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CORRIGENDUM

K. A. Overholser, N. A. Lomangino, T. R. Harris, J. D. Bradley and S. Bosan (1994). Deduction of pulmonary microvascular hematocrit from indicator dilution curves, *Bull. math. Biol.* **56**, 225–247.

Professors Andreas Schwab and Carl Goresky of McGill University have pointed out an error in our recent paper on the deduction of microvascular hematocrit from indicator dilution curves (Overholser *et al.*, 1994). In this letter we correct that mistake and present a revised formula for computation of hematocrit reduction ratio. Revised equations will be denoted by primes; thus, equation (1) in the original paper should be replaced by equation (1)′ as presented here.

The necessary revisions arise from changes in equations (1) and (2) expressing conservation of tracer albumin in phases 1 and 2 of the microvasculature, where phase 1 refers to the intravascular volume accessible to both red cells and plasma and phase 2 denotes a plasma-only volume from which red cells are excluded. The revised equations are:

$$\frac{\partial C_1}{\partial t} + u_1 \frac{\partial C_1}{\partial x} = -\frac{KS}{V_1}\left(\frac{C_1}{1-H_1} - C_2\right) \tag{1$'$}$$

$$\frac{\partial C_2}{\partial t} + u_2 \frac{\partial C_2}{\partial x} = \frac{KS}{V_2}\left(\frac{C_1}{1-H_1} - C_2\right), \tag{2$'$}$$

where $C_1(t, x)$ and $C_2(t, x)$ are concentrations of albumin tracer in phases 1 and 2, respectively, H_1 is the hematocrit in phase 1, u_1 and u_2 are the mean flow velocities of the two phases relative to the stationary vessel walls and KS is the product of an interphase exchange coefficient K and the interphase surface area S. The changes arise on the right-hand side of equations (1)′ and (2)′. In the original paper, we had expressed the driving force for mass transfer as $(C_1 - C_2)$. The revised equations reflect the fact that the driving force from phase 1 should be the concentration in the plasma portion of that phase—thus the expression $C_1/(1-H_1)$ replaces C_1. On the left-hand side of equation (1)′, C_1 remains as in the original version of the equation, since C_1 refers to tracer albumin concentration in the mixture of red cells and plasma, as measured in the laboratory.

As described in the original paper, the above system of equations can be solved under the assumption of infinite K and uniform large-vessel transit time to yield an expression for albumin concentration at the organ outlet. Revised to reflect the changes in equations (1)′ and (2)′, that solution is:

$$C_{1,\text{organ}}[t-\tau]=\frac{1+\dfrac{u_2}{u_1}\dfrac{V_2}{V_1(1-H_1)}}{1+\dfrac{V_2}{V_1(1-H_1)}}\,C_{\text{R,organ}}\left[\frac{(t-\tau)\left(1+\dfrac{u_2}{u_1}\dfrac{V_2}{V_1(1-H_1)}\right)}{1+\dfrac{V_2}{V_1(1-H_2)}}\right] \qquad (24)'$$

(In obtaining equation (24)′, it was necessary to revise equations (11)–(14) and (16) of the original paper to include the factor $(1-H_1)$, but we do not show those revisions here.) Equation (24)′ differs from its counterpart in the earlier paper only in the appearance of the factor $(1-H_1)$ associated with phase 1 volume V_1. The equation may be written in the following form, where h_r' is defined by comparing equations (35)′ and (24)′, $C_{1,\text{organ}}$ and $C_{\text{R,organ}}$ are the concentrations of albumin tracer and non-diffusing reference tracer at the organ outlet, and τ is the tracer appearance time:

$$C_{1,\text{organ}}[t-\tau]=h_r' C_{\text{R,organ}}[(t-\tau)h_r']. \qquad (35)'$$

The parameter h_r' may be found by fitting equation (35)′ to experimental measurements of $C_{1,\text{organ}}$ and $C_{\text{R,organ}}$, just as in the original paper. However, h_r' is no longer equal to the hematocrit reduction ratio. The hematocrit reduction ratio is $h_r \equiv H_{\text{mv}}/H_{\text{lv}}$, the ratio of microvascular to large-vessel hematocrit. It is the quantity we seek. Comparison of equation (34) of the original paper with the present definition of h_r' enables us to deduce the following relationship between h_r and h_r':

$$h_r=\frac{h_r'}{(1+H_{\text{lv}}h_r'-H_{\text{lv}})}. \qquad (53)'$$

The corrected procedure for obtaining the hematocrit reduction ratio is to first

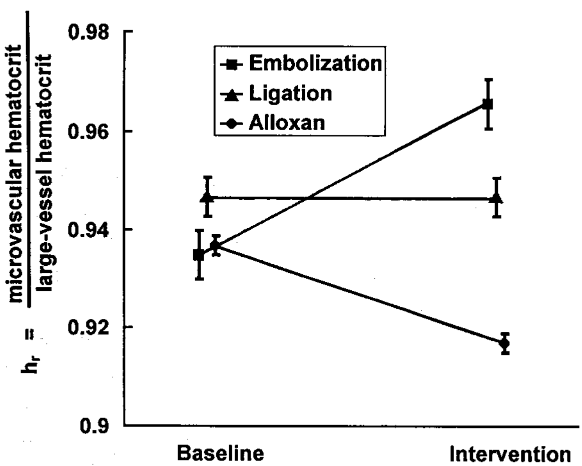

Figure 4′. Corrected hematocrit ratios deduced using the revised transport model, equations (35)′ and (53)′. These values represent the ratio of microvascular to large vessel hematocrit consistent with the observed tracer results. The interpretation of these results is the same as for Figure 4 of Overholser *et al.* (1994).

compute h_r' by fitting equation (35)′ to experimental tracer data, then to use equation (53)′ along with measured values of large-vessel hematocrit H_{lv} to calculate the desired hematocrit reduction ratio h_r. (For example, if the measured h_r' is 0.9 and H_{lv} is 0.40, then the true reduction ratio h_r is 0.937.) Using this procedure, we re-analyzed our own experimental measurements of hematocrit reduction ratio in isolated, perfused dog lungs under baseline conditions and after glass bead embolization, alloxan infusion and ligation of portions of the lung. The results are shown in Fig. 4′. which should replace Fig. 4 of the original paper.

For typical values of large-vessel hematocrit, the revised procedure results in values of h_r somewhat higher than in the original paper and somewhat closer to values of hematocrit reduction ratio calculated from mean transit times. Except for that observation and the corrections described above, the methodology, results and conclusions of the original paper remain the same.

We thank Drs Schwab and Goresky for their help in identifying this problem and in correcting the theory.

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