Chapter 4
Analysis and Design of Members in Fire

Abstract This chapter deals with aspects related to analysis and design of structures in fire. Detailed design procedure of steel members, reinforced concrete members, and steel–concrete composites under fire exposure are discussed. Design examples and solved following code provisions illustrate the presented concepts in detail.

4.1 Material Properties at Elevated Temperatures

Thermal and mechanical properties of steel and concrete play a major role in defining their behavior at elevated temperatures. It is important to note that the properties of steel and concrete at elevated temperature are necessary and important to assess their performance under fire loads. They include yield strength and modulus of elasticity, which are influenced by the temperature range. According to the codal methods for design of structures under fire loads, yield strength and the young's modulus must be estimated at an elevated temperature. Alternatively, if the advanced method of design is used, then stress–strain variations at elevated temperature must be completely known as an input to the designer. In both the design approaches, variation of strength of steel and concrete at elevated temperature, with respect to different strain levels, is very important to compute. Behavior of materials at elevated temperature is handled using strength reduction factor.

Strength reduction factor is a ratio of strength of steel at specific strain level to strength at ambient temperature yield stress. It is given by:

$$k_{y,\theta} = \frac{f_{y,\theta}}{f_y}$$

where $f_{y,\theta}$ is the yield strength of steel at temperature θ and f_y is the yield strength at an ambient temperature (usually 20 °C). Strength reduction factor is influenced by the strain level; higher the value of strain, lower the strength reduction factor is. Changes in strength reduction are significant with respect to increase in temperature. In the design procedure, it is necessary to choose the strain level, which is

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appropriate to the type of loading and cross section. For design purposes, basic strength values at 2% strain can be used. In addition, suitable modification factors are applied to account for the failure cases at the lower strain levels. At an elevated temperature, two main consequences in the steel are: (i) loss of strength and (ii) loss of stiffness. Further, it also leads to the reduction of the modulus of elasticity. Most of the relevant material properties are standardized after several experimental programs and are deliberated in select building codes in detail (EN 1992-1-2, 2008; EN1993-1-2, 2005); codes address fire as a service load for design purposes. While it is expected that the reader will have access to such building codes, a few important properties of concrete and steel are discussed in this chapter with reference to the codes.

4.2 Thermal Properties of Concrete

4.2.1 Specific Heat

Specific heat of concrete is determined using the following relationships:

$$C_\mathrm{c} = 900 \quad \text{for}\, 20\,^\circ\mathrm{C} \le \theta_\mathrm{c} \le 100\,^\circ\mathrm{C} \tag{4.1a}$$

$$C_\mathrm{c} = \{900 + (\theta_c - 100)\} \quad \text{for}\, 100\,^\circ\mathrm{C} \le \theta_c \le 200\,^\circ\mathrm{C} \tag{4.1b}$$

$$C_\mathrm{c} = \{1000 + 0.5(\theta_c - 200)\} \quad \text{for}\, 200\,^\circ\mathrm{C} \le \theta_c \le 400\,^\circ\mathrm{C} \tag{4.1c}$$

$$C_\mathrm{c} = 1100 \quad \text{for}\, 400\,^\circ\mathrm{C} \le \theta_c \le 1200\,^\circ\mathrm{C} \tag{4.1d}$$

where specific heat of concrete (C_c) is measured in (J kg^{-1} K^{-1}) and θ_c is temperature of concrete measured in (°C). Relationship given in Eq. (4.1) applies to concrete constituted of both siliceous and calcareous aggregates. Variation in the range of temperature is due to the fact that presence of moisture (U) alters the specific heat capacity of concrete; this is more significant in the temperature range of 100–200 °C. Figure 4.1 shows the variation of specific heat capacity of concrete with rise in temperature.

4.2.2 Density

Density of concrete alters with temperature primarily due to water loss and is given by the following relationships:

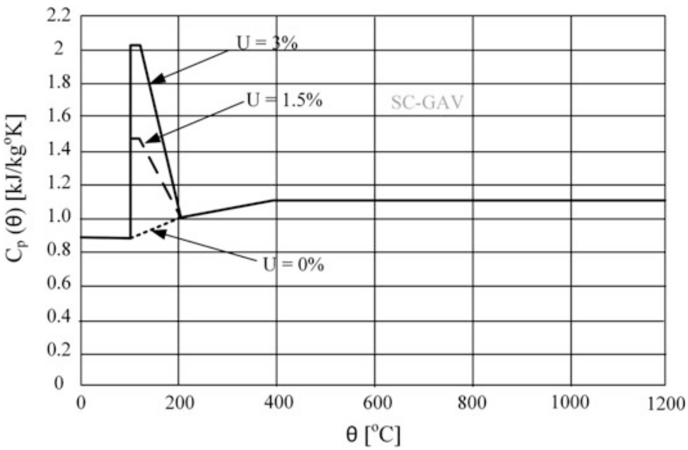


Fig. 4.1 Influence of moisture content on specific heat capacity of concrete

$$\rho_c = \rho_{20} \quad \text{for } 20\,^\circ\text{C} \le \theta_c \le 115\,^\circ\text{C} \tag{4.2a}$$

$$\rho_c = \rho_{20}\left\{1 - \frac{0.02(\theta_c - 115)}{85}\right\} \quad \text{for } 115\,^\circ\text{C} \le \theta_c \le 200\,^\circ\text{C} \tag{4.2b}$$

$$\rho_c = \rho_{20}\left\{0.98 - \frac{0.03(\theta_c - 200)}{200}\right\} \quad \text{for } 200\,^\circ\text{C} \le \theta_c \le 400\,^\circ\text{C} \tag{4.2c}$$

$$\rho_c = \rho_{20}\left\{0.95 - \frac{0.07(\theta_c - 400)}{800}\right\} \quad \text{for } 400\,^\circ\text{C} \le \theta_c \le 1200\,^\circ\text{C} \tag{4.2d}$$

where ρ_{20} is density of concrete at 20 °C, expressed in (kg/m^3), and θ_c is temperature of concrete expressed in (°C), respectively.

4.2.3 Thermal Conductivity

Thermal conductivity (λ_c) of normal concrete is specified through an upper and a lower limit. Typical values of thermal conductivity lie within these limits. Upper and lower limits of thermal conductivity are given by the following relationship:

$$\lambda_c = 2 - 0.2451\left(\frac{\theta_c}{100}\right) + 0.0107\left(\frac{\theta_c}{100}\right)^2 \quad \text{for } 20\,^\circ\text{C} \le \theta_c \le 1200\,^\circ\text{C} \tag{4.3a}$$

$$\lambda_c = 1.36 - 0.136\left(\frac{\theta_c}{100}\right) + 0.0057\left(\frac{\theta_c}{100}\right)^2 \quad \text{for } 20\,^\circ\text{C} \le \theta_c \le 120\,^\circ\text{C} \tag{4.3b}$$

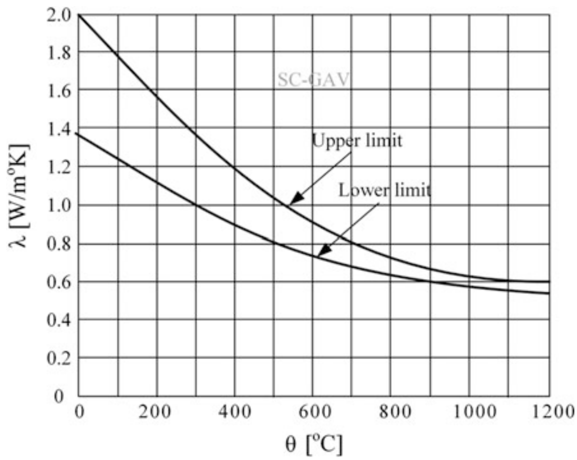


Fig. 4.2 Limits of thermal conductivity of concrete

where λ_c is thermal conductivity of concrete (expressed in Wm^{-1} K^{-1}) and θ_c is temperature of concrete (expressed in °C). A graphical representation of the upper and the lower limits of thermal conductivity is shown in Fig. 4.2.

4.3 Thermal Properties of Steel

Various thermal properties are required to understand steel at different temperatures. They include specific heat, density, thermal conductivity. Time–temperature curve should be obtained from the thermal load estimates on the members. Temperature time history is dependent on the following factors, namely: (i) applied heat load, (ii) shape (size of the member and section dimensions), and (iii) presence of passive fire protection methods/techniques. Fourier equation for heat transfer, in the absence of an internal heat source, is given by:

$$\nabla(a(\nabla\theta)) = \dot{\theta} \tag{4.3c}$$

where θ is the space-dependent temperature, $\dot{\theta} = \mathrm{d}\theta/\mathrm{d}t$, and a is the temperature-dependent thermal diffusivity. Thermal diffusivity is given by:

$$a = \frac{\lambda}{\rho C_v} \tag{4.3d}$$

where λ is thermal conductivity, ρ is density of material, and C_v is specific heat capacity.

4.3.1 Specific Heat and Density

Specific heat (C_s) of steel is recommended at different range of temperature by international code (EN 1994-1-2). They are given by the following relationships:

$$C_s = 425 + 7.73 \times 10^{-1}\theta_s - 1.69 \times 10^{-3}\theta_s^2 + 2.22 \times 10^{-6}\theta_s^3 \quad \text{for } 20\,°\text{C} \leq \theta_s \leq 600\,°\text{C} \tag{4.4a}$$

$$C_s = 666 + \frac{1302}{738 - \theta_s} \quad \text{for } 600\,°\text{C} \leq \theta_s \leq 735\,°\text{C} \tag{4.4b}$$

$$C_s = 545 + \frac{17820}{\theta_s - 731} \quad \text{for } 735\,°\text{C} \leq \theta_s \leq 900\,°\text{C} \tag{4.4c}$$

$$C_s = 650 \quad \text{for } 900\,°\text{C} \leq \theta_s \leq 1200\,°\text{C} \tag{4.4d}$$

where C_s is specific heat (expressed in J kg^{-1} °C) and θ_s is temperature of steel (expressed in °C). Variation of specific heat of carbon steel with rise in temperature is shown in Fig. 4.3. For design purposes, specific heat of steel can be taken as 600 J/kg °C. Density of steel is taken to be constant at 7850 kg m^{-3} at all temperatures. It is inversely proportional to thermal diffusivity.

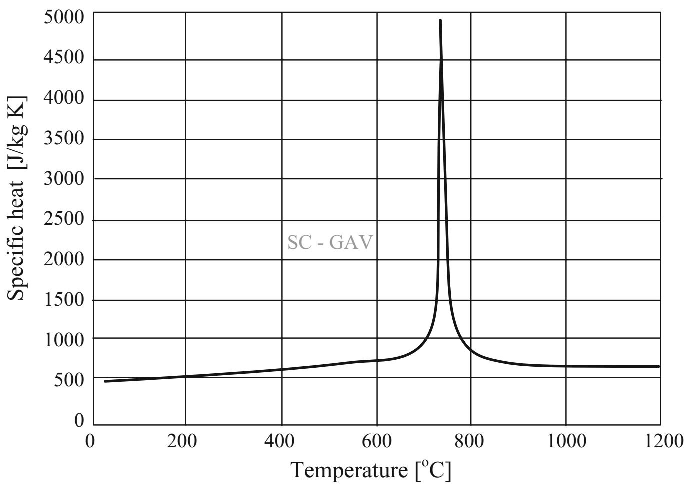


Fig. 4.3 Specific heat of carbon steel

4.3.2 Thermal Conductivity

Thermal conductivity depends on strength of steel, but it is not very significant. Thermal conductivity of steel is determined from the following relationship:

$$\lambda_s = 54 - 3.33 \times 10^{-3}\,\theta_s \quad \text{for}\, 20\,^\circ\mathrm{C} \le \theta_s \le 800\,^\circ\mathrm{C} \tag{4.5a}$$

$$\lambda_s = 27.3 \quad \text{for}\, 800\,^\circ\mathrm{C} \le \theta_s \le 1200\,^\circ\mathrm{C} \tag{4.5b}$$

where λ_s is thermal conductivity (expressed in Wm^{-1} °C^{-1}) and θ_s is temperature of steel (expressed in °C). Variation of thermal conductivity of steel with rise in temperature is shown in Fig. 4.4. For design purposes, thermal conductivity of steel can be taken as 45 W/m °C.

4.3.3 Thermal Diffusivity

Thermal diffusivity shows a linear relationship with temperature up to 750 °C. It is given by:

$$a = 0.87 - 0.84 \times 10^{-3}\,\theta_s \tag{4.5c}$$

where a is thermal diffusivity and θ_s is temperature (°C).

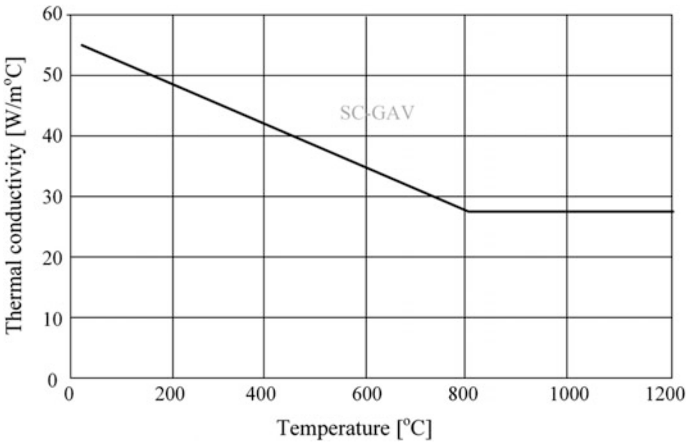


Fig. 4.4 Thermal conductivity of carbon steel

4.4 Mechanical Properties

4.4.1 Concrete

For design of concrete structures under fire, the most important mechanical property is its characteristic compressive strength, f_{ck}. As concrete is exposed to higher temperatures, its characteristic compressive strength reduces. This reduction is often expressed as strength reduction factor $k_{c,\theta} = f_{c,\theta}/f_{\mathrm{ck}}$. There are two more mechanical properties that have a role in design: (i) strain at which peak compressive strength of concrete is attained, $\varepsilon_{c1,\theta}$, and (ii) ultimate crushing strain, $\varepsilon_{\mathrm{cu}1,\theta}$. Contrary to the characteristic compressive strength, limiting values of these strains decrease with rise in temperature. Variations of the strength reduction factor, peak strain, and ultimate strain with temperature are shown in Table 4.1. It is seen from the table that compressive strength of concrete comprising calcareous aggregates is slightly greater than that of the concrete with siliceous aggregates; however, there is no difference in the variation of corresponding strain values. Further, it can also be observed that strength of concrete is reduced by about 50% at 600 °C and loses its entire strength at 1200 °C.

4.4.2 Steel

For design of steel structures, typically two material quantities are required, namely: (i) Young's modulus and (ii) yield stress. For ambient temperature design, Young's modulus of steel remains unchanged and the yield stress defines the grade of steel.

Table 4.1 Reduction factors for concrete

θ_s (°C)	Siliceous aggregates			Calcareous aggregates		
	$k_{c,\theta}$	$\varepsilon_{c1,\theta}$	$\varepsilon_{\mathrm{cu}1,\theta}$	$k_{c,\theta}$	$\varepsilon_{c1,\theta}$	$\varepsilon_{\mathrm{cu}1,\theta}$
20	1.000	0.0025	0.0200	1.000	0.0025	0.0200
100	1.000	0.0040	0.0225	1.000	0.0040	0.0225
200	0.950	0.0055	0.0250	0.970	0.0055	0.0250
300	0.850	0.0070	0.0275	0.910	0.0070	0.0275
400	0.750	0.0100	0.0300	0.850	0.0100	0.0300
500	0.600	0.0150	0.0325	0.740	0.0150	0.0325
600	0.450	0.0250	0.0350	0.600	0.0250	0.0350
700	0.300	0.0250	0.0375	0.430	0.0250	0.0375
800	0.150	0.0250	0.4000	0.270	0.0250	0.4000
900	0.080	0.0250	0.4250	0.150	0.0250	0.4250
1000	0.040	0.0250	0.0450	0.006	0.0250	0.0450
1100	0.010	0.0250	0.0475	0.020	0.0250	0.0475
1200	0.000			0.000		

It is seen that as the temperature of a steel specimen rises, both the Young's modulus and yield stress reduce. A convenient way to represent this reduction is through reduction factors. Reduction in yield stress is given by the following relationship:

$$k_{y,\theta} = f_{y,\theta}/f_y \tag{4.6}$$

Reduction in Young's modulus is given by the following relationship:

$$k_{E,\theta} = E_{s,\theta}/E_s \tag{4.7}$$

Reduction in proof strength is given by the following relationship:

$$k_{p,\theta} = f_{p,\theta}/f_y \tag{4.8}$$

where $f_{y,\theta}, f_{p,\theta}$ and $E_{s,\theta}$ are the yield stress, proof strength, and the Young's modulus at temperature θ, respectively. Values of yield strength and Young's Modulus of steel are corresponding values in the cold conditions (at 20 °C). Variation of these reduction factors with temperature is shown in Fig. 4.5. For design calculations, reduction factors are also given in Table 4.2; values for intermediate temperatures may be calculated by linear interpolation. It is seen from the table that steel loses its entire strength at 1200 °C. However, temperature higher than 500 °C (up to 600 °C) may be considered to be critical as steel loses half of its strength during this temperature range. While the actual critical temperature of a steel member depends on its utilization factor, 600 °C is often considered to be the critical temperature of a steel member in the absence of other data.

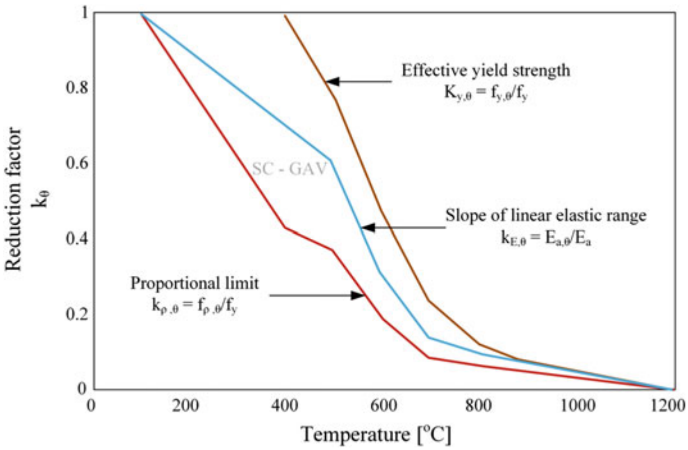


Fig. 4.5 Variation of reduction factors of steel with temperature

Table 4.2 Reduction factors for steel at different temperatures

Steel temperature θ_s (°C)	$k_{y,\theta} = f_{y,\theta}/f_y$	$k_{p,\theta} = f_{p,\theta}/f_y$	$k_{E,\theta} = E_{s,\theta}/E_s$
20	1.000	1.000	1.0000
100	1.000	1.000	1.0000
200	1.000	0.807	0.9000
300	1.000	0.613	0.8000
400	1.000	0.420	0.7000
500	0.780	0.360	0.6000
600	0.470	0.180	0.3100
700	0.230	0.075	0.1300
800	0.110	0.050	0.0900
900	0.060	0.0375	0.0675
1000	0.040	0.0250	0.0450
1100	0.020	0.0125	0.0225
1200	0.000	0.000	0.0000

4.5 Constitutive Models for Concrete and Steel Under Fire

4.5.1 Concrete

Constitutive behavior of concrete, to be used in compression zone for advanced structural analysis under fire, is shown in Fig. 4.6. Functional form of stress–strain curves is given below:

$$\sigma(\theta) = \frac{3\varepsilon f_{c,\theta}}{\varepsilon_{c1,\theta}\left(2 + \left(\frac{\varepsilon}{\varepsilon_{c1,\theta}}\right)^3\right)} \quad \text{for } \varepsilon \leq \varepsilon_{c1,\theta} \tag{4.9}$$

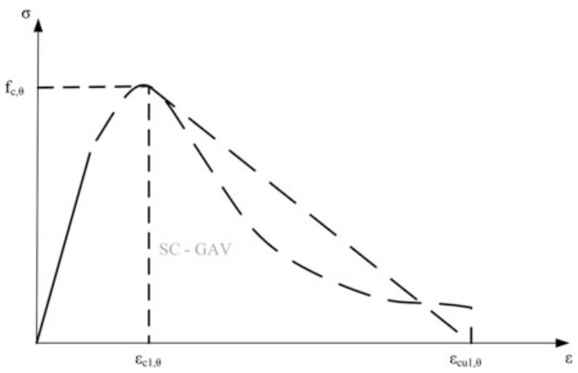


Fig. 4.6 Temperature-dependent stress–strain behavior of concrete in compression

For other range of values like $\varepsilon_{c1,\theta} \leq \varepsilon \leq \varepsilon_{cu1,\theta}$, appropriate descending branch should be adopted as shown in Fig. 4.6. Tensile strength is usually ignored. In case it is required for advanced calculations, following empirical equations for estimating material degradation coefficients under tensile strength are useful:

$$f_{ck,t}(\theta) = k_{c,t}(\theta) f_{ck,t} \tag{4.10}$$

where $f_{c,\theta}$ is temperature-dependent tensile strength, $f_{ck,t}$ represents tensile strength of concrete at room temperature, and $k_{c,t}(\theta)$ is given by:

$$k_{c,t}(\theta) = \begin{cases} 1 & \text{for } 20\,^\circ\text{C} \leq \theta \leq 100\,^\circ\text{C} \\ 1 - \frac{\theta - 100}{500} & \text{for } 100\,^\circ\text{C} \leq \theta \leq 600\,^\circ\text{C} \end{cases} \tag{4.11}$$

4.5.2 Steel

Figure 4.7 gives overall stress–strain behavior of steel, which is useful in estimating resistance against tension, compression, moment and shear. Functional forms of the stress–strain relation for various ranges of strain are given below:

Strain at proportional limit ($\varepsilon_{p,\theta}$) is given by:

$$\epsilon_{p,\theta} = \frac{f_{p,\theta}}{E_{a,\theta}} \tag{4.12}$$

$$\varepsilon_{y,\theta} = 0.02 \tag{4.13}$$

$$\varepsilon_{t,\theta} = 0.15 \tag{4.14}$$

$$\varepsilon_{u,\theta} = 0.20 \tag{4.15}$$

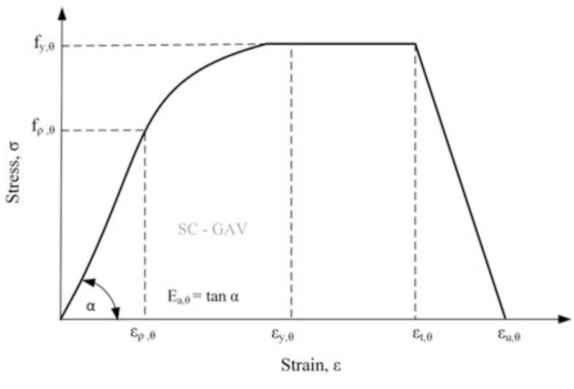


Fig. 4.7 Stress–strain behavior of steel

where $f_{p,\theta}$ is stress at proportional limit, $\varepsilon_{y,\theta}$ is yield strain, $\varepsilon_{t,\theta}$ is limiting strain for yield strength, $\varepsilon_{u,\theta}$ is ultimate strain, and $E_{a,\theta}$ is Young's Modulus of steel, as described in Fig. 4.7.

(a) For strain range $\epsilon \leq \epsilon_{p,\theta}$, stress is given by:

$$\sigma = \epsilon E_{a,\theta} \tag{4.16}$$

where $E_{a,\theta}$ is tangent modulus.

(b) For strain range $\epsilon_{p,\theta} < \epsilon < \epsilon_{p,\theta}$, stress is given by:

$$\sigma = f_{p,\theta} - c + \left\{ \frac{b}{a} \right\} \left[a^2 - \left(\epsilon_{y,\theta} - \epsilon \right)^2 \right]^{0.5} \tag{4.17}$$

Tangent modulus is given by:

$$E_{a,\theta} = \frac{b\left(\epsilon_{y,\theta} - \epsilon\right)}{a\left[a^2 - \left(\epsilon_{y,\theta} - \epsilon\right)^2\right]^{0.5}} \tag{4.18}$$

(c) For strain range $\epsilon_{y,\theta} \leq \epsilon \leq \epsilon_{t,\theta}$, stress is given as $f_{y,\theta}$

(d) For strain range $\epsilon_{t,\theta} < \epsilon < \epsilon_{u,\theta}$, stress is given by:

$$\sigma = f_{y,\theta} \left[1 - \frac{\left(\epsilon - \epsilon_{t,\theta}\right)}{\left(\epsilon_{u,\theta} - \epsilon_{t,\theta}\right)} \right] \tag{4.19}$$

For this specific range of strain, tangent modulus is not defined.

(e) For strain range $\epsilon = \epsilon_{u,\theta}$, stress is zero

where following parameters are useful to determine the stress values as given above:

$$a^2 = \left(\epsilon_{y,\theta} - \epsilon_{p,\theta}\right) \left(\epsilon_{y,\theta} - \epsilon_{p,\theta} + \frac{c}{E_{a,\theta}} \right) \tag{4.20a}$$

$$b^2 = c\left(\epsilon_{y,\theta} - \epsilon_{p,\theta}\right) E_{a,\theta} + c^2 \tag{4.20b}$$

$$c = \frac{\left(f_{y,\theta} - f_{p,\theta}\right)^2}{\left(\epsilon_{y,\theta} - \epsilon_{p,\theta}\right) E_{a,\theta} - 2\left(f_{y,\theta} - f_{p,\theta}\right)} \tag{4.20c}$$

where $f_{y,\theta}$ is effective yield strength and $E_{s,\theta}$ is slope of the stress–strain curve at linear elastic range (Young's Modulus of steel).

4.6 Design of Steel Members Exposed to Fire

It is a usual practice to note that structural design of members under fire is carried out for a predefined fire rating that the structural system must demonstrate during any fire event. Design procedures entail two steps, namely: (i) thermal analysis; and (ii) structural analysis. Usually, unprotected steel members perform very poor even under moderate fire. Hence, the use of exposed steel structures is discouraged. Therefore, design of steel structures against fire also includes design of insulation material to ensure the required fire rating. Fire rating is defined either on temperature-based or on strength-based criterion.

4.6.1 Temperature Assessment

Primary objective of temperature assessment is to compute temperature of the structural members under certain fire conditions. Temperature of the structural member is useful to assess temperature-based fire rating of the member. In addition, it serves as vital input to assess its strength-based fire rating. Heat reaches from source of fire to the surface of the member through convection and radiation. In the initial stages of fire, convection dominates, while after sustained burning, radiation is the major contributor toward heat transfer. Due to very high thermal conductivity of steel, surface temperature quickly penetrates through its entire volume. Hence, thermal gradients within the member become negligible. Under such conditions, lumped capacitance method is useful to assess temperature of structural member. Resultant heat flux is given by the following relationship:

$$h_{\text{net}} = KF\left(\theta_t - \theta_{a,t}\right) \tag{4.21}$$

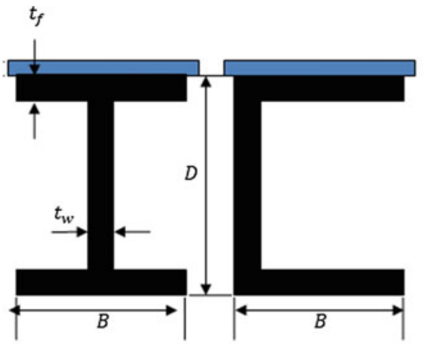
where K is coefficient of total heat transfer, F is surface area of the member exposed to fire, θ_t is the gas temperature, and $\theta_{a,t}$ is the temperature of steel member. Rate of heat flow into the member, in an incremental form, is given by:

$$\mathrm{h}_{\text{net}} = c_a \rho_a V_t \frac{\Delta\theta_{a,t}}{\Delta t} \tag{4.22}$$

where c_a is the specific heat of steel, ρ_a is the density of steel, and V_t is the volume per unit length. Equating the values of heat flow in the above equations, time-variant temperature of the member can be obtained as follows:

$$\Delta\theta_{a,t} = \frac{K}{c_a \rho_a V_t} \frac{A_m}{V} \left(\theta_t - \theta_{a,t}\right)\Delta t \tag{4.23}$$

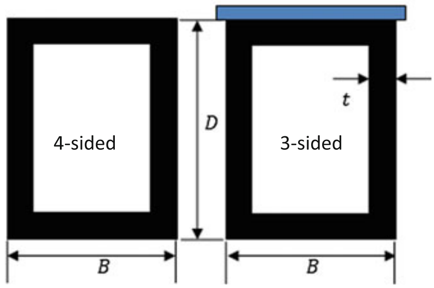
where A_m/V is the heated perimeter per unit volume of the structural member. Calculation procedure for typical steel sections is given below:



$$A_m = 2D + 4B - 2t_w \text{ for 4-sided exposure}$$

$$A_m = 2D + 3B - 2t_w \text{ for 3-sided exposure}$$

$$V = 2Bt_f + (D - 2t_f)t_w \approx 2Bt_f + Dt_w$$



$$A_m = 2D + 2B \text{ for 4-sided exposure}$$

$$A_m = 2D + B \text{ for 3-sided exposure}$$

$$V = BD - (B - 2t)(D - 2t)$$

A_m/V for other configurations can be calculated for other sections similarly or can be found in EN 1993-1-2:2005 (Table 4.2). It is to be noted that usually, the calculated values of A_m/V are rounded up to the nearest 5. Heat flow in uninsulated steelwork is given by:

$$\Delta\theta_{a,t} = \frac{\alpha_c + \alpha_r}{c_a \rho_a} \frac{A_m}{V} \left(\theta_t - \theta_{a,t}\right) \Delta t \tag{4.24}$$

Here, α_c is the coefficient of convective heat transfer (typical value for hydrocarbon fire is 50 W/m^2 °C) and α_r is the coefficient of radiative heat transfer and depends on factors such as the temperature of steel and gas, configuration factor, and emissivity of the structural member. For protected steel members, temperature calculations are based on the concept of thermal resistance of materials. Thermal resistance is given by:

$$R = L/kA \tag{4.25}$$

where L is thickness of the member (taken to be unity for convection and radiation modes), k is the thermal conductivity or heat convection/radiation coefficient, and A is the area of cross section. Note the one-to-one similarity between the concept of thermal and electrical resistance. Consequently, when multiple layers of different materials are encountered in heat transfer (as in protected steel members), the overall heat transfer calculations are performed by considering equivalent thermal resistance of the constituent materials connected in series or parallel, as applicable. Typically, the ECCS method is used to calculate temperature rise in protected steelwork. In case of insulated members with negligible heat capacity, temperature rise is obtained as follows:

$$\Delta\theta_{a,t} = \frac{\frac{\lambda_p}{d_p}}{c_a\rho_a}\frac{A_m}{V}\left(\theta_t - \theta_{a,t}\right) \tag{4.26}$$

Here, λ_p is the thermal conductivity (W/m °C) and d_p is the thickness (m) of the insulation. Heat flow in members with substantial heat capacity is obtained as:

$$\Delta\theta_{a,t} = \frac{\frac{\lambda_p}{d_p}}{c_a\rho_a}\frac{A_m}{V}\frac{\left(\theta_t - \theta_{a,t}\right)\Delta t}{1+\frac{\phi}{2}} - \frac{\Delta\theta_t}{1+\frac{2}{\phi}} \tag{4.27}$$

Here, $\boldsymbol{\phi} = \frac{c_p\rho_p}{c_a\rho_a}\boldsymbol{d_p}\frac{A_m}{V}$ is a parameter used to define whether insulation has substantial heat capacity or not. Insulations with $\boldsymbol{\phi} > 0.5$ are considered to have substantial heat capacity and require (4.27) for temperature calculations, while for others, (4.26) is to be used. $\boldsymbol{c_p}$ and $\boldsymbol{\rho_p}$ are the specific heat and density of the insulating material, respectively.

