Noting that $\mathcal{L} = \Pi(\partial\varphi/\partial t) - \mathcal{H}$, where \mathcal{H} is the Hamiltonian, we obtain

$$dS = \Pi d\varphi - \mathcal{H} dt. \tag{3.13}$$

Note that Π should be rewritten in terms of φ by using eqs. (3.9) and (3.10). Since the energy is conserved for the case of constant m and v , it is given by

$$S(\varphi, t) = S_0(\varphi, E) - Et, \tag{3.14}$$

$$S_0(\varphi, E) = \int \Pi(\varphi', E) d\varphi', \tag{3.15}$$

where we explicitly write E dependence of S_0 for later convenience. Here, we can take $S_0(\varphi, E)$ as a function of I because $E = mI$. In the case of the harmonic oscillator, S_0 is explicitly calculated as

$$S_0(\varphi, I) = \sqrt{2mI} \int \sqrt{1 - \frac{m}{2I}(\varphi' - v)^2} d\varphi' = I \left(\theta + \frac{\sin 2\theta}{2} \right), \quad (3.16)$$

where $\theta = \theta(\varphi, I)$ is given by the inverse of eq. (3.9).

Let us take S_0 as a mother function of a canonical transformation:

$$\frac{dS_0}{dt}(\varphi, I) = \left(\Pi \frac{d\varphi}{dt} - \mathcal{H} \right) - \left(I \frac{d\theta}{dt} - \mathcal{H}' \right) + \frac{d}{dt}(I\theta), \quad (3.17)$$

where \mathcal{H}' is a new Hamiltonian. The last terms comes from the Legendre transformation that change the dependence of the variable θ to I . Then the adiabatic invariant I becomes a new canonical momentum and satisfies

$$\Pi = \frac{\partial S_0(\varphi, I)}{\partial \varphi}, \quad (3.18)$$

$$\theta = \frac{\partial S_0(\varphi, I)}{\partial I}. \quad (3.19)$$

The parameter θ is a new canonical variable. It coincides with the one used in eq. (3.9), as expected. Since the mother function S_0 is independent of time, the new Hamiltonian is identified with the old one, i.e., $\mathcal{H} = E(I)$. The Hamilton equations of motion are thus given by

$$\dot{I} = -\frac{\partial E(I)}{\partial \theta} = 0, \quad (3.20)$$

$$\dot{\theta} = \frac{\partial E(I)}{\partial I} = m, \quad (3.21)$$

which imply that $I = \text{const.}$ and $\theta = mt$ as expected.

3.1.2 Case with time-dependent parameters

Next, we consider a case where the parameters m and v depend on time. We perform a canonical transformation of this system via the mother function of S_0 given in eq. (3.16), where the constant parameters m and v are replaced by the time-dependent ones. Since the mother function depends on time via the parameters $m(t)$ and $v(t)$, the new Hamiltonian is given by

$$\mathcal{H}' = E(I) + \frac{\partial S_0}{\partial t} = E(I) + \Lambda_m \dot{m} + \Lambda_v \dot{v}, \quad (3.22)$$

where

$$\Lambda_m = \left(\frac{\partial S_0}{\partial m} \right)_{\varphi, I} = \frac{I}{2m} \sin 2\theta, \quad (3.23)$$

$$\Lambda_v = \left(\frac{\partial S_0}{\partial v} \right)_{\varphi, I} = -\sqrt{2mI} \cos \theta, \quad (3.24)$$

where $(\partial f(x, y)/\partial x)_y$ is the partial derivative of $f(x, y)$ with respect to x while y fixed.

The new canonical variables I and θ are determined by eqs. (3.18) and (3.19). The equations of motion are now given by

$$\dot{I} = -\frac{\partial \mathcal{H}'}{\partial \theta} = -\left(\frac{\partial \Lambda_m}{\partial \theta}\right)_{I,m,v} \dot{m} - \left(\frac{\partial \Lambda_v}{\partial \theta}\right)_{I,m,v} \dot{v}, \quad (3.25)$$

$$\dot{\theta} = \frac{\partial \mathcal{H}'}{\partial I} = \left(\frac{\partial E}{\partial I}\right)_{m,v} + \left(\frac{\partial \Lambda_m}{\partial I}\right)_{\theta,m,v} \dot{m} + \left(\frac{\partial \Lambda_v}{\partial I}\right)_{\theta,m,v} \dot{v}, \quad (3.26)$$

where $(\partial E/\partial I)_{m,v}$ ($= m$) is the frequency of the harmonic oscillator for the case of time-independent parameters. These are rewritten as

$$\dot{I}(t) = -I \frac{\dot{m}}{m} \cos(2\theta) - \sqrt{2mI} \dot{v} \sin \theta, \quad (3.27)$$

$$\dot{\theta} \simeq m, \quad (3.28)$$

where we neglect the second and third terms in eq. (3.26) because the inverse of the oscillation time scale m is much larger than that of \dot{m}/m and \dot{v}/v .

On some occasions, we are interested in the case where the adiabatic invariant I is absent initially. Then the second term of the right-hand side in eq. (3.27) mainly contributes to the growth of I . Even if I is initially nonzero, we can neglect the first term in the right-hand side for the case of $I \ll (m/\dot{m})^2 (\dot{v}/v)^2 m v^2 \approx m v^2$. For example, suppose that ϕ oscillates at $t = t_0$ with an amplitude of ϕ_0 . Then we have $I_0 \simeq m \phi_0^2$, which is much smaller than $m v^2 \simeq m \phi_0^2 (t/t_0)^{3p}$ for $t \gg t_0$. Therefore, what we need to calculate is the following integral:

$$\sqrt{I(\infty)} - \sqrt{I(t_0)} = -\int_{t_0}^{\infty} dt \sqrt{\frac{m}{2}} \dot{v} \sin \theta, \quad (3.29)$$

where t_0 is an arbitrary time much before $\tilde{m}(t) \simeq m_\phi(t)$. We need to specify a model to calculate this integral.

3.2 Examples

3.2.1 Large Hubble-induced mass

Here let us check that our method reproduces the result in ref. [22], by taking $m_\phi = \text{const.}$ and $\tilde{m}(t) = CH(t)$ with $C \gg 1$. This can be realized when we consider a modulus that has interaction with an inflaton with a cutoff scale below the Planck scale. In this case, we can take the limit of $t_0 \rightarrow 0$ because the axion initially stays at a certain VEV due to the Hubble friction effect (i.e., $I(t_0) = 0$ for $t_0 \rightarrow 0$).

