AN EMPIRICAL MODEL FOR THE 11-YEAR COSMIC-RAY MODULATION

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Abstract. An analysis of monthly data from nine world-wide neutron monitoring stations over the period 1965–1975 is carried out for the study of the long-term cosmic-ray modulation. In an attempt to gain insight into the relationships which exist between solar activity, high-speed solar wind streams and various terrestrial phenomena an empirical relation for the cosmic-ray modulation has been found. Accordingly the modulated cosmic-ray intensity is equal to the galactic cosmic-ray intensity corrected by a few appropriate solar, interplanetary and terrestrial activity indices which causes the disturbances in interplanetary space, multiplying with the corresponding time-lag of cosmic-ray intensity from each of these indices. This relation is well explained by a generalization of the Simpson solar wind model which has been proved by the spherically symmetric diffusion-convection theory.

1. Introduction

It is known that the cosmic-ray intensity observed at the Earth is found to vary with an eleven-year cycle, the cosmic-ray intensity decreasing with increasing solar activity. This solar modulation takes place as the galactic cosmic-rays propagate through the region around the Sun containing the interplanetary medium. In an attempt to study this modulation several researches have expressed the long-term variations of galactic cosmic-ray intensity by appropriate solar and terrestrial indices (Pomerantz and Duggal, 1974; Moraal, 1976; Hatton, 1980; Nagashima and Morishita, 1980a, b, etc.).

In previous work Xanthakis *et al.* (1981) have taken into account the contribution to the cosmic-ray modulation process by more than one solar and geophysical parameter such as sunspot number, proton events and geomagnetic index. These indices were selected as the most proper source functions among various kinds of solar and terrestrial activity indices to simulate the cosmic-ray intensity during the 20th solar cycle. Recently, Mavromichalaki and Petropoulos (1984) have shown that in order to understand the cosmic-ray modulation it is very useful to determine also the structure of the interplanetary medium and its influence on cosmic-ray intensity variations. The possible influence of the interplanetary indices and especially of the solar-wind streams on cosmic-ray intensity was then studied using the neutron monitor data of the Inuvik station. For this purpose the 11-year cosmic-ray

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TABLE I

Stations whose data have been utilized in this analysis

Station (Super NM–64)	Height (m)	Geographic latitude (deg)	Coord. longitude (deg)	Threshold rigidity (GV)
Alert	57	82.50 N	62.33 W	0.00
Thule	260	76.60 N	68.80 W	0.00
Mc Murdo	48	77.90 S	166.60 E	0.01
Inuvik	21	68.35 N	133.72 W	0.18
Goose Bay	46	53.27 N	60.40 W	0.52
Deep River	145	46.10 N	77.50 W	1.02
Kiel	54	54.30 N	10.10 E	2.29
Hermanus	26	34.42 S	19.22 E	4.90
Pic du Midi	2860	42.93 N	0.25 E	5.36

modulation was modelled by treating the sunspot number R, the proton events N_p, the geomagnetic index A_p and the high-speed solar-wind streams S as the input and the cosmic-ray intensity as the output of a linear system.

In this work this model applied successfully to another eight ground-based stations which detected cosmic-rays well distributed over the Earth and so it was established in the study of cosmic-ray modulation. Moreover it has been shown that the function which describes the amount of modulation produced after the disturbance travelled from the Sun can be the time-lag of cosmic-ray intensity with respect to the above mentioned indices.

2. Selection of Data and Data Analysis

In order to study the 11-year modulation in cycle number 20 data of cosmic-ray intensities have been used from nine neutron monitoring stations (super NM–64) extending over the period 1965–1975. The altitude, geographic coordinates and cut-off rigidity of each station are listed in Table I. The data (corrected for pressure) for each station were normalized by the method

$$\frac{I_i - I_{\min}}{I_{\max} - I_{\min}},$$

where $I_{\min}$ and $I_{\max}$ are, respectively, the minimum and maximum intensities of cosmic-rays during the 20th solar cycle and I_i is the corresponding monthly value of cosmic-ray intensity. Thus the intensities at solar minimum (May 1965) are taken equal to 1.00 and at solar maximum (June 1969) are taken equal to zero.