4.6.2 Eurocode Approach (EN 1993-1-2)

This approach uses the relation derived by Wickstorm (1986). Following equations apply:

$$\Delta\theta_{a,t} = \frac{\frac{\lambda_p}{d_p}}{c_a\rho_a}\frac{A_m}{V}\frac{(\theta_t - \theta_{a,t})\Delta t}{1+\frac{\phi}{3}} - \left(\mathrm{e}^{\left(\frac{\phi}{10}\right)} - 1\right)\Delta\theta_t \tag{4.28}$$

Eurocode EN 1993-1-2 places a limit on the minimum value of Δt to be used for computing temperature rise as 30 s (clause 4.2.3.2). Wickstorm (1986) suggested that the limit on Δt should be taken as:

$$\Delta t \leq \frac{c_a\rho_a}{\lambda_p}\frac{V}{A_m}\left(1+\frac{\phi}{3}\right) < 60\,\mathrm{s} \tag{4.29}$$

Above methods for computing temperature are semiempirical in nature and are derived based on the lumped capacitance and thermal resistance concepts.

4.6.3 Bare Steelwork

Twilt and Witteveen (1986) gave the following equation for computing the temperature rise on bare steelwork:

$$t_{\mathrm{fi},d} = 0.54\left(\theta_{a,t} - 50\right)\left(\frac{A_m}{V}\right)^{-0.6} \tag{4.30}$$

where $\theta_{a,t}$ is the temperature in the steel reached at a time t (minutes). Note that the above equation only holds for $10 \le t_{\mathrm{fi},d} \le 80\,\mathrm{min}$; $400 \le \theta_{a,t} \le 600\,^\circ\mathrm{C}$ and $10 \le A_m/V \le 300\,\mathrm{m}^{-1}$.

4.6.4 Protected Steelwork

When steelwork is protected with dry insulation, it is found that the time to failure depends solely on the term $\left(d_p/\lambda_p\right)\left(V_t/A_p\right)$ with the following resultant equation:

$$t_{\mathrm{fi},d} = 40\left(\theta_{a,t} - 140\right)\left(\frac{d_p}{\lambda_p}\frac{V}{A_m}\right)^{0.77} \tag{4.31}$$

The above equation is an empirical relation determined from tests in which insulation material was very light. When the insulation has substantial heat capacity, above equation is revised as below:

$$t_{\mathrm{fi},d} = 40\left(\theta_{a,t} - 140\right)\left(\frac{d_p}{\lambda_p}\left(\frac{V}{A_m} + \frac{d_p\rho_p}{\rho_a}\right)\right)^{0.77} \tag{4.32}$$

4.7 Fire Rating Based on Temperature Criterion

For a member to perform adequately where deflection or stability (buckling) is not critical in a fire, EN 1993-1-2 requires that the structural temperature remains below a critical temperature, i.e.,

$$\theta_a \le \theta_{a,\mathrm{cr}} \tag{4.33}$$

Temperature of structural members can be determined through thermal analysis as detailed above. Value of critical temperature $\theta_{a,\mathrm{cr}}$ is dependent on the degree of utilization μ_0 of the structural member. Relationship between $\theta_{a,\mathrm{cr}}$ and μ_0 has been determined from elementary plasticity theory. Reduction in steel strength with rise in temperature is given by:

$$\theta_{a,\mathrm{cr}} = 39.19\ln\left(\frac{1}{0.9674\,\mu_0^{3.833}} - 1\right) + 48 \text{ subject to the limit } \mu_0 > 0.013 \tag{4.34}$$

The degree of utilization is defined as:

$$\mu_0 = \frac{E_{\mathrm{fi},d}}{R_{\mathrm{fi},d,0}} \tag{4.35}$$

where $R_{\mathrm{fi},d,0}$ is the resistance of the member at time $t = 0$ (i.e., at the beginning of the fire) and $E_{\mathrm{fi},d}$ is the design effect of the structural fire actions.

4.8 Strength Assessment

Strength assessment of structural members under fire conditions is performed by considering the same set of design equations that are usually employed for ambient temperature; only change is to use appropriate strength reduction factors. It is important to note that classification of a structural member may change due to strength and modulus degradation at higher temperatures. For instance, a column member, which was not slender earlier, becomes slender during fire; alternatively, Class 1 section becomes Class 2 and so on. In such cases, design calculations need to be adjusted accordingly. It is usually assumed that end conditions do not change during a fire event.

4.8.1 Fire Rating Based on Strength Criterion

Fire rating, based on strength criterion, is defined as the time up to which the design value of the internal force to be resisted remains below the design resistance during fire. Mathematically, it is given as:

$$E_{\mathrm{fi},d} < R_{\mathrm{fi},d,t}. \tag{4.36}$$

This is usually calculated in accordance with the Code (EN 1993-1-2).

(a) Section classification: This should be carried out in accordance with the code EN 1993-1-1, except the value of ε is modified to allow for the effects of temperature, as given below:

$$\varepsilon = 0.85\left[\frac{235}{f_y}\right]^{0.5} \tag{4.37}$$

(b) Tension members, with a uniform temperature distribution, will have a reduced axial tensile capacity ($N_{\mathrm{fi},\theta,\mathrm{Rd}}$), and it is given by:

$$N_{\mathrm{fi},\theta,\mathrm{Rd}} = k_{y,\theta}N_{\mathrm{Rd}}\left[\frac{\gamma_{M,1}}{\gamma_{M,\mathrm{fi}}}\right] \tag{4.38}$$

where, $k_{y,\theta}$ is the normalized strength reduction factor at a temperature of θ_a and N_{Rd} is the ambient design resistance. For tension members with a non-uniform temperature distribution, axial capacity may be obtained by summing the contributions of discretized areas as $N_{\mathrm{fi},\theta,\mathrm{Rd}} = \sum_{i=1}^{n}\frac{A_i k_{y,\theta,i} f_y}{\gamma_{M,\mathrm{fi}}}$, where A_i and $k_{y,\theta,i}$ are the area and the strength reduction factor of ith area. One can also use (4.38) using the maximum steel temperature reached and assuming constant temperature.

(c) Compression members (Class 1, 2, and 3 cross sections):
The compressive strength in fire conditions is given by:

$$N_{b,\mathrm{fi},t,\mathrm{Rd}} = \chi_{\mathrm{fi}} A k_{y,\theta}\frac{f_y}{\gamma_{M,\mathrm{fi}}} \tag{4.39}$$

where buckling strength reduction factor χ_{fi} is given by:

$$\chi_{\mathrm{fi}} = \frac{1}{\phi_\theta + \sqrt{\phi_\theta^2 - \lambda_\theta^2}} \tag{4.40}$$

with $\phi_\theta = 0.5\left(1 + \alpha\lambda_\theta + \lambda_\theta^2\right)$ and $\alpha = 0.65\sqrt{\frac{235}{f_y}}$. $\lambda_\theta = \lambda\sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}}$ is the non-dimensional slenderness ratio at temperature θ with λ being the normalized slenderness ratio. Buckling length in fire conditions is usually calculated in a similar manner as done for ambient temperatures. For braced frames, buckling length for columns of an intermediate story is taken as 0.5 times the story height, while for columns of top story, it is taken as 0.7 times the story height.

(d) Beams:
In case of Class 1 or 2 sections, with a uniform temperature distribution, moment capacity $M_{\mathrm{fi},\theta,\mathrm{Rd}}$ with no lateral torsional buckling may be calculated using the following relationship:

$$M_{\mathrm{fi},\theta,\mathrm{Rd}} = k_{y,\theta}M_{\mathrm{Rd}}\left[\frac{\gamma_{M,1}}{\gamma_{M,\mathrm{fi}}}\right] \tag{4.41}$$

For $\gamma_{M,1} = \gamma_{M,\text{fi}} = 1.0$, above equation reduces to the following form:

$$M_{\text{fi},\theta,\text{Rd}} = k_{y,\theta} M_{\text{Rd}} \tag{4.42}$$

where $k_{y,\theta}$ is the normalized strength reduction at a temperature of θ_a and M_{Rd} is the ambient design resistance. When lateral torsional buckling can occur, the moment capacity $M_{b,\text{fi},t,\text{Rd}}$ is given by the following relationship:

$$M_{b,\text{fi},t,\text{Rd}} = \chi_{\text{Lt,fi}} W_{\text{pl},y} k_{y,\theta,\text{com}} \frac{f_y}{\gamma_{M,\text{fi}}} \tag{4.43}$$

where $k_{y,\theta,\text{com}}$ is the strength reduction factor for the temperature in the compression flange, which can be conservatively based on the uniform temperature θ_a. For members with non-uniform temperature, the capacity can be computed by discretizing the area into multiple areas and summing over the contributions of individual areas, as was described in case of tension members previously.

4.9 Numerical Examples

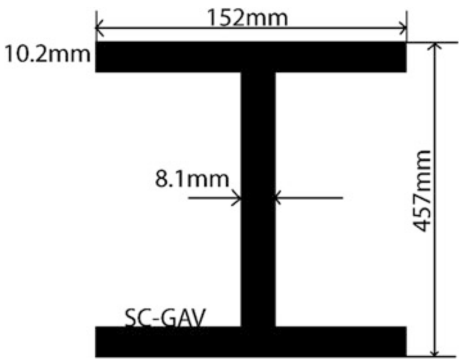
Example 1 Evaluate the temperature profile for an unprotected I section shown in Fig. 4.8, when it is exposed to three-sided standard ISO-834 fire.

Thermal analysis:

Increment in temperature in steel structure when exposed to fire is given by:

$$\Delta\theta_{a,t} = \frac{\alpha}{c_a \rho_a} \frac{A_m}{V_m} \left(\theta_t - \theta_{a,t}\right) \Delta t$$

Fig. 4.8 Steel I section for Example 1



where c_a is the specific heat, ρ_a is the density of the material, A_m is the area of the unit length exposure, V_m is the volume of unit length section, $\theta_{a,t}$ is the temperature in member, θ_t is the gas temperature, Δt is the increment in time, and α is the effective heat transfer coefficient given by:

$$\alpha = 25 + \frac{0.7 \times k_{sh} \times 0.56 \times 10^{-8}}{\theta_t - \theta_{a,t}} \left[(\theta_t + 273)^4 - \left(\theta_{a,t} + 273\right)^4 \right]$$

where k_{sh} is the shielding parameter which is similar to the configuration factor and determines whether all surfaces of an exposed member receive direct radiation or certain portions of the cross-sectional shield other sections. The maximum value of shielding parameter (= $(A_m/V)_b/(A_m/V)$ is unity and can be used in calculations. For I sections, Eurocode 1993-1-2 specifies k_{sh} = 0.9 $(A_m/V)_b/(A_m/V)$.

For this example, gas temperature is given by:

$$\theta_t = 20 + 345\ \log(8t + 1)$$

For the given section,

$$\frac{A_m}{V} = \frac{2D + 3B - 2t_w}{2Bt_f + \left(D - 2t_f\right)t_w} = \frac{2 \times 457 + 3 \times 152 - 2 \times 10.22}{2 \times 152 \times 10.22 + (457 - 2 \times 10.22) \times 8.1} \times 10^3,$$

$$\frac{A_m}{V} = 1000 \times \frac{1349.56}{6643} = 203.2 \approx 205\,\text{m}^{-1}, \text{ while}$$

$$\left(\frac{A_m}{V}\right)_b = \frac{2D + B}{2Bt_f + \left(D - 2t_f\right)t_w} = \frac{2 \times 457 + 152}{2 \times 152 \times 10.22 + (457 - 2 \times 10.22) \times 8.1} \times 10^3,$$

$$\left(\frac{A_m}{V}\right)_b = 1000 \times \frac{1066}{6643} = 160.5 \approx 165\,\text{m}^{-1}.$$

Thus, (A_m/V) = 205 m^{-1} and $(A_m/V)_b$ = 165 m^{-1}. Subsequently, k_{sh} is computed as 0.72. Temperature variation of the member with respect to time is summarized in Table 4.3. Figure 4.9 shows the variation of structural temperature with time.

Example 2 Evaluate the temperature profile for an unprotected C-Channel section of constant thickness shown in Fig. 4.10, when it is exposed to four-sided standard ISO-834 fire.

Thermal analysis:

Increment in temperature in steel structure when exposed to fire is given by:

$$\Delta\theta_{a,t} = \frac{\alpha}{c_a \rho_a} \frac{A_m}{V_m} \left(\theta_t - \theta_{a,t}\right) \Delta t$$

Table 4.3 Temperature in given steel section of Example 1

Time (min)	α	θ_t	$\Delta\theta$	θ_m
0		20.00	0.00	20.00
0.5	25.87	261.14	32.79	52.79
1	26.32	349.21	41.01	93.80
1.5	26.75	404.31	43.66	137.46
2	27.18	444.50	43.86	181.32
2.5	27.61	476.17	42.79	224.10
3	28.05	502.29	41.01	265.11
3.5	28.49	524.53	38.85	303.96
4	28.93	543.89	36.49	340.45
4.5	29.38	561.03	34.06	374.52
5	29.82	576.41	31.65	406.16
5.5	30.25	590.36	29.29	435.45
6	30.68	603.12	27.04	462.49
6.5	31.09	614.88	24.90	487.39
7	31.49	625.78	22.91	510.30
7.5	31.88	635.94	21.05	531.35
8	32.25	645.46	19.34	550.69
8.5	32.60	654.40	17.77	568.47
9	32.95	662.85	16.34	584.81
9.5	33.27	670.84	15.05	599.86
10	33.58	678.43	13.87	613.73
0.5	25.87	261.14	32.79	52.79

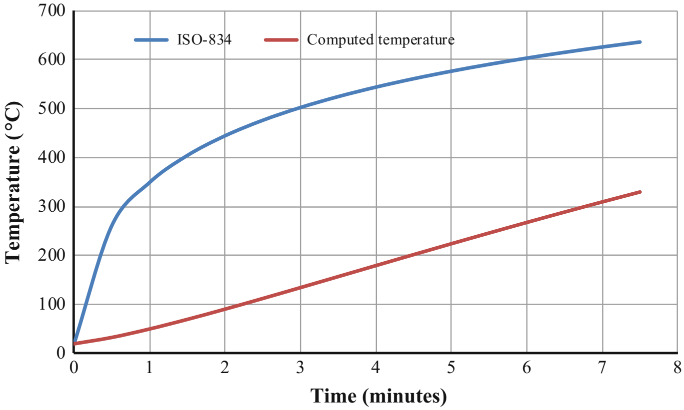
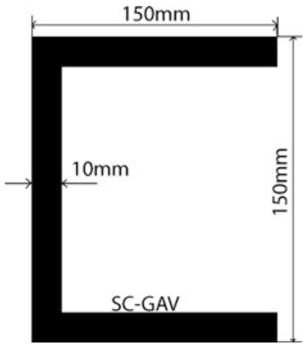


Fig. 4.9 Temperature variation of beam section with time

Fig. 4.10 Steel channel section for Example 2



where c_a is the specific heat, ρ_a is the density of the material, A_m is the area of the unit length exposure, V_m is the volume of unit length section, $\theta_{a,t}$ is the temperature in member, θ_t is the gas temperature, Δt is the increment in time, and α is the effective heat transfer coefficient given by:

$$\alpha = 25 + \frac{0.7 \times k_{sh} \times 0.56 \times 10^{-8}}{\theta_t - \theta_{a,t}} \left[(\theta_t + 273)^4 - (\theta_{a,t} + 273)^4 \right]$$

For the given example, the gas temperature is given by:

$$\theta_t = 20 + 345 \log(8t + 1)$$

For the given section, $(A_m/V) = 205\ \mathrm{m}^{-1}$, $(A_m/V)_b = 140\ \mathrm{m}^{-1}$, and subsequently, k_{sh} is computed as 0.68. Temperature variation of the member with respect to time is summarized in Table 4.4. Figure 4.11 shows the variation of structural temperature with time.

Example 3 Evaluate the temperature profile for I section (same web and flange thicknesses) beam protected with a 20-mm fiberboard as shown in Fig. 4.12, when exposed to a four-sided ISO-834 standard fire.

Thermal analysis:

Increment in temperature in protected steel structure when exposed to fire is given by:

$$\Delta\theta_{a,t} = \frac{\frac{\lambda_p}{d_p}}{c_a \rho_a} \frac{A_m}{V} \frac{(\theta_t - \theta_{a,t})\Delta t}{1 + \frac{\phi}{3}} - \left(e^{\left(\frac{\phi}{10}\right)} - 1 \right) \Delta\theta_t,$$

Table 4.4 Temperature in given steel section of Example 2

Time	α	θ_t	$\Delta\theta$	θ_m
0		20.00	0.00	20.00
0.5	25.82	261.15	8.18	28.20
1	26.18	349.21	11.04	39.20
1.5	26.47	404.31	12.70	51.90
2	26.72	444.51	13.79	65.70
2.5	26.96	476.17	14.54	80.30
3	27.18	502.29	15.08	95.30
3.5	27.40	524.53	15.45	110.80
4	27.61	543.89	15.71	126.50
4.5	27.81	561.03	15.88	142.40
5	28.01	576.41	15.98	158.40
5.5	28.22	590.36	16.02	174.40
6	28.41	603.12	16.01	190.40
6.5	28.61	614.88	15.96	206.30
7	28.81	625.78	15.88	222.20
7.5	29.01	635.94	15.77	238.00
8	29.21	645.46	15.64	253.60
8.5	29.41	654.40	15.49	269.10
9	29.61	662.85	15.32	284.40
9.5	29.81	670.84	15.14	299.60
10	30.01	678.43	14.94	314.50

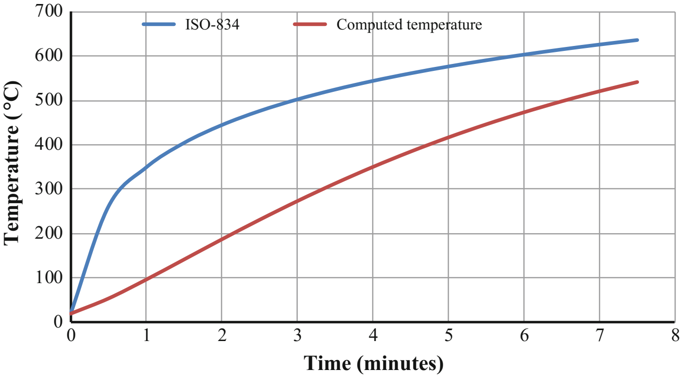


Fig. 4.11 Temperature variation of C-section with time

Material data:

For fiberboard fire protection, $\lambda_p = 0.25$ W/m °C, $\rho_p = 500$ kg/m^3, $p = 2\%$, $c_p = 1500$ J/kg °C, $d_p = 0.020$ m, $A_p/V = 160$ m^{-1}, $\rho_a = 7850$ kg/m^3, and $c_a = 600$ J/kg °C.

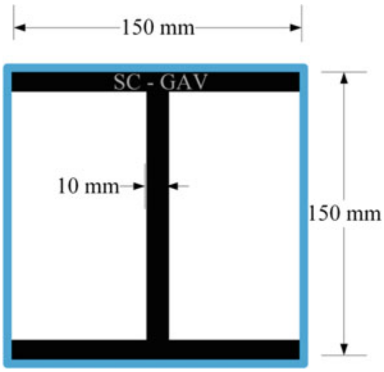


Fig. 4.12 Box-insulated steel beam I section for Example 3

Since $\phi = \frac{c_p \rho_p}{c_a \rho_a} d_p \frac{A_m}{V}$ gives $\phi = 0.50$,

Which gives, $\Delta\theta_{a,t} = 0.0109\left(\theta_t - \theta_{a,t}\right) - 0.0523\Delta\theta_t$.

For the given section, $(A_m/V) = 160\ \text{m}^{-1}$. Temperature variation of the member with respect to time is summarized in Table 4.5. Figure 4.13 shows the variation of structural temperature with time.

Example 4 Evaluate the temperature profile for a C-Channel section protected with a 20-mm fiberboard protection shown in Fig. 4.14, when exposed to a four-sided ISO-834 standard fire.

Table 4.5 Temperature in given steel section of Example 3

Time	θ_t	$\Delta\theta$	θ_m
0	20.00	0.00	20.00
0.5	261.14	2.60	22.63
1	349.21	3.38	26.05
1.5	404.31	3.90	29.99
2	444.50	4.27	34.29
2.5	476.17	4.54	38.88
3	502.29	4.76	43.69
3.5	524.53	4.94	48.67
4	543.89	5.08	53.80
4.5	561.03	5.21	59.06
5	576.41	5.31	64.42
5.5	590.36	5.40	69.87
6	603.12	5.47	75.39
6.5	614.88	5.54	80.97
7	625.78	5.59	86.61
7.5	635.94	5.64	92.30

(continued)

Table 4.5 (continued)

Time	θ_t	$\Delta\theta$	θ_m
8	645.46	5.68	98.03
8.5	654.40	5.71	103.79
9	662.85	5.74	109.57
9.5	670.84	5.76	115.38
10	678.43	5.78	121.21
10.5	685.65	5.79	127.05
11	692.54	5.80	132.91
11.5	699.13	5.81	138.77
12	705.44	5.82	144.63
12.5	711.49	5.82	150.50
13	717.31	5.82	156.36
13.5	722.91	5.82	162.23
14	728.31	5.81	168.08
14.5	733.52	5.81	173.94
15	738.56	5.80	179.78
15.5	743.43	5.79	185.61
16	748.15	5.78	191.43
16.5	752.73	5.76	197.24
17	757.17	5.75	203.03
17.5	761.48	5.74	208.81
18	765.67	5.72	214.57
18.5	769.75	5.70	220.32
19	773.72	5.69	226.05
19.5	777.59	5.67	231.75
20	781.35	5.65	237.44
20.5	785.03	5.63	243.10
21	788.62	5.61	248.75
21.5	792.13	5.59	254.37
22	795.55	5.56	259.97
22.5	798.90	5.54	265.55
23	802.17	5.52	271.10
23.5	805.38	5.49	276.63
24	808.52	5.47	282.13
24.5	811.59	5.44	287.61
25	814.60	5.42	293.06
25.5	817.56	5.39	298.49
26	820.45	5.37	303.89
26.5	823.29	5.34	309.26
27	826.08	5.32	314.61
27.5	828.82	5.29	319.93
28	831.50	5.26	325.22

(continued)

Table 4.5 (continued)

Time	θ_t	$\Delta\theta$	θ_m
28.5	834.14	5.24	330.49
29	836.74	5.21	335.72
29.5	839.29	5.18	340.93
30	841.80	5.15	346.12
30.5	844.26	5.13	351.27
31	846.69	5.10	356.40
31.5	849.08	5.07	361.49
32	851.43	5.04	366.56
32.5	853.74	5.02	371.60
33	856.02	4.99	376.61
33.5	858.26	4.96	381.60
34	860.48	4.93	386.55
34.5	862.66	4.90	391.48
35	864.80	4.88	396.37
35.5	866.92	4.85	401.24
36	869.01	4.82	406.08
36.5	871.07	4.79	410.89
37	873.10	4.76	415.67
37.5	875.11	4.73	420.43
38	877.08	4.71	425.15
38.5	879.04	4.68	429.85
39	880.96	4.65	434.51
39.5	882.87	4.62	439.15
40	884.74	4.59	443.76
40.5	886.60	4.57	448.34
41	888.43	4.54	452.90
41.5	890.24	4.51	457.42
42	892.03	4.48	461.92
42.5	893.80	4.45	466.39
43	895.55	4.43	470.83
43.5	897.27	4.40	475.24
44	898.98	4.37	479.62
44.5	900.67	4.34	483.98
45	902.34	4.32	488.31
45.5	903.99	4.29	492.61
46	905.62	4.26	496.88
46.5	907.24	4.23	501.12
47	908.84	4.21	505.34
47.5	910.42	4.18	509.53
48	911.98	4.15	513.70
48.5	913.53	4.13	517.83

(continued)

Table 4.5 (continued)

Time	θ_t	$\Delta\theta$	θ_m
49	915.07	4.10	521.94
49.5	916.58	4.07	526.03
50	918.08	4.05	530.08
50.5	919.57	4.02	534.11
51	921.04	4.00	538.11
51.5	922.50	3.97	542.09
52	923.95	3.94	546.04
52.5	925.38	3.92	549.97
53	926.79	3.89	553.86
53.5	928.20	3.87	557.74
54	929.59	3.84	561.58
54.5	930.97	3.82	565.40
55	932.33	3.79	569.20
55.5	933.68	3.77	572.97
56	935.03	3.74	576.72
56.5	936.35	3.72	580.44
57	937.67	3.69	584.13
57.5	938.98	3.67	587.80
58	940.27	3.64	591.45
58.5	941.55	3.62	595.07
59	942.83	3.59	598.67
59.5	944.09	3.57	602.24
60	945.34	3.55	605.79
60.5	946.58	3.52	609.32
61	947.81	3.50	612.82
61.5	949.03	3.48	616.30
62	950.24	3.45	619.75
62.5	951.44	3.43	623.18
63	952.64	3.41	626.59
63.5	953.82	3.38	629.98
64	954.99	3.36	633.34
64.5	956.15	3.34	636.68
65	957.31	3.32	640.00
65.5	958.46	3.29	643.29
66	959.59	3.27	646.56
66.5	960.72	3.25	649.81
67	961.84	3.23	653.04
67.5	962.95	3.21	656.25
68	964.06	3.19	659.43
68.5	965.15	3.16	662.60
69	966.24	3.14	665.74

(continued)

Table 4.5 (continued)

Time	θ_t	$\Delta\theta$	θ_m
69.5	967.32	3.12	668.86
70	968.39	3.10	671.96
70.5	969.46	3.08	675.03
71	970.51	3.06	678.09
71.5	971.56	3.04	681.13
72	972.61	3.02	684.14
72.5	973.64	3.00	687.14
73	974.67	2.98	690.11
73.5	975.69	2.96	693.07
74	976.70	2.94	696.00
74.5	977.71	2.92	698.91
75	978.71	2.90	701.81
75.5	979.71	2.88	704.68
76	980.69	2.86	707.54
76.5	981.67	2.84	710.37
77	982.65	2.82	713.19
77.5	983.62	2.80	715.99
78	984.58	2.78	718.77
78.5	985.53	2.76	721.53
79	986.48	2.75	724.27
79.5	987.43	2.73	726.99
80	988.37	2.71	729.69
80.5	989.30	2.69	729.21
81	990.22	2.67	731.88
81.5	991.15	2.66	734.54
82	992.06	2.64	737.17
82.5	992.97	2.62	739.79
83	993.87	2.60	742.40
83.5	994.77	2.58	744.98
84	995.67	2.57	747.55
84.5	996.55	2.55	750.10
85	997.44	2.53	752.63
85.5	998.31	2.52	755.15
86	999.19	2.50	757.65
86.5	1000.05	2.48	760.13
87	1000.92	2.47	762.60
87.5	1001.77	2.45	765.04
88	1002.63	2.43	767.48

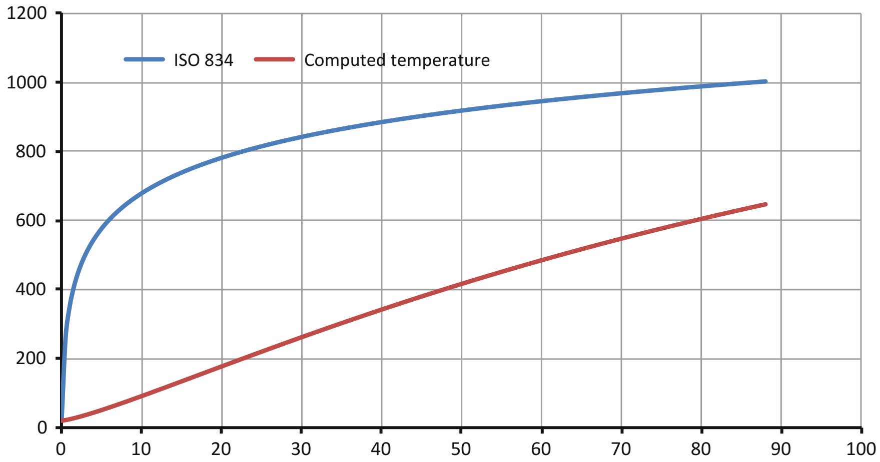
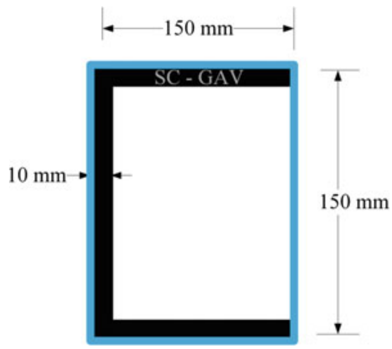


Fig. 4.13 Temperature variation of box-insulated steel beam I section with time

Fig. 4.14 Box-insulated steel beam channel section for Example 4



Thermal analysis:

Increment in temperature in steel structure when exposed to fire is given by:

$$\Delta\theta_{a,t} = \frac{\frac{\lambda_p}{d_p}}{c_a\rho_a}\frac{A_m}{V}\frac{(\theta_t - \theta_{a,t})\Delta t}{1+\frac{\phi}{3}} - \left(e^{\left(\frac{\phi}{10}\right)} - 1\right)\Delta\theta_t$$

Material data:

$\lambda_p = 0.25$ W/m °C, $\rho_p = 500$ kg/m^3, $p = 2\%$, $c_p = 1500$ J/kg °C, $d_p = 0.020$ m, $A_m/V = 160$ m^{-1}, $\rho_a = 7850$ kg/m^3, and $c_a = 600$ J/kg °C.

Thus, $\phi = 0.50$ and, therefore, $\Delta\theta_{a,t} = 0.0109\left(\theta_t - \theta_{a,t}\right) - 0.0523\Delta\theta_t$.

