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Obituary

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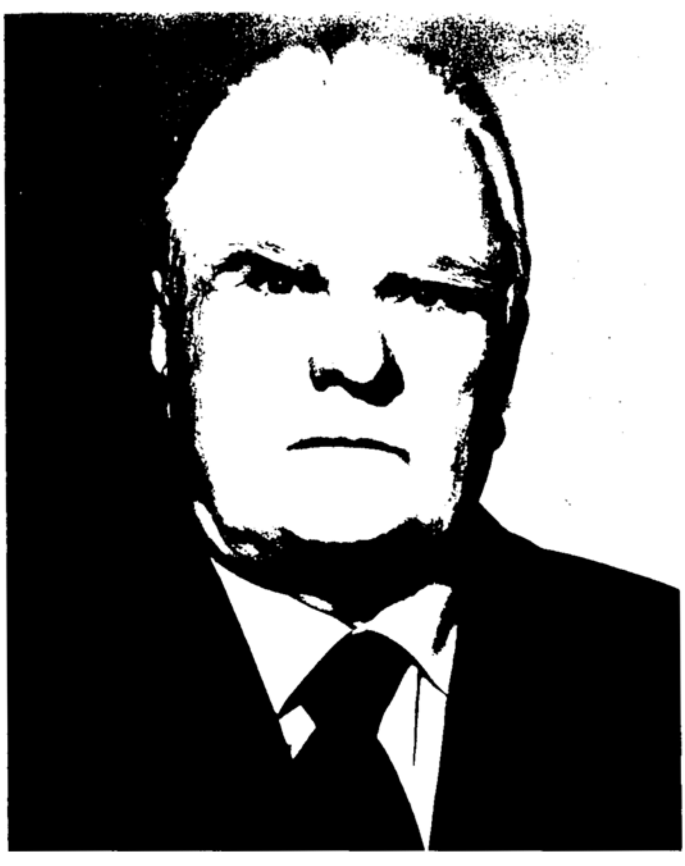
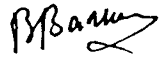
1908–1981





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OBITUARY

VICTOR VLADIMIROVICH VAGNER (1908-1981)

Boris M. Schein

In the night from August 15 to 16, 1981 Professor V. V. Wagner died in Brest, USSR. V. V. Vagner was born on November 4, 1908, in Saratov, the Russian Empire. In 1927 he finished the pedagogical professional school in Balashov (not far from Saratov). After that he worked as a teacher at high schools (including colonies for juvenile delinquents). Due to his social background, he had no access to the higher education. Already at that time he felt a keen interest in both mathematics and theoretical physics, so he learned mathematics and physics himself. After that, he managed to be admitted to final examinations at the Physico-Mathematical Faculty of the Moscow University. He passed the exams in 1930 and received a university diploma.

His main interest was in theoretical physics (mainly, in relativity), and he tried to become a graduate student in Moscow to write his dissertation under Academician E. Tamm. However, at that time, it was established that relativity was a pseudoscience (fortunately for Russian physics that result was forgotten later) and Tamm was not allowed to bring up students in this field. Instead, he had to supervise theses on physics of metals. Seeing Vagner's genuine interest in relativity Tamm gave him good advice to switch to differential geometry. "I hope this craziness will pass over," said Tamm. "I can wait. But you are young and you can't wait, these are your best years. Go to differential geometry to Professor Kagan. The very spirit of modern geometry is close to that of relativity. They think they understand physics and tell us, physicists, what to do. However, even they don't dare to tell mathematicians what to do. The atmosphere is better in mathematics."

In 1932 V. V. Vagner became a graduate student of Professor

1) Of course, the name is Wagner. However, I follow the AMS transliteration of the Cyrillic alphabet.

Veniamin F. Kagan (in 1923 this famous geometer moved to the Moscow University from Odessa, where he was professor of mathematics before the revolution.) His Candidate Dissertation contained important results on differential geometry of nonholonomic manifolds, and the Scientific Council of the Physico-Mathematical Faculty of Moscow University decided the quality of the thesis was far above that of a usual Candidate (the Candidate degree in Russia is a rough equivalent of the Ph.D.), and in 1935 Vagner was awarded the Doctor of Sciences degree (there is no equivalent to the Russian Doctor of Sciences in the West). After that Vagner moved to Saratov University where he organized a chair of geometry. He headed this chair until 1978 when he retired.

