THE SHAPE OF ASTEROIDS: THEORETICAL CONSIDERATIONS

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Abstract. Theoretical consideration and observations by other authors indicate that small asteroids are capable of maintaining irregular shapes, notably the shape of a cigar and even of a dumb-bell. This paper presents a model which describes the changes in the shape of an asteroid due to collisions of smaller objects (meteoroids) with the asteroid. The following assumptions must be approximately valid: (1) collisions are not uncommon; (2) collisions between a (relatively) large asteroid and small objects (meteroids) are more common than collisions between asteroids; (3) the cumulative probability of the collision of a meteoroid on a point on the surface of an asteroid is proportional to the zenith angle of the horizon as seen by that point; (4) obliquities of all but the major asteroids are random, so that there is not a preferred side on which collisions occur; (5) a considerable percentage of collision ejecta achieves escape velocity; and (6) the rate of erosion of each point on the surface of an asteroid is proportional to the cumulative probability of collision.

Generalized conclusions that are obtained from the computer running of the model indicate that both cigars and dumb-bells are possible outcomes. Sharp corners are smoothed away, the radius of curvature of rounded surfaces increases to the point of going from convexity to concavity, and flat surfaces develop into gentle concavities.

Collisions of an asteroid with an object of sufficient size such that the impact causes the breakage of the asteroid or the formation of a large crater, are not discussed in this paper. Previous work, however, suggests that the crater will undergo geomorphological changes of different geometry than a similar crater on the Moon.

1. Introduction

Since Giuseppe Piazzi discovered Ceres at the very beginning of the 19th century a considerable amount of information has been gathered on the orbital and rotational characteristics of almost two thousand asteroids. Several textbooks, such as Hartmann (1973) give extensive treatments to the subject. The situation is quite different concerning the shape of asteroids. Their size and distance are such that direct observations are very difficult and, until these are obtained from space craft missions, theoretical approaches and analysis of the asteroid 'light curves' are the only available methods.

Fujiware and others (1978) have conducted laboratory impact experiments using cylindrical polycarbonate projectiles. They conclude that, as long as the asteroid is sufficiently small so that the internal stress is less than the yield stress (few hundred kilometers in radius) the ideal shape is approximated by the axes having a ratio of $2:\sqrt{2}:1$. By using the method of studying the 'light curves,' it is possible to obtain information not only about their period of revolution, but also on their shapes (Kopal, 1970). Zellner (1976) deduces a shape of Eros of $13 \times 15 \times 36$ km from photometric, radar and occultation data. Other studies seem to indicate roughly either a cylindrical bar with semi-spherical caps at both ends or a triaxial ellipsoid (Dunlap, 1974, 1976; Dunlap and Gehrels, 1969; Sather, 1976).

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Theoretical work by Johnson and McGetchin (1973) deals with the effects of static loading and creep deformation. They conclude that small bodies of non-icey composition could maintain significantly non-spherical shapes. Similar conclusions are reached by Sotar and Harris (1977).

This paper presents some hypothetical shapes of asteroids developed from a model which utilizes the collisional history of the asteroid after its formation.

The Development of the Model

Evidence exists that collisions among asteroids are not uncommon. Hartmann (1973) summarizes a possible history and evolution of the asteroids as follows. Four and a half billion (10^9) years ago planetesimals were growing to become the present planets. In the zone between Mars and Jupiter, however, the process did not reach completion. The planetesimals, instead of accreting, began to destroy each other. The most common diameter at this time was about 60 km. Many of these asteroids collided and fragmented, producing the smaller objects which exist today. Why the accretion regime was replaced by a destruction regime is not clear. One possibility is that Jupiter perturbed the orbits by increasing the orbital velocities. Another possibility is that the nebula dissipated.

Even if the complete picture is not available, it seems generally accepted (Kuiper, 1950) that the more common small asteroids are mostly fragments, and that the larger ones may have escaped collision and thus be original planetesimals.

The total number of small observable asteroids larger than mass m is a power function of m (as reported by Hartmann 1973). The exact functionality does not need to be known, as long as the relationship that smaller asteroids (meteoroids) are much more abundant than larger ones is valid.

This model refers only to the effects of collisions between 'large' asteroids (from now on simply referred to as asteroids) and 'small' asteroids (from now on simply referred to as meteoroids). The asteroid is assumed to be subjected to a 'rain' of meteoroids. Each meteoroid will cause a crater on impact upon the asteroid, but each crater is many orders of magnitude smaller than the asteroid and therefore individually invisible as such. Throughout time, this 'rain' of meteoroids can be considered continuous and the integration of the craters a continuous erosional process.

