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A method to find $\mathcal{N} = 1$ AdS₄ vacua in type IIB

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ABSTRACT: In this paper, we are looking for $\mathcal{N} = 1$, AdS₄ sourceless vacua in type IIB. While several examples exist in type IIA, there exists only one example of such vacua in type IIB. Thanks to the framework of generalized geometry we were able to devise a semi-algorithmical method to look for sourceless vacua. We present this method, which can easily be generalized to more complex cases, and give two new vacua in type IIB.

KEYWORDS: Flux compactifications, Superstring Vacua

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Introduction. Compactification to 4-dimensional anti-de Sitter (AdS_4) are of relevance to several aspects of string theory. In particular, they are central in the $\text{CFT}_3/\text{AdS}_4$ correspondence. They can also be a first step toward obtaining a de Sitter vacuum if one devise a way to break supersymmetry in a controlled way.

In type IIA, several AdS_4 vacua have been found without [1–5] or with [6–14] sources (this is a non exhaustive list of examples). On the contrary, in type IIB, there have been far less studies. Some results have been found with sources [15–18] but only one example without sources [3] (even if the solution is singular in the compactified description). It is to remedy to this state of affairs that we looked for more sourceless vacua in type IIB. This type of vacua also presents two advantages. The first one is, as we already mentioned, their use in the $\text{CFT}_3/\text{AdS}_4$ correspondence. The second one is the validity of such vacua. Indeed, in most known examples with sources, the sources are smeared and one can ask if this assumption is well-founded. Getting rid of the sources also gets rid of this problem.

In order to find sourceless vacua, we use the pure spinors formalism developed in [19–21]. This permits to obtain linear algebraic equations for the SUSY equations. We are left, thanks to the integrability theorem [10, 22, 23], with the Bianchi identities which are quadratic and differential. Since these are not solvable in all generality, one has to devise a way to solve them. Taking inspiration from [24], where parts of the quadratic equations were in fact linear and permitted to solve the whole system of quadratic equations, we put in place a semi-algorithmical method to solve the equations. We also had to take care of

the differential part which was absent from [24]. This method can be easily generalized to all type of problems with the same characteristics. Thanks to it, we were able to recover an example of the known sourceless vacuum [3] and discover two new vacua which are a priori sourceless. A more careful study shows that these solutions are singular and we give for one of these examples a possible interpretation in terms of sources.

This paper is organized as follows. In section 1, we present the supersymmetry conditions in the framework of generalized geometry applied to our specific case. In section 2, we expose the method to solve the quadratic equations. Finally in section 3, we give three examples of vacua, one of them already known that we recover thanks to our method and two new ones.

1 The supersymmetry conditions

We are interested in $\mathcal{N} = 1$ SUSY AdS₄ vacua in type IIB theories. That is to say that the manifold the theory lives on is of the type:

$$ds^2 = e^{2A} ds_{(4)}^2 + ds_{(6)}^2, \tag{1.1}$$

with A the warp factor. As discussed in [15, 25] such solutions are only possible when the compactification manifold have SU(2) structure group. Let us recall that a manifold is said to be of SU(2) structure if it admits a complex one form z , a real and a holomorphic two-form, j and ω , that are globally defined and satisfy

$$z_L \bar{z} = 2, \quad z_L z = \bar{z}_L \bar{z} = 0, \tag{1.2a}$$

$$j \wedge \omega = 0, \tag{1.2b}$$

$$z_L j = z_L \omega = 0, \tag{1.2c}$$

$$j \wedge j = \frac{1}{2} \omega \wedge \bar{\omega}. \tag{1.2d}$$

In order to study $\mathcal{N} = 1$ vacua with non trivial fluxes, it is convenient to use the language of Generalized Complex Geometry [26, 27]. We will give here a lightning review restricted to our specific case, for some more details, see for example [16, 24] and references therein.

The idea is to express the ten-dimensional supersymmetry variations as differential equations on a pair of polyforms defined on the internal manifold. In our case they are

$$\Phi_- = -\frac{e^A}{8} z \wedge (k_{\perp} e^{-ij} + i k_{\parallel} \omega), \tag{1.3}$$

$$\Phi_+ = \frac{e^A e^{i\theta}}{8} e^{z\bar{z}/2} (k_{\parallel} e^{-ij} - i k_{\perp} \omega), \tag{1.4}$$

where z , j and ω are the forms defining the SU(2) structure, A the warp factor and θ a free parameter. The parameters k_{\parallel} and k_{\perp} ($k_{\parallel}^2 + k_{\perp}^2 = 1$) are related to the choice of structure on the internal manifold. When $k_{\parallel} = 0$ and $k_{\perp} = 1$ the structure is strict SU(2), while the general case where both k_{\parallel} and k_{\perp} are non-zero is often referred to as dynamical SU(2)

structure.¹ When k_{\parallel} and k_{\perp} are non zero and constant, we speak of intermediate SU(2) structure rather than dynamical SU(2) structure [28].