Details of temperature variation in the member with respect to time are summarized in Table 4.5. Figure 4.15 shows variation of structural temperature with time (Table 4.6).

Example 5 Evaluate the fire rating of the unprotected I section subjected to a dead load of 17.8 kN/m and live load of 9.375 kN/m, as shown in Fig. 4.16, when exposed to three-sided standard ISO-834 fire.

Structural Analysis:

For the given beam, load combination for fire is considered as per EN 1991-1-2 and is given by:

$$1 \times \mathrm{P.L} + 0.3 \times \mathrm{I.L.}$$

Design moment on the structure, $M_{\mathrm{fi,ED}}$, is computed as follows:

$$= 3.75 \times (17.8 + 0.3 \times 0.375) \times 8^2/8 = 165\,\mathrm{kN/m}.$$

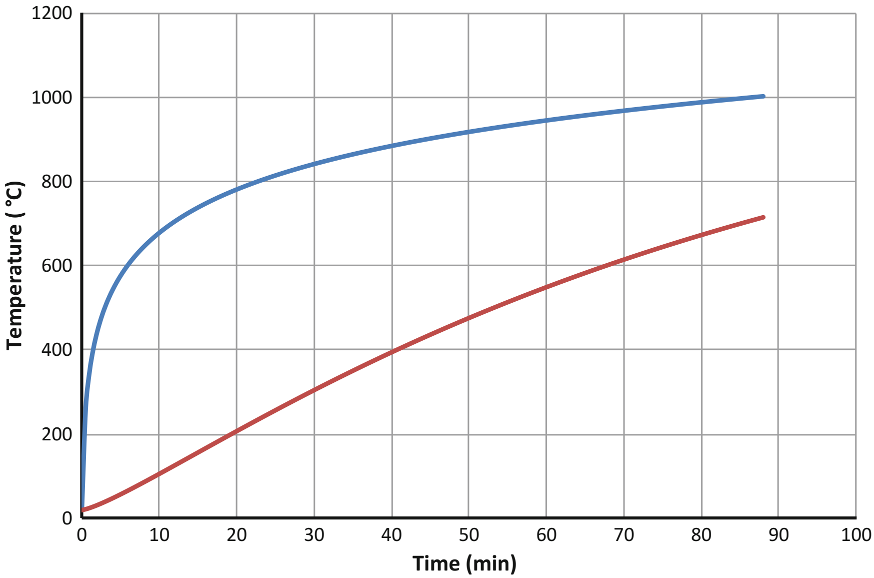


Fig. 4.15 Temperature variation of box-insulated channel section with time

Table 4.6 Temperature in given steel section of Example 4

Time	θ_t	$\Delta\theta$	θ_m
0	**20.00**	**0.00**	**20.00**
0.5	261.14	2.60	22.63
1	349.21	3.38	26.05
1.5	404.31	3.90	29.99
2	444.50	4.27	34.29
2.5	476.17	4.54	38.88
3	502.29	4.76	43.69
3.5	524.53	4.94	48.67
4	543.89	5.08	53.80
4.5	561.03	5.21	59.06
5	576.41	5.31	64.42
5.5	590.36	5.40	69.87
6	603.12	5.47	75.39
6.5	614.88	5.54	80.97
7	625.78	5.59	86.61
7.5	635.94	5.64	92.30
8	645.46	5.68	98.03
8.5	654.40	5.71	103.79
9	662.85	5.74	109.57
9.5	670.84	5.76	115.38
10	678.43	5.78	121.21
10.5	685.65	5.79	127.05
11	692.54	5.80	132.91
11.5	699.13	5.81	138.77
12	705.44	5.82	144.63
12.5	711.49	5.82	150.50
13	717.31	5.82	156.36
13.5	722.91	5.82	162.23
14	728.31	5.81	168.08
14.5	733.52	5.81	173.94
15	738.56	5.80	179.78
15.5	743.43	5.79	185.61
16	748.15	5.78	191.43
16.5	752.73	5.76	197.24
17	757.17	5.75	203.03
17.5	761.48	5.74	208.81
18	765.67	5.72	214.57
18.5	769.75	5.70	220.32
19	773.72	5.69	226.05
19.5	777.59	5.67	231.75
20	781.35	5.65	237.44

(continued)

Table 4.6 (continued)

Time	θ_t	$\Delta\theta$	θ_m
20.5	785.03	5.63	243.10
21	788.62	5.61	248.75
21.5	792.13	5.59	254.37
22	795.55	5.56	259.97
22.5	798.90	5.54	265.55
23	802.17	5.52	271.10
23.5	805.38	5.49	276.63
24	808.52	5.47	282.13
24.5	811.59	5.44	287.61
25	814.60	5.42	293.06
25.5	817.56	5.39	298.49
26	820.45	5.37	303.89
26.5	823.29	5.34	309.26
27	826.08	5.32	314.61
27.5	828.82	5.29	319.93
28	831.50	5.26	325.22
28.5	834.14	5.24	330.49
29	836.74	5.21	335.72
29.5	839.29	5.18	340.93
30	841.80	5.15	346.12
30.5	844.26	5.13	351.27
31	846.69	5.10	356.40
31.5	849.08	5.07	361.49
32	851.43	5.04	366.56
32.5	853.74	5.02	371.60
33	856.02	4.99	376.61
33.5	858.26	4.96	381.60
34	860.48	4.93	386.55
34.5	862.66	4.90	391.48
35	864.80	4.88	396.37
35.5	866.92	4.85	401.24
36	869.01	4.82	406.08
36.5	871.07	4.79	410.89
37	873.10	4.76	415.67
37.5	875.11	4.73	420.43
38	877.08	4.71	425.15
38.5	879.04	4.68	429.85
39	880.96	4.65	434.51
39.5	882.87	4.62	439.15
40	884.74	4.59	443.76
40.5	886.60	4.57	448.34

(continued)

Table 4.6 (continued)

Time	θ_t	$\Delta\theta$	θ_m
41	888.43	4.54	452.90
41.5	890.24	4.51	457.42
42	892.03	4.48	461.92
42.5	893.80	4.45	466.39
43	895.55	4.43	470.83
43.5	897.27	4.40	475.24
44	898.98	4.37	479.62
44.5	900.67	4.34	483.98
45	902.34	4.32	488.31
45.5	903.99	4.29	492.61
46	905.62	4.26	496.88
46.5	907.24	4.23	501.12
47	908.84	4.21	505.34
47.5	910.42	4.18	509.53
48	911.98	4.15	513.70
48.5	913.53	4.13	517.83
49	915.07	4.10	521.94
49.5	916.58	4.07	526.03
50	918.08	4.05	530.08
50.5	919.57	4.02	534.11
51	921.04	4.00	538.11
51.5	922.50	3.97	542.09
52	923.95	3.94	546.04
52.5	925.38	3.92	549.97
53	926.79	3.89	553.86
53.5	928.20	3.87	557.74
54	929.59	3.84	561.58
54.5	930.97	3.82	565.40
55	932.33	3.79	569.20
55.5	933.68	3.77	572.97
56	935.03	3.74	576.72
56.5	936.35	3.72	580.44
57	937.67	3.69	584.13
57.5	938.98	3.67	587.80
58	940.27	3.64	591.45
58.5	941.55	3.62	595.07
59	942.83	3.59	598.67
59.5	944.09	3.57	602.24
60	945.34	3.55	605.79
60.5	946.58	3.52	609.32
61	947.81	3.50	612.82

(continued)

Table 4.6 (continued)

Time	θ_t	$\Delta\theta$	θ_m
61.5	949.03	3.48	616.30
62	950.24	3.45	619.75
62.5	951.44	3.43	623.18
63	952.64	3.41	626.59
63.5	953.82	3.38	629.98
64	954.99	3.36	633.34
64.5	956.15	3.34	636.68
65	957.31	3.32	640.00
65.5	958.46	3.29	643.29
66	959.59	3.27	646.56
66.5	960.72	3.25	649.81
67	961.84	3.23	653.04
67.5	962.95	3.21	656.25
68	964.06	3.19	659.43
68.5	965.15	3.16	662.60
69	966.24	3.14	665.74
69.5	967.32	3.12	668.86
70	968.39	3.10	671.96
70.5	969.46	3.08	675.03
71	970.51	3.06	678.09
71.5	971.56	3.04	681.13
72	972.61	3.02	684.14
72.5	973.64	3.00	687.14
73	974.67	2.98	690.11
73.5	975.69	2.96	693.07
74	976.70	2.94	696.00
74.5	977.71	2.92	698.91
75	978.71	2.90	701.81
75.5	979.71	2.88	704.68
76	980.69	2.86	707.54
76.5	981.67	2.84	710.37
77	982.65	2.82	713.19
77.5	983.62	2.80	715.99
78	984.58	2.78	718.77
78.5	985.53	2.76	721.53
79	986.48	2.75	724.27
79.5	987.43	2.73	726.99
80	988.37	2.71	729.69
80.5	989.30	2.69	729.21
81	990.22	2.67	731.88
81.5	991.15	2.66	734.54

(continued)

Table 4.6 (continued)

Time	θ_t	$\Delta\theta$	θ_m
82	992.06	2.64	737.17
82.5	992.97	2.62	739.79
83	993.87	2.60	742.40
83.5	994.77	2.58	744.98
84	995.67	2.57	747.55
84.5	996.55	2.55	750.10
85	997.44	2.53	752.63
85.5	998.31	2.52	755.15
86	999.19	2.50	757.65
86.5	1000.05	2.48	760.13
87	1000.92	2.47	762.60
87.5	1001.77	2.45	765.04
88	1002.63	2.43	767.48

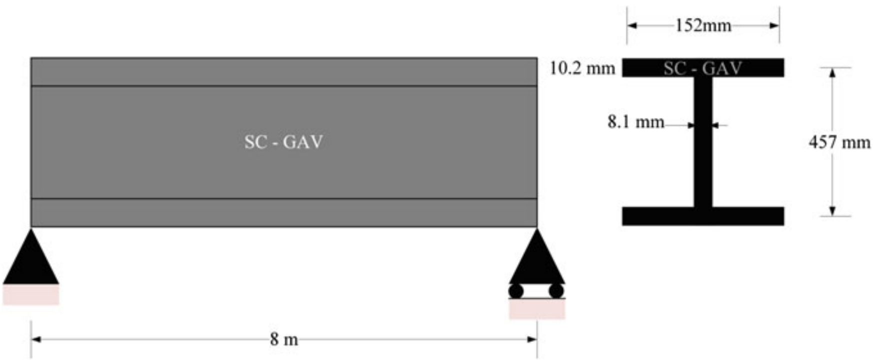


Fig. 4.16 Unprotected steel I section for Example 5

Section classification:

$$\varepsilon = 0.85\left[\frac{235}{f_y}\right]^{0.5} = 0.85\left[\frac{235}{275}\right]^{0.5} = 0.786$$

Class of flange:

$$\varepsilon = 0.85\left[\frac{235}{f_y}\right]^{0.5} = 0.85\left[\frac{235}{275}\right]^{0.5} = 0.786$$

$$c = 0.5[b - 2.t_f - t_w] = 62.6\,\text{mm}$$
$$\frac{c}{t_f} = 4.71$$

Limiting value for Class 1 is $9\varepsilon = 9 \times 0.786 = 7.07$. Hence, flange is Class 1
Class of web:

$$c = 407.6\,\text{mm}$$
$$\frac{c}{t_w} = 50.3$$

Limiting value for Class 1 is $9\varepsilon = 72 \times 0.786 = 56.6$. Hence, web is Class 1.
Evaluation of fire rating in accordance with EN 1993 1-2:
Strength-based fire resistance:
Design moment of resistance $M_{\text{RD}} = 275 \times 1287 \times 10^3/1.0 = 354$ kNm.

$$k_{y,\theta} = \frac{M_{\text{fi,ED}}}{M_{\text{RD}}} = \frac{163}{354} = 0.460$$

As it is a three-sided exposure, effective value of $k_{y,\theta}$ is obtained by further dividing with 0.85 as 0.541. From Table 4.2, it can be found using linear interpolation that this value of $k_{y,\theta}$ is obtained at a temperature of 577 °C.

Thermal analysis:

Increment in temperature in steel structure when exposed to fire is given by:

$$\Delta\theta_{a,t} = \frac{\alpha}{c_a \rho_a} \frac{A_m}{V_m} \left(\theta_t - \theta_{a,t}\right) \Delta t$$

where c_a is the specific heat, ρ_a is the density of the material, A_m is the area of the unit length exposure, V is the volume of unit length section, $\theta_{a,t}$ is the temperature in member, θ_t is the gas temperature, Δt is the increment in time, and α is the effective heat transfer coefficient given by:

$$\alpha = 25 + \frac{0.7 \times k_{\text{sh}} \times 0.56 \times 10^{-8}}{\theta_t - \theta_{a,t}} \left[(\theta_t + 273)^4 - \left(\theta_{a,t} + 273\right)^4 \right]$$

where k_{sh} is the shielding parameter given by $(A_m/V)_m/(A_m/V)$. For the given example, the gas temperature is given by:

$$\theta_t = 20 + 345 \log(8t + 1)$$

For the given section, $(A_m/V_m) = 205\ \text{m}^{-1}$ and $(A_m/V_m)_b = 165\ \text{m}^{-1}$. Subsequently, k_{sh} is computed as 0.72. Detailed member temperature with 0.5-min time interval is shown in Table 4.7. Critical temperature of the beam is 598 °C. Temperature obtained from strength-based criteria for beam is 577 °C. Failure of the beam will happen at lower of the two temperatures, i.e., at 577 °C. Thus, the fire rating of the beam is approximately 20 min as shown in Fig. 4.17 and highlighted in Table 4.7.

Example 6 Consider the same beam and fire scenario as in Example 5 but protected with a 20-mm fiberboard fire protection. Evaluate its fire rating (Fig. 4.18).

Table 4.7 Temperature in the steel section of Example 5

Time	α	θ_t	$\Delta\theta$	θ_m
0.0		20.00	0.00	20.00
0.5	25.963	261.15	8.23	28.23
1.0	26.384	349.21	11.13	39.36
1.5	26.727	404.31	12.82	52.17
2.0	27.029	444.51	13.94	66.11
2.5	27.308	476.17	14.72	80.82
3.0	27.572	502.29	15.27	96.10
3.5	27.825	524.53	15.67	111.76
4.0	28.073	543.89	15.94	127.70
4.5	28.315	561.03	16.12	143.83
5.0	28.555	576.41	16.23	160.06
5.5	28.793	590.36	16.28	176.34
6.0	29.03	603.12	16.28	192.62
6.5	29.266	614.88	16.24	208.86
7.0	29.503	625.78	16.16	225.02
7.5	29.74	635.94	16.06	241.08
8.0	29.977	645.46	15.93	257.01
8.5	30.215	654.40	15.78	272.79
9.0	30.453	662.85	15.61	288.40
9.5	30.693	670.84	15.43	303.83
10.0	30.933	678.43	15.23	319.05
10.5	31.174	685.65	15.02	334.07
11.0	31.415	692.54	14.80	348.87
11.5	31.657	699.13	14.57	363.44
12.0	31.899	705.44	14.34	377.78
12.5	32.142	711.49	14.10	391.87
13.0	32.385	717.31	13.85	405.72
13.5	32.628	722.91	13.60	419.32
14.0	32.871	728.31	13.35	432.67
14.5	33.114	733.52	13.09	445.76
15.0	33.356	738.56	12.83	458.59
15.5	33.598	743.43	12.58	471.17
16.0	33.839	748.15	12.32	483.49
16.5	34.079	752.73	12.06	495.54
17.0	34.318	757.17	11.80	507.34
17.5	34.556	761.48	11.54	518.88
18.0	34.793	765.67	11.28	530.17
18.5	35.029	769.75	11.03	541.20
19.0	35.262	773.72	10.78	551.97
19.5	35.495	777.59	10.52	562.49
20.0	***35.725***	*781.36*	*10.28*	*572.77*

(continued)

Table 4.7 (continued)

Time	α	θ_t	$\Delta\theta$	θ_m
20.5	35.954	785.03	10.03	582.80
21.0	36.181	788.62	9.79	592.58
21.5	36.405	792.13	9.55	602.13
22.0	36.628	795.55	9.31	611.44
22.5	36.848	798.90	9.08	620.52
23.0	37.066	802.174	8.848	629.365
23.5	37.281	805.379	8.623	637.988
24.0	37.494	808.517	8.402	646.390
24.5	37.705	811.591	8.185	654.576
25.0	37.913	814.603	7.973	662.548

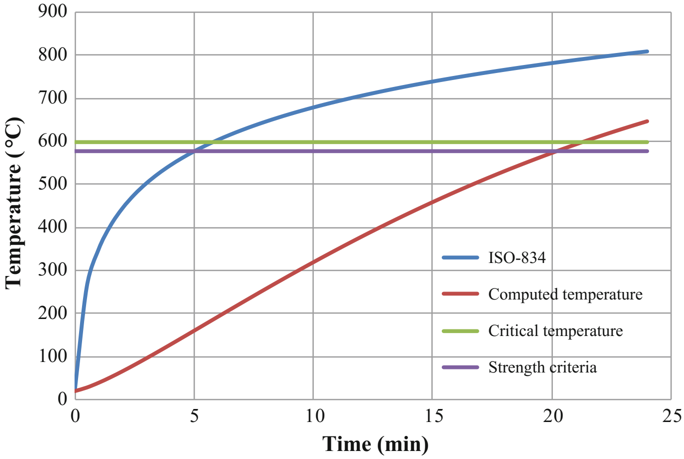


Fig. 4.17 Temperature variation for computing fire rating of steel I section

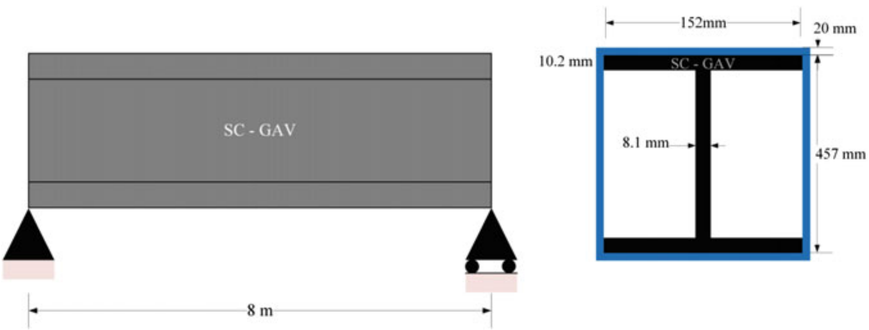


Fig. 4.18 Box-insulated steel I section for Example 6

Structural calculations will be same as that of Example 5, i.e., $M_{\text{fi,ED}} = 165$ kN/m, and both web and flange are of Class 1, $M_{\text{RD}} = 354$ kNm, and $k_{y,\theta} = 0.541$, which is attained at a temperature of 577 °C. Thermal analysis can also be performed in manner similar to previous examples with $\lambda_p = 0.25$ W/m °C, $\rho_p = 500$ kg/m^3, $p = 2\%$, $c_p = 1500$ J/kg °C, $A_p/V = 170$ m^{-1}, $\rho_a = 7850$ kg/m^3, and $c_a = 600$ J/kg °C.

Temperature variation is shown in Fig. 4.19 and in Table 4.8. Similar to Example 5, critical temperature is 577 °C (from strength criterion). Thus, the fire rating is about 54 min. Notice that when the same beam was not protected (in Example 5), its fire rating was 20 min. Adding 20 mm of fiberboard protection has increased the fire rating of the beam by almost 3 times.

Example 7 Evaluate the fire rating for an unprotected column subjected to a dead load of 300 kN and a live load of 2333 kN and having an effective length of 3.5 m with section shown in Fig. 4.20, exposed to four-sided standard ISO-834 fire.

The ambient design gives $\lambda = 0.612$ with $N_{\text{RD}} = 3112$ kN. At ambient, $N_{\text{ED}} = 2850$ kN.

Section classification:

$$\varepsilon = 0.85\left[\frac{235}{f_y}\right]^{0.5} = 0.85\left[\frac{235}{275}\right]^{0.5} = 0.786$$

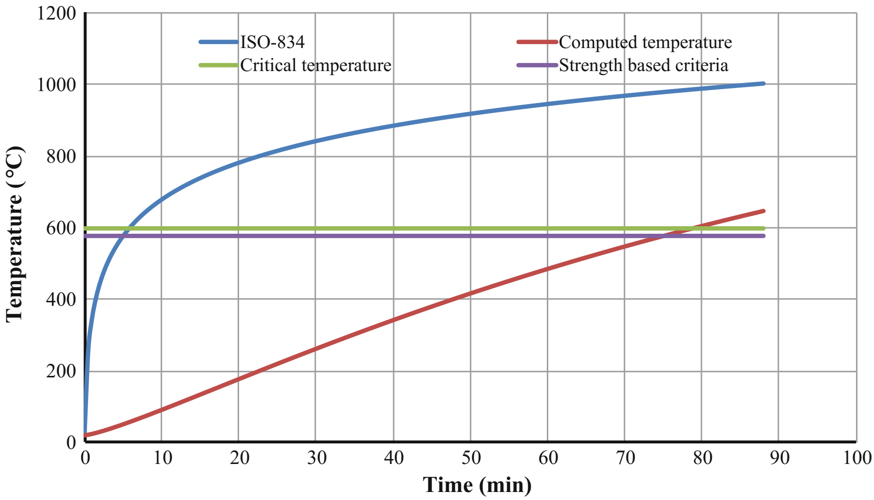


Fig. 4.19 Temperature variation for computing fire rating of box-insulated steel I section

Table 4.8 Temperature in the steel section of Example 6

Time	θ_t	$\Delta\theta$	θ_m
0	20.00	0.00	20.00
0.5	261.14	2.77	22.77
1	349.21	3.59	26.35
1.5	404.31	4.13	30.49
2	444.50	4.52	35.01
2.5	476.17	4.81	39.81
3	502.29	5.04	44.85
3.5	524.53	5.22	50.07
4	543.89	5.37	55.44
4.5	561.03	5.50	60.94
5	576.41	5.60	66.54
5.5	590.36	5.69	72.24
6	603.12	5.77	78.01
6.5	614.88	5.83	83.84
7	625.78	5.89	89.73
7.5	635.94	5.94	95.67
8	645.46	5.97	101.64
8.5	654.40	6.01	107.65
9	662.85	6.03	113.68
9.5	670.84	6.05	119.73
10	678.43	6.07	125.80
10.5	685.65	6.08	131.88
11	692.54	6.09	137.97
11.5	699.13	6.10	144.07
12	705.44	6.10	150.17
12.5	711.49	6.10	156.26
13	717.31	6.09	162.36
13.5	722.91	6.09	168.45
14	728.31	6.08	174.53
14.5	733.52	6.07	180.60
15	738.56	6.06	186.66
15.5	743.43	6.05	192.71
16	748.15	6.03	198.74
16.5	752.73	6.02	204.76
17	757.17	6.00	210.76
17.5	761.48	5.98	216.74
18	765.67	5.96	222.70
18.5	769.75	5.94	228.64
19	773.72	5.92	234.56
19.5	777.59	5.90	240.46
20	781.35	5.87	246.33

(continued)

Table 4.8 (continued)

Time	θ_t	$\Delta\theta$	θ_m
20.5	785.03	5.85	252.18
21	788.62	5.83	258.01
21.5	792.13	5.80	263.81
22	795.55	5.77	269.58
22.5	798.90	5.75	275.33
23	802.17	5.72	281.05
23.5	805.38	5.69	286.74
24	808.52	5.67	292.41
24.5	811.59	5.64	298.05
25	814.60	5.61	303.66
25.5	817.56	5.58	309.24
26	820.45	5.55	314.79
26.5	823.29	5.52	320.31
27	826.08	5.49	325.80
27.5	828.82	5.46	331.26
28	831.50	5.43	336.70
28.5	834.14	5.40	342.10
29	836.74	5.37	347.47
29.5	839.29	5.34	352.81
30	841.80	5.31	358.12
30.5	844.26	5.28	363.40
31	846.69	5.25	368.65
31.5	849.08	5.22	373.86
32	851.43	5.19	379.05
32.5	853.74	5.15	384.20
33	856.02	5.12	389.33
33.5	858.26	5.09	394.42
34	860.48	5.06	399.48
34.5	862.66	5.03	404.51
35	864.80	5.00	409.51
35.5	866.92	4.97	414.47
36	869.01	4.94	419.41
36.5	871.07	4.90	424.31
37	873.10	4.87	429.19
37.5	875.11	4.84	434.03
38	877.08	4.81	438.84
38.5	879.04	4.78	443.62
39	880.96	4.75	448.37
39.5	882.87	4.72	453.09
40	884.74	4.69	457.77
40.5	886.60	4.66	462.43

(continued)

Table 4.8 (continued)

Time	θ_t	$\Delta\theta$	θ_m
41	888.43	4.63	467.06
41.5	890.24	4.60	471.65
42	892.03	4.56	476.22
42.5	893.80	4.53	480.75
43	895.55	4.50	485.25
43.5	897.27	4.47	489.73
44	898.98	4.44	494.17
44.5	900.67	4.41	498.58
45	902.34	4.38	502.97
45.5	903.99	4.35	507.32
46	905.62	4.32	511.65
46.5	907.24	4.30	515.94
47	908.84	4.27	520.21
47.5	910.42	4.24	524.45
48	911.98	4.21	528.65
48.5	913.53	4.18	532.83
49	915.07	4.15	536.98
49.5	916.58	4.12	541.11
50	918.08	4.09	545.20
50.5	919.57	4.06	549.26
51	921.04	4.04	553.30
51.5	922.50	4.01	557.31
52	923.95	3.98	561.29
52.5	925.38	3.95	565.24
53	926.79	3.93	569.17
53.5	928.20	3.90	573.07
54	***929.59***	*3.87*	*576.94*
54.5	930.97	3.84	580.78
55	932.33	3.82	584.60
55.5	933.68	3.79	588.39
56	935.03	3.76	592.15
56.5	936.35	3.74	595.89
57	937.67	3.71	599.60
57.5	938.98	3.68	603.29
58	940.27	3.66	606.95
58.5	941.55	3.63	610.58
59	942.83	3.61	614.19
59.5	944.09	3.58	617.77
60	945.34	3.56	621.33
60.5	946.58	3.53	624.86
61	947.81	3.51	628.36

(continued)

Table 4.8 (continued)

Time	θ_t	$\Delta\theta$	θ_m
61.5	949.03	3.48	631.85
62	950.24	3.46	635.30
62.5	951.44	3.43	638.74
63	952.64	3.41	642.14
63.5	953.82	3.38	645.53
64	954.99	3.36	648.89
64.5	956.15	3.34	652.22
65	957.31	3.31	655.54
65.5	958.46	3.29	658.83
66	959.59	3.27	662.09
66.5	960.72	3.24	665.33
67	961.84	3.22	668.55
67.5	962.95	3.20	671.75
68	964.06	3.17	674.92
68.5	965.15	3.15	678.08
69	966.24	3.13	681.20
69.5	967.32	3.11	684.31
70	968.39	3.08	687.40
70.5	969.46	3.06	690.46
71	970.51	3.04	693.50
71.5	971.56	3.02	696.52
72	972.61	3.00	699.52
72.5	973.64	2.98	702.49
73	974.67	2.96	705.45
73.5	975.69	2.93	708.38
74	976.70	2.91	711.30
74.5	977.71	2.89	714.19
75	978.71	2.87	717.06
75.5	979.71	2.85	719.91
76	980.69	2.83	722.74
76.5	981.67	2.81	725.56
77	982.65	2.79	728.35
77.5	983.62	2.77	731.12
78	984.58	2.75	733.87
78.5	985.53	2.73	736.60
79	986.48	2.71	739.32
79.5	987.43	2.69	742.01
80	988.37	2.67	744.69

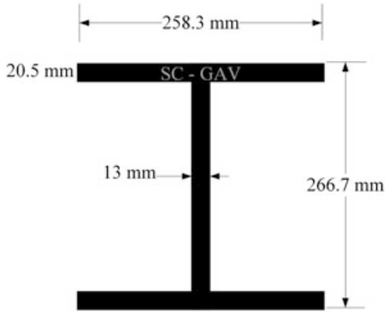


Fig. 4.20 Steel column I section for Example 7

Class of flange:

$$c = 0.5\left[b - 2.t_f - t_w\right] = 110.3 \text{ mm}$$
$$\frac{c}{t_f} = 5.38$$

Limiting value for Class 1 is 9ε = 9 × 0.786 = 7.07. Hence, flange is Class 1
Class of web:

$$c = 200.3\,\text{mm}$$
$$\frac{c}{t_w} = 15.6$$

Limiting value for Class 1 is 9ε = 72 × 0.786 = 56.6. Hence, web is Class 1.

As the exact end conditions are not known, buckling length in the fire limit state is assumed as 3.5 m. Taking ψ = 0.3 on the variable load gives the following value:

$$N_{\text{ED,fi}} = 1.0 \times 770 + 0.3 \times 770 = 1000\,\text{kN}$$

$$k_{y,\theta} = \frac{N_{\text{fi,ED}}}{N_{\text{RD,fi}}} = \frac{1300}{2850} = 0.57$$

Strength-based fire rating:

From Table 4.2, this strength degradation is obtained at a temperature of 565 °C. Degradation in modulus of elasticity at that temperature is given by $k_{E,\theta}$ = 0.4115. Subsequently, slenderness ratio at 565 °C is computed as computed below:

$$\lambda_\theta = \lambda\left[\frac{k_{y,\theta}}{k_{E,\theta}}\right]^{0.5} = 0.725.$$

The buckling strength reduction factor χ_{fi} is determined from the following relationship:

$$\chi_{\text{fi}} = \frac{1}{\phi_\theta + \sqrt{\phi_\theta^2 - \lambda_\theta^2}},$$

$$\phi_\theta = 0.5\left[1 + 0.65\sqrt{\frac{235}{f_y}}\lambda_\theta + \lambda_\theta^2\right].$$

Subsequently, axial resistance at elevated temperatures is computed as given below:

$$N_{b,\text{fi},t,\text{RD}} = \chi_{\text{fi}} A k_{y,\theta} \frac{f_y}{\gamma_{m,\text{fi}}} = 1657\,\text{kN}$$

The detailed calculations are given in Table 4.9 (refer Fig. 4.21 for the temperature variation with respect to time). The fire rating is between 30 and 35 min and can be said to be 30 min, conservatively.

Example 8 Consider the same column and fire scenario as in Example 7 but with a fiberboard protection of 30 mm. Evaluate its fire rating (Fig. 4.22).

Design at ambient temperature gives $\lambda = 0.612$ with $N_{RD} = 3112$ kN. At ambient, $N_{ED} = 2850$ kN.

