PROBLEMS ON DISTORTION UNDER CONFORMAL MAPPINGS

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(1) W. Hayman and J. M. Wu have shown that if Ω is a simply connected domain in the plane and f is a conformal mapping from Ω onto the unit disk Δ then for every circle L one has that $f(\Omega \cap L)$ has finite length and moreover is a regular curve in Ahlfors' sense.

When L is contained in Ω, O. Martio has shown that $f(L)$ is a quasicircle. In the general case, is it true that each component of $f(\Omega \cap L)$ is a quasiline ? .

(2) The Hayman-Wu theorem shows that $f' \in L^1(\Omega \cap L)$; it is natural to conjecture that $f' \in L^p$ for each $p < 2$, or even further that

$$\text{length}\left(f(E)\right) \leqq c \ \text{length}(E)^{1/2}$$

if $E \subset L$. (This is a conjecture of A. Baernstein.) It is true at least when E is an interval.

(3) The following is a stronger version of the so-called Brennan's conjecture : if $A \subset \Omega$ then

$$\text{area}\left(f(A)\right) \leqq c \ \text{area}(A)^{1/2} .$$

This is the case when A is any disk.

(4) A domain G in the plane has uniformly perfect boundary if the following capacitary condition is satisfied :

$$\text{cap}\left(\{z : |z - a| < r\} \cap G\right) \geqq c \cdot r$$

for any $a \in \partial G$, $0 < r < \text{diam}(G)$. Is the Hayman-Wu theorem valid for this domain when f^{-1} is replaced by the universal covering map ? .