For this analysis we have also used the monthly number of solar flares (importance $\geqslant 1$), the monthly averages of relative sunspot number R (Zürich Observatory), the number of significant solar proton events N_p (Shea and Smart, 1977, 1979) and the geomagnetic index A_p (Solar Geophysical Data). Moreover the number of high-speed solar-wind streams and their polarity are taken from the catalogue of Lindblad and Lundstedt (1981). This catalogue is based on a data compilation by J. King available through the National Space Science Data Center (King, 1977).

According the Lindblad and Lundstedt (1981) a possible high-speed solar-wind stream (HSPS) is that in which the difference between a smallest 3-hr velocity value for a given day and the largest 3-hr value for the following day is greater than or equal to 100 km s^{-1}. These HSPS are studied and separated into two basic types: the first one is a long-lasting HSPS emitted by coronal-holes (coronal-hole stream, *S*) and the second one, characterized by lower solar wind speed, is associated with strong active regions emitting solar flares and producing Forbush decreases at the Earth (solar flare streams, *F*) (Iucci *et al.*, 1979; Venkatesan *et al.*, 1982).

A detailed study of these data led us to a new generalized empirical relation. Accordingly the cosmic-ray intensity I which is observed at the Earth on a semi-annual basis can be calculated from the difference between the constant function C and the sum of the most important solar and terrestrial indices which are affected cosmic-ray modulation. The expression is of the form

$$I = C - 10^{-3}(kR + 4N_p + 12A_p + 3S - a), \tag{2}$$

where C is a constant which depends linearly on the cut-off rigidity of each station; k is a coefficient which is also rigidity-dependent and is probably related to the diffusion coefficient of cosmic-rays and its transition in space; and a is a constant which depends on the polarity of the high-speed coronal-hole stream. The physical properties in the modulating region derived from the above constants are discussed below.

The standard deviation between the observed I_{obs} and the calculated by Equation (2) I_{cal} values of the cosmic-ray intensity is of the order of 3–5%. The corresponding standard deviation which was calculated in Paper I (Xanthakis *et al.*, 1981) was of the order of 5–9%. If we substract the calculated I_{cal} from the observed I_{obs} values of cosmic-ray intensity, the difference $\Delta(I_{\text{obs}} - I_{\text{cal}})$ should be independent of the 11-year and short-term variations. Practically, however, the difference $\Delta(I_{\text{obs}} - I_{\text{cal}})$ as presented in Figure 1 shows some short-term variations, especially during the years 1965–1975 due, perhaps, to incomplete elimination by the present indices.

Examining the above relation (2) and applying this to the nine neutron-monitor stations, we observe that the constant C is linearly correlated with the cut-off rigidity of each station. We can derive the relations

$$C = 0.93 + 0.007P, \qquad \bar{k} = 2e^{0.68P}, \tag{3}$$

where P is the cut-off rigidity of each station. It has been shown in Paper I that the coefficient k is a quantity related to the modulation of cosmic-rays travelling through interplanetary space by the solar wind and gives information on their diffusion coefficients.