Section classification:

$$\varepsilon = 0.85\left[\frac{235}{f_y}\right]^{0.5} = 0.85\left[\frac{235}{275}\right]^{0.5} = 0.786$$

Table 4.9 Temperature in the steel section of Example 7

Time (min)	Temperature	$k_{y,\theta}$	$k_{E,\theta}$	λ_θ	ϕ_t	χ_{fi}	N_R	N
0	20.00	1.00	1.00	0.52	0.79	0.72	2674.05	1000
5	101.71	1.00	1.00	0.52	0.79	0.72	2673.00	1000
10	204.92	1.00	0.90	0.55	0.82	0.70	2614.22	1000
15	307.33	1.00	0.79	0.59	0.85	0.68	2544.82	1000
20	403.21	0.99	0.70	0.62	0.88	0.66	2452.08	1000
25	490.36	0.80	0.61	0.60	0.86	0.68	2017.09	1000
30	567.95	0.57	0.40	0.62	0.88	0.67	1408.80	1000
35	636.00	*0.38*	*0.25*	*0.65*	*0.91*	*0.65*	*924.23*	*1000*
40	694.94	0.24	0.14	0.69	0.94	0.63	566.87	1000

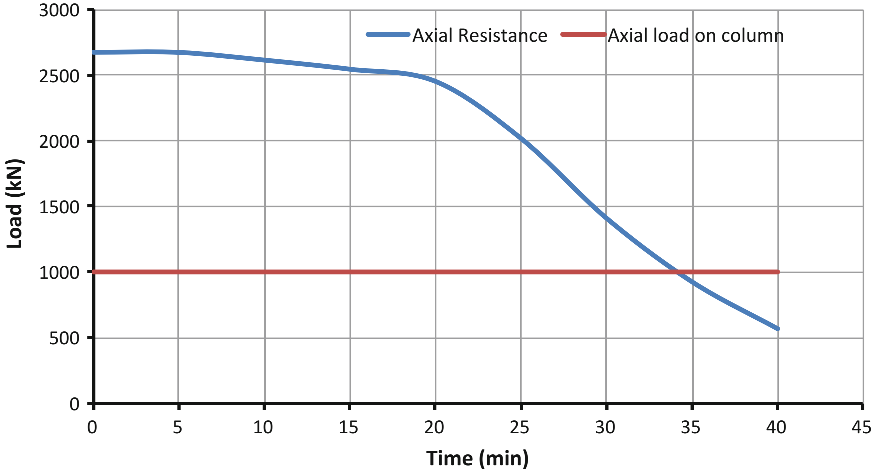


Fig. 4.21 Load versus time graph for unprotected column I section

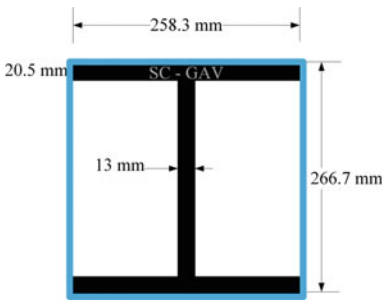


Fig. 4.22 Protected box column I section for Example 8

Class of flange:

$$c = 0.5\left[b - 2.t_f - t_w\right] = 110.3\,\mathrm{mm}$$
$$\frac{c}{t_f} = 5.38$$

Limiting value for Class 1 is 9ε = 9 × 0.786 = 7.07. Hence, flange is Class 1
Class of web:

$$c = 200.3\,\mathrm{mm}$$
$$\frac{c}{t_w} = 15.6$$

Limiting value for Class 1 is 9ε = 72 × 0.786 = 56.6. Hence, web is Class 1.

As the exact end conditions are not known, buckling length in the fire limit state is assumed as 3.5 m. Taking ψ = 0.3 on the variable load gives the following:

$$N_{\mathrm{ED,fi}} = 1.0 \times 770 + 0.3 \times 770 = 1000\,\mathrm{kN}$$

$$k_{y,\theta} = \frac{N_{\mathrm{fi,ED}}}{N_{\mathrm{RD,fi}}} = \frac{1300}{2850} = 0.57$$

Strength-based fire rating:

From Table 4.10, aforementioned strength degradation is obtained at a temperature of 565 °C. The degradation in modulus of elasticity at that temperature is given by $k_{\mathrm{E},\theta} = 0.4115$. Subsequently, slenderness ratio at 565 °C is computed as below:

$$\lambda_\theta = \lambda \left[\frac{k_{y,\theta}}{k_{\mathrm{E},\theta}}\right]^{0.5} = 0.725.$$

The buckling strength reduction factor χ_{fi} is determined from the following relationship:

Table 4.10 Temperature in the steel section of Example 8

Time (min)	Temperature	$k_{y,\theta}$	$k_{E,\theta}$	λ_θ	ϕ_t	χ_{fi}	N_R	N
0	20.00	1.00	1.00	0.52	0.79	0.72	2674.05	1000
5	50.65	1.00	1.00	0.52	0.79	0.72	2674.05	1000
10	91.00	1.00	1.00	0.52	0.79	0.72	2674.05	1000
15	133.74	1.00	0.97	0.53	0.80	0.71	2655.77	1000
20	176.93	1.00	0.92	0.54	0.81	0.71	2629.41	1000
25	219.71	1.00	0.88	0.56	0.82	0.70	2604.79	1000
30	261.58	1.00	0.84	0.57	0.83	0.69	2578.40	1000
35	302.38	1.00	0.80	0.59	0.85	0.69	2547.79	1000
40	341.65	1.00	0.76	0.60	0.86	0.68	2519.38	1000
45	379.59	1.00	0.72	0.62	0.87	0.67	2486.20	1000
50	416.30	0.96	0.68	0.62	0.88	0.67	2380.22	1000
55	451.00	0.89	0.65	0.61	0.87	0.67	2214.10	1000
60	484.57	0.81	0.62	0.60	0.86	0.68	2044.18	1000
65	516.64	0.73	0.55	0.60	0.86	0.68	1836.91	1000
70	547.30	0.63	0.46	0.61	0.87	0.67	1579.95	1000
75	576.59	0.54	0.38	0.63	0.88	0.66	1332.11	1000
80	604.54	0.46	0.30	0.65	0.90	0.65	1116.61	1000
85	631.23	*0.40*	*0.25*	*0.65*	*0.91*	*0.65*	*953.46*	*1000*
90	656.67	0.33	0.21	0.66	0.91	0.65	794.29	1000

$$\chi_{\mathrm{fi}} = \frac{1}{\phi_\theta + \sqrt{\phi_\theta^2 - \lambda_\theta^2}},$$

$$\phi_\theta = 0.5\left[1 + 0.65\sqrt{\frac{235}{f_y}}\lambda_\theta + \lambda_\theta^2\right].$$

Subsequently, axial resistance at elevated temperatures is computed as below:

$$N_{b,\mathrm{fi},t,\mathrm{RD}} = \chi_{\mathrm{fi}} A k_{y,\theta} \frac{f_y}{\gamma_{m,\mathrm{fi}}} = 1657\,\mathrm{kN}$$

The calculations involved are presented in Table 4.10 (refer Fig. 4.23 for temperature variation in the member with respect to time). Fire rating of this column can be said to be 80 min. Notice that addition of fire protection has more than doubled the fire rating of the column.

Example 9 A beam is simply supported over a span of 8 m as shown in Fig. 4.24. It carries a permanent loading of 17.8 kN/m and an imposed load of 9.375 kN/m. Design a material board for this beam for a 60-min fire rating when subjected to ISO-834 standard fire exposure on three sides.

Structural Analysis:

For the given beam, load combination for fire is considered as per EN 1991-1-2 and is given by:

$$= 1 \times \mathrm{P.L} + 0.3 \times \mathrm{I.L.}$$

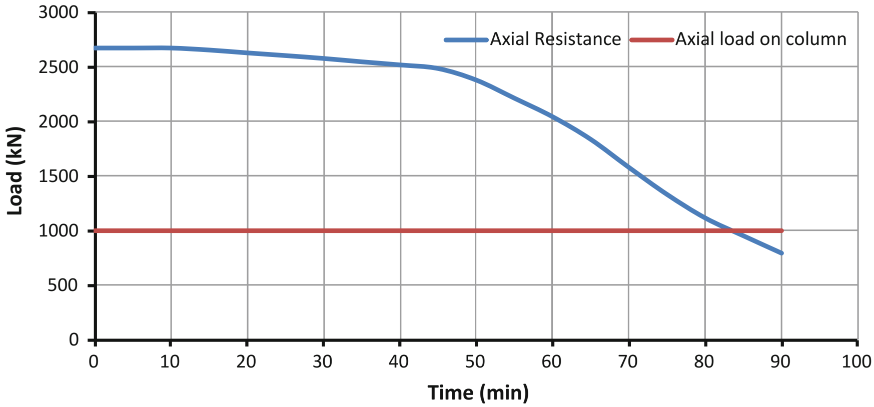


Fig. 4.23 Load versus time graph for protected column I section

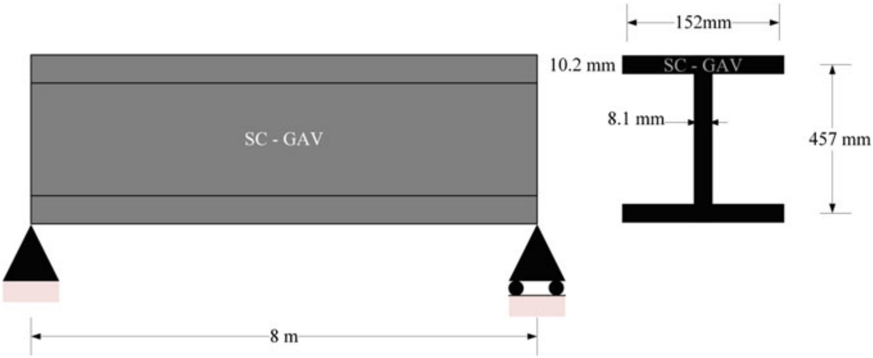


Fig. 4.24 Simply supported I section for Example 9

Design moment on the structure, $M_{\mathrm{fi,ED}}$, is computed as below:

$$= 3.75 \times (17.8 + 0.3 \times 9.375) \times 8^2/8 = 163\,\mathrm{kN/m}.$$

Section classification:

$$\varepsilon = 0.85\left[\frac{235}{f_y}\right]^{0.5} = 0.85\left[\frac{235}{275}\right]^{0.5} = 0.786$$

Class of flange:

$$\varepsilon = 0.85\left[\frac{235}{f_y}\right]^{0.5} = 0.85\left[\frac{235}{275}\right]^{0.5} = 0.786$$

$$c = 0.5\left[b - 2.t_f - t_w\right] = 62.6\,\mathrm{mm}$$
$$\frac{c}{t_f} = 4.71$$

Limiting value for Class 1 is $9\varepsilon = 9 \times 0.786 = 7.07$. Hence, flange is Class 1
Class of web:

$$c = 407.6\,\text{mm}$$
$$\frac{c}{t_w} = 50.3$$

Limiting value for Class 1 is 9ε = 72 × 0.786 = 56.6. Hence, web is Class 1.
Evaluation of fire rating in accordance with EN 1993 1-2:
Strength-based fire resistance:
Design moment of resistance M_{RD} = 275 × 1287 × 10^3/1.0 = 354 kNm.

$$k_{y,\theta} = \frac{M_{\text{fi,ED}}}{M_{\text{RD}}} = \frac{163}{354} = 0.460$$

As it is a three-sided exposure, effective value of $k_{y,\theta}$ is obtained by further dividing with 0.85 as 0.541. From Table 4.2, this strength degradation is obtained at a temperature of 577 °C.

Fire protection data:

Following data can be used for mineral fiber: (i.e., q_p = 500 kg/m^3, p = 2%, c_p = 1500 J/kgC, λ_p = 0.25 W/mC). For box protection, Am/V = 165 m^{-1}. As per Wickstorm's method, thickness of the required protection board is computed as:

$$d_p = \frac{\frac{V}{\lambda_p A_m} + \sqrt{\left(\frac{V}{\lambda_p A_m}\right)^2 + 4\frac{\rho_p}{\rho_a \lambda_p}\left[\frac{t_{\text{fi},d}}{40\left(\theta_{a,t}-140\right)}\right]^{1/3}}}{2\frac{\rho_p}{\rho_a \lambda_p}}$$

where d_p is the thickness of protection, ρ_a is the density of the material, ρ_p is the density of the protection material A_m is the area of the unit length exposure, V is the volume of unit length section, $\theta_{a,t}$ is the temperature in member, λ_p is the coefficient of thermal conductivity of the protection, and $t_{\text{fi},d}$ is the time of exposure. Upon substitution, thickness of protection is found to be 0.021 m. Let us provide 25-mm-thick board for protection. Temperature variation with this protection, as calculated by methods discussed earlier, is given in Table 4.11. It can be observed that steel temperature at 60 min is 540 °C which is lesser than the critical temperature of 577 °C as governed by strength degradation. Hence, the beam with 25 mm protection will have a rating of 60 min.

Example 10 A 254 × 254 × 107 UC beam carries a permanent load and a variable axial load both equal to 1270 kN at ambient conditions. The effective length of the column is 3.5 m. Design box protection is to ensure a 90-min fire rating for the column. Section is shown in Fig. 4.25.

Conventional design at ambient temperature gives λ = 0.612 with N_{RD} = 3112 kN and N_{ED} = 2850 kN.

Table 4.11 Temperature in the steel section of Example 9

Time	θ_t	$\Delta\theta$	θ_m
0	20.00	0.00	20.00
0.5	261.14	2.18	22.18
1	349.21	2.80	24.98
1.5	404.31	3.23	28.21
2	444.50	3.53	31.74
2.5	476.17	3.77	35.51
3	502.29	3.95	39.46
3.5	524.53	4.10	43.56
4	543.89	4.23	47.79
4.5	561.03	4.34	52.13
5	576.41	4.43	56.56
5.5	590.36	4.51	61.07
6	603.12	4.58	65.65
6.5	614.88	4.64	70.29
7	625.78	4.69	74.98
7.5	635.94	4.74	79.71
8	645.46	4.78	84.49
8.5	654.40	4.81	89.30
9	662.85	4.84	94.14
9.5	670.84	4.87	99.01
10	678.43	4.89	103.90
10.5	685.65	4.91	108.81
11	692.54	4.93	113.74
11.5	699.13	4.94	118.68
12	705.44	4.95	123.63
12.5	711.49	4.96	128.59
13	717.31	4.97	133.56
13.5	722.91	4.97	138.53
14	728.31	4.98	143.51
14.5	733.52	4.98	148.49
15	738.56	4.98	153.47
15.5	743.43	4.98	158.44
16	748.15	4.98	163.42
16.5	752.73	4.97	168.39
17	757.17	4.97	173.36
17.5	761.48	4.96	178.32
18	765.67	4.96	183.28
18.5	769.75	4.95	188.22
19	773.72	4.94	193.16
19.5	777.59	4.93	198.09
20	781.35	4.92	203.01

(continued)

Table 4.11 (continued)

Time	θ_t	$\Delta\theta$	θ_m
20.5	785.03	4.91	207.92
21	788.62	4.90	212.82
21.5	792.13	4.89	217.71
22	795.55	4.87	222.59
22.5	798.90	4.86	227.45
23	802.17	4.85	232.30
23.5	805.38	4.83	237.13
24	808.52	4.82	241.95
24.5	811.59	4.81	246.76
25	814.60	4.79	251.55
25.5	817.56	4.77	256.32
26	820.45	4.76	261.08
26.5	823.29	4.74	265.82
27	826.08	4.73	270.55
27.5	828.82	4.71	275.26
28	831.50	4.69	279.95
28.5	834.14	4.67	284.63
29	836.74	4.66	289.28
29.5	839.29	4.64	293.92
30	841.80	4.62	298.54
30.5	844.26	4.60	303.15
31	846.69	4.58	307.73
31.5	849.08	4.57	312.30
32	851.43	4.55	316.85
32.5	853.74	4.53	321.37
33	856.02	4.51	325.88
33.5	858.26	4.49	330.38
34	860.48	4.47	334.85
34.5	862.66	4.45	339.30
35	864.80	4.43	343.73
35.5	866.92	4.41	348.14
36	869.01	4.39	352.54
36.5	871.07	4.37	356.91
37	873.10	4.35	361.27
37.5	875.11	4.33	365.60
38	877.08	4.31	369.91
38.5	879.04	4.29	374.21
39	880.96	4.27	378.48
39.5	882.87	4.25	382.74
40	884.74	4.23	386.97
40.5	886.60	4.21	391.19

(continued)

Table 4.11 (continued)

Time	θ_t	$\Delta\theta$	θ_m
41	888.43	4.19	395.38
41.5	890.24	4.17	399.56
42	892.03	4.15	403.71
42.5	893.80	4.13	407.84
43	895.55	4.11	411.96
43.5	897.27	4.09	416.05
44	898.98	4.07	420.12
44.5	900.67	4.05	424.18
45	902.34	4.03	428.21
45.5	903.99	4.01	432.22
46	905.62	3.99	436.22
46.5	907.24	3.97	440.19
47	908.84	3.95	444.14
47.5	910.42	3.93	448.08
48	911.98	3.91	451.99
48.5	913.53	3.89	455.88
49	915.07	3.87	459.75
49.5	916.58	3.85	463.61
50	918.08	3.83	467.44
50.5	919.57	3.81	471.25
51	921.04	3.79	475.05
51.5	922.50	3.77	478.82
52	923.95	3.75	482.58
52.5	925.38	3.73	486.31
53	926.79	3.72	490.03
53.5	928.20	3.70	493.72
54	929.59	3.68	497.40
54.5	930.97	3.66	501.06
55	932.33	3.64	504.69
55.5	933.68	3.62	508.31
56	935.03	3.60	511.91
56.5	936.35	3.58	515.49
57	937.67	3.56	519.05
57.5	938.98	3.54	522.59
58	940.27	3.52	526.12
58.5	941.55	3.50	529.62
59	942.83	3.49	533.11
59.5	944.09	3.47	536.57
60	945.34	3.45	540.02

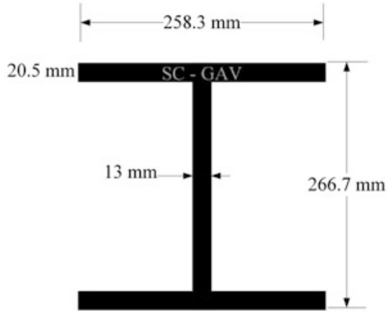


Fig. 4.25 Steel column I section for Example 10

Section classification:

$$\varepsilon = 0.85\left[\frac{235}{f_y}\right]^{0.5} = 0.85\left[\frac{235}{275}\right]^{0.5} = 0.786$$

Class of flange:

$$c = 0.5\left[b - 2.t_f - t_w\right] = 110.3\,\text{mm}$$
$$\frac{c}{t_f} = 5.38$$

Limiting value for Class 1 is 9ε = 9 × 0.786 = 7.07. Hence, flange is Class 1
Class of web:

$$c = 200.3\,\text{mm}$$
$$\frac{c}{t_w} = 15.6$$

Limiting value for Class 1 is 9ε = 72 × 0.786 = 56.6. Hence, web is Class 1.
Taking ψ = 0.3 on the variable, load is computed as:

$$N_{\text{ED,fi}} = 1.0 \times 1270 + 0.3 \times 1270 = 1653\,\text{kN}$$

$$k_{y,\theta} = \frac{N_{\text{fi,ED}}}{N_{\text{RD,fi}}} = \frac{1300}{2850} = 0.57$$

Strength-based fire rating:

From Table 4.2, this strength degradation is obtained at a temperature of 565 °C. Degradation in modulus of elasticity at that temperature is given by $k_{E,\theta} = 0.4115$. Subsequently, slenderness ratio at 565 °C is computed as below:

$$\lambda_\theta = \lambda \left[\frac{k_{y,\theta}}{k_{\mathrm{E},\theta}}\right]^{0.5} = 0.725.$$

The buckling strength reduction factor χ_{fi} is determined from the following relationship:

$$\chi_{fi} = \frac{1}{\phi_\theta + \sqrt{\phi_\theta^2 - \lambda_\theta^2}},$$

$$\phi_\theta = 0.5\left[1 + 0.65\sqrt{\frac{235}{f_y}}\lambda_\theta + \lambda_\theta^2\right].$$

Subsequently, axial resistance at elevated temperatures is computed as below:

$$N_{b,\mathrm{fi},t,\mathrm{RD}} = \chi_{\mathrm{fi}} A k_{y,\theta} \frac{f_y}{\gamma_{m,\mathrm{fi}}} = 1657\,kN$$

Fire protection data:

Mineral fiber (i.e., ρ_p = 500 kg/m^3, p = 2%, c_p = 1500 J/kgC, λ_p = 0.25 W/mC). For a four-sided box, Am/V = 85 m^{-1}. As per Wickstorm's method, thickness of protection required is given by the following relationship:

$$d_p = \frac{\frac{V}{\lambda_p A_m} + \sqrt{\left(\frac{V}{\lambda_p A_m}\right)^2 + 4\frac{\rho_p}{\rho_a \lambda_p}\left[\frac{t_{\mathrm{fi},d}}{40\left(\theta_{a,t} - 140\right)}\right]^{1/3}}}{2\frac{\rho_p}{\rho_a \lambda_p}}$$

where d_p is the thickness of protection, ρ_a is the density of the material, ρ_p is the density of the protection material, A_m is the area of the unit length exposure, V is the volume of unit length section, $\theta_{\mathrm{a,t}}$ is the temperature in member, λ_p is the coefficient of thermal conductivity of the protection, and $t_{\mathrm{fi,d}}$ is the time of exposure. Upon substitution of above, thickness of protection is 0.01984 m. Let us provide 20-mm-thick board. Temperature variation with this protection, as calculated by methods discussed earlier, is given in Table 4.11. It can be observed that steel temperature at 90 min is 517 °C which is lesser than the critical temperature of 565 °C as governed by strength degradation. Hence, the beam with 25 mm protection will have a rating of 90 min (Table 4.12).

Table 4.12 Temperature in the steel section of Example 10

Time	θ_t	$\Delta\theta$	θ_m
0.5	261.14	1.17	21.17
1	349.21	1.56	22.73
1.5	404.31	1.80	24.53
2	444.50	1.98	26.51
2.5	476.17	2.12	28.63
3	502.29	2.23	30.86
3.5	524.53	2.33	33.19
4	543.89	2.40	35.59
4.5	561.03	2.47	38.07
5	576.41	2.53	40.60
5.5	590.36	2.59	43.19
6	603.12	2.64	45.82
6.5	614.88	2.68	48.50
7	625.78	2.72	51.22
7.5	635.94	2.75	53.97
8	645.46	2.78	56.75
8.5	654.40	2.81	59.57
9	662.85	2.84	62.40
9.5	670.84	2.86	65.27
10	678.43	2.88	68.15
10.5	685.65	2.91	71.06
11	692.54	2.92	73.98
11.5	699.13	2.94	76.92
12	705.44	2.96	79.88
12.5	711.49	2.97	82.85
13	717.31	2.98	85.83
13.5	722.91	3.00	88.83
14	728.31	3.01	91.84
14.5	733.52	3.02	94.86
15	738.56	3.03	97.89
15.5	743.43	3.04	100.92
16	748.15	3.04	103.97
16.5	752.73	3.05	107.02
17	757.17	3.06	110.08
17.5	761.48	3.06	113.14
18	765.67	3.07	116.21
18.5	769.75	3.07	119.28
19	773.72	3.08	122.36
19.5	777.59	3.08	125.44
20	781.35	3.09	128.53
20.5	785.03	3.09	131.62

(continued)

Table 4.12 (continued)

Time	θ_t	$\Delta\theta$	θ_m
21	788.62	3.09	134.71
21.5	792.13	3.09	137.80
22	795.55	3.09	140.89
22.5	798.90	3.09	143.99
23	802.17	3.10	147.08
23.5	805.38	3.10	150.18
24	808.52	3.10	153.28
24.5	811.59	3.10	156.37
25	814.60	3.10	159.47
25.5	817.56	3.10	162.57
26	820.45	3.09	165.66
26.5	823.29	3.09	168.75
27	826.08	3.09	171.84
27.5	828.82	3.09	174.93
28	831.50	3.09	178.02
28.5	834.14	3.09	181.11
29	836.74	3.08	184.19
29.5	839.29	3.08	187.27
30	841.80	3.08	190.35
30.5	844.26	3.08	193.43
31	846.69	3.07	196.50
31.5	849.08	3.07	199.57
32	851.43	3.07	202.64
32.5	853.74	3.06	205.70
33	856.02	3.06	208.76
33.5	858.26	3.05	211.81
34	860.48	3.05	214.86
34.5	862.66	3.05	217.91
35	864.80	3.04	220.95
35.5	866.92	3.04	223.99
36	869.01	3.03	227.02
36.5	871.07	3.03	230.05
37	873.10	3.02	233.08
37.5	875.11	3.02	236.10
38	877.08	3.01	239.11
38.5	879.04	3.01	242.12
39	880.96	3.00	245.13
39.5	882.87	3.00	248.13
40	884.74	2.99	251.12
40.5	886.60	2.99	254.11
41	888.43	2.98	257.09

(continued)

Table 4.12 (continued)

Time	θ_t	$\Delta\theta$	θ_m
41.5	890.24	2.98	260.07
42	892.03	2.97	263.04
42.5	893.80	2.97	266.01
43	895.55	2.96	268.97
43.5	897.27	2.96	271.92
44	898.98	2.95	274.87
44.5	900.67	2.94	277.82
45	902.34	2.94	280.75
45.5	903.99	2.93	283.68
46	905.62	2.93	286.61
46.5	907.24	2.92	289.53
47	908.84	2.91	292.44
47.5	910.42	2.91	295.35
48	911.98	2.90	298.25
48.5	913.53	2.89	301.14
49	915.07	2.89	304.03
49.5	916.58	2.88	306.91
50	918.08	2.87	309.78
50.5	919.57	2.87	312.65
51	921.04	2.86	315.51
51.5	922.50	2.85	318.37
52	923.95	2.85	321.22
52.5	925.38	2.84	324.06
53	926.79	2.83	326.89
53.5	928.20	2.83	329.72
54	929.59	2.82	332.54
54.5	930.97	2.81	335.36
55	932.33	2.81	338.16
55.5	933.68	2.80	340.97
56	935.03	2.79	343.76
56.5	936.35	2.79	346.55
57	937.67	2.78	349.33
57.5	938.98	2.77	352.10
58	940.27	2.77	354.87
58.5	941.55	2.76	357.63
59	942.83	2.75	360.38
59.5	944.09	2.75	363.12
60	945.34	2.74	365.86
60.5	946.58	2.73	368.59
61	947.81	2.72	371.32
61.5	949.03	2.72	374.03

(continued)

Table 4.12 (continued)

Time	θ_t	$\Delta\theta$	θ_m
62	950.24	2.71	376.74
62.5	951.44	2.70	379.45
63	952.64	2.70	382.14
63.5	953.82	2.69	384.83
64	954.99	2.68	387.51
64.5	956.15	2.67	390.19
65	957.31	2.67	392.85
65.5	958.46	2.66	395.51
66	959.59	2.65	398.17
66.5	960.72	2.65	400.81
67	961.84	2.64	403.45
67.5	962.95	2.63	406.08
68	964.06	2.62	408.71
68.5	965.15	2.62	411.32
69	966.24	2.61	413.93
69.5	967.32	2.60	416.54
70	968.39	2.60	419.13
70.5	969.46	2.59	421.72
71	970.51	2.58	424.30
71.5	971.56	2.57	426.87
72	972.61	2.57	429.44
72.5	973.64	2.56	432.00
73	974.67	2.55	434.55
73.5	975.69	2.55	437.10
74	976.70	2.54	439.64
74.5	977.71	2.53	442.17
75	978.71	2.52	444.69
75.5	979.71	2.52	447.21
76	980.69	2.51	449.72
76.5	981.67	2.50	452.22
77	982.65	2.49	454.71
77.5	983.62	2.49	457.20
78	984.58	2.48	459.68
78.5	985.53	2.47	462.15
79	986.48	2.47	464.62
79.5	987.43	2.46	467.08
80	988.37	2.45	469.53
80.5	989.30	2.44	471.97
81	990.22	2.44	474.41
81.5	991.15	2.43	476.84
82	992.06	2.42	479.26

(continued)

Table 4.12 (continued)

Time	θ_t	$\Delta\theta$	θ_m
82.5	992.97	2.42	481.68
83	993.87	2.41	484.09
83.5	994.77	2.40	486.49
84	995.67	2.39	488.89
84.5	996.55	2.39	491.27
85	997.44	2.38	493.65
85.5	998.31	2.37	496.03
86	999.19	2.37	498.39
86.5	1000.05	2.36	500.75
87	1000.92	2.35	503.11
87.5	1001.77	2.35	505.45
88	1002.63	2.34	507.79
88.5	1003.47	2.33	510.12
89	1004.32	2.32	512.44
89.5	1005.15	2.32	514.76
90	1005.99	2.31	517.07

Example 11 An I section beam is simply supported over a span of 8 m as with a depth of 452 mm, width of 157 mm, and thickness of 10.2 mm. It carries a permanent loading of 4.75 kN/m^2 and an imposed load of 0.75 kN/m^2. Evaluate the thickness of fire proof board required such that the fire rating when subjected to hydrocarbon fire exposure is 90 min.

Structural Analysis:

For the given beam, load combination for fire is considered as per EN 1991-1-2 and is given by:

$$= 1 \times \text{P.L} + 0.3 \times \text{I.L.}$$

Design moment on the structure, $M_{\text{fi,ED}}$, is computed as below:

$$= 3.75 \times (4.75 + 0.3 \times 2.5) \times 8^2/8 = 163 \text{ kN/m}.$$

Section classification:

$$\varepsilon = 0.85\left[\frac{235}{f_y}\right]^{0.5} = 0.85\left[\frac{235}{275}\right]^{0.5} = 0.786$$

Class of flange:

$$\varepsilon = 0.85\left[\frac{235}{f_y}\right]^{0.5} = 0.85\left[\frac{235}{275}\right]^{0.5} = 0.786$$

$$c = 0.5[b - 2.t_f - t_w] = 62.6\,\text{mm}$$

$$\frac{c}{t_f} = 4.71$$

Limiting value for Class 1 is 9ε = 9 × 0.786 = 7.07. Hence, flange is Class 1
Class of web:

$$c = 407.6\,\text{mm}$$

$$\frac{c}{t_w} = 50.3$$

Limiting value for Class 1 is 9ε = 72 × 0.786 = 56.6. Hence, web is Class 1.
Evaluation of fire rating in accordance with EN 1993 1-2:
Strength-based fire resistance:
Design moment of resistance M_{RD} = 354 kNm and $k_{y,\theta} = \frac{163}{354} = 0.46\,\text{kNm}$

As it is a three-sided exposure, effective value of $k_{y,\theta}$ is obtained by further dividing with 0.85 as 0.54. From Table, aforementioned strength degradation is obtained at a temperature of 577 °C. Hydrocarbon fire curve is given by the following relationship:

$$\text{T} = 20 + 1080\left(1 - 0.325\,\text{e}^{-0.167\,t} - 0.675\,\text{e}^{-2.5\,t}\right)$$

Thickness required for a fire rating of 90 min is obtained as 45 mm from the relation used in previous examples. Results of thermal analysis of the section with 45-mm fiberboard protection are shown below:

Time	(Am/Vm)	θ_t	$\Delta\theta$	θ_m
0	195	20	0	20
5	195	949.4253	3.69	53.63
10	195	1035.803	3.88	91.82
20	195	1089.539	3.78	168.98
30	195	1099.654	3.52	241.99
40	195	1101.558	3.25	309.62
50	195	1101.917	3.00	371.99
60	195	1101.984	2.76	429.46
70	195	1101.997	2.54	482.42
80	195	1101.999	2.34	531.20
90	195	1102	2.16	576.14

It can be observed that at 90 min, temperature of the beam is 576.14 °C which is lesser than the critical temperature of 577 °C. Hence, the protected beam will have the desired 90 min of fire rating.

