XIV Some open problems

a) Problems on general just infinite groups

(i) Assume P is an insoluble hereditarily just infinite group satisfying $w(P) < \infty$. Is every open subgroup Q of P also of width $w(Q) < \infty$?

(ii) Is there a hereditarily just infinite pro-p-group of finite width such that the series $|\gamma_i(P) : \gamma_{i+1}(P)|$ for $i \in \mathbb{N}$ is not periodic?
Can one find such an example as an open subgroup of the Nottingham group?

(iii) Is every hereditarily just infinite pro-p-group of finite width finitely presented as a pro-p-group?

(iv) Is every just infinite pro-p-group of finite width also of finite obliquity? If so, it gives a positive answer to (i).

(v) Does average width 1 imply finite coclass?

(vi) Is it the case that for every just infinite pro-p-group P of finite width, there is a natural number i such that, if Q is any infinite pro-p-group with $Q/\gamma_i(Q) \cong P/\gamma_i(P)$ then Q is just infinite of finite width? Note that an affirmative answer would give an affirmative answer to (iii). Since there are uncountably many just infinite pro-p-groups of finite width, we cannot hope that for large enough i the isomorphism $Q/\gamma_i(Q) \cong P/\gamma_i(P)$ should imply $Q \cong P$, as in the case if P is a $\tilde{p}$-group.

b) Problems on $\tilde{p}$-groups

(i) Find a finite pro-p-presentation for every maximal $\tilde{p}$-group (either algorithmically or theoretically).

(ii) Implement an algorithm to determine, from a sufficiently large quotient of a $\tilde{p}$-group Q, the maximal $\tilde{p}$-group P containing Q as an open subgroup.

(iii) Can one use the Cayley version of the Baker-Campbell-Hausdorff-formula to extend some of the investigations of Chapter VI to the case $p = 2$?