We decompose the oscillation factor such as $\sin \theta = (e^{i\theta} - e^{-i\theta})/2i$, and calculate each term separately. The integrand has poles and branch cuts in the complex t -plane, so that we replace the integration contour in the complex plane as shown in the left panel in figure 1:

$$\frac{i}{2\sqrt{2}} \int_{C_1} \dot{v} \sqrt{m} e^{i\theta} dt = \frac{i}{2\sqrt{2}} \int_{C_2+C_3+C_4+C_5+C_6} \dot{v} \sqrt{m} e^{i\theta} dt. \quad (3.30)$$

The integrand is given as

$$\dot{v} \sqrt{m} e^{i\theta} = \frac{V}{\tilde{C}^{3/2}} \frac{t^{3p/2-3/2}}{\left(1 + \frac{t^2}{z_{\text{pole}}^2}\right)^{7/4}} \left[\frac{3p}{2} + \left(\frac{3p}{2} - 2\right) \frac{t^2}{z_{\text{pole}}^2} \right] e^{i\theta}, \quad (3.31)$$

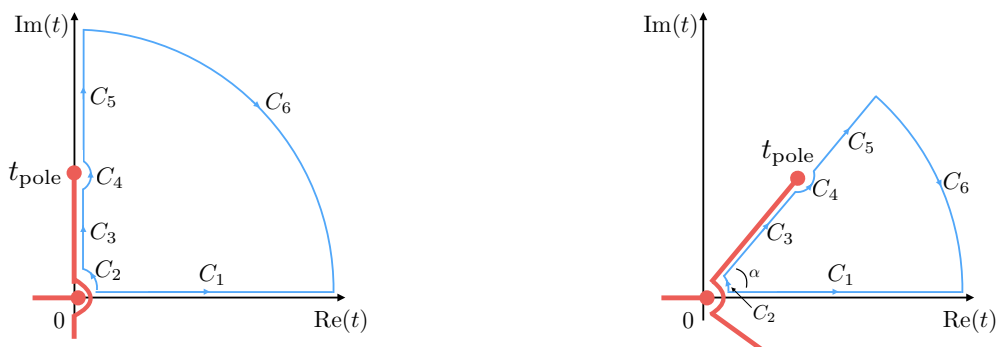


Figure 1. Integration contours in the complex t plane for the cases of $m_a = \text{const.}$ (left panel) and $m_a \propto t^{n/2}$ (right panel). The red lines represent branch cuts and the red points are poles.

where we define

$$z_{\text{pole}} \equiv \tilde{C}/m_\phi \quad (3.32)$$

$$\tilde{C} \equiv p\sqrt{3/2(1/p - 3/2) + C^2} \simeq pC \quad (3.33)$$

$$V \equiv p^2 C^2 \phi_0 t_0^{-3p/2}. \quad (3.34)$$

The parameter θ is calculated from

$$\theta(t) = \int_{t_0}^t m(t') dt'. \quad (3.35)$$

On the imaginary axis of t , it is given by

$$\theta(t) = \begin{cases} \int_{C_2} m(t') dt' - \int_{z_0}^z dz' \sqrt{-m_\phi^2 + \tilde{C}^2/z'^2} & \text{for } z < z_{\text{pole}} \\ \int_{C_2+C_3+C_4} m(t') dt' + i \int_{z_{\text{pole}}}^z dz' \sqrt{m_\phi^2 - \tilde{C}^2/z'^2} & \text{for } z > z_{\text{pole}} \end{cases} \quad (3.36)$$

where $t = iz$ and $t_{\text{pole}} \equiv iz_{\text{pole}}$ is the pole location in the complex t -plane and $z_0 \equiv t_0$. The integral for the contour C_2 can be calculated as

$$\int_{C_2} m(t') dt' = \int_{C_2} \sqrt{m_\phi^2 t'^2 + \tilde{C}^2} \frac{dt'}{t'} \quad (3.37)$$

$$= \frac{\pi \tilde{C}}{2} i. \quad (3.38)$$

The second terms in eq. (3.36) are calculated as

$$- \int_{z_0}^z dz' \sqrt{-m_\phi^2 + \tilde{C}^2/z'^2} = -\tilde{C} \sqrt{1 - z^2/z_{\text{pole}}^2} - \frac{\tilde{C}}{2} \ln \left[\frac{1 - \sqrt{1 - z^2/z_{\text{pole}}^2}}{1 + \sqrt{1 - z^2/z_{\text{pole}}^2}} \right] + \text{const.} \quad (3.39)$$

$$i \int_{z_{\text{pole}}}^z dz' \sqrt{m_\phi^2 - \tilde{C}^2/z'^2} = i\tilde{C} \sqrt{z^2/z_{\text{pole}}^2 - 1} + i\tilde{C} \arctan \left[\frac{1}{\sqrt{z^2/z_{\text{pole}}^2}} \right] + \text{const.}, \quad (3.40)$$

and $\int_{C_4} m(t') dt' = 0$ because $m(t) = 0$ at $t = t_{\text{pole}}$.

Now we can calculate eq. (3.30). First, it is easy to see that the integral of C_2 in eq. (3.30) is proportional to $t_0^{3p/2-1/2}$, so that it can be neglected in the limit of $t_0 \rightarrow 0$ for $p > 1/3$. Since we are interested in the radiation dominated era, where $p = 1/2$, this condition is satisfied.

Second, the integral of the contour C_3 has an oscillation term eq. (3.39) from $e^{i\theta}$. The time scale of this oscillation term ($\propto \tilde{C}$) is much larger than that of the prefactor in the integrand $\dot{v}\sqrt{m}$, so that the integral of C_3 contour is suppressed by the oscillation term. However, when t is sufficiently close to the pole ($|t - iz_{\text{pole}}| \lesssim z_{\text{pole}}/\tilde{C}^{2/3}$), the oscillation term becomes constant, and the integral gives a large contribution.

Finally, the integral of the contour C_5 and C_6 has a damping term eq. (3.40) from $e^{i\theta}$, so that the integral of $\int_{C_5+C_6} e^{i\theta}$ is highly suppressed by the additional exponential factor. However, when t is sufficiently close to the pole ($|t - iz_{\text{pole}}| \lesssim z_{\text{pole}}/\tilde{C}^{2/3}$), the damping term becomes constant, and the integral gives a large contribution.

Therefore, the integral at the vicinity of the pole gives the main contribution of eq. (3.30). We should calculate

$$\int_{|t-iz_{\text{pole}}| < z_{\text{pole}}/\tilde{C}^{2/3}} \dot{v}\sqrt{m}e^{i\theta} dt \tag{3.41}$$

$$\simeq \frac{2V}{\tilde{C}^{3/2}} e^{i\theta(t=iz_{\text{pole}})} (iz_{\text{pole}})^{3p/2-3/2} \int_{|t-iz_{\text{pole}}| < z_{\text{pole}}/\tilde{C}^{2/3}} \left(1 + \frac{t^2}{z_{\text{pole}}^2}\right)^{-7/4} dt. \tag{3.42}$$

where the integral should be taken on the contour $C_3 + C_4 + C_5$. The integral can be calculated as

$$\begin{aligned} & \int_{|t-iz_{\text{pole}}| < z_{\text{pole}}/\tilde{C}^{2/3}} \left(1 + \frac{t^2}{z_{\text{pole}}^2}\right)^{-7/4} dt \\ &= \frac{4}{3} \left(\frac{z_{\text{pole}}}{2}\right)^{7/4} \left(\frac{\tilde{C}^{2/3}}{z_{\text{pole}}}\right)^{3/4} e^{-7/8\pi i} \left(e^{3/8\pi i} - e^{-3/8\pi i}\right). \end{aligned} \tag{3.43}$$

We can calculate $\int \dot{v}\sqrt{m}e^{-i\theta} dt$ in the same way. Combining these results, we obtain the adiabatic invariant such as

$$I \simeq \frac{\tilde{C}^{3p+1}}{m_\phi^{3p-1}} \phi_0^2 t_0^{-3p} e^{-\pi p C} \tag{3.44}$$

where we assume $I(t_0) = 0$ and omit $\mathcal{O}(1)$ factors. Using $I \simeq m_\phi \varphi^2$ for $CH \ll m_\phi$, we obtain

$$\phi \equiv \left(\frac{t_0}{t}\right)^{3p/2} \varphi \simeq p^{3/2} C^{(3p+1)/2} \phi_0 \left(\frac{p}{tm_\phi}\right)^{3p/2} e^{-\pi p C/2}, \tag{3.45}$$

where we use $\tilde{C} \simeq pC$. All parameter dependences are consistent with the one derived by Linde. Thus we can solve the moduli problem when it has an effective mass term $CH(t)$ with $C \sim 30$ [22].