Example 12 A Grade S275 C-Channel section beam is simply supported over a span of 8 m as with a depth of 400 mm, width of 150 mm, and thickness of 10 mm.

It carries a permanent loading of 2.75 kN/m^2 and an imposed load of 0.75 kN/m^2. Evaluate the thickness of fire proof board required such that the fire rating when subjected to hydrocarbon standard fire exposure is 90 min.

Structural Analysis:

For the given beam, load combination for fire is considered as per EN 1991-1-2 and is given by:

$$= 1 \times \text{P.L} + 0.3 \times \text{I.L.}$$

Design moment on the structure, $M_{\text{fi,ED}}$, is computed as

$$= 3.75 \times (2.75 + 0.3 \times 2.5) \times 8^2/8 = 110\,\text{kN/m}.$$

Section classification:

$$\varepsilon = 0.85\left[\frac{235}{f_y}\right]^{0.5} = 0.85\left[\frac{235}{275}\right]^{0.5} = 0.786$$

Class of flange:

$$\varepsilon = 0.85\left[\frac{235}{f_y}\right]^{0.5} = 0.85\left[\frac{235}{275}\right]^{0.5} = 0.786$$

$$c = 0.5\left[b - 2.t_f - t_w\right] = 62.6\,\text{mm}$$
$$\frac{c}{t_f} = 4.71$$

Limiting value for Class 1 is 9ε = 9 × 0.786 = 7.07. Hence, flange is Class 1
Class of web:

$$c = 407.6\text{ mm}$$
$$\frac{c}{t_w} = 50.3$$

Limiting value for Class 1 is 9ε = 72 × 0.786 = 56.6. Hence, web is Class 1.
Evaluation of fire rating in accordance with EN 1993 1-2:
Strength-based fire resistance:
Design moment of resistance M_{RD} = 200.4 kNm and $k_{y,\theta} = \frac{110}{200.4} = 0.55\,\text{kNm}$

As it is a three-sided exposure, effective value of $k_{y,\theta}$ is obtained by further dividing with 0.85 as 0.64. From Table, aforementioned strength degradation is obtained at a temperature of 620 °C.

The hydrocarbon fire curve is given by the following equation:

$$T = 20 + 1080 * \left(1 - 0.325 * e^{-0.167*t} - 0.675 * e^{-2.5*t}\right)$$

After iterative thermal analysis, thickness required for a fire rating of 90 min is obtained as 40 mm. Thermal analysis with 40 mm protection is shown in the table below:

Time	θ_t	$\Delta\theta$	θ_m
0	20.00	0.00	20.00
5	949.42	3.78	54.41
10	1035.80	3.98	93.58
20	1089.54	3.87	172.64
30	1099.65	3.60	247.29
40	1101.56	3.32	316.28
50	1101.92	3.05	379.76
60	1101.98	2.80	438.13
70	1102.00	2.58	491.78
80	1102.00	2.37	541.10
90	1102.00	2.18	586.43

It can be observed that steel temperature at 90 min is lesser than the critical temperature of 620 °C.

4.10 Design of Concrete Members Under Fire

Similar to design of steel structures, fire-resistant design of concrete structures proceeds through temperature and strength assessments. Fire ratings are computed based on both temperature and strength criteria. Temperature criterion is applied on the maximum temperature of any tension rebar of a beam, while strength criterion is applicable to all concrete structures. It provides a measure of the time up to which a structural member can sustain the imposed loads when exposed to a standard fire.

4.10.1 Temperature Analysis

Temperature distribution within any given reinforced cement concrete section (RCC) can be computed using both numerical (finite element method and finite difference method) and empirical methods. Empirical methods are easy to implement than the former ones. Two such available empirical methods for the determination of temperature profile are discussed below. Please note that unlike steel, thermal conductivity of concrete is very low. Hence, highly nonlinear thermal gradients can form within deep concrete members. Obviously, lumped capacity assumption cannot be used for concrete structures. Due to low thermal conductivity, even for members of reasonable cross section, temperatures far from the surface take very long time to rise. This enables idealization of the cross section of the structural member as a semi-infinite body with one face exposed to fire and the other face at infinity. Such an idealization allows analytical closed-form solutions of the heat transfer equation. This approach is used to develop both the semianalytical methods of assessing temperature rise within concrete sections, as detailed below.

4.10.2 Wickstrom's Method

Temperature rise ($\Delta\theta$) above the ambient temperature in a normal weight concrete element is given by the following relationships:

(a) For uniaxial heat flow,

$$\Delta\theta = n_x n_w \Delta\theta_f \tag{4.44}$$

(b) For biaxial heat flow,

$$\Delta\theta_{\mathrm{xy}} = \left(n_w\left(n_x + n_y - 2n_x n_y\right) + n_x n_y\right)\Delta\theta_f \tag{4.45}$$

where n_x, n_y, and n_w are thermal properties of concrete and given by:

$$n_x = 0.18 \ln u_x - 0.81, \quad n_y = 0.18 \ln u_y - 0.81 \tag{4.46}$$

$$n_w = 1 - 0.0616t^{-0.88} \tag{4.47}$$

with

$$u_x = \frac{a}{a_c}\frac{t}{x^2} \tag{4.48}$$

where a is the thermal diffusivity of the concrete and a_c is the reference thermal diffusivity of normal weight concrete given by Wickstrom ($a_c = 0.417 \times 10^{-6}$ m^2/s).

When $a = a_c$, following equations are valid:

$$n_x = 0.18 \ln \frac{t}{x^2} - 0.81 \tag{4.49}$$

with x subject to the limit: $x \geq 2h - 3.6\sqrt{0.0015t}$
where t is time in hours, (x, y) are depths into the member (m), and h is the overall depth of the considered section.

4.10.3 Hertz's Method

The unidirectional time-dependent temperature increment $\Delta\theta(x,t)$ is given by the following relationship:

$$\Delta\theta(x,t) = f_1(x,t) + f_2(x,t) + f_3(x,t) \tag{4.50}$$

where the above functions are solutions to the heat transfer equation for particular boundary conditions, as discussed previously. The functions are given as follows:

$$f_1(x,t) = E\left(1 - \frac{x}{3.363\sqrt{at}}\right)^2 \tag{4.51}$$

$$f_2(x,t) = De^{-x\sqrt{\frac{\pi}{2Ca}}}\sin\left(\frac{\pi t}{C} - x\sqrt{\frac{\pi}{2Ca}}\right) \tag{4.52}$$

$$f_3(x,t) = \frac{D+E}{2(e^{LC}-1)}\left(1 - e^{\left(L(t-C)-x\sqrt{\frac{L}{a}}\right)}\right) \tag{4.53}$$

where a is the thermal conductivity. Above functions are set equal to zero, respectively, if the following conditions are satisfied:

$$1 - \frac{x}{3.363\sqrt{at}} \le 0 \tag{4.54}$$

$$\frac{\pi t}{C} - x\sqrt{\frac{\pi}{2Ca}} \le 0 \tag{4.55}$$

$$L(t-C) - x\sqrt{\frac{L}{a}} \le 0 \tag{4.56}$$

where L is dependent on the temperature curve during cooling and is given by the following relationship:

$$L = \frac{2}{C}\ln\left(\frac{3D}{E-2D}\right) \tag{4.57}$$

where E, D, and C parameters are dependent on the heating regime. It is to be noted that one of the parameters (namely L) does not demand detailed computation as f_3 is always zero when exposed to standard furnace temperature–time curve. Parametric values of C, D, and E are given in Table 4.13, for exposure to standard furnace curve. It is to be noted that C is equal to twice of that of the required time period. Above calculations are for one-dimensional heat flow. For two-dimensional heat flow, temperature increment is given by the following relationship:

Table 4.13 Parameters for temperature analysis of concrete members under standard conditions

Time (h)	C (h)	D (°C)	E (°C)
0.5	1.0	150	600
1.0	2.0	220	600
1.5	3.0	310	600
2.0	4.0	360	600
3.0	6.0	410	600
4.0	8.0	460	600

$$\Delta\theta(x, y, t) = \Delta\theta_0\left(\xi_{\theta,x} + \xi_{\theta,y} - \xi_{\theta,x}\xi_{\theta,y}\right) \tag{4.58}$$

where $\Delta\theta_0$ is the surface temperature rise at time (t), $\xi_{\theta,x}\Delta\theta_0$ is the temperature rise at the point being considered assuming one-dimensional heat flow on the x direction, and $\xi_{\theta,y}\Delta\theta_0$ is the temperature rise for heat flow on the y direction.

4.11 Strength Assessment of Concrete Members Under Fire

EN 1991-1-2 provides two semiempirical methods for determining the section resistance: (i) 500 °C isotherm method and (ii) method of slices given by Hertz. Both the methods effectively utilize the same principles employed for calculation of capacity of structural members at ambient temperatures. Effects of temperature are incorporated through relevant strength reduction factors and related idealizations.

4.11.1 500 °C Isotherm Method

This is also called as reduced section method. Anderberg proposed this method following the analysis of a number of fire tests carried out on flexural reinforced concrete elements. As the name suggests, this method utilizes the isotherm of 500 °C as a basis for design calculations. Isotherms can be computed using results from temperature assessment. Eurocode provides temperature contours for certain cross-sectional dimensions, which can also be utilized to determine the 500 °C isotherm. The following assumptions are made during design calculations:

(1) Concrete within the 500 °C isotherm retains its full strength, while that beyond the 500 °C contour has zero strength.
(2) Consequently, a reduced cross section of the structural member is considered in design calculations.
(3) All rebars are considered for strength calculations with appropriate strength reduction factors as given in Tables 4.14 and 4.15, for strains less than 2% and

Table 4.14 Strength reduction factors for compression and tension reinforcement ($\varepsilon_{s,fi} < 2\%$)

Temperature (°C)	$k_s\ (\theta)$
$0 \le T \le 20$	1
$100 \le T \le 400$	$0.7 - 0.3\left(\frac{\theta-400}{300}\right)$
$400 \le T \le 500$	$0.5 - 0.13\left(\frac{\theta-500}{100}\right)$
$500 \le T \le 700$	$0.1 - 0.47\left(\frac{\theta-700}{200}\right)$
$700 \le T \le 1200$	$0.1\left(\frac{1200-\theta}{500}\right)$

Table 4.15 Strength reduction factors for tension reinforcement ($\varepsilon_{s,fi} > 2\%$)

Temperature (°C)	$k_s\ (\theta)$
$20 \le T \le 400$	1
$400 \le T \le 500$	$0.77 - 0.22\left(\frac{\theta-500}{200}\right)$
$500 \le T \le 600$	$0.47 - 0.31\left(\frac{\theta-600}{100}\right)$
$600 \le T \le 700$	$0.23 - 0.24\left(\frac{\theta-700}{100}\right)$
$700 \le T \le 800$	$0.11 - 0.12\left(\frac{\theta-800}{100}\right)$
$800 \le T \le 1200$	$0.11\left(\frac{1200-\theta}{400}\right)$

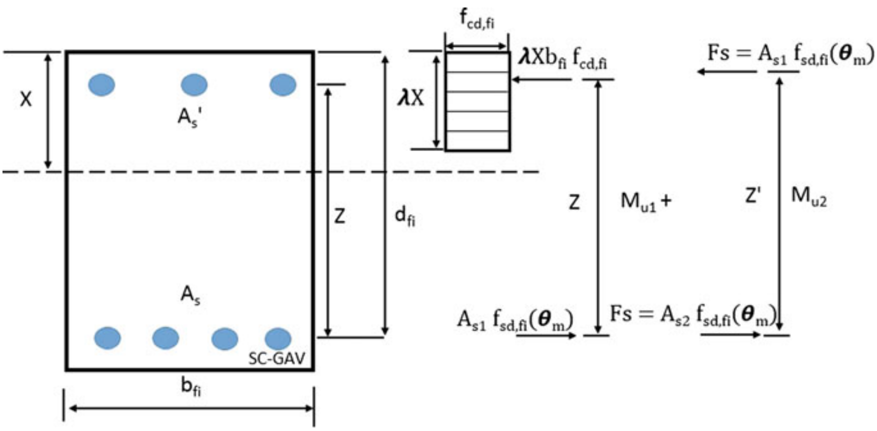


Fig. 4.26 Stress distribution at ultimate limit state for a rectangular cross section with compression reinforcement (EN 1992-1-2)

greater than 2%, respectively. Temperature of rebars is assessed during computation of the 500 °C isotherm for the cross section of the member.

In accordance with EN 1992-1-1, depth of the concrete compressive stress block is taken as λx, as shown in Fig. 4.26, where x is the depth of the neutral axis and λ is given by the following relationship:

$$\lambda = 0.8 - \frac{f_{\mathrm{ck}} - 50}{200} \leq 0.8 \quad (4.59)$$

Strength of concrete is taken as ηf_{cd}, where η is given by:

$$\eta = 1.0 - \frac{f_{\mathrm{ck}} - 50}{200} \leq 1.0 \quad (4.60)$$

Subsequently, moment capacity of the section M_{u} is given by the following relationship:

$$M_u = M_{u1} + M_{u2} \quad (4.61)$$

where M_{u1} is due to the balanced concrete–steel section and M_{u2} is due to the compression reinforcement and the corresponding balancing tension reinforcement. These moment capacities are given as follows:

$$M_{u1} = A_{\mathrm{s1}} f_{\mathrm{sd,fi}}(\theta_m) z \quad (4.62)$$

$$M_{u2} = A_{\mathrm{s2}} f_{\mathrm{scd,fi}}(\theta_m) z' \quad (4.63)$$

Mechanical reinforcement ratio ω_{k} is given by:

$$\omega_{\mathrm{k}} = \frac{A_{\mathrm{s1}} f_{\mathrm{sd,fi}}(\theta_m)}{b_{\mathrm{fi}} d_{\mathrm{fi}} f_{\mathrm{cd,fi}}(20)} \quad (4.64)$$

The total tension steel area $A_{\mathrm{s}} = A_{\mathrm{s1}} + A_{\mathrm{s2}}$. The values shown in the above equation, namely $f_{\mathrm{scd,fi}}(\theta_m)$ and $f_{\mathrm{sd,fi}}(\theta_m)$, are the temperature-reduced strengths of the reinforcement at a mean temperature θ_m in a given layer. Where the reinforcement is in layers, the mean temperature reduced strength is computed as below:

$$k(\phi) f_{\mathrm{sd,fi}} = \frac{\sum \left[k_s(\theta_i) f_{\mathrm{sd},i} A_1\right.}{\sum A_i} \quad (4.65)$$

Effective axis distance a is given by the following relationship:

$$a = \frac{\sum \left[a_i k_s(\theta_i) f_{\mathrm{sd},i} A_i\right]}{k_s(\theta_i) f_{\mathrm{sd},i} A_i} \quad (4.66)$$

4.11.2 Zone Method (Method of Slices)

According to this method, heat-affected concrete is divided into a series of slices and temperature (θ) is determined at the mid-depth of each slice. Compressive strength of concrete in each slice is considered using appropriate strength reduction factors discussed earlier. Mean concrete strength reduction factor ($k_{c,m}$) for all slices is computed as below:

$$k_{c,m} = \frac{1 - \frac{0.2}{n}}{n} \sum_{i=1}^{n} k_c(\theta_i) \tag{4.67}$$

In the above equation, a factor of ($1 - 0.2/n$) is used to compensate for the fact that $k_c(\theta_i)$ is determined at the center of a strip, where n is the total number of slices. Effective width of a uniform stress block is determined by calculating the width of the damage zone a_z and is given by the following relationships:

(a) For columns:

$$a_z = w\left[1 - \left(\frac{k_{c,m}}{k_c(\theta_m)}\right)^{1.3}\right] \tag{4.68}$$

(b) For beams and slabs:

$$a_z = w\left[1 - \left(\frac{k_{c,m}}{k_c(\theta_m)}\right)\right] \tag{4.69}$$

Strength reduction factor $k_c(\theta_m)$ is determined at the center of the member, and w is the half width for exposure on opposite faces (thickness of a slab). For columns, $2w$ is the smaller cross-sectional dimension. Strength reduction factors for reinforcements are the same as those used in the 500 °C isotherm method. Code imposes a restriction on the zone method that it may be used for exposure to the standard fires.

4.12 Numerical Example for Concrete Members Under Fire

Example 1 Evaluate the temperature profile for the given cross section of the beam (160 × 300) exposed to standard fire curve using Wickstorm's method and compare against experimental temperature profile available in EN 1992-1-2 (Fig. 4.27).

Solution The temperature profile at 30 min and 60 min is calculated at certain coordinate points using Wickstorm's method as discussed earlier. Corresponding

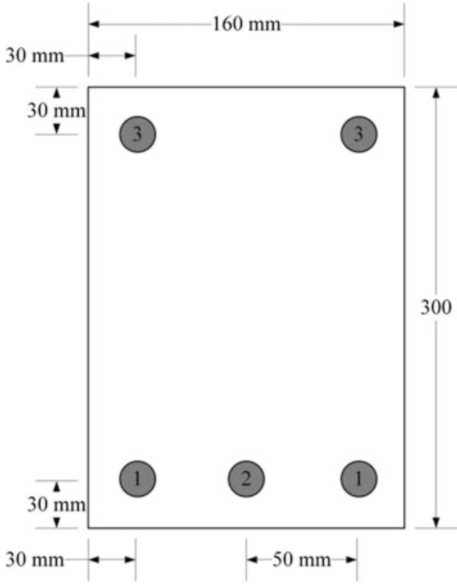


Fig. 4.27 Cross section of RC beam (160 × 300) under standard fire

Table 4.16 Temperature profile for beam section of 160 × 300 mm

X, Y coordinates	EN 1992-1-2		Wickstorm's method	
	60 min	30 min	60 min	30 min
20, 20	760	550	738	533
20, 40	650	420	662	451
40, 20	650	420	640	428
40, 40	500	280	530	318

temperature at those coordinates as available through EN 1992-1-2 is compared and tabulated in Table 4.16 for 30 and 60 min.

Example 2 Evaluate the temperature profile for the given cross section of the beam (300 × 600), exposed to standard fire curve using Wickstorm's method, and compare against temperature profile available in EN 1992-1-2 (Fig. 4.28).

Solution Temperature profile calculated at 60 and 90 min for different coordinate points is summarized in Table 4.17.

Example 3 Evaluate the temperature profile for the given cross section of a column (300 × 300), exposed to standard fire curve using Wickstorm's method, and compare against temperature profile available in EN 1992-1-2 (Fig. 4.29).
The temperature profile calculated at 30 and 60 min at above calculated points is tabulated and compared against experimental temperature profile as in EN 1992-1-2 in Table 4.18.

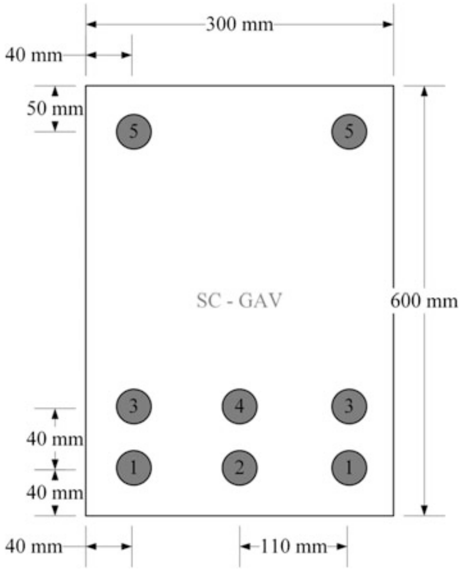


Fig. 4.28 Cross section of RC beam (300 × 600) under standard fire

Table 4.17 Temperature profile for beam section of 300 × 600 mm

X, Y coordinates	EN 1992-1-2		Wickstorm's method	
	60 min	90 min	60 min	90 min
20, 20	760	860	785	891
20, 40	650	780	671	795
40, 20	630	780	700	821
40, 40	500	630	495	625

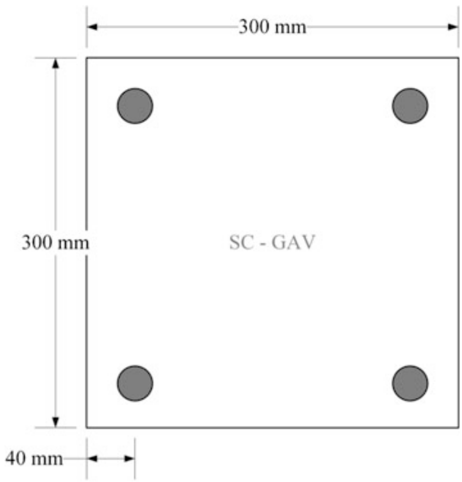


Fig. 4.29 Cross section of RC column (300 × 300) under standard fire

Table 4.18 Temperature profile for column section of 300 × 300 mm

X, Y coordinates	EN 1992-1-2		Wickstorm's method	
	30 min	60 min	30 min	60 min
20, 20	550	750	537	741
20, 40	420	650	469	682
40, 20	420	650	469	682
40, 40	260	500	301	513

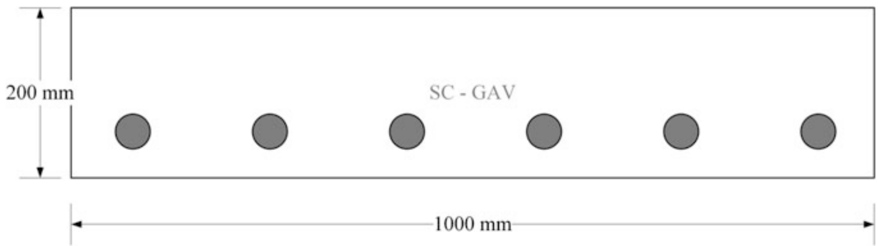


Fig. 4.30 Cross section of RC slab (200 mm thick) under standard fire

Table 4.19 Temperature profile for slab section (200 mm thick)

X, Y coordinates	EN 1992-1-2		Wickstorm's method	
	30 min	60 min	30 min	60 min
20, 20	320	510	306	469
20, 40	160	300	152	286
40, 20	320	510	306	469
40, 40	160	300	152	286

Example 4 Evaluate the temperature profile for the given cross section of a slab (200 mm thick), exposed to standard fire curve using Wickstorm's method, and compare against experimental temperature profile available in EN 1992-1-2 (Fig. 4.30).

The temperature profile calculated at 30 and 60 min at above calculated points is tabulated and compared against temperature profile given in EN 1992-1-2 in Table 4.19.

Example 5 A simply supported concrete slab of dimensions (5 × 2 m) of M25 concrete is located over an underground storage pit, which stores hazardous waste. Other required details of the slab are given in Table 4.20. Determine the fire rating of the given slab when exposed to standard fire curve (Fig. 4.31).

Solution

Ignoring loads that arise from partitions/finishes and no variable load acting on the underground slab, structural loading in the fire limit state is given by:

Table 4.20 Details of the cover slab

Width (b)	1000 mm
Depth (D)	150 mm
Length	2 m
Variable load q_k	0
f_y	500 N/mm^2
f_{ck}	25 N/mm^2
A_{st}	261 mm^2
Thermal diffusivity	0.417×10^{-6} m^2/s
Cover	25 mm
Bar diameter	10 mm
Exposure	Single-sided

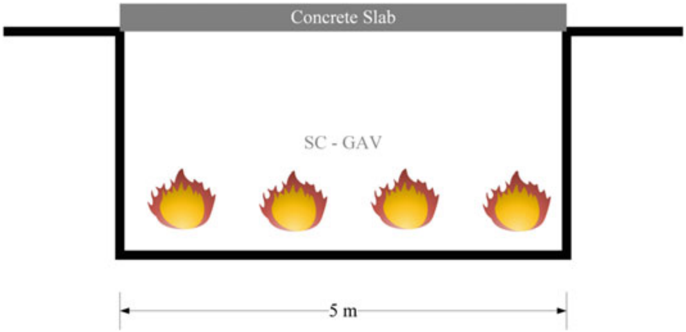


Fig. 4.31 Details of RC slab cover of underground pit

$$0.125 \times 25 + 0.3 \times 0 = 3.75\,\text{kPa}$$

Demand factored moment $M_{\text{Ed,fi}} = 1.35 \times 3.75 \times 2^2/8 = 2.53\,\text{kNm/m}$

Sample calculation of reduced moment capacity for a particular time of fire exposure is presented below:

Using Wickstrom's method to determine temperatures, time of exposure t = 2 h

Temperature in steel:

The centroid of the reinforcement is at 25 + 10/2 = 30 mm

$$n_w = 1 - 0.0616t^{-0.88} = 0.967$$

$$n_x = 0.18 \ln u_x - 0.81 = 0.58$$

$$\Delta\theta_g = 345 \log(480t + 1) = 1029\,°\text{C}$$

$$\Delta\theta = n_x n_w \Delta\theta_g = 574\,°\text{C}, \text{ or}$$

$\theta_s = 574 + 20 = 594\,°\mathrm{C}$ (assuming an ambient temperature of 20 °C)

Strength reduction factor for $\varepsilon_{s,f_i} > 2\%$
From Table 4.3, $k_s = 0.489$.
Tension force F_s is:

$$F_s = 500 \times 261 \times 0.489 = 63{,}762\,\mathrm{N}$$

For Grade M 25 concrete, $\lambda = 0.8$ and $\eta = 1$

$$F_c = \eta f_{\mathrm{cd,fi}}(20)\lambda xb$$

$$F_c/x = 24{,}000$$

$$x(\mathrm{mm}) = 2.66$$

$$M_{u1} = F_s(d - 0.5\lambda x) = 7.58\,\mathrm{kNm}$$

$M_{u1} > M_{\mathrm{Ed,fi}}$ (OK) not exceeded demand

Similarly, calculation of reduced section capacity needs to be done for different time exposure until section is no longer able to resist the demand. Table 4.21 shows values of reduced section capacity for different time of fire exposure. It can be observed that the section capacity becomes lesser than the demand at 3.5 h. This variation is also plotted in Fig. 4.32. The fire rating of the slab can be said to be around 3.4 h.

Example 6 A simply supported slab in a multi-span structure is designed and detailed as given in Table 4.22. The cover provided is that to satisfy durability only. Assume that the usage of structure is as an office and no continuity steel in the top face is provided. Assess the fire rating (Fig. 4.33).

Solution Considering 1 kPa partition and finishes load and variable loading of 3.5 kPa, structural loading in the fire limit state is given by:

$$0.125 \times 25 + 0.3 \times 3.5 + 1 = 5.175\,\mathrm{kPa}$$

Demand factored moment $M_{\mathrm{Ed,fi}} = 1.35 \times 5.175 \times 2.7^2/8 = 6.37$ kNm/m

Table 4.21 Reduced section capacity with increasing fire exposure time

Fire rating		
Time (h)	Reduced moment capacity of section (kNm)	Demand Moment (kNm)
0.5	15.38	2.53
1	13.34	2.53
1.5	11.29	2.53
2	7.58	2.53
2.5	5.15	2.53
3	3.36	2.53
3.5	*2.49*	*2.53*

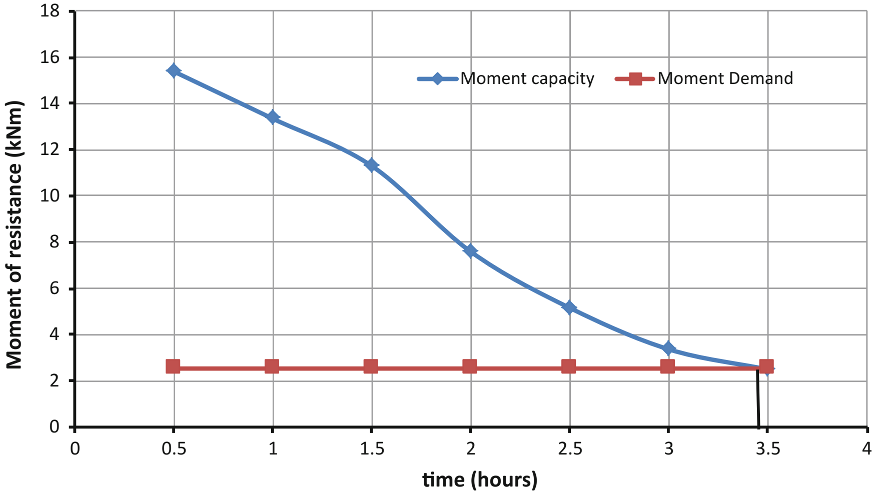


Fig. 4.32 Strength degradation curve for determining fire rating of cover slab

Table 4.22 Details of the simply supported slab

Notation	Value
Width (b)	1000 mm
Depth (D)	125 mm
Length	2700 mm
Variable coefficient	0.3
Variable load q_k	3.5
f_y	415 N/mm^2
f_{ck}	25 N/mm^2
A_{st}	452 mm^2
Thermal diffusivity	0.417×10^{-6} m^2/s
Cover	25 mm
Bar diameter	12 mm
Exposure	Single-sided
Specific weight	25 kN/m^3
Partitions and finishes load	1 kPa

Calculation of the reduced moment-carrying capacity of the section is required, which is similar as shown in Example 1. Table 4.23 shows the values of calculated moment of resistance of section after different times of fire exposure. The same variation is also shown in Fig. 4.34. The fire rating can be said to be just below 2.5 h, say 2.4 h.

Example 7 Determine the load-carrying capacity over a complete range of standard furnace exposures, and check the duration of fire resistance the beam can last. Data for the example is given in Fig. 4.35.