Vagner started his research activity at the time when differential geometry was rapidly developing and providing a part of mathematical apparatus for general relativity. At that time quite a few people believed that the importance of new geometric theories, having a general scientific character, transcended mathematics. All Vagner's research is connected with differential geometry and algebraization of its foundations. Algebraic systems considered by Vagner were usually related to differential geometric structures. His research activities were connected with the Seminar on Vector and Tensor Analysis at Moscow University.

Vagner's first papers are dedicated to theory of curvature of nonholonomic manifolds, special nonholonomic manifolds, and applications to concrete problems of theoretical mechanics. Later he generalized nonholonomic geometry, and these results permit the use of differential geometric methods for studying moving material systems with nonlinear nonholonomic connections.

In 1943-52 Vagner considered geometric methods for solving and investigating various variational problems. The concept of a local indicatrix of a metric determined by a variational problem was the cornerstone of his geometrical theories of variational problems. Vagner created geometric theories for regular variational problems with ordinary and partial first derivatives for unconditional extremum and for corresponding Lagrange problems. While studying local indicatrices, he considered theory of curves and surfaces of certain dimensions in centro-affine, affine, and projective spaces. He was the first to give geometric sufficient conditions for the existence of extremum in the Lagrange problem with first partial derivatives and to create a geometrical theory of the simple n-dimensional singular variational problem.

Since those connections which had been known could not be used for geometrization of calculus of variations, Vagner considered theory of connections in a "compound manifold" (in a differentiable foliation, if one uses more modern terminology) with all fibers having a same dimension. Up to now this is the most general theory of connections among those used in differential geometry (e.g. theory of connections for locally trivial foliations with a structural Lie group created by Charles Ehresmann is a particular case of Vagner's theory).

Vagner's papers on calculus of variations are connected with his papers on geometric theory of partial differential equations. Vagner gave clear and explicit definitions of tangent spaces of higher dimensions and of geometric objects, developed their general theory, and clarified an important role these concepts had in differential geometry. In 1945 (using almost forgotten results of an Italian mathematician P. Medolaghi), he established an interesting connection between differential geometric objects and Lie pseudogroups of transformations. Using this theory he described all simple differential geometric objects and gave a general method for constructing all geometric objects.

Vagner's papers devoted to algebra started to appear from 1950-1951. In 1949 he published a long appendix to the Russian translation of "The Foundations of Differential Geometry" by Oswald Veblen and J. H. C. Whitehead (the appendix is almost as long as the book itself; by the way, Vagner had nothing to do with the preface to the Russian translation, where "vicious, depraved, and idealistic" opinions of the authors on the subject of geometry are "unmasked"). Work on this appendix was decisive for the crystallization of the concept of inverse semigroup which was introduced later by Vagner. In the book, one of the most important concepts is that of a pseudogroup of transformations. A pseudogroup of transformations is a set Φ of non-empty partial one-to-one transformations of a set (or a space) such that for every $\phi\varepsilon\Phi$ the inverse transformation ϕ^{-1} also belongs to Φ, and whenever $\phi,\psi\varepsilon\Phi$ and the product $\phi\psi$ is defined, $\phi\psi\varepsilon\Phi$. While the product $\phi\psi$ was always a natural composition (or "superposition") of ϕ and ψ, different authors defined the domain of this operation in different ways. For example, $\phi\psi$ was defined to exist exactly when the image of ϕ was a subset of the domain of ψ. In the book of Veblen and Whitehead, $\phi\psi$ exists precisely when the image of ϕ coincides with the domain of ψ. Thus, from an abstract algebraic point of view, a pseudogroup was an algebra with a partial binary operation. Theory of algebras with partial operations was not a very well developed branch of algebra and this fact hindered the development of algebraic theory of pseudogroups.

Vagner was the first to overcome this psychological obstacle - considering the product $\phi\psi$ only if $\mathrm{Im}\phi \subseteq \mathrm{Dom}\psi$. He introduced the natural product of any two partial transformations - exactly in the sense as we understand it now. In fact, this multiplication of partial transformations was a special case of multiplication of binary relations, the fact which Vagner has, of course, acknowledged.