To sum up, the exponential factor comes from eq. (3.38) and the other factors should be calculated around the vicinity of the pole $t = iz_{\text{pole}}$. These facts allow us to easily estimate the resulting adiabatic invariant.

Finally, we rewrite the result in terms of the number density to entropy density ratio for later convenience:

$$\frac{n_\phi}{s} \equiv \frac{m_\phi \phi^2/2}{s} \Big|_{t \gg z_{\text{pole}}} \quad (3.46)$$

$$= \frac{45}{2\pi^2 g_{*s}} \left(\frac{4\pi^2 g_*}{90M_{\text{pl}}^2} \right)^{3/4} \times t_0^{3/2} I, \quad (3.47)$$

where we use $p = 1/2$, and g_{*s} and g_* are effective relativistic degrees of freedom for entropy and energy density, respectively.

3.2.2 Application to an axion model

Now we calculate the abundance in the case where m_ϕ depends on time such as the case of QCD axion, where $m_\phi(t) \equiv At^{pn/2}$ [see eq. (2.5)]. In this subsection, we assume $\tilde{m}(t) = CH(t)$, which is considered in refs. [40, 48]. This can be realized when we introduce a term like

$$\mathcal{L} \supset c_R^2 R M_{\text{pl}}^2 \cos\left(\frac{a}{f_a} - \theta_R\right), \quad (3.48)$$

where R is the Ricci scalar. In the radiation dominated era, this term gives $\tilde{m}(t) = CH(t)$ with $C = 1.6c_R\alpha_s M_{\text{pl}}/f_a$, where α_s is the fine-structure constant of $SU(3)_c$ gauge interaction.

As shown in the right panel in figure 1, the pole location is given by $t = e^{i\alpha} z_{\text{pole}}$, where

$$\alpha = \pi/(2 + pn) \quad (3.49)$$

$$z_{\text{pole}} = \left(\frac{\tilde{C}}{A}\right)^{2/(2+pn)} \quad (3.50)$$

$$\tilde{C} \equiv p\sqrt{3/2(1/p - 3/2) + C^2} \simeq pC. \quad (3.51)$$

In this case, $i\theta(t = e^{i\alpha} z_{\text{pole}})$ is calculated as

$$i\theta(t = e^{i\alpha} z_{\text{pole}}) = -\alpha pC + (\text{imaginary part}). \quad (3.52)$$

The integral of eq. (3.30) is again dominated by the contribution in the vicinity of the pole: $|t - e^{i\alpha} z_{\text{pole}}| < z_{\text{pole}}/(\tilde{C}\sqrt{2 + pn})^{2/3}$. Then we estimate the adiabatic invariant such as

$$I \simeq \frac{\tilde{C}^2 z_{\text{pole}}^{3p-1}}{pn+2} t_0^{-3p} \phi_0^2 e^{-2\alpha pC} \quad (3.53)$$

$$\simeq \frac{\tilde{C}^{2(pn+1+3p)/(2+pn)}}{pn+2} A^{2(1-3p)/(2+pn)} t_0^{-3p} \phi_0^2 e^{-2\alpha pC}. \quad (3.54)$$

Therefore, an effective mass of $\tilde{m}(t) = CH(t)$ suppresses the axion abundance efficiently when $C \gg 1$.

3.2.3 Application to a generic model

Finally, we calculate the abundance in the case of $m_\phi(t) = At^{pn/2}$ and $\tilde{m}(t) = C't^{-d}$, where d is a constant parameter. The following calculation should reproduce the previous result if we take $d = 1$ and $C' = Cp$.

In this case, $m^2(t)$ is given by

$$m^2(t) = A^2t^{pn} + \frac{3}{2} \left(\frac{1}{p} - \frac{3}{2} \right) H^2(t) + C'^2t^{-2d}. \quad (3.55)$$

We neglect the $H^2(t)$ term because we are interested in the change of adiabatic invariant at a time around $A^2t^{pn} \sim C'^2t^{-2d} \gg H^2(t)$. Note that t_0 cannot be taken to be 0 because C'^2t^{-2d} is larger than $H^2(t)$ only after a certain time. Thus we take, say, $t_0 \approx z_{\text{pole}}/10$, though our result is independent of this value for $t_0 \ll z_{\text{pole}}$. Here, z_{pole} is given by

$$z_{\text{pole}} \equiv \left(\frac{C'}{A} \right)^{2/(2d+pn)}. \quad (3.56)$$

The pole location is given by $t = e^{i\alpha}z_{\text{pole}}$, where

$$\alpha = \frac{\pi}{2d+pn}. \quad (3.57)$$

First, let us calculate the imaginary part of $\theta(t_{\text{pole}})$. The integral on the contours C_2 and C_3 is calculated as

$$\begin{aligned} \text{Im}[\theta(t_{\text{pole}})] &= \text{Im} \int_{C_2+C_3} m(t') dt' \\ &\simeq C' z_{\text{pole}}^{1-d} \text{Im} \left[\int_{t_0/z_{\text{pole}}}^{t_0/z_{\text{pole}} e^{i\alpha}} dt' t'^{-d} + e^{i(1-d)\alpha} \int_{t_0/z_{\text{pole}}}^1 dt' \sqrt{t'^{-2d} - t'^{pn}} \right] \\ &= \alpha C' z_{\text{pole}}^{1-d} D \frac{\sin(1-d)\alpha}{(1-d)\alpha}, \end{aligned} \quad (3.58)$$

where in the second line we define

$$D \equiv \left[\left(\frac{t_0}{z_{\text{pole}}} \right)^{1-d} + (1-d) \int_{t_0/z_{\text{pole}}}^1 dt' \sqrt{t'^{-2d} - t'^{pn}} \right]. \quad (3.59)$$

We can evaluate D numerically for given parameters and we find that it is almost independent of t_0/z_{pole} for $t_0/z_{\text{pole}} \ll 1$ and is about unity for $(2d+pn) \gg 1$.

The integral of eq. (3.30) is again dominated by the contribution in the vicinity of the pole:

$$|t - e^{i\alpha}z_{\text{pole}}| < z_{\text{pole}} \left(\frac{z_{\text{pole}}^{d-1}}{C' \sqrt{2d+pn}} \right)^{2/3}. \quad (3.60)$$

Then we estimate the adiabatic invariant such as

$$I \simeq \frac{C'^2}{2d+pn} z_{\text{pole}}^{3p-2d+1} t_0^{-3p} \phi_0^2 e^{-2\text{Im}[\theta(t_{\text{pole}})]}. \quad (3.61)$$

It is convenient to rewrite this result as the number density to entropy density ratio:

$$\frac{n_\phi}{s} \equiv \frac{m_\phi \phi^2 / 2}{s} \Big|_{t \gg z_{\text{pole}}} \tag{3.62}$$

$$= \frac{45}{2\pi^2 g_{*s}} \left(\frac{4\pi^2 g_*}{90 M_{\text{pl}}^2} \right)^{3/4} \frac{2C'^2}{n + 4d} z_{\text{pole}}^{5/2-2d} \phi_0^2 e^{-2\text{Im}[\theta(t_{\text{pole}})]} \tag{3.63}$$

where we use $p = 1/2$, and g_{*s} and g_* are effective relativistic degrees of freedom for entropy and energy density, respectively. Note that $\text{Im}[\theta(t_{\text{pole}})] \propto C' z_{\text{pole}}^{1-d} \simeq \tilde{m}/H(t = z_{\text{pole}}) \gg 1$.