As shown in [20], for type IIB compactifications to AdS₄ the ten-dimensional supersymmetry variations are equivalent to the following set of equations on the pure spinors Φ_{\pm}

$$(d - H \wedge)(e^{2A-\phi} \Phi_-) = -2\mu e^{A-\phi} \text{Re } \Phi_+, \tag{1.5a}$$

$$(d - H \wedge)(e^{A-\phi} \text{Re } \Phi_+) = 0, \tag{1.5b}$$

$$(d - H \wedge)(e^{3A-\phi} \text{Im } \Phi_+) = -3e^{2A-\phi} \text{Im}(\bar{\mu} \Phi_-) - \frac{1}{8} e^{4A} * \lambda(F), \tag{1.5c}$$

where ϕ is the dilaton and F is the sum of the RR field strength on M , $F = F_1 + F_3 + F_5$ and where λ acts on a form as the transposition of all indices

$$\lambda(\omega_p) = (-)^{\lfloor p/2 \rfloor} \omega_p. \tag{1.6}$$

The ten-dimensional fluxes are defined in terms of F by

$$F_{(10)} = \text{vol}_4 \wedge \lambda(*F) + F. \tag{1.7}$$

The complex number μ determines the size of the AdS₄ cosmological constant

$$\Lambda = -3|\mu|^2. \tag{1.8}$$

It is convenient to introduce the rescaled forms

$$\hat{\omega} = e^{i\theta} \omega, \tag{1.9}$$

$$\hat{z} = \frac{\bar{\mu}}{|\mu|} z, \tag{1.10}$$

but for simplicity of notation, we will drop the $\hat{}$ symbols in the rest of the paper.

Plugging the explicit form of (1.3) and (1.4), into the SUSY variations (1.5a)–(1.5c), one can deduce the general conditions for AdS₄ $\mathcal{N} = 1$ SUSY vacua in terms of the forms z, ω, j and the fluxes. As discussed in [16], (1.5a) implies

$$k_{\parallel} = 0 \quad \text{or} \quad \cos \theta = 0. \tag{1.11}$$

We will choose the first case namely a strict SU(2) structure. In this case, the equations (1.5a)–(1.5c) become:

$$(d - H \wedge)(e^{3A-\phi} z \wedge e^{-ij}) = 2|\mu| e^{2A-\phi} (\omega_I - z_R z_I \omega_R) \tag{1.12a}$$

$$(d - H \wedge)(e^{2A-\phi} (\omega_I - z_R z_I \omega_I)) = 0 \tag{1.12b}$$

$$(d - H \wedge)(e^{4A-\phi} (\omega_R + z_R z_I \omega_I)) = -3|\mu| e^{3A-\phi} \text{Im}(z \wedge e^{ij}) + e^{4A} * \lambda(F) \tag{1.12c}$$

with R and I denoting the real and imaginary part.

¹When $k_{\parallel} = 1$ and $k_{\perp} = 0$ the internal manifold is said to be of SU(3) structure. We will not consider this case here.

2 Description of the method

In this section, we present the semi-algorithmical method used to find new sourceless vacua in $\mathcal{N} = 1$, AdS₄ in type IIB. In fact this method can be extended to all problems where parts of the equations are linear, i.e. of the type (2.5), and parts of the equations are quadratic/differential, i.e. of the type (2.6).

2.1 Step 0: definitions

Let e^i be a 6D vielbein on the internal manifold. Define:

$$z_1 = e^A(e^1 + ie^2) \quad z_2 = e^A(e^3 + ie^4) \quad z_3 = e^A(e^5 + ie^6) \quad (2.1)$$

Then

$$z = z_1 \quad (2.2)$$

$$j = \frac{i}{2}(z_2 \wedge \bar{z}_2 + z_3 \wedge \bar{z}_3) = e^{2A}(e^{34} + e^{56}) \quad (2.3)$$

$$\omega = z_2 \wedge z_3 = e^{2A}(e^{35} - e^{46} + i(e^{36} + e^{45})) \quad (2.4)$$

define a SU(2) structure on the internal manifold. Moreover define

$$\begin{aligned} de^i &= -\frac{1}{2}f^i_{jk}e^{jk} & dA &= dA_i e^i & d\phi &= d\phi_i e^i & F_1 &= F_{1i} e^i \\ F_3 &= F_{3i} \omega_3^i & F_5 &= F_{5i} \omega_5^i & H &= H_i \omega_3^i \end{aligned}$$

where ω_k^i are the canonical real basis of k -forms on a 6-dimensional manifold (for example $\omega_{3i} = \{e^{123}, e^{124}, \dots\}$).