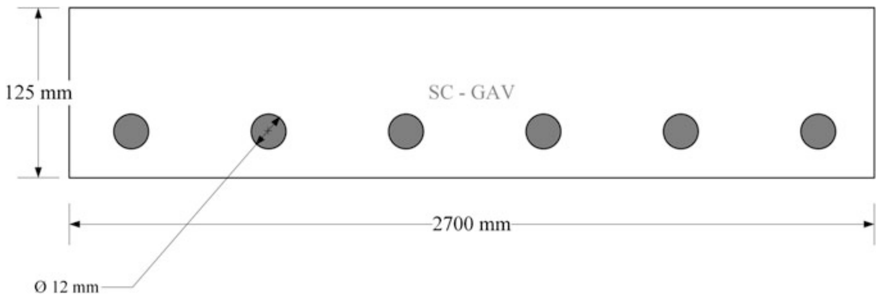


Fig. 4.33 Details of simply supported slab for determining fire rating

Table 4.23 Reduced section capacity with increasing fire exposure time

Fire rating		
Time (h)	Reduced moment capacity of section (kNm)	Demand moment (kNm)
0.5	17.23	6.37
1	14.99	6.37
1.5	12.7	6.37
2	8.57	6.37
2.5	*5.83*	*6.37*
3	3.81	6.37

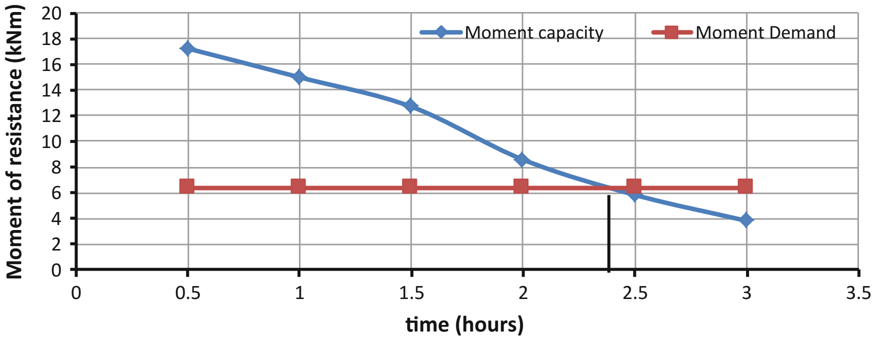


Fig. 4.34 Strength degradation curve for determining fire rating of simply supported slab

Input data: grade of concrete—c40/37; density of concrete—2500 kg/m^3; type of structure—office building; diameter of tension reinforcement—16 mm; diameter of compression reinforcement—12 mm; grade of steel—Fe 415.

Solution

Load combination LC: 1 × D.L + 0.3 × L.L

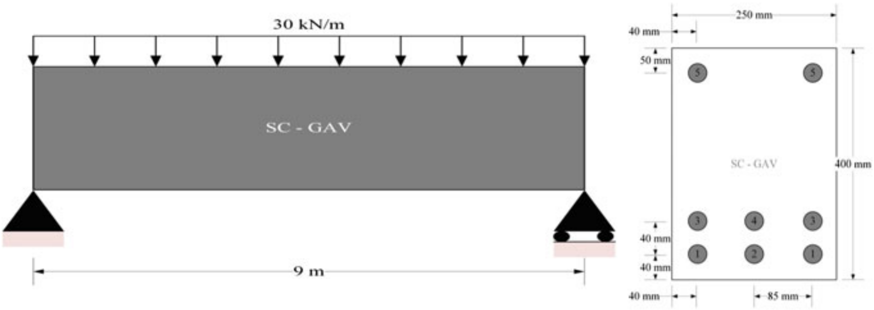


Fig. 4.35 Simply supported beam section for 9 m span for Example 7

Table 4.24 Temperatures and strength reduction factors for rebars

Time (h)	Rebar 1		Rebar 2		Rebar 3		Rebar 4		Rebar 5	
	T (°C)	k_s (T)	T (°C)	k_s (T)	T (°C)	k_s (T)	T (°C)	k_s (T)	T (°C)	k_s (T)
0.25	83	1	30	1	73	1	20	1	73	1
0.5	263	1	124	1	179	1	20	1	179	0.921
0.75	385	1	196	1	259	1	24	1	256	0.844
1	476	0.7964	253	1	351	1	69	1	318	0.782
1.25	547	0.6343	300	1	425	0.8525	108	1	368	0.732
1.5	606	0.4556	355	1	486	0.7854	159	1	412	0.6844
1.75	656	0.3356	408	0.8712	538	0.6622	212	1	451	0.6337
2	699	0.2324	455	0.8195	584	0.5196	260	1	485	0.5895
2.25	737	0.1856	497	0.7733	624	0.4124	302	1	516	0.468
2.5	771	0.1448	535	0.6715	661	0.3236	340	1	545	0.41
2.75	802	0.10945	569	0.5661	693	0.2468	375	1	571	0.358
3	829	0.102025	600	0.47	723	0.2024	407	0.8723	595	0.31

Permanent load $w_d = 0.25 \times 0.4 \times 25 = 2.5$ kN/m
$M_d = 2.5 \times 9^2/8 = 25.3125$ kNm
Live load $w_l = 0.3 \times 30 = 9$ kN/m
$M_l = 9 \times 9^2/8 = 91.125$ kNm

Total applied moment = 116.4375 kNm

Using 500 °C isotherm method, temperature in reinforcement is determined. Values are given in Table 4.24. As all the reinforcement is the same size, strength reduction factor reduces to the following equation:

$$k(\phi) = \frac{1}{n}\sum k_s(\theta_i)$$

The above equation is expanded as per the cross-sectional details:

$$k(\phi) = \frac{k_s(\theta_1) + k_s(\theta_2) + k_s(\theta_3) + k_s(\theta_4)}{4}$$

Effective axis distance a reduces to the following relationship:

$$a = \frac{\sum [a_i k_s(\theta_i)]}{\sum k_s(\theta_i)}$$

Equation for effective axis distance expands to the following relationship:

$$\begin{aligned}
a &= \frac{68[k_s(\theta_1) + k_s(\theta_3)] + 13{,}268[k_s(\theta_2) + k_s(\theta_4)]}{4k(\varphi)} \\
b_{\mathrm{fi}} &= b - 2x_{500} = 250 - 2x_{500} \\
F_s &= A_s f_{\mathrm{scd,fi}}(\theta_m) = A_{\mathrm{sc}} k(\theta_5) f_{\mathrm{yk}} = 226 \times 415\, k(\theta_5) = 93{,}790\, k(\theta_5) \\
A_{\mathrm{si}} f_{\mathrm{s,fi}}(\theta) &= A_s f_{\mathrm{Sd,fi}}(\theta_m) - F_{\mathrm{SC}} = 1608 \times 415 k(\phi) = 667{,}320\, k(\phi)\mathrm{MN} \\
x &= A_{\mathrm{si}} f_{\mathrm{s,fi}}(\theta)/(0.8 \times f_{\mathrm{cd}}(20) \times b_{\mathrm{fi}}) \\
&= A_{\mathrm{si}} f_{\mathrm{s,fi}}(\theta)/(0.8 \times 40 \times b_{\mathrm{fi}}) = A_{\mathrm{si}} f_{\mathrm{s,fi}}(\theta)/(24 \times b_{\mathrm{fi}}) \\
M_{u1} &= A_{\mathrm{si}} f_{\mathrm{s,fi}}(\theta)[h - a - 0,4x] = A_{\mathrm{si}} f_{\mathrm{s,fi}}(\theta)[400 - a - 0,4x] \\
M_{u2} &= F_{\mathrm{sc}}[h - a - 85] = F_{\mathrm{sc}}[315 - a].
\end{aligned}$$

The values of M_{u1}, M_{u2}, and M are determined in Table 4.25. As seen from the table, moment-carrying capacity of the section reduces with increasing fire exposure time. Figure 4.36 shows the reduced capacity versus demand curve (strength degradation curve) which gives fire rating of about 1.75 h.

Example 8 Determine the load-carrying capacity history of 10 m span beam over the complete range of standard furnace exposures, and check the duration the beam can last (refer Fig. 4.37 for data).

Input data: grade of concrete—c30/37; density of concrete—2400 kg/m^3; diameter of tension reinforcement—16 mm; diameter of compression reinforcement—10 mm; grade of steel—Fe 415.

Solution Load combination LC: 1 × D.L + 0.3 × L.L

Permanent load	w_d = 0.16 × 0.3 × 24 = 1.152 kN/m
	M_d = 1.152 × 10^2/8 = 14.4 kNm
Live load	w_l = 0.3 × 20 = 6 kN/m
	M_l = 6 × 10^2/8 = 75 kNm

Total applied moment = 89.4 kNm

Determination of reinforcement temperatures is done in the similar way as in Example 3 and is summarized in Table 4.26. Values of moments M_{u1}, M_{u2}, and M are determined and summarized in Table 4.27.

Table 4.25 Moment of resistance of section with increasing fire exposure

Time (t) (h)	k (T)	a (mm)	x_{500}	b_{fi} (mm)	f_s (MN)	f_{SC} (MN)	f_{ST} (MN)	x (mm)	M_{u1} (kNm)	M_{u2} (kNm)	M_r (kNm)
0.250	1.000	60.000	6.250	237.500	0.334	0.094	0.240	31.565	126.513	42.430	168.942
0.500	1.000	60.000	12.500	135.000	0.334	0.086	0.247	57.247	127.883	39.078	166.961
0.750	1.000	60.000	18.750	122.500	0.334	0.079	0.255	64.933	130.838	35.811	166.649
1.000	0.949	61.073	25.000	110.000	0.317	0.073	0.243	69.138	124.427	33.101	157.528
1.250	0.872	61.252	25.000	110.000	0.291	0.069	0.222	63.133	114.113	30.972	145.085
1.500	0.810	62.035	31.250	97.500	0.270	0.064	0.206	66.085	105.471	28.908	134.379
1.750	*0.717*	*63.175*	*37.500*	*85.000*	*0.239*	*0.059*	*0.180*	*66.142*	*91.818*	*26.699*	*118.517*
2.000	0.643	63.638	37.500	85.000	0.215	0.055	0.159	58.541	81.677	24.811	106.488
2.250	0.593	63.825	43.750	72.500	0.198	0.044	0.154	66.350	78.449	19.689	98.138
2.500	0.535	64.741	43.750	72.500	0.179	0.038	0.140	60.374	71.590	17.214	88.804
2.750	0.481	65.943	50.000	60.000	0.160	0.034	0.127	66.040	64.367	14.990	79.357
3.000	0.412	66.105	50.000	60.000	0.137	0.029	0.108	56.408	55.379	12.976	68.355

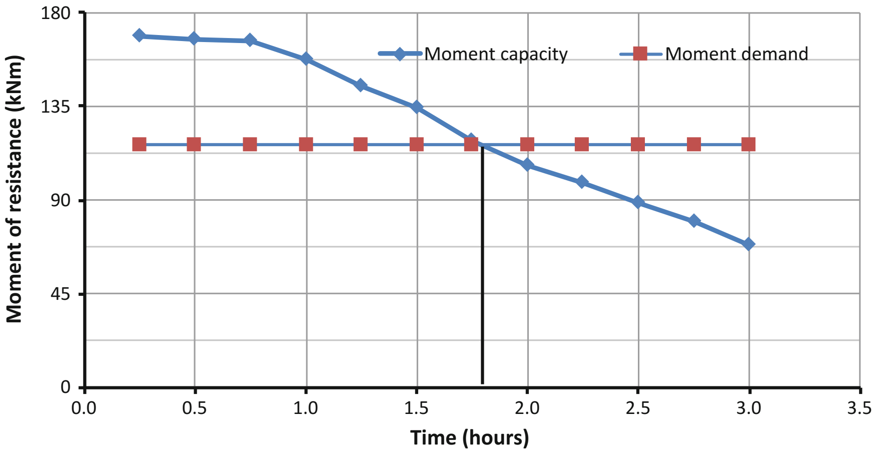


Fig. 4.36 Strength degradation curve for determining fire rating of 9 m span beam

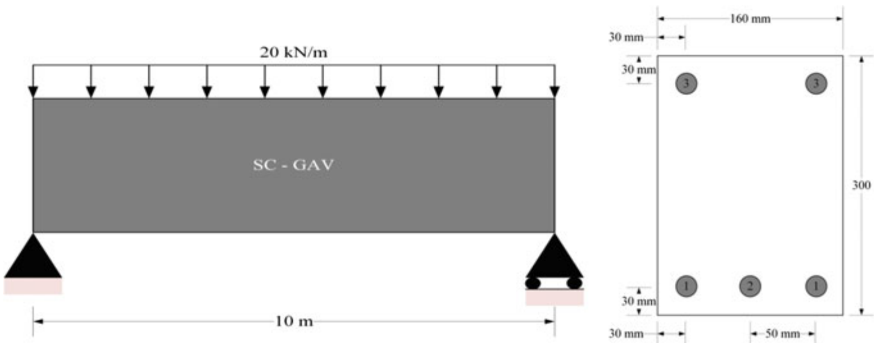


Fig. 4.37 Simply supported beam section of 10 m span

Table 4.26 Temperatures and strength reduction factors for rebars

Time (h)	Rebar 1		Rebar 2		Rebar 3	
	T (°C)	k_s (T)	T (°C)	k_s (T)	T (°C)	k_s (T)
0.25	44	1	20	1	29	1
0.5	157	1	20	1	139	0.961
0.75	238	1	20	1	218	0.882
1	302	1	64	1	281	0.819
1.25	412	0.802	163	1	376	0.724
1.5	520	0.513	274	1	450	0.547

Table 4.27 Moment of resistance of section with increasing fire exposure

Time (t) (h)	k (T)	a (mm)	x_{500}	b_{fi} (mm)	f_s (MN)	f_{SC} (MN)	f_{ST} (MN)	x (mm)	M_{u1} (kNm)	M_{u2} (kNm)	M_r (kNm)
0.250	1.000	30.000	4.000	152.000	0.334	0.065	0.269	73.622	145.178	31.421	176.599
0.500	1.000	30.000	12.000	136.000	0.334	0.058	0.276	84.481	147.857	27.964	175.821
0.750	0.894	30.000	16.000	128.000	0.298	0.053	0.246	80.021	132.251	25.388	157.639
1.000	0.802	30.000	24.000	112.000	0.268	0.049	0.219	81.474	117.694	23.408	141.102
1.250	0.603	30.000	28.000	104.000	0.201	0.045	0.156	62.649	85.213	21.668	106.881
1.500	*0.487*	*30.000*	*32.000*	*96.000*	*0.162*	*0.041*	*0.121*	*52.624*	*66.558*	*19.830*	86.388
1.750	0.365	30.000	36.000	88.000	0.122	0.038	0.084	39.881	46.667	18.196	64.862
2.000	0.285	30.000	40.000	80.000	0.095	0.033	0.062	32.488	34.744	15.842	50.587
2.250	0.221	30.000	44.000	72.000	0.074	0.025	0.049	28.389	27.405	11.940	39.345
2.500	0.175	30.000	48.000	64.000	0.058	0.021	0.038	24.415	21.010	10.117	31.127
2.750	0.144	30.000	52.000	56.000	0.048	0.017	0.031	22.799	17.186	8.421	25.607
3.000	0.126	30.000	56.000	48.000	0.042	0.014	0.028	24.283	15.673	6.787	22.460

As seen from the table, moment-carrying capacity of the section reduces with the increase in fire exposure time. Figure 4.38 shows the reduced capacity versus demand curve (strength degradation curve) which indicates fire rating of about 1.5 h.

Example 9 Determine the fire resistance of a short reinforced concrete column (300 × 300) reinforced with 4–16-mm-diameter bars having a cover of 40 mm (Fig. 4.27).

Input data: The concrete is Grade c50/60; density of concrete—2400 kg/m^3; grade of steel—Fe 415; A_c = 89,196 mm^2 (Fig. 4.39).

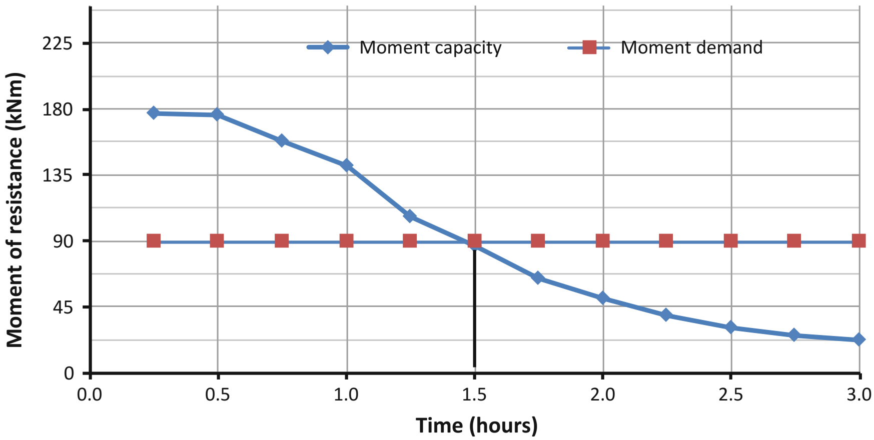


Fig. 4.38 Strength degradation curve for determining fire rating of 10 m span beam

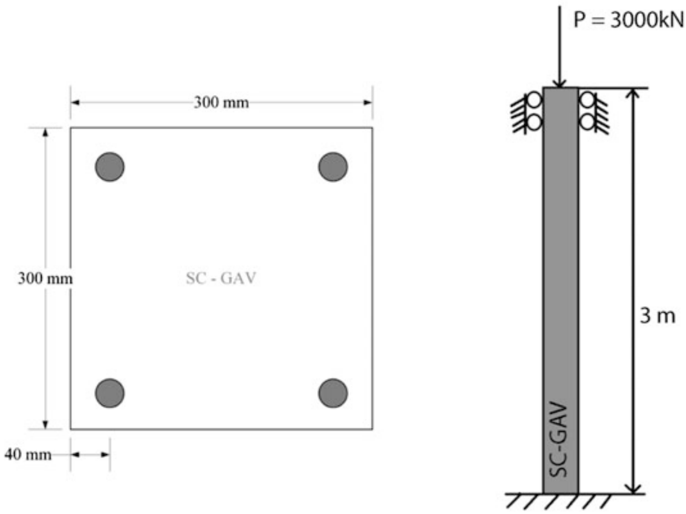


Fig. 4.39 RC column (300 × 300) with axial load demand of 3000 kN

Solution Axial load demand is 3000 kN. For determining reduced axial capacity of the section, values of x_{500} are calculated using Wickstrom's method. Temperatures in the reinforcement are computed and summarized in Table 4.28; rounding effects of isotherms are neglected (Table 4.29).

Results of load-carrying capacity are summarized in Table 4.30. As both α_{cc} and γ_{mc} are equal to 1.0 in the fire limit state, concrete capacity in compression effectively becomes as follows:

$$N_{c,\mathrm{fi}} = b_{\mathrm{fi}} d_{\mathrm{fi}} f_{\mathrm{ck}}$$

Table 4.28 Temperature of reinforcement and strength reduction factor

Rebar-1		
Time (h)	T (°C)	$k_s(T)$
0.25	51	1
0.5	227	0.873
0.75	348	0.752
1	439	0.6493
1.25	511	0.478
1.5	570	0.36
1.75	621	0.258
2	664	0.172
2.25	703	0.0994
2.5	737	0.0926
2.75	768	0.0864
3	797	0.0806

Table 4.29 Axial load-carrying capacity of the section with increasing fire exposure time

Time (h)	b_{fi} (mm)	d_{fi} (mm)	$N_{\mathrm{c,fi}}$ (kN)	$N_{\mathrm{s,fi}}$ (kN)	$N_{\mathrm{rd,fi}}$ (kN)
0.25	285	285	4061.25	402.12	4463.37
0.5	270	270	3645.00	351.05	3996.05
0.75	255	255	3251.25	302.40	3553.65
1	240	240	2880.00	261.10	3141.10
1.25	*225*	*225*	*2531.25*	*192.22*	*2723.46*
1.5	210	210	2205.00	144.76	2349.76
1.75	195	195	1901.25	103.75	2005.00
2	180	180	1620	69.16	1689.16
2.25	165	165	1361.25	39.97	1401.22
2.5	135	135	911.25	37.24	948.49
2.75	120	120	720	34.74	754.74
3	90	90	405	32.41	437.411

Table 4.30 Temperature of reinforcement bars and strength reduction factor

Time (h)	Rebar 1		Rebar 2	
	T (°C)	$k_s(T)$	T (°C)	$k_s(T)$
0.25	28	1	28	1
0.5	202	0.898	121	0.979
0.75	322	0.778	192	0.908
1	413	0.6831	249	0.851
1.25	485	0.5895	296	0.804
1.5	545	0.41	337	0.763
1.75	595	0.31	374	0.726
2	640	0.22	406	0.6922
2.25	678	0.144	436	0.6532
2.5	713	0.0974	463	0.6181
2.75	745	0.091	488	0.5856
3	773	0.0854	511	0.478

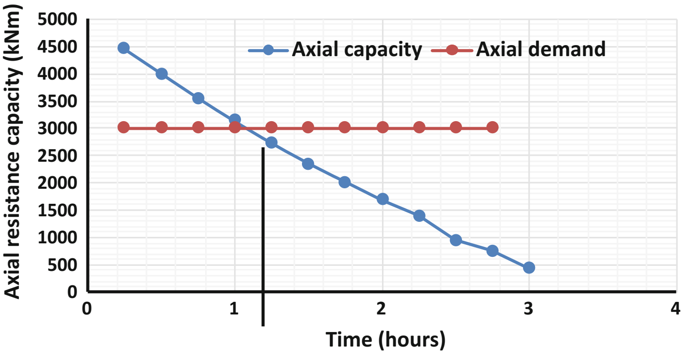


Fig. 4.40 Strength degradation curve for determining fire rating of column (300 × 300)

Compression capacity of the reinforcement with $\gamma_{m,s}$ = 1.0 is given by:

$$N_{s,fi} = 804 \times 500(k_s(\theta_1) + k_s(\theta_2)) = 402,000(k_s(\theta_1) + k_s(\theta_2))$$

Reduced capacity of the column section is calculated as $N_{Rd,fi} = N_{c,fi} + N_{s,fi}$ at all time exposure. Figure 4.40 shows the strength degradation curve, which provides fire rating as 1.15 h.

Example 10 Determine the fire resistance of a short reinforced concrete column (600 × 450), reinforced with a cover of 45 mm (Fig. 4.41).

Input data: The concrete is Grade c60/60; density of concrete—2500 kg/m^3; grade of steel—Fe 500; A_c = 266,781 mm^2

Solution Axial load demand is 12,000 kN. Determine the reduced axial capacity of the section. Values of x_{500} are calculated using Wickstrom's method, and

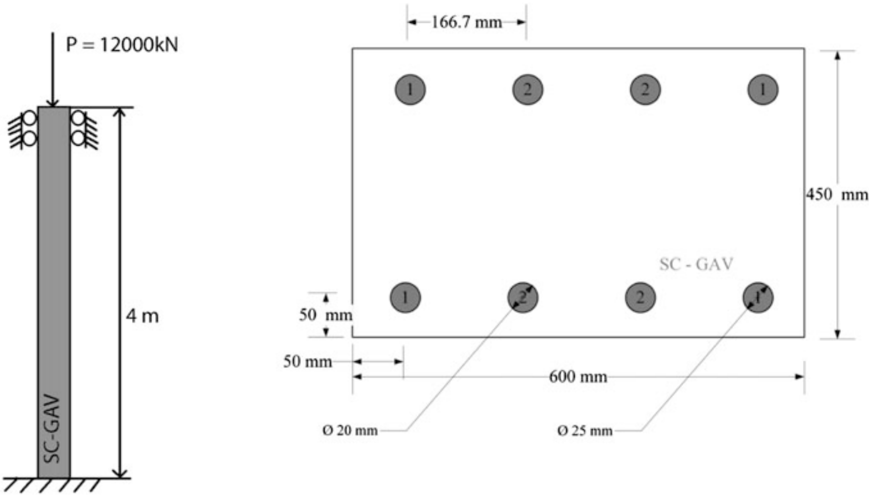


Fig. 4.41 RC column (600 × 450) with axial load demand of 12,000 kN

Table 4.31 Axial load-carrying capacity of column (600 × 450) with fire exposure

Time (h)	b_{fi} (mm)	d_{fi} (mm)	$N_{c,fi}$ (kN)	$N_{s,fi}$ (kN)	$N_{rd,fi}$ (kN)
0.25	600	450	16,200	1610.07	17810.07
0.5	570	427.5	14620.5	1496.73	16117.23
0.75	570	405	13,851	1334.31	15185.31
1	540	405	13,122	1205.33	14327.33
1.25	540	405	13,122	1083.91	14205.91
1.5	540	382.5	12,393	881.92	13274.92
1.75	510	382.5	11704.5	760.50	12465.00
2	510	382.5	11704.5	650.91	12355.41
2.25	*510*	*360*	*11,016*	*551.79*	*11567.79*
2.5	480	360	10,368	483.99	10851.99
2.75	480	360	10,368	457.28	10825.28
3	450	337.5	9112.5	384.18	9496.68

temperatures in the reinforcement are summarized in Table 4.30. Rounding effects of isotherms are neglected.

Results of the determination of load-carrying capacity are summarized in Table 4.31. As both α_{cc} and γ_{mc} are unity in the fire limit state, concrete capacity in compression effectively becomes as follows:

$$N_{c,fi} = b_{fi} d_{fi} f_{ck}$$

Compression capacity of the reinforcement with $\gamma_{m,s}$ = 1.0 is given by:

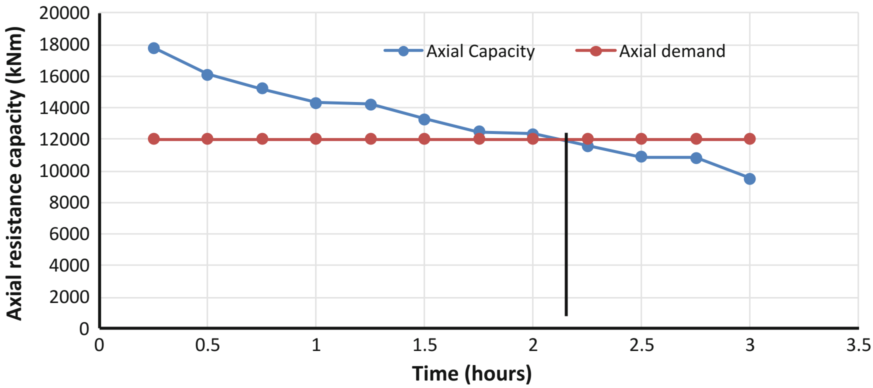


Fig. 4.42 Strength degradation curve for determining fire rating of column (600 × 450)

$$N_{S,fi} = 500(1256 \times k_s(\theta_1) + 1962 \times k_s(\theta_2))$$

Reduced capacity of the column section is calculated as $N_{Rd,fi} = N_{c,fi} + N_{s,fi}$ at all time exposure. Figure 4.42 shows the strength degradation curve, which provides fire rating of 2.2 h.

4.13 Composite Structures Under Fire

This section presents detailed discussions on the design of steel–concrete composite structures under fire. It covers mainly two types of composite structures, namely: (i) composite beams where the composite action is obtained from welded shear studs on to the top flange of beam and (ii) concrete-filled steel columns. Structural design of these structures for fire is carried out for the required fire rating, which is evaluated from two criteria, namely: (i) critical temperature and (ii) strength-based approach.

4.13.1 Composite Beams

In this section, two types of composite beams are considered as shown in Fig. 4.43. They are designated as trapezoidal decking and dovetail decking composite beam, respectively. For evaluation of fire rating, EN 1994-1-2 considers two approaches, namely: (i) critical temperature-based approach and (ii) full moment capacity approach. In both the approaches, thermal analysis utilizes empirical equations developed for non-composite steelworks that assume no thermal gradients.

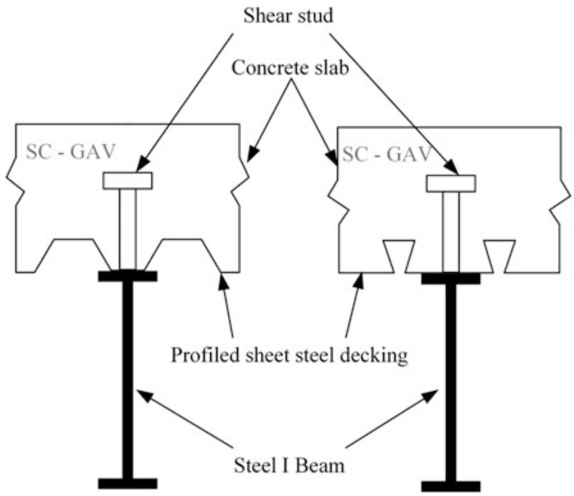


Fig. 4.43 Composite beams. **a** Trapezoidal profile steel decking and **b** dovetail profile steel decking

4.13.2 Critical Temperature-Based Approach

This approach can be employed under certain conditions: (i) when depth of the beam is lesser than 500 mm; (ii) depth of slab h_c is greater than 120 mm; and (iii) beam is simply supported. Critical temperature θ_{cr} for a fire resistance period of R30 is evaluated corresponding to a critical strength of steel $f_{\mathrm{ay},\theta_{\mathrm{cr}}}$ and is given by:

$$0.9\eta_{\mathrm{fi},t} = \frac{f_{\mathrm{ay},\theta_{\mathrm{cr}}}}{f_{\mathrm{ay}}}, \tag{4.44}$$

For cases not covered in the assumptions mentioned above, the following relationship is valid:

$$\eta_{\mathrm{fi},t} = \frac{f_{\mathrm{ay},\theta_{\mathrm{cr}}}}{f_{\mathrm{ay}}} \tag{4.45}$$

where f_{ay} is the ambient yield strength, and load level $\eta_{\mathrm{fi},t}$ is given by:

$$\eta_{\mathrm{fi},t} = \frac{E_{\mathrm{fi},d,t}}{R_d} \tag{4.46}$$

where R_d is the resistance of the member at time t = 0 and $E_{\mathrm{fi},d,t}$ is the design effect of the structural fire actions.