Thus, pseudogroups became algebras with an everywhere defined associative multiplication, i.e. they were semigroups. This is what led Vagner to his semigroup studies. In 1951 he had already found an abstract ("up to isomorphism") characterization of his pseudogroups (he called them "generalized groups" instead). They were regular semigroups with commuting idempotents. In 1952 he published his famous short note, "Generalized groups", where this new class of semigroups was introduced for the first time. In 1954 the same class of semigroups was independently introduced by G. B. Preston who called them "inverse semigroups."

Vagner's first publication on inverse semigroups in "Doklady Akademii Nauk" had to be short (the journal published papers not longer than 4 pages). However, at that time, he already had a fairly developed theory of inverse semigroups and lots of important results on general semigroups of transformations. In 1953 he published a long paper on inverse semigroups in "Matematičeskiĭ Sbornik." In my opinion, even now, almost 30 years after its first publication, this paper is grossly under-estimated (probably because a good many of researchers in inverse semigroups cannot read this paper in its original Russian). This paper contains a wealth of results, some (but only some!) of which have been rediscovered later (importance of the natural order relation, the smallest group congruence, the greatest indempotent-separating congruence, etc). Now the theory of inverse semigroups is one of the most vital and important parts of semigroup theory. [By the way, inverse semigroups, or rather a special class of them important for differential geometry, was later rediscovered by Ehresmann, who called them "inductive groupoids with pseudomultiplication." As I have already mentioned, Vagner and Ehresmann were interested in very similar problems. For example, Ehresmann was probably the first to recognize the importance of category theory for differential geometry. Later, Vagner came to very much the same conclusions and, from the mid-sixties, he was considering categories important for differential geometry.]

Together with inverse semigroups Vagner introduced the so-called "generalized grouds." If M is an n-dimensional differentiable manifold, then it has its "coordinate atlas" A which is a set of partial one-to-one maps of M into R^n, the n-dimensional arithmetic space. Each map $\kappa \in A$ is a local system of coordinates; if $m \in M$, then $\kappa(m) = (r_1 r_2, \ldots, r_n) \in R^n$ are coordinates of m in κ. The atlas A satisfies simple and natural conditions. It follows from them that if $\kappa, \lambda, \mu \in A$, then the product $\kappa \circ \lambda^{-1} \circ \mu$, which is a partial one-to-one map of M into R^n, can also be considered as a local system of coordinates. From this point of view, A is closed under the ternary operation

$$[\kappa\lambda\mu] = \kappa \circ \lambda^{-1} \circ \mu.$$

Any abstract algebra with a single ternary operation, which is isomorphic to a set of partial one-to-one maps between two sets closed under the operation $[\ldots]$, is called a generalized groud. Vagner gave a simple system of axioms for generalized grouds. They are characterized by the following identities:

$$[[x_1 x_2 x_3] x_4 x_5] = [x_1 [x_4 x_3 x_2] x_5] = [x_1 x_2 [x_3 x_4 x_5]],$$

$$[xxx] = x,$$

$$[[xyy] zz] = [[xzz] yy], \quad [xx[yyz]] = [yy[xxz]].$$

An example is any inverse semigroup considered under the operation $[xyz] = x \cdot y^{-1} \cdot z$. Such operations on groups were considered by R. Baer in 1929. The resulting system was called "Schar" by Baer. In his pioneering works on semigroup theory, A. K. Suschkewitsch in Russia translated "Schar" as "gruda" (which means "pile", "heap" in Russian). Obviously, Suschkewitsch exploited phonetic similarity between Russian "gruppa" (=group) and "gruda". Thus Vagner already had a coined term "gruda" when he considered his systems. He called these systems "generalized heaps." I try to use "generalized groud" instead. The word "groud" means nothing in English, sounds almost like "group", and has no connotations (which "heap" has).

At about the same time, Vagner developed a general theory of representations of semigroups by partial transformations. He submitted this paper to "Izvestiya Akademii Nauk SSSR." Vagner used symbolism of mathematical logic there (quantifiers, connectives of the sentential calculus, etc.). However, as the times were uncertain then, it was not clear if symbolic logic was in limbo or had ceased to be something very much akin to bourgeois pseudosciences (as genetics, cybernetics, etc. were). A contributing editor of the journal, a famous specialist in symbolic logic himself, suggested that Vagner replace logical symbols by plain words. After that, Vagner withdrew his paper and submitted it to "Matematičeskiĭ Sbornik" where (at that time) the atmosphere was more liberal. This paper appeared in 1956 and rendered a deep influence on the development of the theory of transformation semigroups. After that Vagner published a few other papers on inverse semigroups and general semigroups of partial transformations. From the beginning of the sixties (1962-63), he turned to other algebras connected with foundations of differential geometry. He considered algebraic systems with a partial binary (or ternary) operation, the so-called groudoids, generalized groudoids, semigroupoids, generalized groupoids, categories and semicategories. In the beginning of the seventies he returned to semigroups again considering the so-called "antigroups" (they are exactly fundamental inverse semigroups) and their ternary analogs. His last papers are dedicated to the so-called "filtral algebras" and to ordered sets which are akin to direct systems of sets and mappings.