4 QCD axion and adiabatic suppression mechanism

The QCD axion potential is protected by the PQ symmetry, and therefore, it is difficult to give the axion a time-dependent effective mass term, which is needed to realize the adiabatic suppression mechanism. In this section, we consider the cosmological history of the QCD axion or string axion that acquires an effective mass due to the Witten effect, using the result given in the previous section.

4.1 Axion dynamics with monopole DM

Here we consider a cosmological history of the axion in the presence of hidden monopoles of $U(1)_H$. We assume that the PQ symmetry is anomalous under the hidden $U(1)_H$, which then implies that the axion acquires an effective mass due to the Witten effect (see appendix A). In this section, we consider the case where $U(1)_H$ is not broken, and monopole is stable and can be DM [63].³

Suppose that monopoles are produced at a temperature of T_m ($T_m \gg \Lambda_{\text{QCD}}$) with an initial number density of $n_M(T_m)$. Its number density decreases with time due to the cosmic expansion, so that the ratio to the entropy density is constant:

$$Y_M \equiv \frac{n_M}{s} = \text{const.} \tag{4.1}$$

In the presence of monopoles, the Witten effect gives an effective mass of the axion [see eq. (A.19)] [32, 33]:

$$m_{a,M}^2(T) = 2\beta \frac{n_M(T)}{f_{a,H}}, \tag{4.2}$$

where β is a constant defined by eq. (A.17) or eq. (A.18). Here, the axion decay constant associated with $U(1)_H$ is denoted as $f_{a,H}$, which may be different from f_a by a rational number because of the difference of periodicity of the axion (or so-called the domain wall number).⁴ Specifically, they are related to each other by $f_{a,H} = (N_{\text{DW}}/N_{a,H})f_a$, where N_{DW} and $N_{a,H}$ are the domain wall numbers of the axion in terms of the QCD instanton

³A scenario where 't Hooft-Polyakov monopole accounts for DM was discussed in refs. [64, 65], where massive gauge bosons are another component of DM.

⁴ $f_{a,H}$ can be enhanced by many orders of magnitude with respect to f_a in the clockwork mechanism [66, 67].

effect and the Witten effect, respectively. This implies that the ratio between the effective axion mass and the Hubble parameter increases with time such as

$$\frac{m_{a,M}^2(T)}{H^2(T)} = 2\beta Y_M \frac{s}{H^2 f_{a,H}}, \quad (4.3)$$

which is approximately proportional to T^{-1} in the radiation dominated era. The ratio becomes unity at a temperature of $T = T_{\text{osc},1}$ and time $t = t_{\text{osc},1}$, which is given by

$$T_{\text{osc},1} \simeq Y_M \frac{8\beta M_{\text{Pl}}^2}{f_{a,H}} \quad (4.4)$$

$$\simeq 2 \times 10^9 \text{ GeV} \beta \left(\frac{m_M}{1 \text{ TeV}} \right)^{-1} \left(\frac{\Omega_M h^2}{0.12} \right) \left(\frac{f_{a,H}}{10^{16} \text{ GeV}} \right)^{-1}, \quad (4.5)$$

where Ω_M is the relic density of monopole:

$$\Omega_M h^2 \simeq \frac{m_M Y_M}{3.6 \text{ eV}}. \quad (4.6)$$

In the case of the 't Hooft-Polyakov monopole, β is given by eq. (A.18) ($\beta = \alpha_H / (32\pi^2 r_c f_{a,H})$), where α_H is the fine-structure constant of $U(1)_H$ and r_c is an electric screening scale of an electrically charged particle.⁵ We may take $r_c = m_W^{-1} = 1/(ev) \simeq 1/\alpha_H m_M$, where m_W is the mass of $SU(2)$ gauge bosons, e is the hidden gauge coupling constant, and v is the $SU(2)$ breaking scale. Then we obtain

$$T_{\text{osc},1} \simeq 65 \text{ GeV} \alpha_H^2 \left(\frac{\Omega_M h^2}{0.12} \right) \left(\frac{f_{a,H}}{10^{12} \text{ GeV}} \right)^{-2}. \quad (4.7)$$

In this paper, we require that the ratio becomes unity before the QCD phase transition, i.e., $T_{\text{osc},1} \gtrsim T_{\text{osc},0}$, which condition is rewritten as

$$f_a \lesssim 9 \times 10^{12} \text{ GeV} \alpha_H^{1.1} \left(\frac{N_{a,H}}{N_{\text{DW}}} \right)^{1.1} \left(\frac{\Omega_M h^2}{0.12} \right)^{0.55}, \quad (4.8)$$

for the case of the 't Hooft-Polyakov monopole. Note that f_a may be different from $f_{a,H}$ by a rational factor.

The Witten effect should be sufficiently small so that the axion VEV cancels the undesired strong CP phase at present. We can check it as follows:

$$\left. \frac{m_{a,M}^2}{m_a^2} \right|_{t_p} \simeq 10^{-29} \beta \left(\frac{\Omega_M h^2}{0.12} \right) \left(\frac{m_M}{1 \text{ TeV}} \right)^{-1} \left(\frac{f_a}{10^{16} \text{ GeV}} \right), \quad (4.9)$$

where the left hand side is evaluated at the present time t_p . Note that $\beta \ll 1$. Since this fraction is much smaller than 10^{-10} at present, the strong CP problem is solved even in the presence of monopole.

⁵The monopole may be a fundamental particle as considered in refs. [68–70]. In this case, we should use eq. (A.17) instead of eq. (A.18), which leads to a larger Witten effect.

4.2 Axion abundance

We consider a scenario where the PQ symmetry is broken before inflation. In this case, the axion stays at a certain phase during and after inflation. After inflation ends and monopoles are produced, the ratio of eq. (4.3) increases and reaches about unity at $T = T_{\text{osc},1}$. Then the axion starts to oscillate around the minimum (i.e., $\langle a \rangle = \theta$), which is determined by the hidden sector.

The axion starts to oscillate by the Witten effect before the QCD phase transition when eq. (4.8) is satisfied. The resulting axion energy density is given by $H^2(T_{\text{osc},1})\theta_{\text{ini},H}^2 f_{a,H}^2/2$ at the onset of oscillation, where $(\theta_{\text{ini},H} + \theta)$ is the initial misalignment angle of axion. This gives the initial value of adiabatic invariant:

$$I_0 = \frac{1}{2} H(T_{\text{osc},1}) \theta_{\text{ini},H}^2 f_{a,H}^2 \left(\frac{t_{\text{osc},1}}{t_0} \right)^{3/2}, \quad (4.10)$$

which represents the comoving number density of axion normalized at $t = t_0$. Note that we include the factor of $(t_{\text{osc},1}/t_0)^{3/2}$ because we normalize the adiabatic invariant at $t = t_0$.