4.14 Moment Capacity-Based Approach

Moment capacity is determined from plastic theory for all classifications of sections except Class 4. In case of slabs with shear connectors, compression flange is taken as Class 1. Temperatures across the steel member are computed assuming both flanges are at uniform temperatures and compression zone of slab is thermally inert. For evaluating the temperature profile of unprotected members, the section factor A_m/V is computed as discussed below (Fig. 4.44):

(1) Top flange with 85% of the concrete slab is in contact with the upper flange and voids filled with non-combustible material:

$$\frac{A}{V} = \frac{b_2 + 2e_2}{b_2 e_2} \tag{4.47}$$

(2) Top flange with less than 85% of concrete slab is in contact:

$$\frac{A}{V} = 2\frac{b_2 + e_2}{b_2 e_2} \tag{4.48}$$

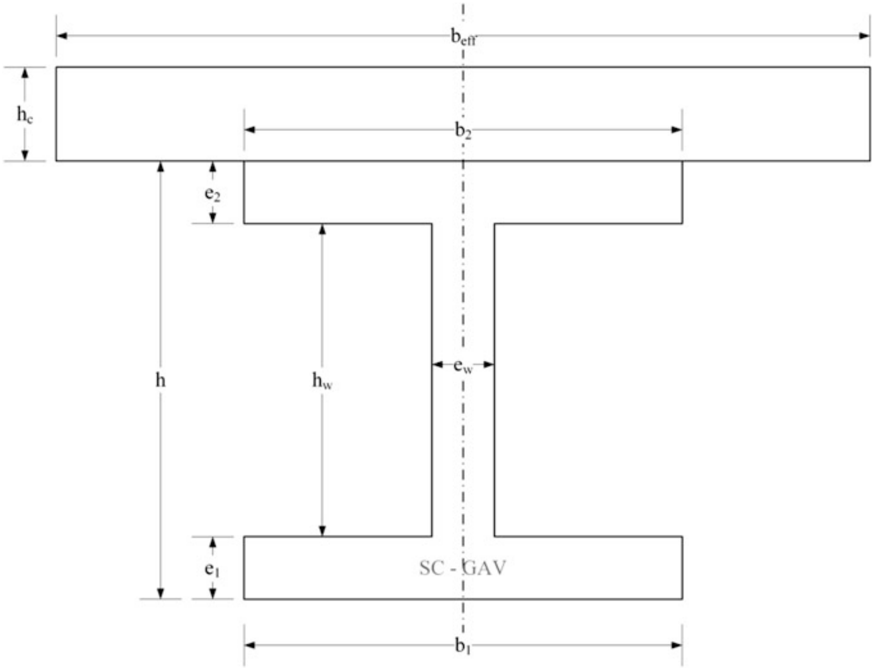


Fig. 4.44 Composite beam under consideration

(3) Bottom flange is:

$$\frac{A}{V} = 2\frac{b_1 + e_1}{b_1 e_1} \tag{4.49}$$

where (b_1, e_1, b_2, e_2) are widths and thickness, corresponding to the bottom and top flanges. When overall depth is lesser than 500 mm, web temperature is taken equal to that of the flange. For box protection, a uniform temperature is assumed over entire cross section with the A/V value taken same as that for the box. For unprotected composite steel beam, shadow factor k_{sh} is computed as:

$$k_{\mathrm{sh}} = 0.9\frac{e_1 + e_2 + \frac{b_1}{2} + \sqrt{h_w^2 + \frac{(b_1 - b_2)^2}{4}}}{h_w + b_1 + \frac{b_2}{2} + e_1 + e_2 - e_w} \tag{4.50}$$

The tensile capacity of the steel beam is calculated as:

$$T = \frac{f_{\mathrm{ay},\theta_1} b_1 e_1 + f_{\mathrm{ay},\theta\mathrm{w}} b_w e_w + f_{\mathrm{ay},\theta_2} b_2 e_2}{\gamma_{m,\mathrm{fi},a}} \tag{4.51}$$

The lever arm of the tensile force from the bottom of the beam is calculated as:

$$y_T = \frac{f_{\mathrm{ay},\theta_1} \frac{b_1 e_1^2}{2} + f_{\mathrm{ay},\theta_w} b_w e_w \left(e_1 + \frac{h_w}{2}\right) + f_{\mathrm{ay},\theta_2} b_2 e_2 \left(h - \frac{e_2}{2}\right)}{T\gamma_{m,\mathrm{fi},a}} \tag{4.52}$$

Limiting value of the tensile force T is given by:

$$T \leq NP_{\mathrm{fi,Rd}} \tag{4.53}$$

where N is the number of shear connectors and $P_{\mathrm{fi,Rd}}$ is the reduced capacity of the shear connecters, which is dependent on temperature. The tensile force is resisted by force in compression regime of concrete, and depth of compression zone is given by:

$$h_u = \frac{T\gamma_{m,\mathrm{fi},c}}{b_{\mathrm{eff}} f_{\mathrm{ck}}} \tag{4.54}$$

where b_{eff} is effective width of slab. Subsequently, h_{cr} is computed as per the code (EN-194-1-2). If $h_c - h_u > h_{\mathrm{cr}}$, temperature in concrete is lesser than 250 °C and full strength can be taken for design consideration. Alternatively, some layers of concrete have temperature greater than 250 °C and calculation of T becomes iterative. This is given by:

$$T = \frac{b_{\text{eff}}(h_c - h_{\text{cr}}) + \sum_{i=2}^{n-1} 10 b_{\text{eff}} f_{\text{cf},\theta i} + h_{u,n} f_{\text{ck},\theta n}}{\gamma_{m,\text{fi},c}} \quad (4.55)$$

$$h_u = (h_c - h_{\text{cr}}) + 10(n-2) + h_{u,n}$$

Subsequently, moment-carrying capacity is defined as:

$$M_{\text{fi,Rd}} = T\left(h + h_c - y_T - \frac{h_u}{2}\right) \quad (4.56)$$

The shear stud capacity must be checked, and design shear stud capacity is minimum of the following relationships:

$$P_{\text{fi,Rd}} = \begin{cases} 0.8 k_{u,\theta} P_{\text{Rd}} \\ k_{c,\theta} P_{\text{Rd}} \end{cases} \quad (4.57)$$

where $k_{u,\theta}$ is degradation factor for ultimate strength of steel, $k_{c,\theta}$ is degradation factor for characteristic compressive strength of concrete and P_{Rd} is calculated according to EN-1994-1-2.

4.15 Numerical Example on Composite Section Under Fire

Example 1 Evaluate the thickness of gypsum plaster protection for 90-min fire resistance of composite beam of span 4 m as shown in Fig. 4.45 for an ISO-834 exposure. It is to be noted that dovetail decking is running normal to the beam span.
Permanent Load:

Dead load: 2.75 (self-weight) + 2 (finishes) (kN/m^2); live load: 4 (kN/m^2)

(a) Moment capacity-based approach

For the given beam, load combination for fire is taken form EN 1991-1-2 as:

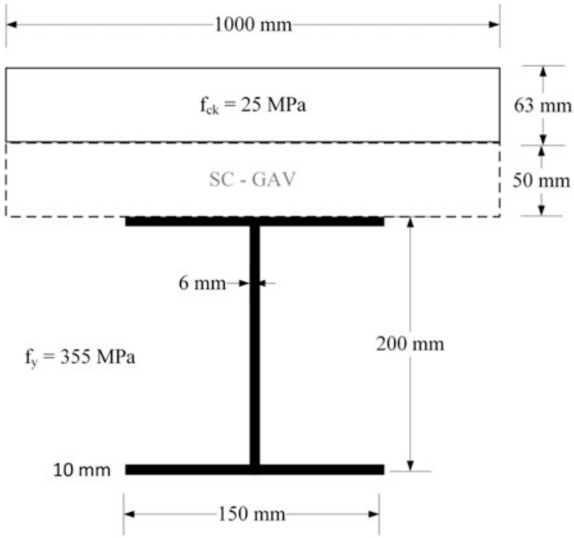
$$= 1 \times \text{P.L} + 0.3 \times \text{I.L.}$$

Design moment on the structure, $M_{\text{fi,ED},}$ is computed as:

$$\begin{aligned} &= 2.98 \times (4.75 + 0.3 \times 4) \times 4^2/8 \\ &= 33.3\ \text{kN/m}. \end{aligned}$$

A/*V* for bottom flange = 213 m^{-1} and *A*/*V* for top flange = 113 m^{-1}. Since the beam depth is lesser than 500 mm, web temperature can be taken to be same as that

Fig. 4.45 Composite beam under consideration for Example 1



of the lower (bottom) flange temperatures. A protection board of thickness 15 mm is assumed to be provided with material properties such as ρ_p = 500 kg/m^3, p = 2%, c_p = 1500 J/kg °C, λ_p = 0.25 W/m °C. Subsequently, thermal analysis is performed. At 90 min of exposure, temperature of bottom flange, web, and top flange is obtained as 800, 800, and 657 °C, respectively.

Tension in steel is computed as:

$$T = \frac{f_{ay,\theta_1} b_1 e_1 + f_{ay,\theta w} b_w e_w + f_{ay,\theta_2} b_2 e_2}{\gamma_{m,fi,a}}$$
$$= \frac{355 \times 150 \times 10 \times 0.091 + 355 \times 180 \times 6 \times 0.091 + 355 \times 150 \times 10 \times 0.253}{1} = 219\,\text{kN}$$

Lever arm y_T is computed as:

$$y_T = 355 \times \frac{0.091 \frac{150 \times 10^2}{2} + 0.091 \times 180 \times 6\left(10 + \frac{180}{2}\right) + 0.25 \times 150 \times 10\left(200 - \frac{10}{2}\right)}{219{,}000}$$
$$= 135\,\text{mm}$$

Taking effective width of slab as 1000 mm h_u is computed as:

$$h_u = \frac{T\gamma_{m,fi,c}}{b_{eff} f_{ck}} = \frac{219{,}000}{1000 \times 25} = 8.76\,\text{mm}$$

$h_c - h_u$ *is* computed as 91 mm and is greater than h_{cr} which is computed as 37 mm from the code. Subsequently, moment of resistance at 90 min is computed as:

$$M_{\mathrm{fi,Rd}} = T\left(h + h_c - y_T - \frac{h_u}{2}\right) = 36\,\mathrm{kNm}$$

Moment capacity of composite beam is greater than the applied moment, so it is safe. In the ambient design, 19-mm studs were placed in each trough at a spacing of 150 mm. Temperature in the stud is taken as 80% of that of the top flange temperature, i.e., 0.8 × 657 = 525 °C. Concrete temperature is taken as 40%, i.e., 0.4 × 657 = 268 °C. Shear stud capacity is computed as 50.1 kN and limiting value of T as 333 kN. Hence, it is seen that the section capacity is not limited due to shear studs.

Critical temperature-based approach cannot be applied to the present example as minimum thickness of concrete slab for this approach is 120 mm.

4.16 Composite Columns

Eurocode (EN-1994-1-2) provides simple calculation methods for design of composite columns. From Annexure H (1), design axial load on composite columns is given by:

$$\begin{aligned} N_{\mathrm{fi,Rd}} &= N_{\mathrm{fi,cr}} \le N_{\mathrm{fi,pl,Rd}} \\ N_{\mathrm{fi,cr}} &= \left(\frac{\pi}{l_\theta}\right)^2 \left[E_{\mathrm{a},\theta,\sigma} I_{\mathrm{a}} + E_{\mathrm{c},\theta,\sigma} I_{\mathrm{c}} + E_{\mathrm{r},\theta,\sigma} I_r\right] \end{aligned} \tag{4.58}$$

where l_θ is the effective length of composite column; $E_{\mathrm{a},\theta,\sigma}$, $E_{\mathrm{c},\theta,\sigma}$, and $E_{\mathrm{r},\theta,\sigma}$ are the tangent modulus of steel, concrete, and reinforcement at stress level σ; and I_{a}, I_{c}, and I_{r} represent moment of inertia of steel, concrete, and reinforcement. The plastic axial load capacity is given by:

$$N_{\mathrm{fi,pl,Rd}} = A_{\mathrm{a}} \frac{\sigma_{\mathrm{a},\theta}}{\gamma_{\mathrm{M,fi,a}}} + A_{\mathrm{c}} \frac{\sigma_{\mathrm{c},\theta}}{\gamma_{\mathrm{M,fi,c}}} + A_r \frac{\sigma_{\mathrm{r},\theta}}{\gamma_{\mathrm{M,fi,r}}} \tag{4.59}$$

where A_{a}, A_{c}, and A_{r} represent areas of steel, concrete, and reinforcement and $\sigma_{a,\theta}$, $\sigma_{c,\theta}$, and $\sigma_{r,\theta}$ represent temperature-dependent stress in constituent materials. Above-mentioned design method needs to be aided with numerical tools like FEA or FDM for the evaluation of temperature response history. Empirical methods have also been applied for simplified design of composite columns subjected to fire. Kodur and Lie (2) suggested empirical equation for fire endurance of rectangular and circular composite columns as:

$$t_{fi,Rd} = f_1 \frac{f_{ck} - 20}{l_\theta - 1000} \frac{D^{2.5}}{\sqrt{N_{fi,Ed}}} \tag{4.60}$$

where l_θ is the buckling length (mm), D is the diameter of a circular column or side length of a square column (mm), $N_{\mathrm{fi,Ed}}$ is the applied load in the fire limit state (kN), and f_1 is a factor. Table 4.32 shows values for f_1 factor.

Example 1 Evaluate the fire endurance of concrete-filled steel column as shown in Fig. 4.46, cast with siliceous aggregates and steel box section of yield strength 400 MPa.

For the given beam, load combination for fire is taken form EN 1991-1-2 as:

$$= 1 \times \mathrm{P.L} + 0.3 \times \mathrm{I.L.}$$

Design axial force on the structure $N_{\mathrm{fi,Ed}}$ is computed as 1340 kN.

Fire endurance is computed as below:

$$t_{\mathrm{fi,Rd}} = f_1 \frac{f_{\mathrm{ck}} - 20}{l_\theta - 1000} \frac{D^{2.5}}{\sqrt{N_{\mathrm{fi,Ed}}}}$$

$$t_{\mathrm{fi,Rd}} = 0.075 \frac{25-20}{1900-1000} \frac{420^{2.5}}{\sqrt{1340}} = 41 \text{ min}$$

Table 4.32 f_1 factor for composite columns

Aggregate type	Column type	Plain concrete	Fiber concrete	Reinforcement ratio and cover			
				<3%		>3%	
				<25	>25	<25	>25
Siliceous	CHS	0.07	0.075	0.075	0.08	0.08	0.085
	SHS	0.06	0.065	0.065	0.07	0.07	0.075
Carbonate	CHS	0.08	0.07	0.085	0.09	0.09	0.095
	SHS	0.06	0.065	0.075	0.08	0.08	0.085

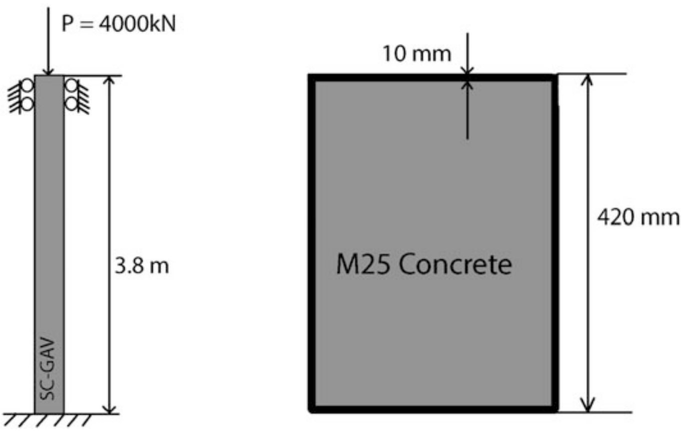


Fig. 4.46 Concrete-filled steel column (Example 2)

Model exam paper-1

Design aids for offshore structures under special environmental loads including fire resistance

Time: 3 h Max marks: 100

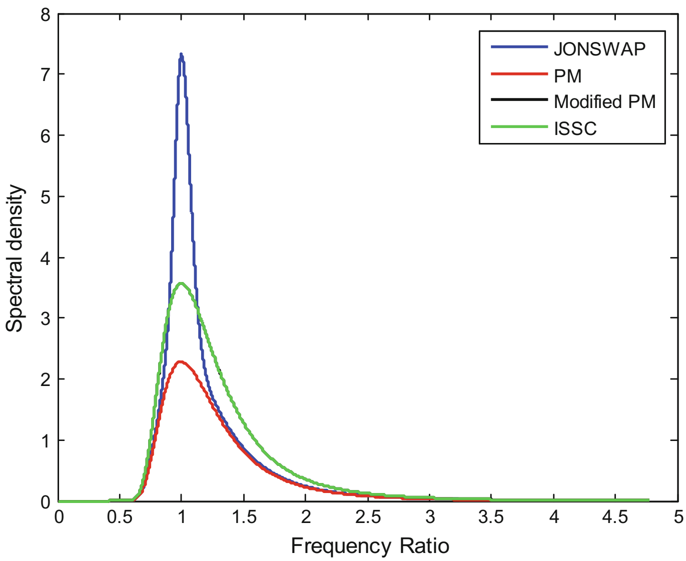
Section A: Each question carries one mark. Use appropriate key words to answer

1. Offshore structures are preferably function-dominant. State true/false and validate the statement.
2. Compliant structures and fixed structures resist the lateral loads by __________ and ____________, respectively.
3. Continuous rotation of the spud-can arrangement in the guyed towers will lead to _____________.
4. Give the expression for the resisting moment in articulated towers.
5. Ball joints restrain ____________ and allow _________ from the buoyant leg to the deck.
6. When the mean square value is unique across the ensemble, the process is said to be ____________.
7. Ice loads on conical structures cause less response. State true/false and validate the statement.
8. The interaction of ice on the structure will cause _____________, which is measured by monitoring __________________.
9. __________________ is used for the analysis of the offshore structures under earthquake loads.
10. _____________ is caused by second-order waves in mild and severe sea state; and ___________ is caused by extremely high steep waves.
11. Non-impact waves cause __________ response on TLP which will lead to ___________.
12. When the trace of the plane of applied moment does not coincide with any of the principal axes of inertia, then the bending is said to be _____________.
13. Write the expression for finding the shear stress on the section.
14. Winkler–Bach equation is useful in estimating the stresses in the curved beams with large initial curvature only at _______________.
15. When risers are exposed to the fluid flow, ______________ takes place which results in the formation of _____________.
16. _____________ is required to maintain the stability of the riser, when installed in greater water depth.
17. ______________ is the lowest temperature at which the liquid gives up enough vapor to maintain the continuous flame.

18. ____________ reduces the risk of flame propagation.
19. High-strength steel does not have a ____________ and the yield region is ____________.
20. A turbulent diffusion of flame resulting from the combustion of the fuel and propagating in a particular direction with significant momentum is called ________.

Section B: Each question carries Two marks. Answer briefly

1. List the uncertainties in estimating the environmental loads.
2. Does the lateral load acting on the compliant platforms gets reduced by the structural action of the platform? Explain.
3. Write the following degrees of freedom in descending order based on the time period of the Spar platform: Pitch, heave, surge.
4. Find the period of a Tension Leg Platform in surge degree of freedom. The platform has eight tethers made up of steel, with 1.5 MN initial pretension each and 250 m long, 0.9 m diameter. The mass of TLP in surge is 1.5×10^7 kg.
5. Name the essential features of buoyant leg structure that makes the structure insensitive to ultra-deep waters.
6. Compare the different wave spectra and write the inferences based on the following spectral density curves.

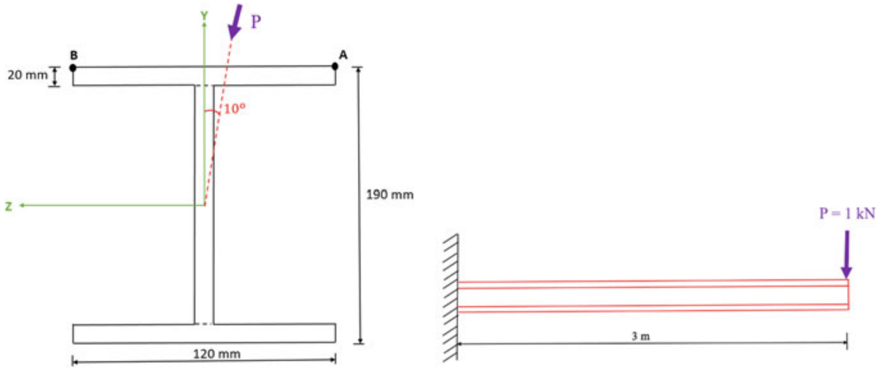


7. List the various ice conditions.
8. What are the factors that affect the dynamic modulus of elasticity?
9. How the orientation of the platform is chosen based on the wave approach angle?

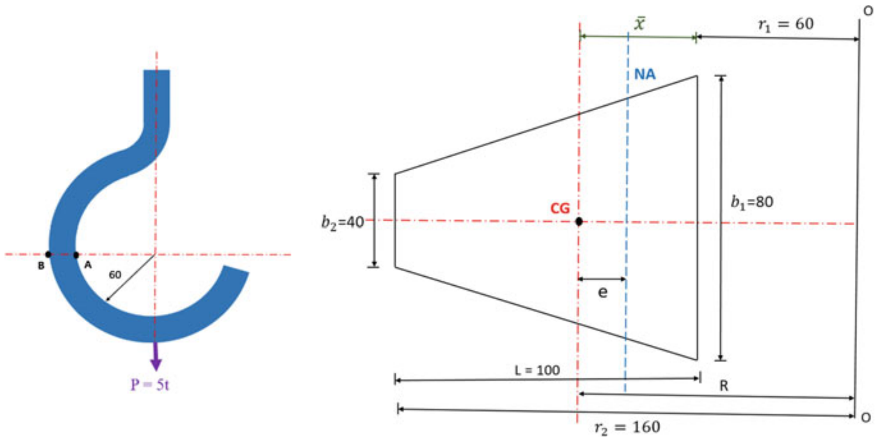
10. What is the effect of springing and ringing waves on the structures?
11. When symmetrical bending occurs in the members?
12. How do you classify the curved beams? What is the condition for the classification?
13. What are the types of failure that may occur on the chain links used as moorings in the offshore structures?
14. List the different group of risers.
15. What are the consequences of vortex-induced vibration on the structure?
16. What are the common mistakes that lead to fire accidents in offshore platforms?
17. Define BLEVE and what are the primary causes of it?
18. Explain blast wave structure interaction.
19. Differentiate the terms blast proof and blast resistant.
20. Name the principal parameters of the blast wave that has to be defined to estimate the blast load.

Section C: Each question carries five marks. Answer in detail. Draw figures, wherever necessary to support your answer

1. Elaborate the following: FEED, FPSO, FSRU, BLSRP, JONSWAP, AIT, CVCE, TNT, NFPA, and PSV.
2. Mention the advantages and disadvantages of Spar platform.
3. How do you calculate the total wave force acting on the pile of the jacket leg platform?
4. Find the stresses on the cantilever beam (l = 3 m) of I section at points A and B, shown in the figure below. The load of 1 kN acts at angle of 10° from the vertical axis.



5. A 10-ton crane hook is used to lift an object during commissioning of an offshore deck. Find the stresses at the intrados and extrados using Winkler–Bach equation and Straight Beam formula. The details of the cross section are shown in the figure below:



6. Describe vortex-induced vibration (ViV) and the ways to suppress the effect of it on the structure.
7. List the best practices for fire safety in offshore platforms.
8. Describe the different types of blast waves.

Key to model exam paper-1

Section A:

1. Offshore structures are preferably function-dominant. State true/false and validate the statement.
   False. Offshore structures are form-dominant.
2. Compliant structures and fixed structures resist the lateral loads by *self-weight* and *relative displacement,* respectively.
3. Continuous rotation of the spud-can arrangement in the guyed towers will lead to *fatigue failure of the joint*.
4. Give the expression for the resisting moment in articulated towers.
   $Resisting\,moment = \{[(B\rho - M_B)gl_B] - [M_D g l_D]\}\theta$
   where
   B = Buoyancy provided by the buoyancy tank.
   l_D *= Distance of CG of the deck from the articulated joint.*
   l_B *= Distance of center of buoyancy from the articulated joint.*
   Θ *= degree of freedom.*
   M_B *= Mass of the buoyancy tank.*
   M_D *= Mass of the deck.*
5. Ball joints restrain *rotation* and allow translation from the buoyant leg to the deck.
6. When the mean square value is unique across the ensemble, the process is said to be *stationary process.*
7. Ice loads on conical structures cause less response. State true/false and validate the statement.
   True. Well-defined cone can change the ice failure mode from crushing to bending.
8. The interaction of ice on the structure will cause *ice-induced vibration*, which is measured by monitoring *the acceleration of the deck*.
9. *Kanai*-Tajimi *power spectrum* is used for the analysis of the offshore structures under earthquake loads.
10. *Springing response* is caused by second-order waves in mild and severe sea state, and *ringing response* is caused by extremely high steep waves.
11. Non-impact waves cause *heave* response on TLP which will lead to *fatigue failure*.
12. When the trace of the plane of applied moment does not coincide with any of the principal axes of inertia, then the bending is said to be *unsymmetrical*.
13. Write the expression for finding the shear stress on the section.

$$\tau = \frac{Va\bar{y}}{It}$$

14. Winkler-Bach equation is useful in estimating the stresses in the curved beams with large initial curvature only at *the extreme skin layer*.

15. When risers are exposed to the fluid flow, *flow separation* takes place which results in the formation of *vortex shedding*.
16. *Top tensioning* is required to maintain the stability of the riser, when installed in greater water depth.
17. *Flash point* is the lowest temperature at which the liquid gives up enough vapor to maintain the continuous flame.
18. *Deflagration arresters* reduce the risk of flame propagation.
19. High-strength steel does not have a *well-developed yield point*, and the yield region is *nonlinear*.
20. A turbulent diffusion of flame resulting from the combustion of the fuel and propagating in a particular direction with significant momentum is called *jet fire*.

Section B:

1. List the uncertainties in estimating the environmental loads.

   - *Estimation of direction of approach.*
   - *Estimation of magnitude of loads.*
   - *Return period of the maximum event.*
   - *Non-availability of data.*

2. Does the lateral load acting on the compliant platforms get reduced by the structural action of the platform? Explain.
   The lateral loads get reduced. Since the compliant platforms are position restrained by tethers or tendons, the horizontal component of the tension in the deflected tendon will reduce the lateral load acting on the platform.
3. Write the following degrees of freedom in descending order based on the time period of the Spar platform: pitch, heave, surge

   - *Surge*: 100–120 s
   - *Pitch*: 60 s
   - *Heave*: 25–40 s

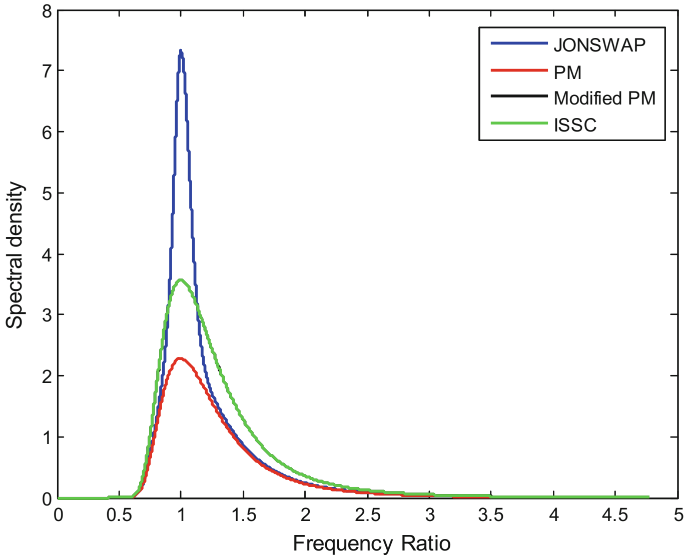
4. Find the time period of a Tension Leg Platform in surge degree of freedom. The platform has 8 tethers made up of steel, with 1.5 MN initial pretension each and 250 m long, 0.9 m diameter. The mass of TLP in surge is 1.5×10^7 kg.

$$(T)_{surge} = 2\pi\sqrt{\frac{m_{surge} l_t}{nT_t}} = 111.07\,\text{s}$$

5. Name the essential features of buoyant leg structure that makes the structure insensitive to ultra-deep waters.

   - *Deep draft*
   - *High stability*

6. Compare the different wave spectra and write the inferences based on the following spectral density curves.



- *The peak value of all the spectrum lies on the same frequency.*
- *JONSWAP spectrum exhibits the highest peak value.*
- *PM spectrum describes a fully developed sea condition and it has the lowest energy.*

7. List the various ice conditions.
   - *Level ice*
   - *Broken ice*
   - *Ice ridges*
   - *Ice bergs*
8. What are the factors that affect the dynamic modulus of elasticity?
   - *Material on which the experiment is carried out.*
   - *Type of test used to find the dynamic modulus of elasticity.*
   - *Shape of the specimen used in the experimental setup.*
9. How the orientation of the platform is chosen based on the wave approach angle?
   The orientation is chosen in such a way that the members should have less projected area on the encountered wave direction.
10. What is the effect of springing and ringing waves on the structures?
    - *Generation of high-frequency force.*
    - *Builds up resonance-type responses within the period of TLP.*
    - *The frequency of the ringing waves seems closer to the natural frequency of the Tension Leg Platforms.*

11. When symmetrical bending occurs in the members?
    - *The plane containing one of the principal axes of inertia, plane of applied moment, and plane of deflection should coincide.*
    - *The neutral axis should coincide with the principal axis of inertia.*
12. How do you classify the curved beams? What is the condition for the classification?
    - *Beam with small initial curvature.*
    - *Beam with large initial curvature.*
    - *For the beams with small initial curvature, the ratio of the initial radius of curvature to the depth of the section should be greater than 10.*
13. What are the types of failure that may occur on the chain links used as moorings in the offshore structures?
    - *Failure due to fatigue crack at the inter-grip area.*
    - *Distortion in the dimensions of the chain along the length.*
    - *Fatigue damage at the weld.*
    - *Erosion at the inter-grip area.*
14. List the different group of risers.
    - *Bundled risers.*
    - *Flexible risers.*
    - *Top Tensioned Risers.*
    - *Steel catenary risers.*
    - *Hybrid risers.*
15. What are the consequences of vortex-induced vibration?
    - *They can cause alternating pressure field on the surface of the riser.*
    - *Reduction of service life of the system.*
    - *Fatigue damage.*
    - *Induce large transverse motion.*
    - *Cause operational difficulties.*
16. Write the expression for Reynolds number and reduced velocity.
    $\mathrm{Re} = \frac{\rho u D}{\mu}$
    $\text{Reduced Velocity} = \frac{uT}{D}$
17. Define BLEVE and what are the primary causes of it?
    BLEVE—Boiling Liquid Expanding Vapor Explosion.
    This is caused due to the sudden release of large amount of vapor through a narrow opening under pressurized conditions.
18. Explain blast wave structure interaction.
    When the blast wave hits the structure, the following occurs:
    - *Reflection of waves.*
    - *Refraction of waves.*

- *Vortex formation.*
- *Diffraction of waves.*

19. Differentiate the terms blast proof and blast resistant.

    - *Blast proof is the non-realistic term, and it is difficult to provide a system with complete blast protection.*
    - *Blast resistance should aim to protect the functional requirements of the critical systems from any irreparable damage that leads to catastrophic damage.*
    - *The platform should remain functional even after the blast event.*

20. Name the principal parameters of the blast wave that has to be defined to estimate the blast load.

    - *Peak side-on overpressure.*
    - *Phase duration.*
    - *Impulse.*
    - *Peak side-on negative pressure (suction).*
    - *Negative phase duration.*
    - *Associated negative impulse.*

Section C:

1. Elaborate the following:
   FEED—*Front End Engineering Design.*
   FPSO—*Floating Production Storage and Offloading system.*
   FSRU—*Floating, Storage, Regasification Unit.*
   BLSRP—*Buoyant Leg Storage and Regasification Unit.*
   JONSWAP—*Joint North Sea Wave Project.*
   AIT—*Auto-Ignition Temperature.*
   CVCE—*Confined Vapor Cloud Explosion.*
   TNT—*Tri-Nitro-Toluene.*
   PSV—*Platform Support Vessels.*
   NFPA—*National Fire Protection Association.*
2. Mention the advantages and disadvantages of Spar platform.
   Advantages:

   - *It is a free floating system more or less, and the response in heave and pitch is lesser.*
   - *The drilling risers are protected by the cylinder of the Spar from the wave action.*
   - *Simple fabrication.*
   - *Unconditional stability because the mass center is always located lower than the center of buoyancy.*
   - *Stability is ensured even when the mooring lines are disconnected.*

Disadvantages:

- *Installation is difficult compared to other platform, because the topside of the platform can only be placed after upending the cylinder of the Spar.*
- *Little storage capacity.*
- *Spar can be used only for storage and offloading.*
- *It is prone to corrosion.*

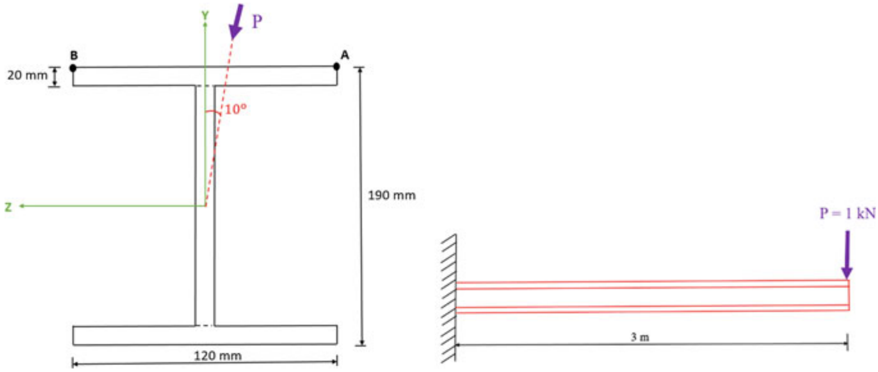
3. How do you calculate the total wave force acting on the pile of the jacket leg platform?