More than 40 Ph.D. dissertations were written under Vagner's guidance. Formerly they were in differential geometry and calculus of variations, later in algebra. The author of these lines was the first one who got his Ph.D. in algebra (in 1962) under Vagner's supervision. Later we could organize two specializations at the chair of geometry: in differential geometry and general algebra, so our students could get their university diploma in mathematics (a rough equivalent of M.A.) having a fairly extensive knowledge of mathematical logic, axiomatic set theory, general algebra, groups, rings, lattices, and, of course, semigroups as an addition to their general mathematics background (advanced calculus, ordinary and partial differential equations, analytic and differential geometry, functional analysis, analytic functions, functions of real variable, numerical mathematics, theoretical mechanics, etc.). So our students did very well subsequently and were appreciated at graduate schools of various universities. The government appreciated Vagner's outstanding contribution, giving him the Lenin Order (the highest-ranking Soviet decoration) and even allowing him a few short visits to free countries to lecture (to the International Congress of Mathematicians at Edinburgh, to Paris and Rome).

In 1970-1975 Vagner served as a contributing editor of "Semigroup Forum". Due to efforts of Vagner and his former students, Saratov became one of the major centers of general algebra in the USSR. Then all that started to change due to general change of atmosphere in Soviet mathematics, but that is another story and I skip it here.

Personally, Vagner was a very agreeable man. What always surprised his interlocutors was his vast erudition. He could read in practically all European languages and even managed somehow to get books in foreign languages. Even specialists sought for Vagner's advice in various questions of philosophy, history, linguistics, literature.

I remember how, when I was a sophomore (and later), I was always accompanying Vagner, who used to walk home after his classes, and I visited his bachelor's apartment almost weekly. The amount of topics we discussed was innumerable. It was from him that I heard first about Freudian psychology (a taboo topic; S. Freud's books were on the index and could not be borrowed in the libraries), K. Menger's dimension theory (luckily, Menger's works might be read and I did that), "Viennese Circle" of philosophers, R. M. Rilke's poetry, German expressionist fiction, Schopenhauer's philosophy, comparative studies of Indo-European languages, and many many other things. I often wondered how it was in the pre-Gutenberg era, before printing presses were invented. I imagine that then such conversations as ours were the principal way of transfer of the knowledge from teachers to their disciples.

And also Vagner was a very decent man. To some people this statement may sound shallow, trivial, almost ridiculous. Blessed are the people who have never lived in circumstances when common human decency almost amounts to a heroic deed.

Farewell, Victor Vladimirovich. Thank you.

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LIST OF PUBLICATIONS OF V.V. VAGNER

