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Erratum: New Lower Bounds on the Self-Avoiding-Walk Connective Constant[1]

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Due to a coding error in our calculation of the integral

$$C_0\left(0, x; \frac{1}{2d}\right) = \int_{[-\pi,\pi]^d} \frac{d^d k}{(2\pi)^d} \frac{e^{ik \cdot x}}{1 - d^{-1} \sum_{j=1}^{d} \cos k_j} \tag{1}$$

the values given in the second column of Table IV contain errors in their final four digits. The correct values are given here in Table I. The error is negligible ($<10^{-17}$) for $d \geqslant 4$. These errors do not affect other numerical values quoted, apart from a change in the final digit of the $d=3$, $(0, 0)$ entry of Table II, whose correct value is slightly improved to 3.956776.

The integral $C_0(0, 0; 1/(2d))$ is equal to the expected number of returns to the origin of the simple random walk in $\mathbb{Z}^d$. For $d=3$, this has been computed exactly in terms of an elliptic integral in ref. 1 and in terms of the gamma function in ref. 2. The latter contains an erroneous numerical prefactor, which when corrected as indicated in ref. 3 yields the exact expression

$$C_0\left(0, 0; \frac{1}{6}\right) = \frac{\sqrt{6}}{32\pi^3} \Gamma\left(\frac{1}{24}\right) \Gamma\left(\frac{5}{24}\right) \Gamma\left(\frac{7}{24}\right) \Gamma\left(\frac{11}{24}\right) \qquad (d=3) \tag{2}$$

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Table I. Corrected Values of $C_0(0, x; 1/(2d))$ for $d=3$, Rounded to Ten Digits after the Decimal

x	$C_0(0, x; 1/(2d))$ for $d=3$
(0, 0, 0)	1.5163860592
(1, 0, 0)	0.5163860592
(1, 1, 0)	0.3311486021
(1, 1, 1)	0.2614701264
(2, 0, 0)	0.2573358873
(2, 1, 0)	0.2155896208
(2, 1, 1)	0.1917916506
(2, 2, 0)	0.1683310356
(2, 2, 1)	0.1569524128
(3, 0, 0)	0.1652707810
(3, 1, 0)	0.1531388988
(3, 1, 1)	0.1441957103
(3, 2, 0)	0.1324510731
(4, 0, 0)	0.1217332037
(4, 1, 0)	0.1171304972
(5, 0, 0)	0.0966064520

Evaluation of the right side using both Mathematica and Maple gives 1.51638605915197801815..., which agrees with our entry here in Table I and with the value 1.516386059 from ref. 1 but disagrees with the final digits of the value 1.516386059137... quoted on p. 126 of ref. 3.

Finally, in an unrelated matter, there is a typographical error in the $d=3$, $(2, 2)_{\rm opt}$ entry of Table II, whose correct value is 4.476141.

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