We are looking for a sourceless solution in type IIB with a strict SU(2) structure internal manifold. That is to say that we have to solve for (1.12a)–(1.12c) and for the sourceless Bianchi identities $dH = 0$ and $d_H F = 0$. We will also require that $d(d(e^i)) = 0$ in order to constrain more the system and be sure to obtain a well-defined manifold at the end of the day.

We also define the following set of variables:

$$T_i = \{f^i_{jk}, dA_i, d\phi_i, F_{1i}, F_{3i}, F_{5i}, H_{3i}\}$$

to which we will add the parameter $T_0 = |\mu|$. It is important to put this parameter with the variables in order to obtain fully linear (2.5) and fully quadratic (2.6) equations. We can claim we have a solution when we find a set of T_i that solves the aforementioned equations. Indeed, all the equations of motion are solved in this case (see for example [19] and references therein).

2.2 Step 1: obtaining linear constraints

The equations (1.12a)–(1.12c) are linear in the T_i 's and of the form:

$$E_{1i} = \left\{ \sum_k \alpha_i^k T_k = 0 \right\} \quad (2.5)$$

We can easily solve for them and thus eliminate some of the T_i 's.

2.3 Step 2: eliminating the derivative in the quadratic equations

The rest of the equations are quadratic in the T_i 's and are of the form:

$$E_{2i} = \left\{ \sum_{j,k} \alpha_i^{jk} (dT)_{jk} + \sum_{j,k} \beta_i^{jk} T_j T_k = 0 \right\} \quad (2.6)$$

where we defined $d(T_i) = (dT)_{ij} e^j$. If these equations are quadratic in the T_i 's, they are linear in the $(dT)_{ij}$'s so we can “solve” for them to simplify the system and obtain two sets of equations of the type:

$$E_{3i} = \left\{ \sum_{j,k} \alpha_i^{jk} (dT)_{jk} + \sum_{j,k} \beta_i^{jk} T_j T_k = 0 \right\} \quad (2.7)$$

$$E_{4i} = \left\{ \sum_{kl} \alpha_i^{kl} T_k T_l = 0 \right\} \quad (2.8)$$

Maybe it can be better explained with an example. Assume the system E_{2i} is composed of two equations $(dT)_{12} + T_1 T_2 = 0$ and $(dT)_{12} + (T_2)^2 = 0$, “solving” for $(dT)_{12}$ means keeping one of the two equations unchanged, and replace $(dT)_{12}$ in the other one to obtain the system E_{4i} : $T_1 T_2 = (T_2)^2$. In other words, we are splitting E_{2i} in two, one part, E_{3i} with all the $(dT)_{ij}$'s and the other, E_{4i} with only T_i 's.

2.4 Step 3: simplifying the leftover quadratic equations

We can still simplify a bit the system of equations E_{4i} (2.8). Indeed, in general, all the equations are not independent and there exists a simple trick to easily get a minimal system. Simply define $(TT)_{ij} = T_i T_j$, with $i \leq j$. Then the system is linear in these new variables and by solving it, one obtains a minimal system in the $(TT)_{ij}$'s. One just has to go back to the T_i 's to have simplified E_{4i} . Moreover while solving for the $(TT)_{ij}$'s, we can make it so that $(TT)_{0j}$ appears as much as possible. It will help us get simpler equations for step 4.

2.5 Step 4: adding linear constraints

The goal of this step is to obtain a linear constraint from the set of quadratic constraints to simplify the original problem. This is inspired by [24] where some linear conditions were hidden in the quadratic constraints and permitted to fully solve these equations.

Having simplified the system in steps 2 and 3, some equations may immediately give such a linear constraint:

- One of the equation can be of the form $\sum_i (\sum_k \alpha_i^k T_k)^2 = 0$. Then the linear constraints are $\sum_k \alpha_i^k T_k = 0$ for all i .
- One of the equations can be of the form $T_0 \sum_k \alpha^k T_k = 0$. Since $T_0 = |\mu|$, which is non-zero since the external manifold is AdS, one can conclude $\sum_k \alpha^k T_k = 0$ which is linear.

If one is not in one of the case above, one has to make an assumption. The system can often give an hint on what is a sensible assumption or not. Indeed, some equations are simpler than other and help make a choice. But how can one find these simpler equations in a system which can be quite complicated? The answer is to look at the eigenvalues of α^{kl} seen as a matrix in (2.8). The equations with a small number of non-zero eigenvalues are usually sufficiently simple to make sensible assumptions (see section 3.1 for an explicit example).

2.6 Step 5: going back to step 1

We are now going back to step 1 with the additional linear constraints obtained in step 4. We are forced to do all the work again for the following reason. Assume you had for example the equation $(dT)_{11} = (T_2)^2$ in the system E_{3i} (2.7) and that you found $T_1 = 0$ as a linear constraint in step 4. Then it implies $(dT)_{11} = 0$ and so $(T_2)^2 = 0$ in step 2 which will give the linear constraint $T_2 = 0$ in step 4. This is the strength of the method: simplify sufficiently the quadratic constraints to spot the linear constraints hidden in them to be able to discover even more linear constraints.