1. Sur la géométrie différentielle des multiplicités anholonomes. Труды семин. по векторн. и тензорн. анализу, 2—3 (1935), 269—318.
2. Двухмерное пространство с кубической метрикой. Саратов, Учен. зап. ун-та, сер. физ.-матем., 1 (14) : 1 (1938), 29—40.
3. Геометрия пространства конфигураций твердого тела, вращающегося вокруг неподвижной точки. Саратов, Учен. зап. ун-та, сер. физ.-матем., 1 (14) : 2 (1938), 34—59.
4. Неголономные многообразия, для которых дифференциальные уравнения линий стационарной длины имеют первый линейный интеграл. Саратов, Учен. зап. ун-та, сер. физ.-матем., 1 (14) : 2 (1938), 60—66.
5. A generalization of non-holonomic manifolds in Finslerian Space. Саратов, Учен. зап. ун-та, сер. физ.-матем., 1 (14): 2 (1938), 67—97.
6. Римановы пространства с постоянными символами Кристоффеля. Саратов, Учен. зап. ун-та, сер. физ.-матем., 1 (14) : 2 (1938), 98—104.
7. Über Berwald'sche Räume. Матем. сб., 3 (45) (1938), 655—662.
8. Über V_3^2 von der Krümmung Null in R_3. Матем. сб., 4 (46) (1938), 333—338.
9. On the geometrical interpretation of the curvature vector of a non-holonomic V_3^2 in the three-dimensional Euclidean space. Матем. сб., 4 (46) (1938), 339—356.
10. Дифференциальная геометрия неголономных многообразий. Казань, VIII Международн. конкурс на соискание премий им. Лобачевского (1940), 195—262.
11. Differential geometry of non-linear non-holonomic manifolds in the three-dimensional Eucladean space. Матем. сб., 8 (50) (1940), 3—40.
12. Геометрия $(n-1)$-мерного неголономного многообразия в n-мерном пространстве. Труды семин. по векторн. и тензорн. анализу, 5 (1941), 173—225.
13. К вопросу об определении инвариантной характеристики поверхностей Лиувилля. Труды семин. по векторн. и тензорн. анализу, 5 (1941), 246—249.
14. Теория конгруэнций кругов и геометрия неголономного $V_3^2R_3$. Труды семин. по векторн. и тензорн. анализу, 5 (1941), 271—283.
15. Геометрическая интерпретация движения неголономных динамических систем. Труды семин. по векторн. и тензорн. анализу, 5 (1941), 301—327.
16. О группе голономии Картана для поверхностей. ДАН, 37 (1942), 7—10.
17. Differential geometry of the family of $R_k$sin R_n and of the family of totally geodesic S_{k-1} sin S_{n-1} of positive curvature. Матем. сб., 10 (52) (1942), 165—212.
18. Об обобщенных пространствах Бервальда. ДАН, 39 (1943), 3—5.
19. Двухмерные пространства Финслера с конечными непрерывными группами голономии. ДАН, 39 (1943), 223—226.
20. Абсолютная производная поля локального геометрического обьекта в составном многообразии. ДАН, 40 (1943), 99—102.
21. The inner geometry of non-linear non-holonomic manifolds. Матем. сб., 13 (55) (1943), 135—167.
22. Гомологические преобразования метрики Финслера. ДАН, 46 (1945), 287—290.
23. Обобщение тождеств Риччи и Бианки для связности в составном многообразии. ДАН, 46 (1945), 335—338.
24. Теория геометрических обьектов и теория конечных и бесконечных непрерывных групп преобразований. ДАН, 46 (1945), 383—386.
25. Геометрия поля локальных центральных плоских кривых в X_3. ДАН, 48 (1945), 245—248.
26. Геометрия поля локальных кривых в X_3 и простейший случай задачи Лагранжа в вариационном исчислении. ДАН, 48 (1945), 383—386.
27. Геометрическая теория кратных интегралов. УМН, 1 : 5—6 (1946), 239—240.
28. Постоянные поля локальных геометрических обьектов в составном многообразии с линейной связностью. ДАН, 53 (1946), 187—190.
29. О достаточном условии в задаче Лагранжа для кратных интегралов. ДАН, 54 (1946), 483—486.
30. Геометрия пространства с ареальной метрикой и ее приложения к вариационному исчислению. Матем. сб., 19 (61) (1946), 341—404.
31. О геометрической интерпретации экстремальных поверхностей в задаче Лагранжа для кратных интегралов. ДАН, 55 (1947), 91—94.
32. **Геометрия n-мерного пространства с m-мерной римановой метрикой и ее приложения к вариационному исчислению. Матем. сб., 20(62) (1947), 3—25.**
33. **Геометрическая теория простейшей n-мерной сингулярной задачи вариационного исчисления. Матем. сб., 21 (63) (1947), 321—362 .**