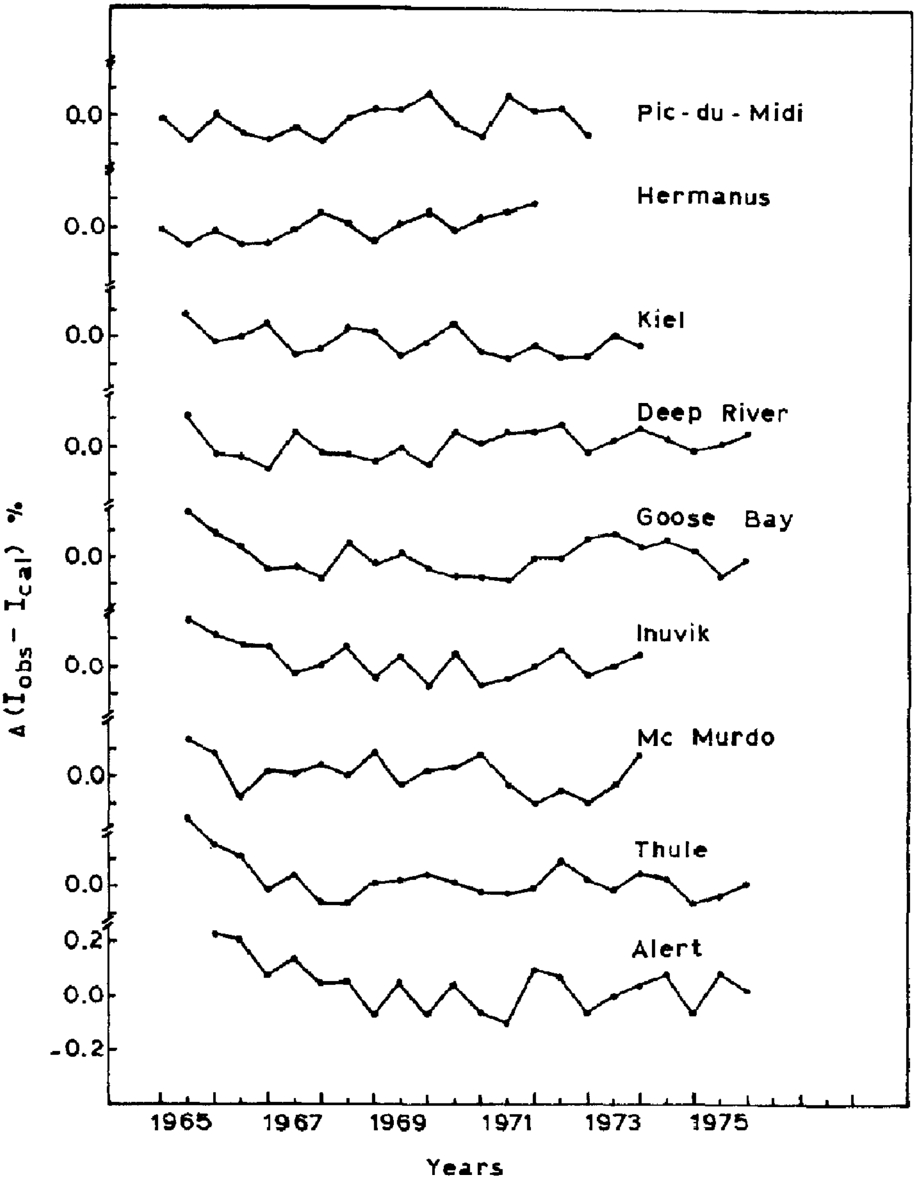

Fig. 1. Differences between the observed cosmic-ray intensities I_{obs} and those calculated by relation (2) I_{cal} for each station from 1965 to 1975.

3. Coronal-Hole Streams

In Paper I it was suggested that the major contribution to solar modulation during solar cycle 20 may be attributed to sunspot number, to solar flare generated