The surface of the asteroid may be covered by a regolith, but this will not stop the overall erosion, provided that a sufficient percentage of the material ejected reaches escape velocity.

The next assumption that must be made is that the asteroids have random obliquities and periods of rotation, so that the flux of meteoroids, *as seen by the asteroid*, is isotropic. Even if bodies are concentrated on the plane of the ecliptic, under the above circumstances no preferred direction of erosion should exist.

If the above assumptions are accepted, at least as a first approximation, then the shape of the asteroid will be modified on the basis of how much meteoroidal flux impinges on any point of its surface, and the flux will be a function of the solid angle of the sky seen

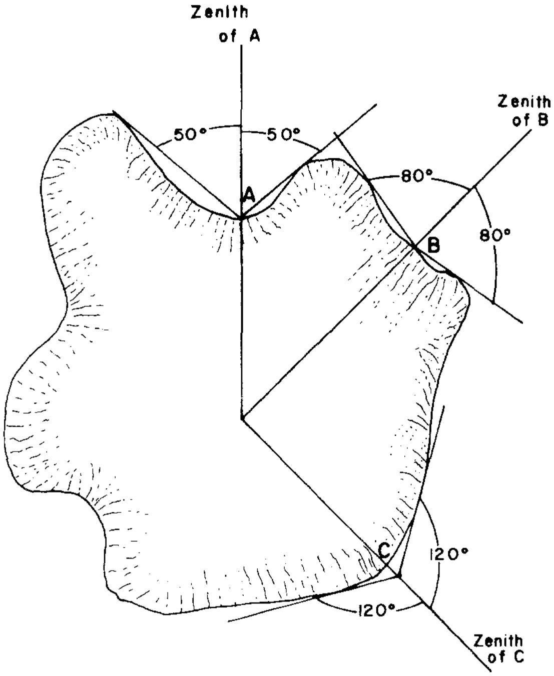

Fig. 1. The determination of the zenith angle of the horizon for any point at the surface of an asteroid. Points A, B and C have respectively zenith angles of the horizon of 50°, 80° and 120°. In cases for which the clockwise zenith angle differs from the counter-clockwise angle, the average of the two is the final zenith angle. This applies also when a third dimension is introduced. The cumulative probability of impact in each point is assumed to be a monotonic function of the zenith angle of the horizon of the point.

by each point. The approach is an extension of previous work concerned with geomorphological changes of airless bodies (Ronca and Furlong, 1977, 1978).

Figure 1 shows an idealized asteroid. Reducing the problem to two dimensions, point A has a zenith angle of the horizon of 50°, point B of 80° and point C of 120°. The assumption is made that the meteoroidal flux, and therefore the erosion of A, B and C, is some monotonic function of 50°, 80° and 120°.

The value of each point i (i from 1 to n) at the surface of the asteroid (in two dimensions) are entered as polar coordinates r_i and θ_i, with the origin at the center of gravity. The distance L between each point i and all the others is calculated by

$$L^2_{k,i+k} = r^2_{i+k} + r^2_i - 2r_{i+k}r_i \cos\theta_{i,i+k}, \tag{1}$$

where $k = 1, 2, \ldots n$. Then the angle α is calculated from

$$\alpha_{i,i+k} = \Pi - \cos^{-1}\left(\frac{L^2_{i,i+k} + r^2_i - r^2_{i+k}}{2L_{i,i+k}r_i}\right). \tag{2}$$

For each point i, the smallest $\alpha_{i,i+k}$ is chosen. This will be the angle between the zenith of point i and the horizon of point i, or what we called the zenith angle. The same operation is performed for the other sides of the zenith of i. The final zenith angle of i is considered to be the average of the zenith angle in each direction.

The next step is to find a relationship between the rate of erosion $-\mathrm{d}r/\mathrm{d}t$ and the zenith angle β.

Gault and Wedeking (1978) reasoned that "the differential probability $\mathrm{d}P$ for an object to impact a spherical body at an angle β is $\mathrm{d}P = 2\sin\beta\cos\beta\,\mathrm{d}\beta$." Then the cumulative probability of impact at angles from 0° to β° will be

$$P_{\mathrm{cum}} = \int_0^\beta 2\sin\beta\cos\beta\,\mathrm{d}\beta = \sin^2\beta. \tag{3}$$

This expression will be assumed to apply also to non-spherical bodies, with some modifications.