Although the effective mass of the axion eq. (A.19) decreases with time, its comoving number density, which is an adiabatic invariant, is approximately conserved after the onset of oscillation. In particular, the effect of the QCD instanton around the time of QCD phase transition does not affect the number density of axion so much. The resulting amount of induced axion at the QCD phase transition is given by eq. (3.61) with $d = 3p/2$:

$$\Delta I \simeq \frac{2C'^2}{n+3} z_{\text{pole}} t_0^{-3/2} \phi_0^2 e^{-2\alpha C' z_{\text{pole}}^{1/4} D}. \quad (4.11)$$

where $D (\simeq 1)$ is given by eq. (3.59) and

$$\phi_0 = \theta f_{a,H} \quad (4.12)$$

$$C' = H(t_{\text{osc},1}) t_{\text{osc},1}^{3/4} \quad (4.13)$$

$$z_{\text{pole}} = \left(\frac{H^2(t_{\text{osc},1})}{m_a^2(t_{\text{osc},1})} \right)^{2/(n+3)} t_{\text{osc},1} \quad (4.14)$$

$$\alpha = \frac{2\pi}{n+3}. \quad (4.15)$$

Here and hereafter we use $p = 1/2$. The induced ΔI is smaller than I_0 when

$$\frac{2C'^2 z_{\text{pole}}^{1/2}}{n+3} \left(\frac{z_{\text{pole}}}{t_{\text{osc},1}} \right)^{1/2} e^{-2\alpha C' z_{\text{pole}}^{1/4}} \lesssim 1 \quad (4.16)$$

$$\Leftrightarrow \frac{H^2(t_{\text{osc},1})}{m_a^2(t_{\text{osc},1})} \gtrsim \left[\frac{1}{\pi} \ln \left(\frac{H^2(t_{\text{osc},1})}{m_a^2(t_{\text{osc},1})} \right) \right]^{2(n+3)}, \quad (4.17)$$

where we use $\alpha = 2\pi/(n+3)$ and assume $\theta_{\text{ini},H} \sim \theta (= \mathcal{O}(1))$. This inequality is usually satisfied. However, we should note that we assume $H^2(t_{\text{osc},1}) \gg m_a^2(t_{\text{osc},1})$ in the above calculation. Therefore, we conclude that the resulting axion abundance is determined by I_0 when $H(t_{\text{osc},1}) \gtrsim m_a(t_{\text{osc},1})$.

When the adiabatic suppression mechanism works, the present axion number density is determined by I_0 and is given as

$$\frac{n_a}{s} \simeq \frac{H_{\text{osc},1} \theta_{\text{ini},H}^2 f_{a,H}^2 / 2}{s(T_{\text{osc},1})} \quad (4.18)$$

$$\simeq \sqrt{\frac{45}{32\pi^2 g_*}} \frac{\theta_{\text{ini},H}^2 f_{a,H}^2}{T_{\text{osc},1} M_{\text{Pl}}}. \quad (4.19)$$

The axion density parameter is thus given by

$$\Omega_a h^2 \simeq 2 \times 10^{-14} \theta_{\text{ini},H}^2 \frac{f_{a,H}}{T_{\text{osc},1}} \left(\frac{N_{\text{DW}}}{N_{a,H}} \right) \quad (4.20)$$

$$\simeq 3 \times 10^{-4} \frac{\theta_{\text{ini},H}^2}{\alpha_H^2} \left(\frac{N_{\text{DW}}}{N_{a,H}} \right) \left(\frac{0.12}{\Omega_M h^2} \right) \left(\frac{f_{a,H}}{10^{12} \text{ GeV}} \right)^3. \quad (4.21)$$

Since the total abundance should be smaller than the observed DM abundance, we obtain an upper bound on the axion decay constant:

$$f_{a,H} \lesssim 4 \times 10^{12} \text{ GeV} \left(\frac{\alpha_H}{\theta_{\text{ini},H}} \right)^{2/3} \left(\frac{N_{a,H}}{N_{\text{DW}}} \right)^{1/3}, \quad (4.22)$$

where we used $(\Omega_a h^2)(\Omega_M h^2) \leq (\Omega_a h^2 + \Omega_M h^2)^2 / 4 \lesssim 0.12^2 / 4$.

We show contours of the axion and monopole abundance in figure 2. We take $f_a = 1 \times 10^{12} \text{ GeV}$ (green curve), $2 \times 10^{12} \text{ GeV}$ (blue curve), and $4 \times 10^{12} \text{ GeV}$ (red curve). We assume $\theta_{\text{ini},H} = 1$, $\alpha_H = 1$, and $N_{\text{DW}} = N_{a,H}$. The observed DM abundance can be explained at the intersection point between the contour curve and the diagonal magenta dashed line for each value of f_a . Note that the adiabatic suppression mechanism works for the case of $T_{\text{osc},1} \gtrsim T_{\text{osc},0}$, so that we should restrict ourselves to the case of

$$\Omega_a h^2 \lesssim 0.22 \theta_{\text{ini},H}^2 \alpha_H^{1,3} \left(\frac{N_{a,H}}{N_{\text{DW}}} \right)^{-0.69} \left(\frac{\Omega_M h^2}{0.12} \right)^{0.65}. \quad (4.23)$$

This is shown in figure 2 as the unshaded region. The adiabatic suppression mechanism does not work in the light-gray shaded region, in which case the coherent oscillation of axion is induced at the QCD phase transition and the resulting axion abundance is given by the sum of eqs. (2.8) and (4.21).

Finally, we comment on the excited states of the monopole (i.e., Julia-Zee dyons). At the monopole production process, some excited states of a monopole with nonzero hidden-electric charges may be produced [71]. This is considered to be the case especially if the axion field takes different values over the space, e.g., around the axionic cosmic string. In the scenario considered in this subsection, however, the effective theta parameter of the hidden $U(1)_H$ (i.e., axion VEV) is constant in the whole Universe at the monopole production process, so that we expect that only monopoles are produced at that time. Thus the axion acquires the space-independent quadratic mass term.

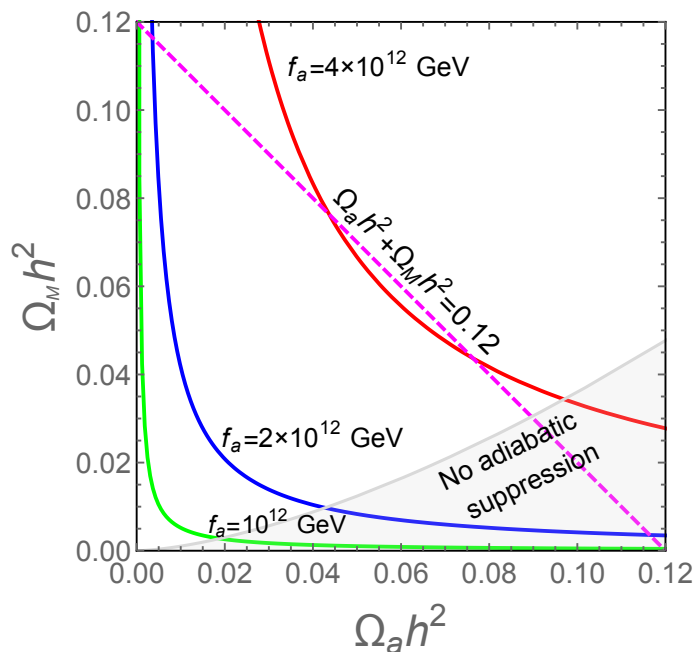


Figure 2. Relation between the axion and monopole abundances. We take $f_a = 1 \times 10^{12} \text{ GeV}^3$ (green curve), $2 \times 10^{12} \text{ GeV}$ (blue curve), and $4 \times 10^{12} \text{ GeV}$ (red curve). We assume $\theta_{\text{ini},H} = 1$, $\alpha_H = 1$, and $N_{\text{DW}} = N_{a,H}$. The diagonal (magenta) dashed line represents the observed DM abundance. In the light-gray shaded region, the adiabatic suppression does not work and the results should not be trusted.