34. О понятии индикатрисы в теории дифференциальных уравнений. ДАН, 57 (1947), 219—222.
35. О понятии индикатрисы в теории дифференциальных уравнений в частных производных. УМН, 2 : 2 (18) (1947), 188—189.
36. Theory of field of local (n-2)-dimensional surfaces X_n and its application to the problem of Lagrange in the calculus of variations. Ann. of Math., 49 (1948), 141—188.
37. Геометрическая теория простейшей n-мерной сингулярной задачи вариационного исчисления. Матем. сб., 21 (63) (1947), 321—364.
38. Теория поля локальных конических кривых и локальных конических поверхностей в X_3 и ее приложение к вариационному исчислению и теории дифференциальных уравнений в частных производных. Труды семин. по векторн. и тензорн. анализу 6 (1948), 257—364.
39. Теория дифференциальных объектов и основания дифференциальной геометрии. УМН, 3 : 4 (26) (1948), 153—154.
40. Геометрия Финслера как теория поля локальных гиперповерхностей в X_n. Труды семин. по векторн. и тензорн. анализу 7 (1949), 65—166.
41. Классификация линейных связностей в составном многообразии $X_{n+(1)}$ по их группам голономии. Труды семин. по векторн. и тензорн. анализу, 7 (1949), 205—226.
42. О вмещении поля локальных поверхностей в X_n в постоянное поле поверхностей в аффинном пространстве. ДАН, 66 (1949), 785—788.
43. Теория поля локальных гиперполос в X_n и ее приложения к механике системы с нелинейными неголономными связями. ДАН, 66 (1949), 1033—1036.
44. Классификация простых геометрических дифференциальных обьектов. ДАН, 69 (1949), 293—296.
45. Теория дифференциальных объектов и основания дифференциальной геометрии (Дополнение к кн. «О. Веблен и Дж. Уайтхед. Основания дифференциальной геометрии»). М. (1949), 135—223.
46. Теория составного многообразия. Труды семин. по векторн. и тензорн. анализу, 8 (1950), 11—72.
47. Геометрия пространства с гипераревльной метрикой как теория поля локальных гиперповерхностей в составном многообразии. Труды семин. по векторн. и тензорн. анализу, 8 (1950), 144—196.
48. Теория поля локальных гиперполос. Труды семин. по векторн. и тензорн. анализу, 8 (1950), 197—272.
49. К теории псевдогрупп преобразований. ДАН, 72 (1950), 453—456.
50. Классификация простых геометрических дифференциальных объектов. УМН, 5 : 1 (35) (1950), 213—214.
51. Геометрия обобщенных пространств Картана и теория геометрических дифференциальных обьектов. ДАН, 77 (1951), 777—780.
52. Алгебраическая теория дифференциальных групп. ДАН, 80 (1951), 845—848.
53. Тернарная алгебраическая операция в теории координатных структур. ДАН, 81 (1951), 981—984.
54. К теории частичных преобразований. ДАН, 84 (1952), 653—656.
55. Обобщенные группы. УМН, 7 : 2 (48) (1952), 146.
56. Обобщенные группы. ДАН, 84 (1952), 1119—1122.
57. Общая аффинная и центрально-проективная геометрия гиперповерхности в центрально-аффинном пространстве и ее приложения к геометрической теории преобразований Каратеодори в вариационном исчислении. Труды семин. по векторн. и тензорн. анализу, 9 (1952), 75—145.
58. Теория обобщенных груд и обобщенных групп. Матем. сб., 32 (74) (1953), 545—632.
59. Алгебраические вопросы оснований дифференциальной геометрии. Казань, Учен. зап. ун-та, 115 : 10 (1955), 3—4.
60. Дифференциально-геометрические методы в вариационном исчислении. Казань, Учен. зап. ун-та, 115: 10 (1956), 4—7.
61. Алгебраическая теория касательных пространств высших порядков. Труды семин. по векторн. и тензорн. анализу, 10 (1956), 31—88.
62. Обобщенные груды, приводимые к обобщенным группам. УМЖ, 8 (1956), 235—253.
63. Обобщенные груды, приводимые к обобщенным группам. Научный ежегодник Саратовского ун-та за 1954 г. Саратов (1955). 668—669.
64. Обобщенные груды и обобщенные группы. Труды 3-го Всесоюзного матем. съезда, т. 1, М. (1956), 18—20.
65. Теория поля локальных поверхностей. Труды 3-го Всесоюзного матем. съезда, т. 2. М. (1956). 57—60.
66. Представление упорядоченных полугрупп. Матем. сб., 38 (80) (1956), 203—240.
67. Полугруппы частичных преобразований с симметричным отношением транзитивности. Изв. высших учебных заведений, Математика, 1 (1957), 81—88.
68. Вариационное исчисление как теория поля центральных полуконусов. Саратов, Науч. ежегодник ун-та за 1955 г., Мех.-матем. фак. (1959), 27—34.
69. Представление обобщенных груд. УМЖ, 11:3 (1959), 231—242.
70. Полугруппы, ассоциированные с обобщенной грудой. Матем. сб., 52:1 (1960), 597—628.
71. Трансформативные полугруппы. Изв. вузов, Математика, 4 (1960), 36—48.
72. Обобщенные груды и обобщенные группы с транзитивным отношением совместности. Саратов, Уч. зап. ун-та, 70 (1961), 25—39.
73. Рестриктивные полугруппы. Изв. вузов, Математика, 6 (1962), 19—27.
74. Generalised groups of partial transformations. Abstracts of short communications. Intern. Congress Math. Stockholm (1962), 3.31.