Let us look at the three points A, B and C on the surface of the asteroid depicted in Figure 1. It is evident that the angle β of Equation (3) is 50° for point A, 80° for point B and 120° for point C. Intuitively the probability of impact should increase monotonically with the angle. Equation (3), however, shows a decrease in probabilities for $\beta > 90°$. In order to expand Equation (3) to angles larger than 90° we can assume that we are dealing with the opposite tail of another β angle, as shown in Figure 2. Then we can write

$$\mathrm{d}P^*(0 < \beta \leqslant 90°) = 2\sin\beta\cos\beta\,\mathrm{d}\beta, \tag{4}$$

$$\mathrm{d}P^*(90° < \beta \leqslant 180°) = 2\sin\gamma\cos\gamma\,\mathrm{d}\gamma, \tag{5}$$

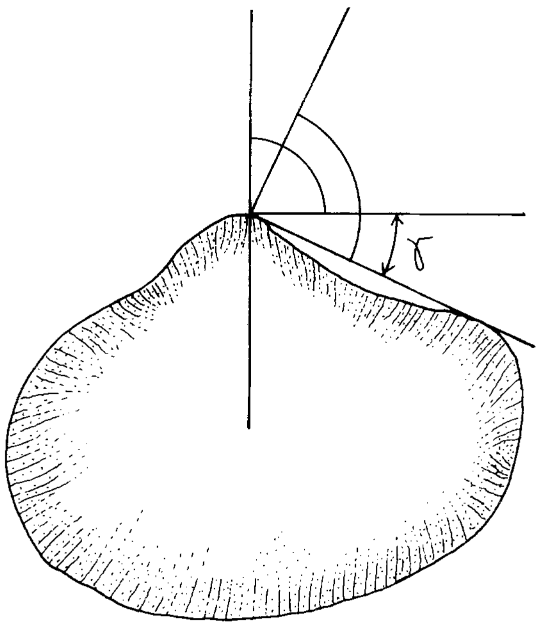

Fig. 2. The extension of the impact cumulative probability to zenith angles of the horizon larger than 90°. See text.

where $\gamma = \beta - 90^\circ$ and the asterisk means that the probability is not normalized. Then

$$P^*_{\text{cum}}(0^\circ \leqslant \beta \leqslant 90^\circ) = \sin^2\beta, \tag{6}$$

$$P^*_{\text{cum}}(90^\circ < \beta \leqslant 180^\circ) = 1 + \sin^2\gamma = 1 + \sin^2(\beta - 90^\circ). \tag{7}$$

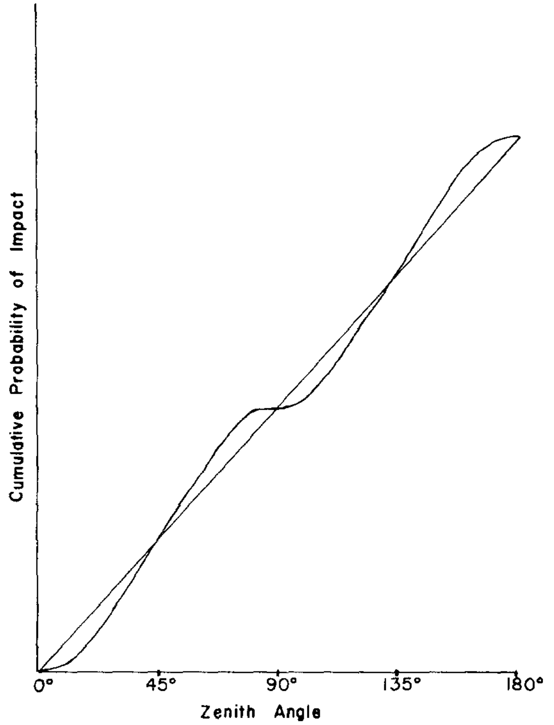

Fig. 3. The cumulative probability of impact versus the zenith angle of the horizon. The serpentine curve represents the function as obtained in the text. It is evident that not much error will be introduced by substituting it with the straight line.