4.3 Isocurvature problem

When the PQ symmetry is broken before inflation, the axion acquires quantum fluctuations during inflation [72]. This is quantitatively written in terms of fluctuations of initial misalignment angle $\delta\theta_{\text{ini},H}$ such as

$$\delta\theta_{\text{ini},H} \approx \frac{H_{\text{inf}}}{2\pi f_{a,H}}, \tag{4.24}$$

where H_{inf} is the Hubble parameter during inflation. This results in isocurvature modes in density perturbations whose amplitude is given by

$$P_{\text{iso}} \simeq \left(\frac{\Omega_a}{\Omega_{\text{DM}}} \right)^2 \left(\frac{H_{\text{inf}}}{\pi f_{a,H} \theta_{\text{ini},H}} \right)^2. \tag{4.25}$$

Since the CMB temperature anisotropies are predominantly adiabatic, the Planck observation puts the constraint on the amplitude of isocurvature perturbations:

$$\frac{P_{\text{iso}}}{P_{\text{ad}}} \lesssim 0.037, \tag{4.26}$$

where the amplitude of adiabatic perturbations is measured as $P_{\text{ad}} \simeq 2.2 \times 10^{-9}$ [73]. As a result, the axion decay constant should satisfy

$$f_{a,H} \gtrsim 3.4 \times 10^4 H_{\text{inf}} \frac{\Omega_a}{\Omega_{\text{DM}}} \frac{1}{\theta_{\text{ini},H}}, \tag{4.27}$$

which can be rewritten by eq. (4.21) such as

$$H_{\text{inf}} \lesssim 1.2 \times 10^{10} \text{ GeV} \alpha_H^2 \theta_{\text{ini,H}}^{-1} \left(\frac{N_{a,H}}{N_{\text{DW}}} \right) \left(\frac{\Omega_M h^2}{0.12} \right) \left(\frac{f_{a,H}}{10^{12} \text{ GeV}} \right)^{-2}. \quad (4.28)$$

One can see that the isocurvature constraint on the inflation scale is greatly relaxed for given f_a and the initial misalignment [31]. For instance, in the ordinary scenario without the Witten effect, the upper bound on H_{inf} is of order 10^7 GeV for $f_a = 10^{12}$ GeV and $\theta = O(1)$. This is because the axion abundance is suppressed by the Witten effect and it can be smaller than the observed DM abundance without fine-tuning the initial misalignment angle. The hidden monopoles are a prime candidate for the major DM component. Note that, in this scenario, the perturbation of axion energy density, $\delta\Omega_a/\Omega_a$, is not suppressed, and therefore, this effect cannot be mimicked by making the initial displacement smaller since it would enhance $\delta\Omega_a/\Omega_a$.

4.4 Constraints on monopole DM

When the hidden $U(1)_H$ is unbroken, the monopole can be DM [63, 64].⁶ Since monopoles interact with hidden photons, they are interacting massive DM. The cross section is calculated as [65]

$$\sigma_T = \frac{16\pi\alpha_M^2}{m_M^2 v^4} \log \left(1 + \frac{m_M^4 v^4}{8\pi\alpha_M^2 \rho_M} \right) \quad (4.29)$$

$$\simeq 0.4 \text{ cm}^2/\text{g} m_M \alpha_M^2 \left(\frac{v}{10 \text{ km/s}} \right)^{-4} \left(\frac{m_M}{1 \text{ PeV}} \right)^{-3} \quad (4.30)$$

for monopole DM, where $\alpha_M \equiv g^2/(4\pi)$ ($= 1/\alpha_H$) is the fine-structure constant for the magnetic charge and we take the log factor as ≈ 40 in the second line.

There are several constraints on the self-scattering cross section depending on the DM velocity v . The Bullet cluster gives the upper bound such as $\sigma_T/m_M < 1.25 \text{ cm}^2/\text{g}$ for $v \sim 1000 \text{ km/s}$ [74–76]. The most stringent constraint comes from the ellipticity of DM halo, where observations indicate that DM halos are somewhat elliptical. Since the self-interaction of DM results in a spherical DM halo, the cross section is restricted above by this consideration such as $\sigma_T/m_M \lesssim 0.1 - 1 \text{ cm}^2/\text{g}$ for $v \sim 200 \text{ km/s}$ [77, 78].

The self-scattering of DM is well motivated to solve astrophysical problems: core-cusp problem and too-big-to-fail problem [79]. These can be solved when $\sigma_T/m_M < 0.1 - 10 \text{ cm}^2/\text{g}$ for $v \sim 10 - 30 \text{ km/s}$ [80]. Also, the recent observation of Abel 3827 indicates $\sigma_T/m_M = 1.5 \text{ cm}^2/\text{g}$, which also motivates us to consider the self-interacting monopole DM [81] (see also ref. [82]). These can be addressed when the mass of monopole is of order PeV scale for $\alpha_H = 1$.

So far we have not specified the nature of the monopole or the mechanism to generate it. Here we briefly comment on them for the case of 't Hooft-Polyakov monopole

⁶When we consider 't Hooft-Polyakov monopole originated from the SSB of $SU(2)_H$ to $U(1)_H$, there are massive gauge bosons as well as monopoles. This theory has been investigated in refs. [64, 65], where they found that the massive gauge bosons are a dominant component of DM.

originated from the spontaneous symmetry breaking (SSB) of $SU(2)_H$ to $U(1)_H$, following refs. [64, 65], where both monopoles and heavy gauge bosons are stable and are DM candidates. Monopoles are produced by the Kibble-Zurek mechanism at the SSB of $SU(2)$ gauge symmetry and their abundance strongly depends on the critical exponents associated with the phase transition. The produced monopoles experience the Brownian motion due to collisions with heavy-gauge bosons while the attractive Coulomb force between the monopoles and anti-monopoles makes them drift towards each other. Eventually the monopoles and anti-monopoles may be captured by the attractive force and annihilate into heavy-gauge bosons. The resulting monopole abundance becomes comparable to that of heavy gauge bosons for the case of the hidden gauge coupling strength $\alpha_H = \mathcal{O}(0.1 - 1)$. The total abundance of the monopoles and heavy gauge bosons can explain the observed DM abundance for the case where the SSB of $SU(2)_H$ occurs at the energy scale of order 10^5 GeV. This is actually a reasonable parameter that can also address the astrophysical problems such as the core-cusp problem and too-big-to-fail problem.

5 String axion with monopole annihilation

In this section, we consider the case that the monopoles disappear at a temperature of T_{ann} after the QCD phase transition but before the BBN epoch (i.e., $1 \text{ MeV} \lesssim T_{\text{ann}} < \Lambda_{\text{QCD}}$). This can be realized by the SSB of $U(1)_H$ at the temperature of T_{ann} because each monopole and anti-monopole pair is attached by a cosmic string associated with the SSB of $U(1)_H$ and annihilate with each other [83]. In this case, the Witten effect turns off and the axion becomes massless at T_{ann} . Since there is no monopole in the present Universe,⁷ we are interested in the case that the axion is the dominant component of DM.

When we require that the monopoles do not dominate the Universe before they disappear, we should satisfy

$$\left. \frac{\rho_M}{\rho_{\text{tot}}} \right|_{T=T_{\text{ann}}} \lesssim 1 \tag{5.1}$$

where

$$\left. \frac{\rho_M}{\rho_{\text{tot}}} \right|_{T=T_{\text{ann}}} \simeq \left(\frac{\Omega_M h^2}{2 \times 10^7} \right) \left(\frac{T_{\text{ann}}}{100 \text{ MeV}} \right)^{-1}, \tag{5.2}$$

where we use $g_* \approx g_{s*}$ and define $\Omega_M h^2$ via the relation of eq. (4.6). Or, we should satisfy

$$\left(\frac{T_{\text{osc},1}}{T_{\text{ann}}} \right) \lesssim 1.2 \times 10^4 \alpha_H^2 \left(\frac{f_a}{10^{16} \text{ GeV}} \right)^{-2}. \tag{5.3}$$

where we assume 't Hooft-Polyakov monopole and use $\beta = \beta|_{\text{with cutoff}}$ with $r_c = 1/(\alpha_H m_M)$ [see eq. (A.18)].

