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CORRECTION AND COMPLEMENTS TO "ON THE SYMMETRIC AND REES ALGEBRAS OF AN IDEAL"

GIUSEPPE VALLA

In [2, pag. 248] I gave the definition of "generalized almost complete intersection ideal" and I proved some properties of such ideals (Lemma 3.3, Lemma 3.4 and Proposition 3.5). Unfortunately, for the proof of Lemma 3.3 I referred to a result of Vasconcelos (see [4]) which is in fact correct; but not all assumptions needed in Vasconcelos' paper were fullfilled. Moreover, Lemma 3.3, Lemma3.4 and Proposition 3.5 are not true.

To fill in the gap, I must replace the above definition and results by the following.

Definition 3.3. *A prime ideal* I *is said to be a "generalized almost complete intersection" (g.a.c.i.) if* $I=(J,x)$ *where* J *is an ideal generated by a regular sequence such that* $J:x=J:x^2$.

The above definition is motivated by the following theorem where $\mu(I)$ denotes the smallest number of elements that can generate the ideal I.

Theorem 3.4. *Let* I *be a prime ideal of the ring* R *such that* $\mu(I)\leq \mathrm{grade}(I)+1$.

If a) $h.d._R I<\infty$

or

b) R *is a Cohen-Macaulay ring and* R_I *is regular,*

then I *is a generalized almost complete intersection.*

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Proof. Let $t=\text{grade}(I)$, then $t\le \text{height}(I)\le \mu(I)\le t+1$; hence if $\text{height}(I)=\mu(I)$, I is a complete intersection (see [3, Theorem 2.6]), and in any case we have $\text{height}(I)=t$. Furthermore, since R_I is a regular local ring of dimension t, using a result of Lazard (see [1, Theorem 1]), we can find an ideal J generated by a regular sequences of t elements and an element x in R such that $I=(J,x)$ and $IR_I=JR_I$. We are going to prove that $J=I\cap(J:I)$ and then the conclusion easily follows. Let $a\in I\cap(J:I)$ and $\mathfrak{q}$ be a $\mathfrak{p}$-primary component of J; if $I\not\subseteq\mathfrak{p}$ and $b\in I$, $b\notin\mathfrak{p}$, we have $ab\in J\subseteq\mathfrak{q}$, hence $a\in\mathfrak{q}$. Therefore it suffices to prove that if $\mathfrak{p}\supseteq I$ is an associated prime of J, then $\mathfrak{p}=I$. This is obvious if R is Cohen-Macaulay, since in this case every ideal generated by a regular sequences is unmixed. Hence assume that $\text{h.d.}_R I<\infty$; then $\text{h.d.}_{R_\mathfrak{p}}(IR_\mathfrak{p})<\infty$ and we have $\text{depth } R_\mathfrak{p}=\text{h.d.}_{R_\mathfrak{p}}(R_\mathfrak{p}/IR_\mathfrak{p})+\text{depth}(R_\mathfrak{p}/IR_\mathfrak{p})$. But since $\mathfrak{p}$ is an associated prime of an ideal generated by a regular sequences of t elements, we have $t=\text{depth } R_\mathfrak{p}=\text{grade}(IR_\mathfrak{p})$; furthermore $\text{grade}(IR_\mathfrak{p})\le\text{h.d.}_{R_\mathfrak{p}}(R_\mathfrak{p}/IR_\mathfrak{p})$. Hence we get $\text{depth}(R_\mathfrak{p}/IR_\mathfrak{p})=0$ and this clearly implies $\mathfrak{p}=I$.

Proposition 3.5. Let I be a g.a.c.i.. Then $S_R(I)$ is isomorphic to $\text{pow}_R(I)$

Proof. Let $I=(J,x)$ where J is generated by a regular sequences and $J:x=J:x^2$. By repeated use of Theorem 2.1 of [2], it is enough to prove that $S_{R/J}(I/J)\simeq\text{pow}_{R/J}(I/J)$; but it is almost trivial that, for a principal ideal (x) in a ring A, we have $S_A(x)\simeq\text{pow}_A(x)$ if and only if $0:x=0:x^2$. This gives the conclusion of the proposition.

REFERENCES

[1] D.Lazard, Suites regulieres dans les ideaux determinantiels; Comm. in Al. 4 (1976), 327-340

[2] G.Valla, On the symmetric and Rees algebras of an ideal, Man.Math. 30 (1980), 239-255

[3] G.Valla, Remarks on generalized analytic independence, Math. Proc. Cam. Phil. Soc. 85 (1979), 281-289

[4] W.Vasconcelos, On the homology of I/I^2, Comm. in Al. 6 (1978), 1801-1809

GIUSEPPE VALLA

ISTITUTO DI MATEMATICA

UNIVERSITA DI GENOVA

GENOVA, ITALIA

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