Convergence in Probability and Allied Results

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Abstract

Let (X,S,P) be a probability space, a d f, f_n , n= 1,2, ... real random variables. Let x be any continuity point of f with respect to P.,i.e P(f=x) = 0. Let E_n^x and E^x denote the sets $f_n^{-1}(-\infty, x]$ and $f^{-1}(-\infty, x]$ respectively. The indicator of any set A will be denoted by I_A.

Let s denote anyone of the following modes of convergence; uniform convergence, convergence in the mean, convergence in probability, convergence almost everywhere and convergence in distribution.

Consider the following statements.

1. f_n ---> f in the sense s .
2. $I_{E_n^x}$ ----> I_{E^x} in the sense s .
3. $g(f_n)$ ---> g(f) in the sense s for any continuous function g with compact support on the line.

Theorem:- These statements are equivalent, whenever s denotes convergence almost everywhere, or convergence in probability, or convergence in distribution. They are not equivalent, if s denotes uniform convergence or convergence in the mean.

Corollary. Convergence in probability is simply convergence in distribution over the class of equivalent measures.