Thus we are going from step 1 to step 4 to step 1 again until one of the three following things happen:

- The system has no solution: it means that one of the assumptions made in step 4 is wrong and should be discarded or that there is no solution within the ansatz one was given.
- E_{4i} (2.8) is empty then one can go to the final step.
- E_{4i} (2.8) is not empty but is sufficiently simple to be able to find a non linear solution of it. Then one can go to the final step.

2.7 Final step: solving the last equations

Ideally at this point both E_{3i} and E_{4i} defined in step 2 are empty but this is often not the case. Nevertheless, they are usually sufficiently simple to be solved by traditional methods. To sum up, the above steps take care of the linear parts of the equations and of some of the quadratic constraints by assuming some linear constraints. What is left are the differential and quadratic parts. An explicit example of this step will be given in section 3.1.

3 Examples of new vacua

In this section we give some examples of solutions found by the above method. One of them is already known as a Lüst-Tsimpis solution [3]. The other two, as far as the author knows, are two new vacua in type IIB.

3.1 An example of Lüst-Tsimpis solution

We will give an example of a Lüst-Tsimpis solution [3]. In this section we will also give a detailed account of how the method works in this particular case.

First of all we assume that there is no vector or tensor in the torsion classes as they do in [3]. These are linear constraints in our variables T_i 's so can already be put in step 1. We will also require $de^2 = de^3 = 0$ that is to say, we want $e^2 = dx^2$ and $e^3 = dx^3$, x^2 and x^3 being coordinates. This requirement is also linear and can be put in step 1. Finally, we will require that all the variables are functions of only x^2 and x^3 . Part of this requirement is linear (for example, $dA_j = 0$ for $j \neq 2, 3$). The other part is differential and means that $(dT)_{ij} = 0$ for $j \neq 2, 3$ and appears in step 2.

We run the algorithm from step 1 to step 3 and take a look at the resulting system E_{4i} (2.8). It contains several simple equations: $|\mu|(f_{15}^4 - \frac{5|\mu|}{2}) = |\mu|f_{45}^4 = |\mu|f_{46}^4 = |\mu|(f_{35}^5 - f_{34}^4) = |\mu|f_{45}^5 = 0$. We add these linear (since $|\mu| \neq 0$) constraints to step 1.

Then we rerun the algorithm from step 1. In step 4, we obtain only one equation in E_{4i} namely: $(f_{25}^4)^2 + 4(dA_2)^2 - \frac{5|\mu|^2}{4} = 0$. We are in the case where there is no obvious linear constraint. So we will make a choice: $f_{25}^4 = 0$ and $dA_2 = \frac{\sqrt{5}|\mu|}{4}$ to solve it.

We rerun the algorithm from step 1 and find that E_{4i} is empty. So we go to the final step and take a look at E_{3i} (2.7). There are 4 equations in it (the projections on e^2 and e^3 of the two following expressions):

$$d(f_{34}^4) = (2(f_{34}^4)^2 - f_{34}^4 f_{36}^6) e^3 \tag{3.1}$$

$$d(f_{36}^6) = (5|\mu|^2 - 2(f_{34}^4)^2 + 2f_{34}^4 f_{36}^6 + (f_{36}^6)^2) e^3 \tag{3.2}$$

There exists a simple solution to this system: $f_{34}^4 = 0$ and $f_{36}^6(x^3) = \sqrt{5}|\mu| \tan(\sqrt{5}|\mu|(x^3 - x_0))$ with x_0 an integration constant. With this, E_{3i} and E_{4i} are empty which means we have successfully solved all the relevant equations.

Let's now give explicitly the results. We have:

$$\begin{aligned} de^1 &= 2|\mu|(e^{36} + e^{45}) & de^2 &= d(dx^2) = 0 & de^3 &= d(dx^3) = 0 & (3.3) \\ de^4 &= -\frac{5}{2}|\mu|e^{15} - f_{36}^6 e^{56} & de^5 &= \frac{5}{2}|\mu|e^{14} + f_{36}^6 e^{46} & de^6 &= -f_{36}^6 e^{36} \end{aligned}$$

The fluxes, dilaton and warp factor being:

$$F_1 = 0 \tag{3.4a}$$

$$F_3 = \frac{|\mu|e^{-2A}}{2} (\sqrt{5}(-e^{135} + e^{146}) - e^{234} - e^{256}) \tag{3.4b}$$

$$F_5 = 3|\mu|e^{13456} \tag{3.4c}$$

$$H = \frac{|\mu|e^{2A}}{2} (\sqrt{5}(e^{134} + e^{156}) + e^{235} - e^{246}) \tag{3.4d}$$

$$\phi = 4A = \sqrt{5}|\mu|x^2 \tag{3.4e}$$

with $f_{36}^6(x^3) = \sqrt{5}|\mu| \tan(\sqrt{5}|\mu|(x^3 - x_0))$ with x_0 an integration constant. This is a solution of the $\mathcal{N} = 1$ SUSY equations, the sourceless Bianchi identities and $d(d(e^i)) = 0$ and so of all the equations of motion. Moreover, there are no vectors and no tensors in the torsion classes.