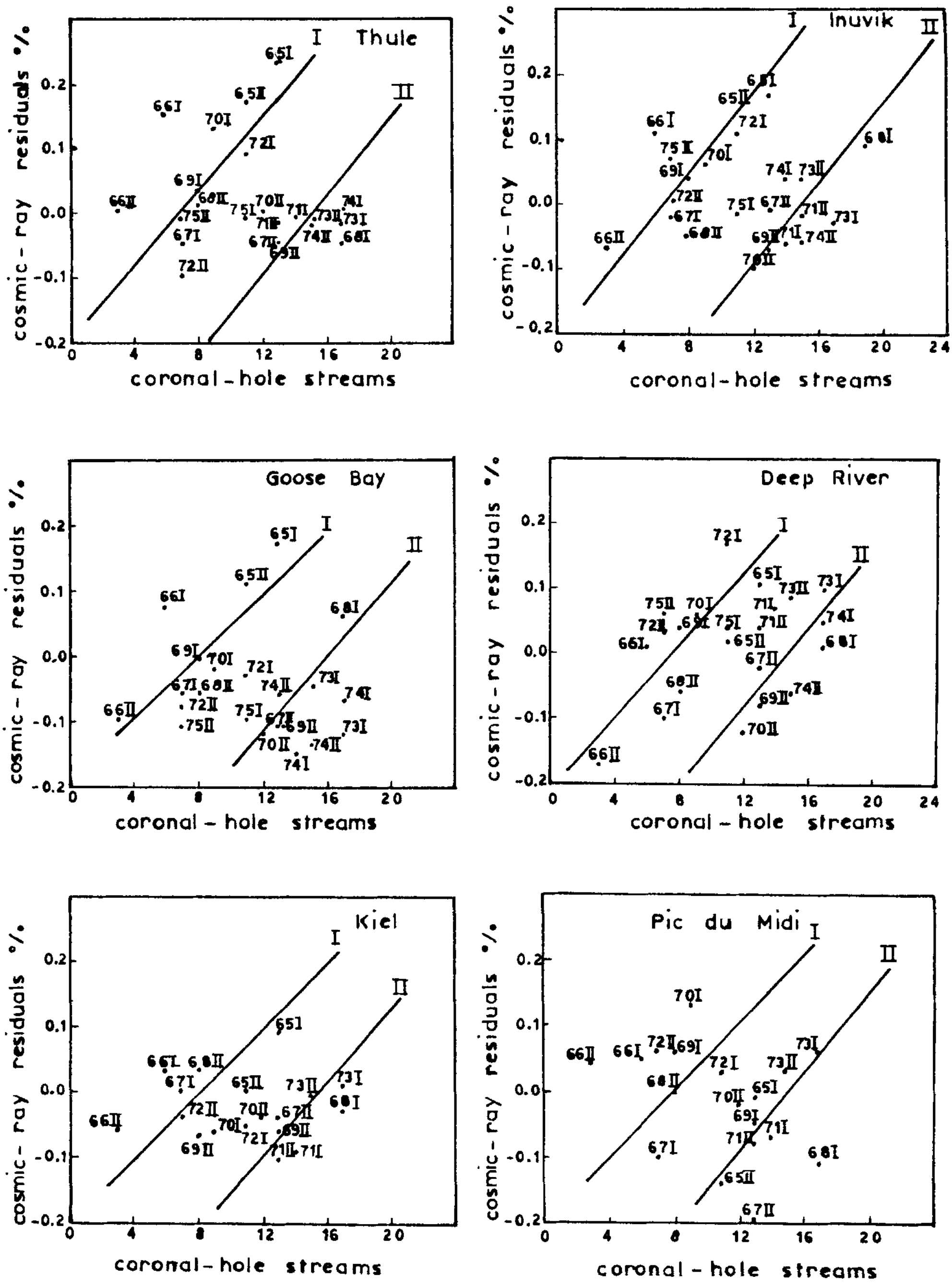

Fig. 2. Correlation diagram between cosmic-ray residuals and coronal-hole streams for six neutron-monitoring stations.

disturbances and to geomagnetic index A_p according to the equation

$$I = C - 10^{-3}(kR + 4N_p + 12A_p). \tag{4}$$

In this work we compared the cosmic-ray residuals (observed and calculated by Equation (4)) with the solar wind speed and the two types of fast solar-wind streams: coronal-hole streams and solar-flare generated streams. From all these parameters it was observed that there is a good agreement between the cosmic-ray residuals and the coronal-hole streams especially at solar minimum. It is known that these streams are characterized by greater solar-wind speed than the streams of the active regions and are observed at solar minimum in the absence of solar flares (Hatton, 1980; Simon, 1979; Hundhausen *et al.*, 1980). These observations led us to take into account the presence of coronol-hole streams in the study of long-term cosmic-ray modulation (Venkatesan *et al.*, 1982; Mavromichalaki and Petropoulos, 1984).

