$$\mathrm{ind}_a(E, D) = \sum_{o}^{\ell} (-1)^i \dim_{\mathbb{C}} H^i(E, D) \quad .$$

The index theorem states that $\mathrm{ind}_t = \mathrm{ind}_a$. Hence 8.1 is just 8.3.

Remark 8.4. This result can also be obtained as a special case of a fixed-point formula on flat manifolds [40]: Assume a compact cyclic group H acts on the complex (E, d), hence on X, compatible with the flat G-structure on X. The equivariant index theorem of Atiyah-Singer [4], [6] was used in [40] to show that the equivariant index $\mathrm{ind}_H(E, d) \in R(H)$ (R(H) the representation ring of H) depends only on the isolated fixed points of the H-action on X (the positive dimensional fixed point manifolds do not contribute to the index). For $H = \{1\}$, there are no isolated fixed points and 8.1 follows.

9. Problems.

After all the preceding work, the following is still an open

Problem. Let X be a closed flat manifold. Does the Euler characteristic of X vanish?

The real Pontrjagin classes of flat manifolds are trivial, hence the Pontrjagin numbers are zero. In [11] there is an example of a flat Riemannian manifold with nonzero second Stiefel-Whitney class. However, in the oriented case the top Stiefel-Whitney class $w_n(X), n = \dim X$ is zero by (6.13). One might study the

Problem. Let X be a closed flat manifold. Do all Stiefel-Whitney numbers vanish, i.e. do (oriented) closed flat manifolds (orientably) bound compact manifolds?