In order to understand more this solution, it is useful to give a coordinate expression of the metric or at least identify each part of the space. In that regard, one can take the

following change of variables:

$$\begin{aligned}
 e^1 &= \frac{2\tilde{e}^1}{5|\mu|} + \frac{2f(x^3)dx^6}{5|\mu|\sqrt{5|\mu|^2 + f(x^3)^2}} & e^2 &= dx^2 & e^3 &= dx^3 \\
 e^4 &= \frac{\tilde{e}^4}{\sqrt{5|\mu|}} & e^5 &= \frac{\tilde{e}^5}{\sqrt{5|\mu|}} & e^6 &= \frac{dx^6}{\sqrt{5|\mu|^2 + f(x^3)^2}}
 \end{aligned} \tag{3.5}$$

with $f(x^3) = \sqrt{5}|\mu| \tan(\sqrt{5}|\mu|(x^3 - x_0))$ with x_0 an integration constant and the triplet $\{\tilde{e}^1, \tilde{e}^4, \tilde{e}^5\}$ parametrizing a $SU(2)$ $d\tilde{e}^1 = \tilde{e}^{45}$, $d\tilde{e}^4 = -\tilde{e}^{15}$, $d\tilde{e}^5 = \tilde{e}^{14}$. If one wants to see explicitly the squashed Sasaki-Einstein of Lüst-Tsimpis [3], we now give the correspondence with their objects (note that for us $W_{LT} = |\mu|$ and $c_{LT} = 0$):

$$\begin{aligned}
 u_{LT} &= \frac{5|\mu|}{6}e^1 \\
 dt_{LT} &= e^2 \\
 \gamma_{LT} &= -\frac{5|\mu|^2}{6}(e^{36} + e^{45}) \\
 \alpha_{LT} &= \frac{5}{6}|\mu|^2(\sin(\theta_{LT})(e^{34} + e^{56}) + \cos(\theta_{LT})(e^{35} - e^{46})) \\
 \beta_{LT} &= \frac{5}{6}|\mu|^2(\cos(\theta_{LT})(e^{34} + e^{56}) - \sin(\theta_{LT})(e^{35} - e^{46}))
 \end{aligned}$$

with θ_{LT} a constant.

3.2 A new solution with constant dilaton

Applying the method to more complex cases, we were able to identify two new vacua. Here we present the first one which has the particularity to have a constant dilaton. We will make an ansatz on the solution to make the method converge more rapidly (this ansatz has been found by trial and error from the general case). We will assume that $de^3 = dx_3$ and that all the variables depend on x_3 only. We will also assume that $de^2 = -f_{23}^2 e^{23}$, $de^4 = -f_{34}^4 e^{34}$ and $de^5 = -f_{35}^5 e^{35}$. Then some iterations of the algorithm give the following algebra:

$$de^1 = (4dA_3(x_3) - d\phi_3(x_3))e^{13} + 2|\mu|(e^{36} + e^{45}) \tag{3.6a}$$

$$de^2 = (4dA_3(x_3) - d\phi_3(x_3))e^{23} \tag{3.6b}$$

$$de^3 = d(dx_3) = 0 \tag{3.6c}$$

$$de^4 = -f_{34}^4(x_3)e^{34} \tag{3.6d}$$

$$de^5 = -(d\phi_3(x_3) + f_{34}^4(x_3))e^{35} \tag{3.6e}$$

$$\begin{aligned}
 de^6 &= 5|\mu|e^{13} - f_{23}^6(x_3)e^{23} - (2dA_3(x_3) - d\phi_3(x_3) - f_{34}^4(x_3))e^{36} \\
 &\quad + (4dA_3(x_3) - 2d\phi_3(x_3) - 2f_{34}^4(x_3))e^{45}
 \end{aligned} \tag{3.6f}$$

with $dA = dA_3(x_3)e^3$ and $d\phi = d\phi_3(x_3)e^3$. Moreover, in order to verify the Bianchi identities and $d(d(e^i)) = 0$, the four functions verify the following equations (which are the

system E_{3i} (2.7) in this case):

$$(dA_3)' = \frac{5|\mu|^2}{2} + 6(dA_3)^2 + dA_3 f_{34}^4 + \frac{1}{2}(f_{23}^6)^2 \quad (3.7a)$$