A correlation analysis between the cosmic-ray residuals ΔI of each station on a semi-annual basis and the number of coronal-hole streams was carried out. Some examples are given in Figure 2. The analytical expression between them is

$$\Delta I = 10^{-3}(3S - a), \tag{5}$$

where

$$a = 20e^{1.25\delta},$$

with $\delta = 0$ for the line I of Figure 2 and $\delta = 1$ for the line II of the same figure.

Examining these two groups of values it is found that these lines can be related to the interplanetary magnetic field polarity which appears in each stream (Lindblad and Lundstedt, 1981). The first group corresponds to positive polarity and the second one to negative polarity. This is attributed to a different modulation process of the cosmic-ray intensity from the coronal-hole streams depending on their polarity.

It is known that the solar wind plasma moves radially outward from the solar corona carrying with it the frozen-in interplanetary magnetic field. As viewed from the Earth the magnetic fields is organized in large scale sectors. Thus one observes the field directed inwards (negative sector) or outwards (positive sector) for a few days and then the direction changes on a short time scale. The two-sector structure is the dominant feature of the interplanetary magnetic field associated with the high-speed solar-wind streams (Lindblad, 1981).

4. Choice of the Coefficients

It is known that the time-lag between cosmic-ray intensity and solar activity varies from several to 12 months depending on the solar cycle and on the activity index adopted (Dorman *et al.*, 1977; Nagashima and Morishita, 1980a, b). Simpson (1963) attributed this time-lag to the dynamics of the build up and subsequent delayed relaxation of the modulating region. So the hysteresis effect of the Sun on the cosmic-ray flux arriving from the Galaxy to the Earth's orbit can be used to give information

TABLE II

Cross correlation coefficients and the corresponding time-lags for the period 1965–1975

Indices	r	Lag (months)
Sunspot	− 0.88	2
Solar flares ≥ 1	− 0.76	4
Proton events	− 0.48	4
Streams	− 0.30	3
Index A_p	− 0.20	0
	+ 0.33	12

about the size of the modulating region, the variations of the sunspot heliolatitude, the time of galactic cosmic-ray diffusion to the modulating region, etc. (Dorman and Soliman, 1979).

For this correlation analysis between the monthly mean cosmic-ray intensity of each station used in this work and the monthly values of sunspot number, solar flares (importance $\geqslant 1$), proton events and geomagnetic index A_p as a function of the lag of cosmic-ray intensity with respect to these parameters was carried out. The best correlation coefficient of the cosmic-ray intensity with respect to each one of these indices gives the corresponding time-lag.

Since periods of higher than average solar-wind velocity are followed by decreases in the cosmic-ray intensity the same correlation analysis between the monthly cosmic-ray intensity and the monthly number of high-speed solar-wind streams was carried out with interesting results. The correlation coefficient is maximum when a lag of three months is introduced into the streams data. This is consistent with Hatton's (1980) result that the time-lag between cosmic-ray residuals (observed and simulated by solar flare data) and solar wind velocity is three months (Hatton and Bowe, 1981).

The correlation coefficients between the monthly cosmic-ray intensity and sunspot number, solar flares, proton events, index A_p and high-speed streams for different time-lags for a representative station (Inuvik) are presented in Figure 3. The best correlation coefficient and the corresponding time-lags for solar cycle 20 are given in Table II. The same results were obtained for all nine stations.

It is interesting to note that the coefficient of each parameter (R, N_p, A_p, S) of the Equation (2) was chosen to be the calculated time-lags of cosmic-ray intensity with respect to these indices. Therefore a useful empirical model for the calculation of cosmic-ray intensity is established in this work.