75. К теории струй Эресмана. ДАН СССР, 152:1 (1963), 17—19.
76. Основания дифференциальной геометрии и современная алгебра. Тр. 4-го Всесоюзн. матем. съезда, 1 (1963), 17—29.
77. Полупрямое произведение бинарных отношений и бинарно разложимые полугруппы. Изв. вузов, Математика, 1 (1963), 21—32.
78. К теории грудоидов. Изв. вузов, Математика, 5 (1965), 31—42.
79. Сдвиги в грудоиде. Изв. вузов, Математика, 6 (1965), 37—47.
80. Теория отношений и алгебра частичных отображений. В сб. «Теория полугрупп и ее прилож.», 1, Саратов (1965), 3—178.
81. Geometria del calcolo delle variazioni. II Centro Intern. Matem. Estivo (C.I.M.E.), Roma (1965), 172-XVIII.
82. К алгебраической теории координатных атласов. М., Тр. семинара по векторн. и тензорн. анализу, 13 (1966), 510—563.
83. К теории обобщенных грудоидов. Изв. вузов, Математика, 6 (1966), 25—39.
84. Теория грудоидов и ее приложения. Тезисы кр. науч. сообщений Междунар. конгресса математиков, Секция 2, М. (1966), 35.
85. Диаграммируемые полугруппоиды и обобщенные группоиды. Изв. вузов, Математика, 10 (1967), 11—23.
86. К теории регулярных бинарных отношений между элементами частичных оперативов. Изв. вузов, Математика, 4 (1967), 26—39.
87. Обобщенные груды и канонически упорядоченные грудоиды. Изв. вузов, Математика. 3 (1967). 8—19.
88. **Алгебраические вопросы общей теории частичных связностей в расслоённых пространствах. Изв. вузов, Математика,** 1968, no. 11, 26-40.
89. **К алгебраической теории координатных атласов, §§3-4. Тр.семинара по векторн. и тензорн. анализу,** 14(1968), 229-281.
90. **Бинарно разложимые категории. Изв. вузов, Математика,** 1969, no.4, 3-13.
91. **Локализованные категории. Изв. вузов, Математика** , 1969, no.11, 24-36.
92. **К теории антигрупп. Изв. вузов, Математика** , 1971, no. 4, 3-15.
93. b **-простые представления антигрупп. Изв. вузов, Математика** , 1971, no. 9, 18-29.
94. **К теории инволютированных полугрупп. Изв. вузов, Математика** , 1971, no. 10, 24-35.
95. **К теории антигруд. Изв. вузов, Математика** , 1972, no. 4, 18-31.
96. b **-простые представления антигруд. Изв. вузов, Математика** , 1972, no. 4, 18-31.
97. **Псевдополугруппы и полугруппы с преобразованиями, Изв. вузов, Математика,** 1973, no. 4, 8-15.
98. **Фильтральные алгебры. Изв. вузов, Математика** , 1973, no. 6, 8-18.
99. **Интроретрактивные факторизованные и бифакторизованные упорядоченные множества. Изв. вузов, Математика** 1978, no.6, 36-50.
100. **Интроретрактивные факторизованные упорядоченные множества и проективные системы множеств. Изв. вузов, Математика** , 1978, no. 12, 18-32.
101. **Ретрактивные суммы внешних оперативов и их естественное расширение. Изв. вузов, Математика,** 1979, no. 2, 7-24.
102. **К теории обобщённых колец, модулей и линейных алгебр. Изв. вузов, Математика,** 1979, no.3, 12-27.
103. **Алгебра бинарных отношений и её приложения в дифференциальной геометрии. Дифференц. Геометрия, Саратов,** 4(1979), 15-131.

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