$$(d\phi_3)' = 10dA_3 d\phi_3 - 2(d\phi_3)^2 + d\phi_3 f_{34}^4 + (f_{23}^6)^2 \quad (3.7b)$$

$$(f_{34}^4)' = 10|\mu|^2 + 16(dA_3)^2 - 16dA_3 d\phi_3 + 4(d\phi_3)^2 - 6dA_3 f_{34}^4 + 4d\phi_3 f_{34}^4 + 3(f_{34}^4)^2 \quad (3.7c)$$

$$(f_{23}^6)' = 4dA_3 f_{23}^6 \quad (3.7d)$$

Unfortunately, the author hasn't been able to solve these equations in all generality. But there exists the following more simple solution (which is the above one with $d\phi = 0$, $f_{34}^4 = 4dA_3$ and $f_{23}^6 = 0$):

$$de^1 = 4dA_3(x_3)e^{13} + 2|\mu|(e^{36} + e^{45}) \quad (3.8a)$$

$$de^2 = 4dA_3(x_3)e^{23} \quad (3.8b)$$

$$de^3 = d(dx_3) = 0 \quad (3.8c)$$

$$de^4 = -4dA_3(x_3)e^{34} \quad (3.8d)$$

$$de^5 = -4dA_3(x_3)e^{35} \quad (3.8e)$$

$$de^6 = 5|\mu|e^{13} + 2dA_3e^{36} - 4dA_3e^{45} \quad (3.8f)$$

The fluxes, dilaton and warp factor being:

$$F_1 = 0 \quad (3.9a)$$

$$F_3 = e^{2A}(4dA_3(x_3)e^{125} + |\mu|(-3e^{234} + 2e^{256})) \quad (3.9b)$$

$$F_5 = 3e^{4A}|\mu|e^{13456} \quad (3.9c)$$

$$H = e^{2A}(4dA_3(x_3)e^{124} + |\mu|(3e^{235} + 2e^{246})) \quad (3.9d)$$

$$\phi = 0 \quad (3.9e)$$

$$dA_3(x_3) = \frac{|\mu|}{2} \tan(5|\mu|(x_3 - x_0)) \quad (3.9f)$$

$$A(x_3) = -\frac{1}{10} \log(\cos(5|\mu|(x_3 - x_0))) \quad (3.9g)$$

with x_0 an integration constant. This is a solution of the $\mathcal{N} = 1$ SUSY equations, the sourceless Bianchi identities and $d(d(e^i)) = 0$ and so of all the equations of motion.

We then put its expression in coordinates by the following change of variables:

$$e^1 = 2|\mu|f(x_3)^{\frac{2}{5}}\tilde{e}^1 + \frac{2|\mu|}{dA_3(x_3)f(x_3)^{\frac{3}{10}}}dx^6 \quad (3.10a)$$

$$e^2 = f(x_3)^{\frac{1}{5}}dx^2 \quad (3.10b)$$

$$e^3 = dx_3 = dA_3(x_3)f(x_3)^{\frac{17}{10}}dx'_3 \quad (3.10c)$$

$$e^4 = f(x_3)^{\frac{1}{5}}dx^4 \quad (3.10d)$$

$$e^5 = f(x_3)^{\frac{1}{5}}dx^5 \quad (3.10e)$$

$$e^6 = 4dA_3(x_3)f(x_3)^{\frac{2}{5}}\tilde{e}^1 + \frac{10}{f(x_3)^{\frac{3}{10}}}dx^6 \quad (3.10f)$$

with $d\tilde{e}^1 = dx'_3 \wedge dx^6 + dx^4 \wedge dx^5$ and $f(x_3) = \frac{1}{\frac{5|\mu|^2}{2} + 10dA_3(x_3)^2}$. Unfortunately, the author has not been able to obtain an explicit change of variables to go from x_3 to x'_3 .

3.3 A new solution with non constant dilaton

3.3.1 The solution

Another solution arose from the method described, one with non constant dilaton. Once again to make the method converge more rapidly one takes an ansatz (this ansatz has been found by trial and error from the general case). We will assume that $de^3 = dx_3$ and that all the variables depend on x_3 only. We will also assume that $de^4 = -f_{34}^4 e^{34}$ and $H = 0$. After some iterations of the algorithm, one obtains:

$$de^1 = dA_3(x_3)e^{13} + 2|\mu|(e^{36} + e^{45}) \tag{3.11a}$$

$$de^2 = dA_3(x_3)e^{23} \tag{3.11b}$$

$$de^3 = d(dx_3) = 0 \tag{3.11c}$$

$$de^4 = \frac{|\mu|^2}{dA_3(x_3)}e^{34} \tag{3.11d}$$

$$de^5 = 2|\mu|e^{14} - dA_3(x_3)e^{35} - \frac{|\mu|^2}{dA_3(x_3)}e^{46} \tag{3.11e}$$

$$de^6 = 2|\mu|e^{13} - dA_3(x_3)e^{36} + \frac{|\mu|^2}{dA_3(x_3)}e^{45} \tag{3.11f}$$