5. The Model

The modulation of cosmic-rays studied here can be described by the following integral equation which is derived from a generalization of Simpson's coasting solar wind model (1963) as

$$I(t) = I_\infty - \int f(\tau) S(t - \tau) \mathrm{d}\tau, \qquad (6)$$

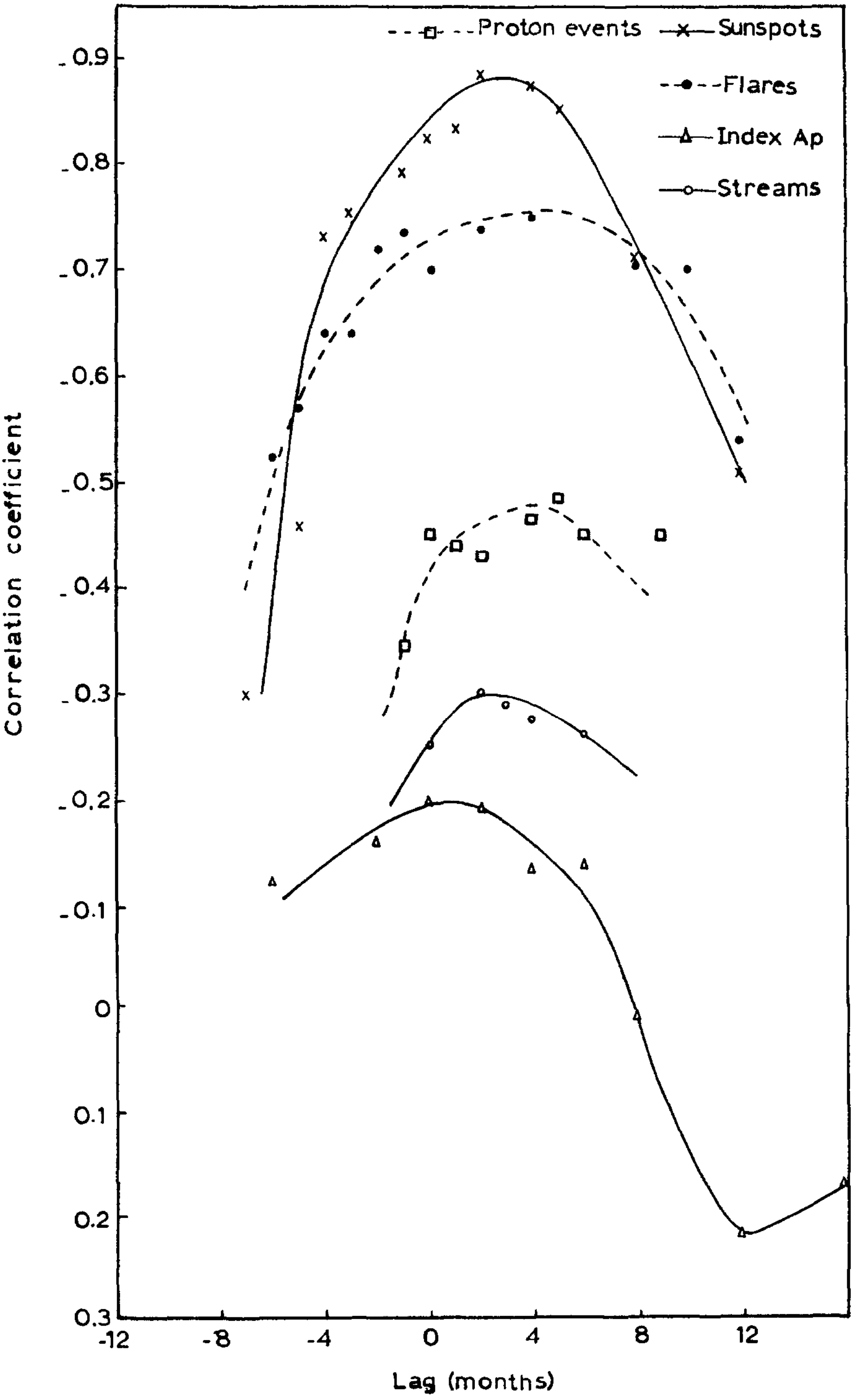

Fig. 3. Correlation coefficient between the monthly cosmic-ray intensity and sunspot number, solar flares, proton events, index A_p and high-speed solar-wind streams as a function of cosmic-ray intensity lag with respect to these indices for the 20th solar cycle.

where I_∞ and $I(t)$ are, respectively, the galactic and modulated cosmic-ray intensities, $S(t-\tau)$ is the source function representing some proper solar activity index at a time $f-\tau$ $(\tau \geqslant 0)$ and $f(\tau)$ is the characteristic function which expresses the time dependence of solar disturbances represented by $S(t-\tau)$.

