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Frank D. Grosshans

Algebraic Homogeneous Spaces and Invariant Theory

Errata

My thanks to Nazih Nahlus, Walter Ferrer Santos, and especially Michel Brion for the errata listed below.

page 8, line −7: "$T(G(x))_x$" should be $T(G \cdot x)_x$.

page 13, line −7: "we adapt" should be "we adopt".

page 15, line 15: "(ω, α)". The notation (,) refers to a $\mathbb{Q}$-valued inner product on $X(T)$ which is invariant under the Weyl group of G.

page 20, statement of Theorem 4.2: "$k[X'] = k[X]$ if and only if $\operatorname{codim}(X - X') \geq 2$". This would be better stated as "$k[X'] = k[X]$ if and only if each irreducible component of $X - X'$ has codimension ≥ 2 in X".

page 27, line −2: "dimensions of the irreducible components of the fibers" should be "dimensions of the irreducible components of the non-empty fibers".

page 28, Theorem 5.3: the assumption that X be affine is not necessary.

page 29, line −4: The sentence, "Embeddings of G/U, where U is maximal unipotent in a semi-simple group G, were completely described in [112] in case char $k = 0$", should be replaced by the following. "The affine embeddings of G/U, where U is maximal unipotent in a semi-simple group G, were completely described in [112] in case char $k = 0$. A general theory of embeddings of homogeneous spaces was developed by Luna and Vust in [70]. An exposition of the Luna-Vust theory may be found in [60]."

page 39, line 18: the bibliographic citation should be [Ferrer Santos, Walter Ricardo, A note on affine quotients. J. London Math. Soc. (2) 31 (1985), no. 2, 292–294].

page 44, line −1: "there is non-negative" should be "there is a non-negative".

page 49, line 5: "Lemma 3 is, in fact" should be "Lemma 8.5 is, in fact".

page 50, line −6: "Then $\Phi(a) \neq 0$" should be "If $a \neq 0$, then $\Phi(a) \neq 0$".

page 59, line −13: "a subgroup H" should be "an observable subgroup H".

page 63, line 1: "$SL(n, \mathbb{C})$" should be "$SL(n, \mathbb{C}), n \geq 2$".

page 66, line 7: "the group acts on S_n" should be "the group G acts on S_n".

page 73, line 10. The sentence beginning "Since $s_o\omega \leq \chi \leq \omega$" should be replaced by the following. "Since $\omega \geq \chi$ and $\chi + \sum e_\alpha \alpha = s_o\omega$, we have $\sum e_\alpha \alpha = s_o\omega - \chi \geq s_o\omega - \omega$. Hence, $s_o\omega - \omega \leq \sum e_\alpha \alpha \leq 0$."

page 93, proof of Theorem 15.14. Michel Brion has shown me an easy way to prove that D is a free $k[x]$-module. Since free modules are flat, this eliminates the need to use Lemma 15.9. The proof is as follows. For each $n \geq 0$, choose a subspace V_n of A_n so that as vector spaces over k, $A_n = V_n \oplus A_{n-1}$; choose a basis for V_n over k say, $\{v_{n1}, v_{n2}, \ldots\}$. Let $F = \{v_{01}, \ldots, v_{11}, \ldots\}$. Note that any $a \in A_n$ can be written as a k-linear combination of the v_{ij}. We now show that D is a free $k[x]$-module with F as a basis. First, we prove by induction on degree that any $f \in D$ is a $k[x]$-linear combination of elements in F. When $f \in A_0$, this is immediate since $A_{-1} = \{0\}$. In general, let $f = \sum_{n=0}^{N} a_n x^n$ where a_N is a non-zero element in A_N. Choose (finitely many) scalars $c_{ij} \in k$ so that $a_N = \sum c_{ij} v_{ij}$. Then, $f - \sum c_{ij} v_{ij} x^N \in \bigoplus_{n=0}^{N-1} A_n x^n$ and we may apply the induction hypothesis. Second, suppose that $\sum c_{ij}(x)\, v_{ij} = 0$ where each $c_{ij}(x)$ is a polynomial in $k[x]$. We equate the coefficient of each power of x to 0 to obtain equations of the form $\sum d_{ij}\, v_{ij} = 0$ where each $d_{ij} \in k$. Since the v_{ij} are linearly independent over k, each $d_{ij} = 0$. Then, each $c_{ij}(x) = 0$.

page 105, line 3: "$D(m, n, r)$"should be "$k[D(m, n, r)]$".

page 107, line −5. The following comment should be placed right after the definition. "This definition makes sense when $G = B$. Lemmas 19.7, 19.8, and Theorem 20.2 also are valid in case $G = B$ and are used in that way in the Example on p. 116."

page 108, Lemma 19.8: the lemma holds without the assumption that X be quasi-affine [Knop, Friedrich: On the set of orbits for a Borel subgroup. Comment. Math. Helv. 70 (1995), no. 2, 285–309].

page 117, line 13: "commutator" should be "centralizer".

page 121, line −19: "fourteen different" should be "thirteen different".