On the other hand, we can also consider the case that the monopoles dominate the Universe before they annihilate. In this case, the annihilation of monopole generates entropy

⁷We assume that the massive hidden photon decays into the SM particles through a kinetic mixing between the hidden $U(1)_H$ and hypercharge soon after the monopole annihilation.

and dilute SM plasma. We define the dilution factor Δ by

$$\Delta \equiv \frac{s_{\text{after}}}{s_{\text{before}}} = \text{Max} \left[\frac{\rho_M}{\rho_{\text{tot}}} \Big|_{T=T_{\text{ann}}}, 1 \right], \quad (5.4)$$

When the dilution factor is larger than unity, the axion abundance is diluted by a factor of Δ^{-1} at $T = T_{\text{ann}}$.

In the case that monopoles disappear after the QCD phase transition, the adiabatic suppression mechanism works around the QCD phase transition, so that the axion abundance is determined by eq. (4.21) with a dilution factor:

$$\Omega_a h^2 \simeq 0.4 \Delta^{-1} \frac{\theta_{\text{ini,H}}^2}{\alpha_H^2} \left(\frac{N_{\text{DW}}}{N_{a,H}} \right) \left(\frac{10^8}{\Omega_M h^2} \right) \left(\frac{f_a}{10^{16} \text{ GeV}} \right)^3. \quad (5.5)$$

Note that in this case, the monopole abundance can be larger than the DM abundance so that the condition of eq. (4.22) can be avoided. In fact, the axion with a decay constant of order the GUT scale can be consistent with the observed DM abundance without a fine-tuning of axion misalignment angle.

The isocurvature constraint is the same as eq. (4.27) with $\Omega_a = \Omega_{\text{DM}}$. It can be rewritten as

$$H_{\text{inf}} \lesssim 3.0 \times 10^{12} \text{ GeV} \theta_{\text{ini,H}} \left(\frac{f_{a,H}}{10^{16} \text{ GeV}} \right). \quad (5.6)$$

Finally, let us check that the annihilation products of the monopoles give only a negligible effect on cosmology. The annihilation products are massive hidden photons, and its energy density would dominate the Universe unless it decays into the SM particles. We assume a nonzero kinetic mixing between the hidden $U(1)_H$ and hypercharge to make the massive hidden photon decay into the SM particles. Denoting the kinetic mixing angle as χ , one can estimate the lifetime of the hidden photon as

$$\tau_{\gamma'} \sim 10^{-2} \text{ sec} \left(\frac{\chi}{10^{-9}} \right)^{-2} \left(\frac{m_{\gamma'}}{100 \text{ MeV}} \right)^{-1}. \quad (5.7)$$

Supernova cooling argument puts a significant constraint on the kinetic mixing such as $\chi \lesssim 4 \times 10^{-9}$ for $m_{\gamma'} \lesssim 5 \times 10 \text{ GeV}$ [84, 85]. The beam dump experiment E137 puts a weaker constraint but is relevant for a slightly heavier hidden photon: $\chi \lesssim 10^{-7}$ for $m_{\gamma'} \lesssim 4 \times 10^2 \text{ GeV}$ [85, 86]. The lifetime of the hidden photon can be much shorter than 1 sec while satisfying these constraints, so that they can decay well before the BBN epoch and do not give any significant effect in the subsequent cosmology. Since there should be an adjoint Higgs field ϕ_a in the hidden sector to break $SU(2)_H$ down to $U(1)_H$, the kinetic mixing originates from the following higher-dimensional operator [87, 88]:

$$\frac{\phi_a}{M} G_a^{\mu\nu} F_{\mu\nu} \supset \frac{v}{M} G_3^{\mu\nu} F_{\mu\nu}, \quad (5.8)$$

where $G_a^{\mu\nu}$ and $F_{\mu\nu}$ are $SU(2)_H$ and $U(1)_Y$ gauge fields, respectively. The parameter M is a cutoff scale. This operator leads to the kinetic mixing of v/M , which is of order 10^{-9} for $v = 10^5 \text{ GeV}$ and $M = 10^{14} \text{ GeV}$.

6 Discussion and conclusions

The adiabatic suppression mechanism is a novel mechanism to solve the moduli problem [22]. For the mechanism to work, the moduli must have a time-dependent mass much larger than the Hubble parameter. In this case, the moduli follow the time-dependent minimum adiabatically, and thus the resultant oscillation amplitude is exponentially suppressed if one neglects the initial abundance generated by the effect of the inflaton oscillations [29, 30].

In this paper, we have provided an analytic method to calculate the resultant adiabatic invariant which describes the comoving number density of homogeneous scalar field in the adiabatic suppression mechanism. In particular, we have seen that the exponential suppression comes from a pole of the mass of the scalar field in the complex plane. The parameter dependence of our results are consistent with the result of ref. [22], and moreover, our method can be used to calculate the number density in more generic models.

Then we apply the result to axions. We have considered a model in which the axion obtains an effective mass due to the Witten effect in the presence of monopoles, and as a result, it starts to oscillate much before the epoch of QCD phase transition. This implies that the energy density of axion coherent oscillation can be much smaller than the case without the early oscillation. We have found that the axion energy density can be consistent with the observed DM abundance even in the case that the axion decay constant is as large as the GUT scale. Due to the early oscillation, both the axion energy density and its fluctuations are suppressed, so that the isocurvature problem can be ameliorated.

If $U(1)_H$ symmetry is not broken, the monopole is stable and also contribute to DM. Since monopoles interact with themselves via $U(1)_H$ gauge interaction, they have a sizable velocity-dependent self-interaction cross section. Such a self-interacting DM may be observationally preferred since it can relax the tension between the observed DM density profile and the prediction of Λ CDM model. Note however that, if stable, the monopole abundance has to be smaller than or equal to the observed DM density, so that the Witten effect on the axion is limited in this case. Thus we also considered the case that $U(1)_H$ symmetry is spontaneously broken at an intermediate scale so that the monopoles annihilate due to the tension of cosmic strings associated with $U(1)_H$ breaking. In this case, the monopole abundance in the early Universe is not related to the DM abundance, and so, the Witten effect can be much more significant. In particular, the axion density can be sufficiently suppressed even if its decay constant is as large as GUT scale.

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A The Witten effect on the QCD axion

In this appendix we explain the Witten effect on the axion and calculate its effective mass in the presence of monopole.

We introduce a hidden Abelian gauge symmetry $U(1)_H$ with a monopole with mass of m_g and a magnetic charge g . The Lagrangian of gauge fields is written as

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{e^2\theta}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad (\text{A.1})$$

where $\tilde{F}^{\mu\nu} \equiv 1/2\epsilon^{\mu\nu\sigma\rho}F_{\sigma\rho}$ and e is the gauge coupling constant of the hidden gauge theory. In the presence of monopoles, the Maxwell's equations are given by

$$\partial_\mu \tilde{F}^{\mu\nu} = j_M^\nu \quad (\text{A.2})$$

$$\partial_\mu \left[F^{\mu\nu} + \frac{e^2\theta}{8\pi^2}\tilde{F}^{\mu\nu} \right] = 0, \quad (\text{A.3})$$

where j_M^ν is a monopole current. In the presence of monopoles, i.e., in the case of $j_M^\nu \neq 0$, there is no electromagnetic potential A_μ defined in the whole region. We can define it only in those regions where $j_M^\nu = 0$. The topology of these regions is nontrivial and in fact A_μ is singular in the presence of monopoles. This implies that the θ -term in the Abelian gauge theory cannot be eliminated by integrating by parts in the presence of monopoles. As we see below, the θ -term is physical and leads to an effect known as the Witten effect [32].

