

2. Notation and Conventions.

Groups G will always act on the left. If M is a G -module and $\sigma \in G$, this action will be written either as $m \mapsto \sigma(m)$ or $m \mapsto \sigma m$. We let M^G denote the sub-module of G -invariants. If M and N are G -modules, so is $\text{Hom}(M, N)$: the action of σ on a homomorphism $f: M \rightarrow N$ is given by

$$\sigma(f)(m) = \sigma(f(\sigma^{-1}m)) .$$

Rings will also act on the left. If M is an R -module and $r \in R$ we let $M_r = \{m \in M : rm = 0\}$ denote the sub-module of "r-torsion."

If A and B are elliptic curves (or, more generally, abelian varieties) defined over the field F , we let $\text{Hom}_F(A, B)$ be the group of algebraic homomorphisms $\phi: A \rightarrow B$ which are defined over F . If S is any F -algebra, we let $A(S)$ denote the abelian group of all S -rational points of A . If σ is any automorphism of F we write ${}^\sigma A$ and ${}^\sigma B$ for the conjugate varieties, and ${}^\sigma \phi$ for the conjugate homomorphism from ${}^\sigma A$ to ${}^\sigma B$.

If F is a field, \overline{F} denotes an algebraic closure of it. We shall always use the isomorphism of local class field theory which takes a uniformizing parameter to an arithmetic Frobenius in the Galois group.