The same procedure can be used for angles larger than 180°. Figure 3 shows the plot of P^* versus β for Equations (6) and (7). It is evident that not much error will be introduced by considering the relationship as linear with zero intercept

$$P_{\text{cum}} = \text{const} \times \beta. \tag{8}$$

The next assumption is that the rate of erosion is, at least approximately, linearly proportional to the probability of impact. Then

$$-\mathrm{d}r/\mathrm{d}t = \text{const} \times \beta. \tag{9}$$

The constant will be referred to as the constant of erosion.

Results from the Model

As long as no close-up pictures of asteroids are available, only idealized shapes can be used. The model was applied to spherical, ellipsoidal, and cubical shapes, as well as irregular shapes obtained by splitting the regular shapes. Also some completely irregular figures were used.

The choice of a numerical value for the constant of erosion is arbitrary, but it happens that it is not critical. In order to run the model on a computer, the infinitesimal dr must be made finite. The smaller the chosen value of dr, the larger the number of iterations necessary to see a change in the shape of the asteroid. It was found that a constant of erosion sufficiently small so that no difference is seen in one iteration in the computer-plotted shape is as small as the constant needs to be. The computer is then instructed to plot the shape every ten or more iterations. The use of smaller values of the constant causes no significant variation in the sequence of shapes and just increases computer time. Using other relationships besides linearity between rate of erosion and probability of impact also had insignificant effects, due to the small Δr of each step.

Several shapes and their erosional progressions are presented in Figures 4 to 9. Although they cannot be taken literally, some of the properties of the sequence may be of interest, especially in relation to the recent work by Hartmann and Cruikshank on the asteroid 624 Hector (as reported in *Scientific American*, May 1979; pp. 96–98). The light curve of this asteroid indicates that it has the shape of either an elongated cigar or perhaps a dumb-bell. One possible explanation is that Hector is two asteroids in contact. The present model suggests another possibility.

Figure 4 shows the change in shape of a spherical asteroid having coincidental center of gravity and center of figure. The spherical shape is maintained as the asteroid erodes to a smaller size. Figure 5 shows a spherical asteroid with the center of gravity away from

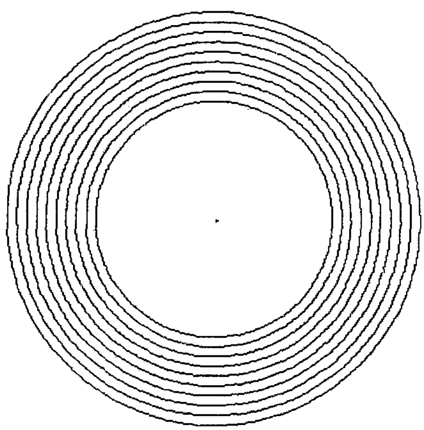

Fig. 4. Erosion of a spherical asteroid having coincidental center of gravity and center of figure. Initial conditions are the outermost circle. Sphericity is maintained.

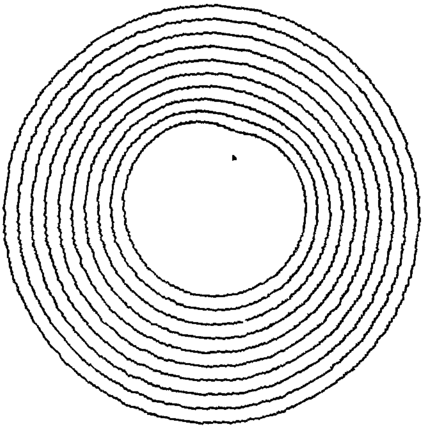

Fig. 5. Erosion of a spherical asteroid having center of gravity (indicated by the mark) separated from the center of figure. The side nearer to the center of gravity shows an increase in the radius of curvature and sphericity is not maintained.

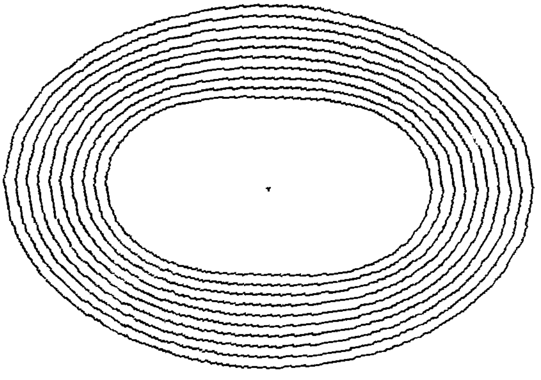

Fig. 6. Erosion of an ellipsoidal asteroid with coincident centers of gravity and figure. The trend is toward the shape of an elongated cigar.

the center of the figure. Figure 6 shows an ellipsoidal asteroid with coinciding centers. As erosion progresses, the asteroid becomes more elongated and ellipticity is not maintained. Figure 7 shows a cubic asteroid. The flat sides of the cube develop concavities. Figure 8 shows a sphere which has been broken and the new center of gravity has been moved accordingly. A concavity develops quickly on the flat face of the broken sphere, but also the opposite side changes from convex to concave as erosion progresses. Figure 9 shows the same broken sphere, but this time the center of gravity is kept near the flat face. In this case only the flat face develops a concavity.