The fluxes, dilaton and warp factor being:

$$F_1 = e^{-\phi}(|\mu|e^1 - 2dA_3(x_3)e^6) \tag{3.12a}$$

$$F_3 = e^{2A-\phi}(dA_3(x_3)e^{125} - 3|\mu|e^{234} + |\mu|e^{256}) \tag{3.12b}$$

$$F_5 = 3|\mu|e^{13456} \tag{3.12c}$$

$$H = 0 \tag{3.12d}$$

$$dA = dA_3(x_3)e^3 = \frac{|\mu|}{2} \tan(2|\mu|(x_3 - x_0))e^3 \tag{3.12e}$$

$$A = -\frac{1}{4} \log\left(\cos(2|\mu|(x_3 - x_0))\right) \tag{3.12f}$$

$$d\phi = d\phi_3 e^3 = 3dA_3(x_3)e^3 \tag{3.12g}$$

$$\phi = 3A + \text{cst} \tag{3.12h}$$

with x_0 an integration constant. This is a solution of the $\mathcal{N} = 1$ SUSY equations, the sourceless Bianchi identities and $d(d(e^i)) = 0$ and so of all the equations of motion.

Once again a coordinate expression is useful. Do the following change of variables:

$$e^1 = \cos(2|\mu|x_4) \cos(X) \sin(X)^{\frac{1}{4}} dx_1 + \sin(2|\mu|x_4) \cos(X) \sin(X)^{\frac{1}{4}} dx_5 + \sin(X)^{\frac{5}{4}} dx_6 \quad (3.13a)$$

$$e^2 = \sin(X)^{\frac{1}{4}} dx_2 \quad (3.13b)$$

$$e^3 = \frac{dX}{2|\mu|} \quad (3.13c)$$

$$e^4 = \cos(X) dx_4 \quad (3.13d)$$

$$e^5 = -\sin(2|\mu|x_4) \sin(X)^{\frac{1}{4}} dx_1 + \cos(2|\mu|x_4) \sin(X)^{\frac{1}{4}} dx_5 \quad (3.13e)$$

$$e^6 = -\cos(2|\mu|x_4) \sin(X)^{\frac{5}{4}} dx_1 - \sin(2|\mu|x_4) \sin(X)^{\frac{5}{4}} dx_5 + \cos(X) \sin(X)^{\frac{1}{4}} dx_6 \quad (3.13f)$$

with $X = (2|\mu|(x_3 - x_0)) + \frac{\pi}{2}$. Note that $e^A = \sin(X)^{-\frac{1}{4}}$. We give the expression of the metric in the $(x_1, x_2, X, x_4, x_5, x_6)$ system of coordinates:

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4|\mu|^2 \sqrt{\sin(X)}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\cos(X)^2}{\sqrt{\sin(X)}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (3.14)$$

One can also calculate the Ricci scalar: $R = |\mu|^2 \frac{1-3\cos(2X)}{(\sin(X))^{\frac{3}{2}}}$ which goes to infinity when X goes to 0. This shows that, a priori, this space is singular. Around 0, this metric doesn't have the form of the D-brane metric so one has to better understand this singularity. In order to do that, let's look at the ten dimensional metric around $X = 0$ at first order:

$$ds^2 = \frac{1}{\sqrt{X}} ds_{(4)}^2 + (dx_1^2 + dx_2^2 + dx_5^2 + dx_6^2) + \frac{dX^2}{4|\mu|^2 \sqrt{X}} + \frac{dx_4^2}{\sqrt{X}} \quad (3.15)$$

Then define $\tilde{x} = \frac{\sqrt{X}}{|\mu|}$, the metric around 0 becomes:

$$ds^2 = \frac{1}{|\mu| \tilde{x}} (ds_{(4)}^2 + dx_4^2) + (dx_1^2 + dx_2^2 + dx_5^2 + dx_6^2) + |\mu| \tilde{x} d\tilde{x}^2 \quad (3.16)$$

This shows that this system can be mapped to a D5-D7 intersecting system which are delocalized in the $\{1, 2, 5, 6\}$ directions. For example D5 along x^1, x^4 and D7 along x^2, x^4, x^5, x^6 . Indeed, we are in the case of a system similar to (10) of [29] with only one transverse direction for both branes (the \tilde{x} direction), and $H_5 = H_7 = |\mu| \tilde{x}$ being the associated harmonic function. Similarly according to equation (478) of [30], one has $e^{-2\phi} = H_5 H_7^2 = |\mu|^3 \tilde{x}^3$ which corresponds to the dilaton value on the solution around X equal zero: $e^{-2\phi} = e^{-6A} = \sin(X)^{\frac{3}{2}} =_{X \rightarrow 0} X^{\frac{3}{2}} = |\mu|^3 \tilde{x}^3$.

