THE DIFFERENTIAL CORRECTION ALGORITHM FOR RATIONAL L_∞ APPROXIMATION

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SUMMARY*

Given a set of function values $f(x_t)$ $(t=1,2,\ldots,N)$, we consider the problem of calculating the rational function $R(x) = P(x)/Q(x)$ that minimizes the quantity

$$\max_t |f(x_t) - R(x_t)|,$$

where $P(x)$ and $Q(x)$ are polynomials of prescribed degrees. To solve this problem Cheney and Loeb [2] proposed a "differential correction algorithm", ODC say, but in a subsequent paper [3] they modified their algorithm, and now the modified algorithm, DC say, is nearly always used in place of ODC. The purpose of this paper is to direct attention back to the original algorithm, because in practice ODC seems to be much better.

The modified algorithm is usually preferred because it has been proved that it has sure convergence properties, see Cheney [1] for example. However now we show that the convergence of ODC is equally reliable. Moreover we prove that the rate of convergence of ODC is usually quadratic, but the rate of convergence of DC is only linear.

Some numerical examples are given to compare the two versions of the differential correction algorithm, and they confirm that ODC is faster and more accurate than DC.

References

[1] Cheney, E. W. "Introduction to approximation theory", McGraw-Hill (1966).

[2] Cheney, E. W. and Loeb, H. L. Numer. Math. 3, 72-75 (1961).

[3] Cheney, E. W. and Loeb, H. L. Numer. Math. 4, 124-127 (1962).

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