Hans G. Kellerer: Extension of stationary processes

Let a_t, $0 \leq t \leq t_o$, be a relatively stationary stochastic process, i.e. the distributions of $a_{t_1},\dots,a_{t_n}$ and $a_{t_1+h},\dots,a_{t_n+h}$ are identical if $0 \leq t_i, t_i+h \leq t_o$ for $1 \leq i \leq n$ (n arbitrary). Then under the additional assumption

(*) a_t weakly continuous for $0 \leq t \leq t_o$,

by a simple passage to the limit a stationary extension a_t, $t \geq 0$, can be constructed from solutions of the analogous discrete time problem. Giving up the condition (*) the existence of such a process can be proved using the following general theorems:

1. If $\mathcal{T}$ is the class of all finite subsets T of $[0,\infty)$ with $\max T - \min T \leq t_o$, then to every family of consistent distributions p_T over R^T, $T \in \mathcal{T}$, there exists a stochastic process a_t, $t \geq 0$, such that the random variables a_t, $t \in T$, have the distribution p_T ($T \in \mathcal{T}$ arbitrary).

2. If a_t, $t \geq 0$, is any stochastic process, then under a boundedness condition (which for instance is fulfilled in the case of identically distributed a_t) there exists a stochastic process $\tilde{a}_t$, $t \geq 0$, such that the expectation of a bounded continuous function $f(a_t;\ t \geq 0)$ is always confined by the lower and upper limit of $E(f(a_{t+h};\ t \geq 0))$ for $h \to \infty$.