The Gauss's laws are now modified as

$$\nabla \cdot \mathbf{B} = j_M^0, \quad (\text{A.4})$$

$$\nabla \cdot \mathbf{E} + \frac{e^2}{8\pi^2}\nabla \cdot (\theta\mathbf{B}) = 0, \quad (\text{A.5})$$

where $E_i \equiv F_{0i}$ and $B_i \equiv -1/2\epsilon_{ijk}F^{jk}$. Let us emphasize that \mathbf{E} and \mathbf{B} are of the hidden electric and magnetic fields. We rewrite j_M^0 as

$$j_M^0 = g(n_{M^+} - n_{M^-}), \quad (\text{A.6})$$

where n_{M^+} (n_{M^-}) is the number density of (anti)monopoles. Then, eq. (A.5) implies that the monopole also carries an electric charge q , which is proportional to θ :

$$q = -\frac{e^2g}{8\pi^2}\theta. \quad (\text{A.7})$$

This implies that the usual charge quantization condition, $q/e = n$, is extended to

$$\frac{q}{e} + \frac{eg}{8\pi^2}\theta = n, \quad (\text{A.8})$$

where n is an integer that is nonzero for dyons. Therefore, the monopole is a dyon in a theory with a nonzero value of θ , which is known as the Witten effect. Note that when we replace $\theta \rightarrow \theta + 2\pi$, the value of n changes from n to $n + 1$ due to the Dirac's quantization

condition: $g = 4\pi/e$.⁸ This means that the periodicity of θ is modified in the presence of monopole (dyon) such as $\theta \rightarrow \theta + 2\pi$ and $n \rightarrow n + 1$.

Now we assume that PQ symmetry is anomalous in terms of $U(1)_H$ and the QCD axion a couples with $U(1)_H$. Thus we promote the theta angle of the hidden $U(1)_H$ theory to an axion by the PQ mechanism such as $\theta \rightarrow a/f_{a,H} - \theta$ [33]:

$$\mathcal{L}_\theta = -\frac{e^2}{32\pi^2} \left(\frac{a}{f_{a,H}} - \theta \right) F_{\mu\nu} \tilde{F}^{\mu\nu}. \quad (\text{A.9})$$

where $f_{a,H}$ is the axion decay constant associated with $U(1)_H$. Suppose that a and $a + 2\pi n f_{a,H}$ are physically identical where n is a positive integer. Then, the smallest n is called the domain wall number, $N_{a,H}$. Let us first consider a monopole located at the origin of a coordinate and calculate the effect of the axion on the energy density of electromagnetic field. The Maxwell's equation of eq. (A.4) can be of course solved such as

$$\mathbf{B} = \frac{g}{4\pi} \frac{\hat{r}}{r^2}, \quad (\text{A.10})$$

where \hat{r} is a normal vector along the radial direction. Together with eq. (A.5), this implies that the electric field is given by

$$\mathbf{E} = -\frac{e}{16\pi^2} \frac{eg}{2\pi} \left(\frac{a(r)}{f_{a,H}} - \theta \right) \frac{\hat{r}}{r^2}, \quad (\text{A.11})$$

where $a(r=0) = \theta f_{a,H}$. This means that a nonzero axion field value carries a large cost in the electrostatic field energy:

$$V = \int d^3r \left[\frac{1}{2} (\nabla a)^2 + \frac{1}{2} \mathbf{E}^2 \right], \quad (\text{A.12})$$

$$= \int 4\pi r^2 dr \left[\frac{1}{2} \left(\frac{\partial a}{\partial r} \right)^2 + \frac{e^2}{512\pi^4} \left(\frac{a}{f_{a,H}} - \theta \right)^2 \frac{1}{r^4} \right], \quad (\text{A.13})$$

where we use $eg = 2\pi$ in the second line.

We obtain the following axion configuration that minimizes the total energy of eq. (A.13):

$$a(r) = a_0 \exp(-r_0/r) + \theta f_{a,H}, \quad (\text{A.14})$$

$$r_0 = \frac{e}{16\pi^2 f_{a,H}}, \quad (\text{A.15})$$

where $(a_0 + \theta f_{a,H})$ denotes the asymptotic field value of the axion. The total energy is then given by

$$V_M = \beta f_{a,H} \left(\frac{a}{f_{a,H}} - \theta \right)^2, \quad (\text{A.16})$$

$$\beta|_{\text{w/o cutoff}} = \frac{e}{8\pi}, \quad (\text{A.17})$$

⁸In our convention, half-integer electric charges are allowed.

where we integrate eq. (A.13) from $r = 0$ to ∞ . However, the integral of eq. (A.13) may have to be taken in the interval of $[r_c, \infty)$ where r_c is a cutoff scale due to, e.g., an electric screening effect of an electrically charged particle. This is actually true for a $U(1)_H$ gauge theory with a 't Hooft-Polyakov monopole, where there are charged massive gauge bosons after the SSB of $SU(2)$ gauge symmetry. In that case, r_c is given by the inverse of the mass of the charged gauge bosons. Then the parameter β is given by

$$\beta|_{\text{with cutoff}} = \frac{\alpha_H}{32\pi^2} \frac{1}{r_c f_{a,H}}, \tag{A.18}$$

where $\alpha_H \equiv e^2/4\pi$.

Note that the resulting energy eq. (A.16) is positive whether the monopole charge is positive or negative. As a result, the energy density of the axion ground state in a plasma with monopoles and antimonopoles is given by $U = n_M V_0$, where $n_M = n_{M+} + n_{M-}$.⁹ This implies that the axion obtains an effective mass of

$$m_{a,M}^2(T) = 2\beta \frac{n_M(T)}{f_{a,H}}, \tag{A.19}$$

and has a VEV of θ at the minimum of the potential in a plasma with monopoles due to the Witten effect. Here we explicitly write the temperature dependence of n_M due to, say, the expansion of the Universe.

Note that the axion VEV is determined such that the electric charge of monopole (dyon) is absent. As we can see from eq. (A.8), the periodicity of θ is absent if we fix the value of n . This may imply that the axion does not have periodic potential but has a mass term of eq. (A.16) due to the Witten effect. However, monopoles (dyons) can have excited states with electric charges equal to or more than unity, which are known as Julia-Zee dyons [71]. Such excited states can be produced at the monopole production process if, e.g., the PQ symmetry is broken after inflation and cosmic strings form at the SSB. In this case, the effective θ parameter (or axion VEV) changes $2\pi N_{a,H}$ around the cosmic strings, where $N_{a,H}$ is the domain wall number. Then, at the monopole production process the Julia-Zee dyons with charge n ($\leq N_{a,H} - 1$) may form in the domain of $\theta \in (2\pi n - \pi, 2\pi n + \pi)$. As a result, the domain wall may form at the boundary of nearby domains due to the Witten effect. In this paper, we do not consider this case and focus on the case where the PQ symmetry is broken before inflation.

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⁹We define n_M as the sum of the number densities of monopoles and anti-monopoles, so that the axion mass squared is different from the one in ref. [33] by a factor two.

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