Generalizing the erosional sequence of shapes, a tendency toward shapes defined by some previous authors is apparent. The tendency toward a cylindrical object with half-spheres on its ends or the elongated cigar is shown by Figure 6. A tendency toward a dumb-bell configuration appears in Figures 7, 8 and 9.

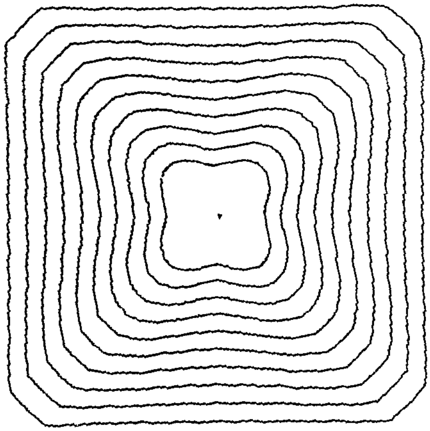

Fig. 7. Erosion of a cubical asteroid. Concavities develop on the sides of the cube.

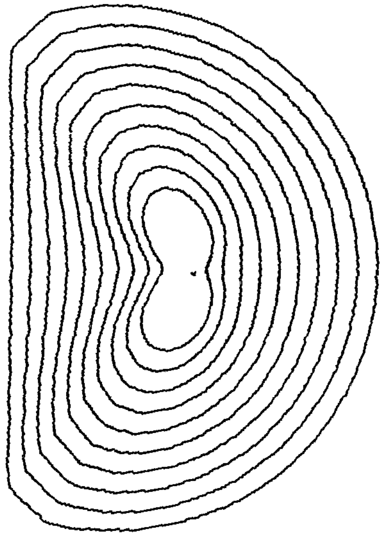

Fig. 8. Erosion of a broken sphere with coincident centers of gravity and figure. A concavity develops quickly on the flat surface. The side nearer to the center of gravity shows a reversal in the sign of the radius of curvature, which changes from a convexity to a concavity. The trend is toward a dumb-bell shape.

It is not the purpose of this paper to speculate on the effects of collisions between objects of comparable size. Breakage of the asteroid may follow in some cases, or the formation of discernible craters in others. It is convenient to repeat here some conclusions that were presented in a previous paper, concerned with the development of the shape of craters on the surface of airless bodies (Ronca and Furlong, 1977; addenda 1978). Under

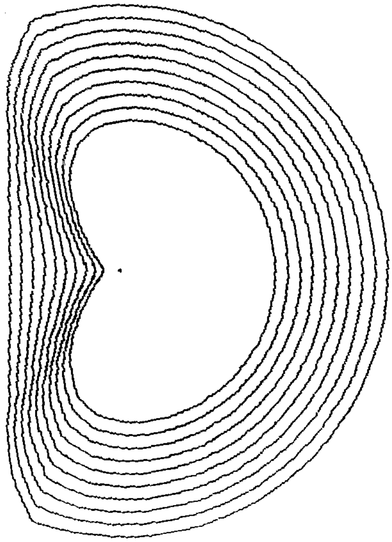

Fig. 9. Erosion of a broken sphere with the center of gravity coincidental with the center of the sphere before the sphere was broken. A concavity develops only on the flat surface.

conditions of insignificant amount of ejecta reaching escape velocity, the rims of craters, subjected to meteoroidal erosion, disappear and the crater becomes bowl-shaped, with the center of the bowl below the level of the surrounding plain. Under conditions of a large percentage of ejecta reaching escape velocity, the trend is toward a circular low hill surrounding a bowl, with the center of the bowl at the same elevation as the surrounding plain.

It is hoped that the space agencies of the U.S.A., U.S.S.R. or the new European Space Agency will undertake a mission to the asteroids. Only then will the overall shape of these bodies and the profiles of their craters be unquestionably known.

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