3.3.2 T-dual solution

One can see that there exists several isometric directions for this solution (at first sight dx_1, dx_2, dx_5, dx_6). To illustrate this, we will explicitly give the T-dual along the $dx_2 = e^A e^2$ direction. The resulting solution in IIA is:

$$de^1 = dA_3(x_3)e^{13} + 2|\mu|(e^{36} + e^{45}) \quad (3.17a)$$

$$de^2 = dA_3(x_3)e^{23} \quad (3.17b)$$

$$de^3 = d(dx_3) = 0 \quad (3.17c)$$

$$de^4 = \frac{|\mu|^2}{dA_3(x_3)}e^{34} \quad (3.17d)$$

$$de^5 = 2|\mu|e^{14} - dA_3(x_3)e^{35} - \frac{|\mu|^2}{dA_3(x_3)}e^{46} \quad (3.17e)$$

$$de^6 = 2|\mu|e^{13} - dA_3(x_3)e^{36} + \frac{|\mu|^2}{dA_3(x_3)}e^{45} \quad (3.17f)$$

The fluxes, dilaton and warp factor being:

$$F_0 = F_4 = H = 0 \quad (3.18a)$$

$$F_2 = e^{A-\phi}(|\mu|e^{12} - 2dA_3e^{15} + 2dA_3e^{26} - 3|\mu|e^{34} + |\mu|e^{56}) \quad (3.18b)$$

$$F_6 = 3e^{5A-\phi}|\mu|e^{123456} \quad (3.18c)$$

$$dA = dA_3(x_3)e^3 = \frac{|\mu|}{2} \tan(2|\mu|(x_3 - x_0))e^3 \quad (3.18d)$$

$$A = -\frac{1}{4} \log\left(\cos(2|\mu|(x_3 - x_0))\right) \quad (3.18e)$$

$$d\phi = d\phi_3 e^3 = 3dA_3(x_3)e^3 \quad (3.18f)$$

$$\phi = 3A + \text{cst} \quad (3.18g)$$

Note that the space the solution lives on is the same in both IIA and IIB. But in IIA, contrary to IIB, we have, the following SU(3) structure:

$$z_1 = e^A(ie^1 - e^2) \quad z_2 = e^A(e^3 + ie^4) \quad z_3 = e^A(ie^5 - e^6) \quad (3.19)$$

$$z = z_1 \quad (3.20a)$$

$$j = \frac{i}{2}(z_2 \wedge \bar{z}_2 + z_3 \wedge \bar{z}_3) = e^{2A}(e^{34} + e^{56}) \quad (3.20b)$$

$$\omega = z_2 \wedge z_3 = e^{2A}(-e^{35} + e^{46} + i(-e^{36} - e^{45})) \quad (3.20c)$$

$$\Omega = z \wedge \omega \quad (3.20d)$$

$$J = \frac{i}{2}z \wedge \bar{z} + j \quad (3.20e)$$

$$\Phi_+ = -\frac{ie^A}{8}e^{-iJ} \quad (3.20f)$$

$$\Phi_- = -\frac{ie^A}{8}\Omega \quad (3.20g)$$

Conclusion and outlooks. In this paper, we managed to identify two new vacua in type IIB which are explicit. It is a step forward in identifying the web of vacua in type II. We also have been able to discover a new IIA solution by applying T-duality along an isometric direction on one of the solutions. One caveat should be pointed out: these solutions are indeed sourceless if the space is smooth which is not guaranteed by the analysis. Indeed, one could find localized sources (or partially localized sources as we did for the second example) but it is not in the scope of this paper.

To obtain these new vacua, we devised a semi-algorithmical method which can be applied to lots of other similar situations. Indeed, one can apply it to type IIA to discover new vacua (and we should be able to easily recover the one we found here), or to type IIB with dynamic $SU(2)$ structure instead of the strict $SU(2)$ structure we restricted to in this paper. More generally, one can apply it to all problems with a linear part and a quadratic/differential part of the type (2.5), (2.6). In that respect, one can see this paper as a proof of concept for the method.

There is also lots of room for improvement for the method depending on which problems one applies it to. Indeed, in this paper we restricted to having only one parameter which had to be non zero $|\mu|$. In fact it is quite common to find other linear combinations of variables to be non zero. Then one can modify step 3 and step 4 to take that into account and be provided with even more linear constraints. Another improvement concerns the automatization. In step 4, it is quite common to have constraints of the type $(\sum_i \alpha^i T_i)(\sum \beta^i T_i) = 0$. One can incorporate this case in the algorithm to build a tree of assumptions (here one branch is given by $(\sum_i \alpha^i T_i) = 0$ and the other by $(\sum \beta^i T_i) = 0$) instead of just choosing one.

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