

Corrigendum to Self-Synchronization of Populations of Nonlinear Oscillators in the Thermodynamic Limit¹

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1. Due to a numerical instability, the true linear stability line for the stationary solution (2.3) differs near the A axis from the dashed curve in Fig. 1. For small θ , (2.6a) is approximately given by $A + 1 = \theta^2 - \frac{1}{2}\theta^4 + O(\theta^6)$. This implies that the dashed and solid lines in Fig. 1 should confound each other near the minimum $A = -1$, $\theta = 0$. All phase transitions in the paper are thus second order.

2. The Liapunov functional for the time-periodic probability density $p_0(t, \mathbf{x})$ can be constructed by using Shiino's results.⁽¹⁾ Instead of the relative entropy $H(p, p_0)$ as in Eq. (2.29), the Liapunov functional is $H(p, Q)$, where

$$Q(t, \mathbf{x}) = \exp\{-\phi(\mathbf{x}) + JF^{-1}[2\mathbf{x} \cdot \langle \mathbf{x}(t) \rangle - \langle \mathbf{x}(t) \rangle^2]\}$$

Here $\langle \mathbf{x}(t) \rangle$ is the mean value corresponding to $p(t, \mathbf{x})$, the solution of Eq. (1.2). The term $H(p, Q)$ is a constant for $p = p_s, p_0$ and it satisfies

$$H(p, Q) \leq \int \exp[(\alpha - \mathbf{x}^2/2) \mathbf{x}^2/F] d\mathbf{x} - 1$$
$$dH(p, Q)/dt = \frac{1}{2} F \int p(\nabla \ln p/Q)^2 d\mathbf{x} \geq 0$$

3. Minor corrections are: In Eq. (2.6a) replace θ by $|\theta|$. In Eq. (2.28) replace ψ_1 by ϕ .

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References 2 and 3 also deal with mean-field models having stable time-periodic probability densities. Shiino⁽²⁾ also studies our Eq. (1.3), while Schentzow⁽³⁾ analyzes the case of a mean-field Brusselator with additive white noise. We are grateful to Prof. D. Dawson for bringing these works to our attention.

REFERENCES

1. M. Shiino, *Phys. Lett.* **112A**:302 (1985).
2. M. Shiino, *Phys. Lett.* **111A**:396 (1985).
3. M. Schentzow, *Prob. Theory Rel. Fields* **72**:425 (1986).