In this work it is pointed out that the modulations during solar cycle 20 can be described by the source function which is expressed by the linear combination of four indices: the sunspot number R, the proton events N_p, the index A_p and the high-speed streams which are emanating from coronal holes. So, we can write

$$\begin{aligned} f(\tau)S(t-\tau) &= f_R(\tau)R(t-\tau) + f_N(\tau)N_p(t-\tau) + \\ &\quad + f_A(\tau)A_p(t-\tau) + f_S(\tau)S(t-\tau). \end{aligned} \tag{7}$$

The time-lag τ between the cosmic-ray intensity and each of the above indices can be neglected, because it is shorter than six months. We recall that in our analysis we have used semi-annual values. Substituting Equation (7) into the general equation and identifying with the empirical Relation (2) we derive for all neutron-monitor stations

$$\begin{aligned} I_\infty &= C + \int_0^\infty f_R(\tau)\,\mathrm{d}\tau = 2 \times 10^{-3} \times e^{0.68P}, \\ &\int_0^\infty f_N(\tau)\,\mathrm{d}\tau = 4 \times 10^{-3}, \\ &\int_0^\infty f_A(\tau)\,\mathrm{d}\tau = 12 \times 10^{-3}, \\ &\int_0^\infty f_S(\tau)\,\mathrm{d}\tau = 3 \times 10^{-3}. \end{aligned} \tag{8}$$

It is interesting to note that the characteristic function $f(\tau)$ of each index R, N_p, A_p, and S has a value which is, respectively, equal to the time lag of cosmic-ray intensity with respect to this index. These values can be explained if we choose a simple form for $f(\tau) : f(\tau) = 1$ for $0 \leqslant \tau \leqslant T$ and $f(\tau) = 0$ for $t < 0$ and $\tau > T$, i.e., the effectiveness of the disturbance in modulating cosmic-rays is independent of distance out to the radius of the heliosphere.

6. Conclusions

From the above analysis we conclude the following. We can describe the cosmic-ray modulation according to a fundamental relation applying to nine ground-based stations detecting cosmic-rays. According to this relation the modulated cosmic-ray intensity is equal to the galactic cosmic-ray intensity (unmodulated) at a finite distance, corrected by a few appropriate solar, interplanetary and terrestrial activity indices, which cause the disturbances in interplanetary space. Using the sunspot number R, the proton events N_p, the geomagnetic index A_p, and the high-speed solar-wind streams emanating from coronal holes S, the cosmic-ray intensity that is measured by a ground based neutron monitor station can be calculated from proper values of the constant C and the coefficients of the above indices. It has been shown in this work that the constant C is dependent on the cut-off rigidity of seach station, while the coefficient of each index has been selected to be the corresponding time-lag of the cosmic-ray intensity with respect to this index during the solar cycle examined. We believe that a further study of this model in the next solar cycles (there are no neutron monitor data before the year 1964) will help towards a better understanding of the relations between coronal structure, interplanetary structure and cosmic-rays. Also because of the fact that this model reproduces to a certain degree the cosmic-ray modulation it will be very useful to cosmic-ray research.

Another interesting conclusion is that the small time-lag between cosmic-ray intensity and solar activity as well as between cosmic-ray intensity and interplanetary activity during solar cycle 20- confirms the fact that the solar activity of this cycle was less than previously. This means that the dimensions of the heliosphere are not constant during a given solar cycle.

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