- *On a pile member, combined inertia and drag forces will act.*
- *Forces on the pile member arise due to velocity and acceleration components of the water particles.*
- *The forces vary with time.*
- *The combined inertia and drag forces can be found out from the Morison's equation.*
- *In order to find the maximum forces, the phase angle at which this will occur should be calculated first, which can be calculated by:*

$$\frac{\mathrm{d}F_t}{\mathrm{d}\theta} = 0$$

- *The maximum force can now be calculated from Morison's equation for the above value of phase angle.*

4. Find the stresses on the cantilever beam (l = 3 m) of I section at points A and B, as shown in the figure below. The load of 1 KN acts at angle of 10° from the vertical axis.



(i) ***Calculation of I_Z, I_Y, and I_{ZY}***:

$$\bar{y} = \frac{\sum ay}{\sum a} = 95\,\text{mm}$$

$$\bar{z} = \frac{\sum az}{\sum a} = 60\,\text{mm}$$

$$I_Y = \frac{2\times 20\times 120^3}{12} + \frac{150\times 20^3}{12} = 0.586\times 10^7\,\text{mm}^4$$

$$I_Z = 2\left[\frac{120\times 20^3}{12} + 120\times 20\times (85)^2\right] + \frac{20\times 150^3}{12} = 4.046\times 10^7\,\text{mm}^4$$

(ii) ***To locate principal axes (u, v)***:

Since I section is symmetrical, ZZ and YY axes are the principal axes of UU and VV, respectively.

$$I_U = 0.586\times 10^7\,\text{mm}^4$$

$$I_V = 4.046\times 10^7\,\text{mm}^4$$

(iii) ***To find the stresses***:

Moment about Z-axis = load × perpendicular distance = 1 × 3 = −3 kNm.

$$M_U = M_Z\cos\alpha = -2.954\,\text{kNm}$$

$$M_V = -M_Z\sin\alpha = 0.521\,\text{kNm}$$

$$\sigma_X = -\left(\frac{M_U}{I_U}v - \frac{M_V}{I_V}u\right)$$

For point A,

$$u_A = -60\,\text{mm}$$

$$v_A = +95\,\text{mm}$$

$$\sigma_A = -\left(\frac{M_U}{I_U}v_A - \frac{M_V}{I_V}u_A\right) = 1.602\,\text{N/mm}^2\ \text{(tension)}$$

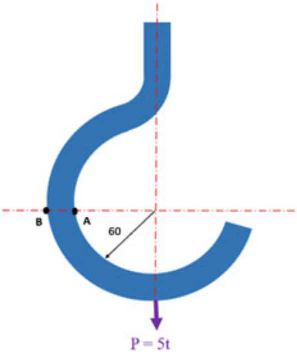
Similarly for point B,

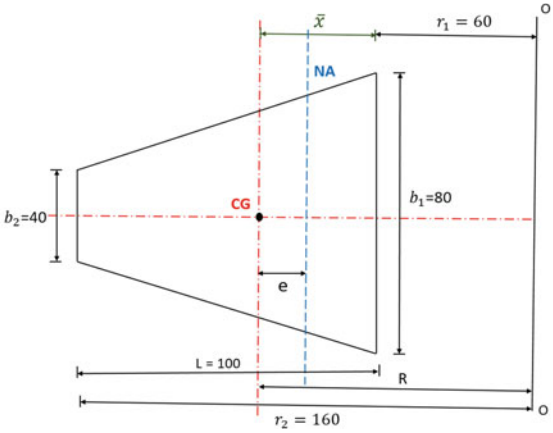
$$u_B = 60\,\text{mm}$$

$$v_B = 95\,\text{mm}$$

$$\sigma_B = -\left(\frac{M_U}{I_U}v_B - \frac{M_V}{I_V}u_B\right) = 12.28\,\text{N/mm}^2\ \text{(tension)}$$

5. A 10-ton crane hook is used to lift an object during commissioning of an offshore deck. Find the stresses at the intrados and extrados using Winkler–Bach equation and Straight Beam formula. The details of the cross section are shown in the figure below.





A. ***Winkler–Bach Equation***

(i) ***Calculation of Geometric properties***:

$b_3 = 20\,\text{mm}$

Location of the neutral axis, $\bar{x} = \frac{\sum A\bar{x}}{\sum A}$ = 44.44 mm

$h_i = x = 44.44\,\text{mm}$

$h_o = h - x = 55.56\,\text{mm}$

1. *Radius of the curved beam*, $R = r_1 + x$ = 104.44 mm
2. *Outer radius of the curved beam*, $r_2 = r_1 + h$ = 160 mm
3. *CS area of the section*, A = 6000 mm^2
4. *Sectional property*, m
   $m = 1 - \left(\frac{R}{A}\right)\left\{\left[b_2 + \frac{r_2(b_1-b_2)}{(r_2-r_1)}\right].\ln\left(\frac{r2}{r1}\right) - (b_1 - b_2)\right\}$ = −0.0794 *(no unit)*
5. *Eccentricity*, $e = \left(\frac{m}{m-1}\right)R$ = 7.679 mm *(The positive value of 'e' indicates that the Neutral axis will shift toward the center of curvature)*
6. *Moment of Inertia*, $I = 4.814 \times 10^6$ mm^4

(ii) ***Section AB***:

$r_i = r_1 = 60\,\text{mm}$

$r_o = r_2 = 100\,\text{mm}$

(a) *Direct Stress*, $\sigma_d = \frac{P}{A} = 8.333$ N/mm^2

(b) *Moment at CG*, $M = -P \times R$ = −5.222 kNm

(c) *Stress at intrados*, $\sigma_i = -\frac{M}{Ae}\left(\frac{h_i - e}{r_i}\right)$ = 69.442 N/mm^2 *(tensile)*

(d) *Stress at extrados*, $\sigma_i = \frac{M}{Ae}\left(\frac{h_s - e}{r_s}\right)$ = −44.79 N/mm^2 *(compressive)*

(e) *Total stress at intrados,* $\sigma_A = \sigma_d + \sigma_i$ = 77.775 N/mm^2 *(tensile)*
(f) *Total stress at extrados,* $\sigma_B = \sigma_d + \sigma_o$ = −36.457 N/mm^2 *(compressive)*

B. ***Straight Beam Formula***:
$\sigma = \frac{M}{I} y$

$$y_i = 44.44\,\text{mm}$$
$$y_s = 55.56\,\text{mm}$$

(a) *Stress at intrados,* $\sigma_i = \frac{M}{I} y_i$ = 48.205 N/mm^2 *(tensile)*
(b) *Stress at extrados,* $\sigma_s = \frac{M}{I} y_s$ = −60.256 N/mm^2 *(compressive)*
(c) *Total stress at intrados,* $\sigma_A = \sigma_d + \sigma_i$ = 56.539 N/mm^2 *(tensile)*
(d) *Total stress at extrados,* $\sigma_B = \sigma_d + \sigma_s$ = −51.923 N/mm^2 *(compressive)*

6. Describe vortex-induced vibration and the ways to suppress the effect of it on the structure.

- *As the fluid passes the body, flow separation occurs. This results in the formation of vortex behind the body.*
- *Vortex shedding frequency tries to lock-in with the natural frequency of the riser, which leads to resonating condition.*

Consequences of VIV:

- *They can cause alternating pressure field on the surface of the riser.*
- *Reduction of service life of the system.*
- *Fatigue damage.*
- *Induce large transverse motion.*
- *Cause operational difficulties.*

VIV suppression:

- *By providing surface protrusion—helical strakes.*
- *By attaching the projected plates called near-wake stabilizers.*
- *By providing shroud, which is a perforated cover around the member.*
- *Changing the geometric shape of the platform.*
- *By ensuring smooth surface of the members.*
- *By connecting external dampers.*

7. List the best practices for fire safety in offshore platforms.

- *Fluid should be used above the flash point temperature and the fire point temperature, but not above AIT.*
- *Fluid can be used up to their maximum bulk temperature, which is higher than the flash and fire point temperature of the fluid.*
- *Avoid designing confined space with the presence of ignition source, because that can result in flash fire easily.*

- *Through proper system design, one can ensure that no oxygen content is available at the heat source.*
- *Fluid should be well contained within the system.*
- *Fluid containment should not have direct contact with the external ignition source.*

8. Describe the different types of blast waves.
   Shock wave:

   - *Leads to instantaneous shoot up of pressure above the ambient temperature.*
   - *The peak side-on pressure then gradually returns to the ambient conditions.*
   - *This also results in negative pressure wave, following the positive phase of the shock wave.*

   Pressure wave:

   - *Gradual rise in the pressure to reach the peak side-on overpressure.*
   - *It then gradually decays without any negative phase.*
   - *Shock waves are the consequences of extremely energetic vapor cloud explosion.*
   - *Vapor cloud deflagration will result in rise of pressure waves in the near field.*
   - *They will further propagate to the far field as a shock wave.*
   - *Negative phase of the shock wave is weaker, but sometimes a suction pressure can also cause significant damage.*

Model exam paper-2

Design aids for offshore structures under special environmental loads including fire resistance

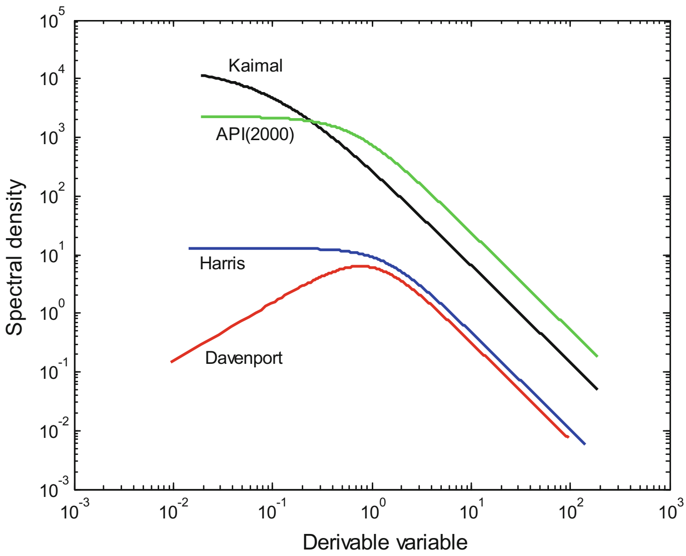
Time: 3 h Max marks: 100

Section A: Each question carries one mark. Use appropriate key words to answer

1. Offshore structures are unique by ___________ and ________________.
2. Fixed structures trigger __________ mode of failure.
3. In Spar platform, helical strakes are provided around the cylinder to reduce _____________.
4. Wave direction does not influence the response of triceratops platform. State true/false.
5. Airy's theory is valid only up to ___________ and stretching modifications should be accounted for __________________.
6. The wind spectra applicable for the offshore structures are ____________ and _____________.
7. If the return period of the event is reduced, the probability of exceedance of the event will ___________.
8. Any sample from the process which represents the average statistical properties of the entire process is said to be ___________.
9. Earthquake loads affect the superstructure of the Tension Leg Platforms indirectly by inducing _________________.
10. Triangular geometry of TLP is advantageous compared to square geometry. Why?
11. Pitch response of triangular TLP is ____________ than the square TLP.
12. ______________ is used for the simulation of the extreme waves in square TLP.
13. Due to unsymmetrical bending, thin sections will undergo _________ under transverse loads.
14. If a section has one axis of symmetry, the shear center will lie on _____________.
15. Write the Winkler–Bach equation
16. ______________ should be introduced in the chain link when the stress in the links is greater than the permissible limits.
17. The inner metal surface of the layered riser is called ___________.
18. AIT is also known ____________ and ______________.
19. If the ignition source is present, and the vessel is in contact with the ignition source, then it will result in the formation of ___________.
20. Rear wall load tends to ___________ the overall blast wave force.

Section B: Each question carries two marks. Answer briefly

1. Elaborate the following:
   - DPS
   - BLS
   - VIV
   - BLEVE
2. Explain the structural action of the spud-can arrangement with neat sketch.
3. Find the stiffness and time period of the Spar platform of 30 m diameter and mass in heave degree of freedom is 60,000*t*.
4. Platforms' remaining stiff in heave degree of freedom is advisable. Why?
5. Write the expression for horizontal and vertical water particle velocities based on Airy's theory.
6. Why aerodynamic admittance function is used? State the reasons.
7. State the inferences for the following wind spectra.



8. How uncertainties in the analysis and design of the structures are grouped?
9. What are the assumptions made in the earthquake analysis of the offshore structures?
10. Write the classification of loads.
11. Mention the effect of impact waves and non-impact waves on TLP.
12. What are the consequences of transverse loading on the structural members?
13. What is the advantage of using buoyancy modules in the risers?
14. What are the ways through which VIV suppression can be achieved in the structure?

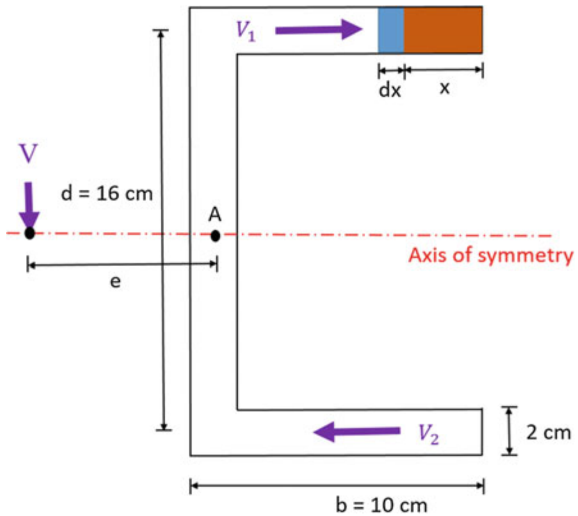
15. Differentiate fire and explosion.
16. Arrange the following in ascending order based on their temperature. Oxyhydrogen fire, simple candle, air acetylene fire, blow torch under welding operation.
17. How the consequences due to the explosion damage can be measured?
18. List the general design requirements of fire protection in the offshore platforms.
19. Define peak-reflected pressure and peak dynamic pressure.
20. Name the different types of fire.

Section C: Each question carries five marks. Answer in detail. Draw figures, wherever necessary to support your answer

1. Match the following:

1	Fixed structures	A	Exploratory drilling
2	Articulated towers	B	Deck-isolated platforms
3	Tethers	C	Turret mooring
4	Guyed towers	D	Tethered Spar
5	Compliant structures	E	Attract more forces
6	FPSO	F	DPS
7	Jack-up rig	G	Skirt piles
8	Drill ships	H	Position restraint
9	Gravity-based structure	I	Recentering is gentle
10	Jacket platforms	J	Spud-can
11	BLS	K	Seabed scouring
12	Triceratops	L	Better under cyclic loads

2. Explain the structural action of TLP.
3. Locate the shear center for the channel section as shown in figure.



4. Write a short note on different types of risers.
5. Describe the steps involved in the identification of explosion damage.
6. Mention the basic problems in the offshore platforms that make fire protection very difficult.
7. Compute the front wall loading due to blast waves traveling horizontally on the building module of offshore platform. Length = 15 m, breadth = 30 m, height = 5 m, and the blast wave approaches the building in the direction parallel to the length of the building. The peak side-on overpressure is 50 kPa for the duration of 0.06 s.
8. Write a case study on the piper alpha accident and deepwater horizon accident.

Key to model exam paper-2

Section A:

1. Offshore structures are unique by *innovative design* and *functional requirements*.
2. Fixed structures triggers *brittle* mode of failure.
3. In Spar platform, helical strakes are provided around the cylinder to reduce *vortex-induced vibration*.
4. Wave direction does not influence the response of triceratops platform. State true/false.
   True because the legs of the platform are symmetrically placed.
5. Airy's theory is valid only up to *mean sea level* and stretching modifications should be accounted for *the actual level of submergence*.
6. The wind spectra applicable for the offshore structures are *Kaimal spectrum and API spectrum*.
7. If the return period of the event is reduced, the probability of exceedance of the event will *increase*.
8. Any sample from the process which represents the average statistical properties of the entire process is said to be *ergodic*.
9. Earthquake loads affect the superstructure of the Tension Leg Platforms indirectly by inducing *tether tension variation*.
10. Triangular geometry of TLP is advantageous compared to square geometry. Why?
    - *Increased tolerance for the position of foundation.*
    - *Increased drought and heat tolerance.*
11. Pitch response of triangular TLP is *lesser* than the square TLP.
12. *Freak wave model* is used for the simulation of the extreme waves in square TLP.
13. Due to unsymmetrical bending, thin sections will undergo *twisting* under transverse loads.
14. If a section has one axis of symmetry, the shear center will lie on *the axis of the section*.
15. Write the Winkler–Bach equation.

$$\sigma = \frac{M}{AR}\left[1 - \frac{1}{m}\left(\frac{y}{R+y}\right)\right]$$

16. *Central stud* should be introduced in the chain link when the stress in the links is greater than the permissible limits.
17. The inner metal surface of the layered riser is called *carcass*.
18. AIT is also known *as self-ignition temperature and kindling point of the material*.
19. If the ignition source is present, and the vessel is in contact with the ignition source, then it will result in the formation of *fire ball*.

20. Rear wall load tends to <u>*reduce*</u> the overall blast wave force.

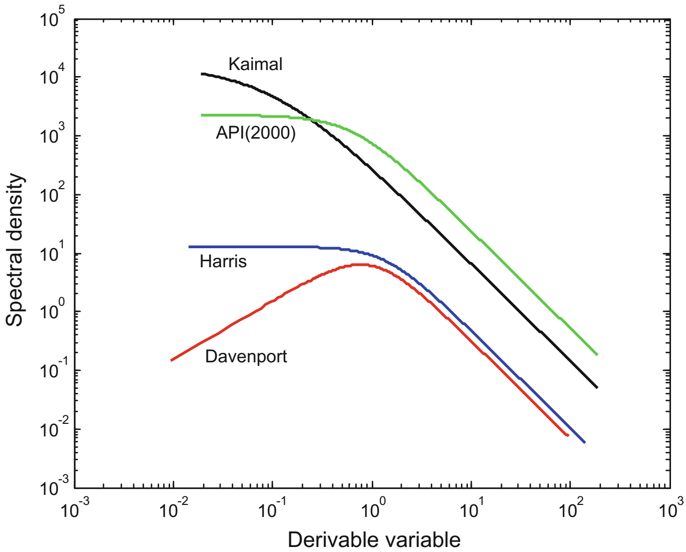
<u>Section B</u>:

1. Elaborate the following:
   DPS—*Dynamic Positioning System.*
   BLS—*Buoyant Leg Structure.*
   VIV—*Vortex-Induced Vibration.*
   BLEVE—*Boiling Liquid Expanding Vapor Explosion.*
2. Explain the structural action of the spud-can arrangement with neat sketch.
   - *Spud-can is similar to the inverted cone placed under suction.*
   - *When the spud-can arrangement is placed on the seabed with high pressure, partial vacuum will develop inside the cone arrangement.*
   - *The partial vacuum space is the filled by the soil particles surrounding the spud-can, and hence, high pressure is required to remove the spud-can from its position.*
3. Find the stiffness and time period of the Spar platform of 30 m diameter and mass in heave degree of freedom is 60,000t.
   Stiffness $= \rho g \pi R^2 = 1025 \times 9.81 \times \pi \times (15)^2 = 7.1 \times 10^6\,\mathrm{N/m}$
$$m = 6 \times 10^7\,\mathrm{kg}$$
$$\omega = \sqrt{k/m} = 0.344$$
$$T = \tfrac{1}{\omega} = 2.907\ \mathrm{s}$$
4. Platforms' remaining stiff in heave degree of freedom is advisable. Why?
   - *For operational convenience.*
   - *To reduce the change in tension in the tethers and to avoid fatigue failure.*
   - *To reduce the consequences of change in buoyancy by the added mass.*
5. Write the expression for horizontal and vertical water particle velocities based on Airy's theory.
$$\dot{u}(x,t) = \frac{\omega H}{2}\frac{\cos\ h\,ky}{\sin\ h\,kd}\cos(kx - \omega t)$$
$$\dot{v}(x,t) = \frac{\omega H}{2}\frac{\sin\ h\,ky}{\sin\ h\,kd}\sin(kx - \omega t)$$
   where
   - $\dot{u}$ *horizontal water particle velocity.*
   - $\dot{v}$ *vertical water particle velocity.*
   - k *wave number = 2π/ω.*
   - H *wave amplitude.*
   - d *water depth.*
   - Y *depth at which the velocity is measured*

6. Why aerodynamic admittance function is used? State the reasons.

   - *Aerodynamic admittance function is used to find the equivalent total wind load acting on the members.*
   - *It is used to bypass the rigorous random variable.*
   - *The value can be found experimentally.*

7. State the inferences for the following wind spectra.



   - *The variation among the spectrum is considerable at lower frequencies.*
   - *The variation between API and Kaimal spectra is considerably low.*
   - *Davenport spectrum and Harris spectrum are not applicable to offshore structures, and they found to have lower peak value.*

8. How uncertainties in the analysis and design of the structures are grouped?

   - Group I: related to material characteristics.
   - Group II: related to load estimation.
   - Group III: arises due to mathematical modeling and method of analysis.

9. What are the assumptions made in the earthquake analysis of the offshore structures?

   - *Seabed movement is horizontal.*
   - *The movement of seabed is very low, and second-order forces generated due to the seabed movement is generally neglected.*
   - *Radiation damping is neglected for the slender structures.*

10. Write the classification of loads.
    - *P class loads—permanent loads.*
    - *L class loads—live loads.*
    - *D class loads—deformation loads.*
    - *E class loads—environmental loads.*
    - *A class loads—accidental loads.*
11. Mention the effect of impact waves and non-impact waves on TLP.
    Impact waves:
    - *Cause ringing response.*
    - *Pitch dof is influenced.*
    - *Challenge the operability of the platform.*
    - *Lead to tether pull-out and stability issues.*

    Non-impact waves:
    - *Cause springing response.*
    - *Heave dof is influenced.*
    - *Lead to fatigue failure of tethers.*
12. What are the consequences of transverse loading on the structural members?
    - *Members will have premature failure due to twisting before bending.*
    - *Plasticization of twisted members.*
13. What is the advantage of using buoyancy modules in the risers?
    - *They are helpful in reducing the top tension force, which is required in the installation of the risers.*
    - *They make the risers neutrally buoyant.*
14. What are the ways through which VIV suppression can be achieved in the structure?
    - *Surface protrusion—helical strakes.*
    - *Helically wounded wire.*
    - *Projected plates.*
    - *Shrouds.*
    - *Proper design.*
    - *Smooth surface.*
15. Differentiate fire and explosion.
    Fire:
    - *It is a rapid exothermal oxidation of the ignition fuel.*

    Explosion:
    - *Rapid expansion of gases resulting from the pressure waves or shock waves.*
    - *Explosion releases energy very rapidly.*

16. Arrange the following in ascending order based on their temperature.
    Oxyhydrogen fire, simple candle, air acetylene fire, blow torch under welding operation.
    2-4-3-1
17. How the consequences due to the explosion damage can be measured?
    - *One of the common methods used to measure the consequences of explosion damage is TNT equivalence method.*
    - *TNT is an important explosive, which rapidly changes its form from solid to hot expanding gas.*
18. List the general design requirements of fire protection in the offshore platforms.
    - *They should be compact in size and weight.*
    - *They should be easy to operate.*
    - *They should be rapid activation systems.*
    - *They should be accessible.*
19. Define peak-reflected pressure and peak dynamic pressure.
    Peak-reflected pressure:
    - *When the blast wave hits the surface of the bluff body, it reflects back.*
    - *The effect of this reflection depends upon the surface characteristics of the body.*
    - *Surface will experience more pressure than the incident side-on pressure.*
    - $P_r = C_r P_{so}$

    where
    C_r is the reflection coefficient.
    Peak dynamic pressure:
    - *Blast wave moves due to the air movement.*
    - *Wind pressure depends upon the magnitude of peak over pressure of the blast wave.*
20. Name the different types of fire.
    - *Pool fire.*
    - *Jet fire.*
    - *Fire ball.*
    - *Flash fire.*

Section C:

1. Match the following:

1	Fixed structures	A	Exploratory drilling
2	Articulated towers	B	Deck-isolated platforms
3	Tethers	C	Turret mooring
4	Guyed towers	D	Tethered Spar
5	Compliant structures	E	Attract more forces
6	FPSO	F	DPS
7	Jack-up rig	G	Skirt piles
8	Drill ships	H	Position restraint
9	Gravity-based structure	I	Recentering is gentle
10	Jacket platforms	J	Spud-can
11	BLS	K	Seabed scouring
12	Triceratops	L	Better under cyclic loads

1-e, 2-i, 3-h, 4-j, 5-b, 6-c, 7-a, 8-f, 9-k, 10-g, 11-d, 12-b

2. Explain the structural action of TLP.

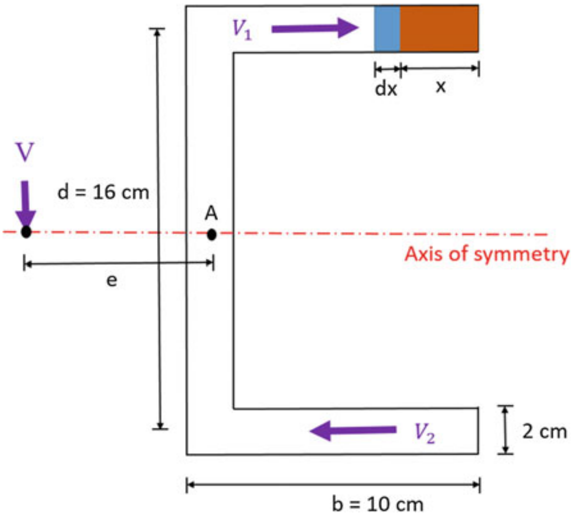
- *The structure is vertically restrained while it is compliant in the horizontal direction, permitting surge, sway, and yaw motion. The structural action results in low vertical force in rough seas, which is the key design factor.*

$W \lll F_B$
$W + T_s = F_B$
where

W *weight of the platform.*
T_o *initial axial pretension which was set initially in the structure to hold the platform down (20% of total weight).*
F_B *buoyancy force*

- *Due to lateral forces, the platform moves along the wave direction. Horizontal movement is called* ***offset****.*
- *Due to horizontal movement, the platform also has the tendency to have increased immerse volume of members. Thus, the platform will undergo* ***set-down*** *effect.*
- *The lateral movement increases the tension in the tethers. The horizontal component of tensile force counteracts the wave action, and the vertical component increases the weight which will balance the additional weight imposed by set down.*

3. Locate the shear center for the channel section as shown in the figure below:



Consider a section of thickness 'dx' on the flange at a distance 'x' from the end.

$$V_1 = \int \tau da$$

$V_1 = \int \frac{VA\bar{y}}{It} da$
where $A = t\,x$
$da = dx\,t$
$\bar{y} = d/2$

$$V_1 = \frac{V}{It}\int_0^b (tx)(dxt)\frac{d}{2} = \left[\frac{V}{It}\frac{t^2 d}{2}\frac{x^2}{2}\right]_0^b$$

$$= \frac{Vtb^2 d}{4I}$$

By symmetry, $V_1 = V_2 = \frac{Vtb^2 d}{4I}$
Neglecting the shear taken by the web and taking moment about the point A on the web,
$Ve = V_1 \frac{d}{2} + V_2 \frac{d}{2}$
$Ve = \frac{Vtb^2 d}{4I} \times d$
$e = \frac{tb^2 d^2}{2I}$
Moment of inertia, $I = 3.031 \times 10^7$ mm^4
$e = 8.44$ cm.

4. Write a short note on different types of risers.
Low-pressure risers:

- *Large diameter risers that are open to the atmospheric pressure at the top end.*
- *They are useful for drilling.*

- *They also have a lot of peripheral lines.*
- *Kill and choke lines are useful to circulate the fluid when kick occurs.*
- *They are also useful in communicating with the well about the closure of BOP.*
- *Booster lines are useful to inject fluid at the lower end and to accelerate the flow.*
- *When the risers are installed at greater water depth, they need top tensioning.*

High-pressure risers:

- *These risers are installed when the blow-out preventer is located closer to the surface.*
- *These risers do not require any additional peripheral lines which are essentially useful for communicating with the BOP.*
- *These risers are designed to operate at full pressure.*

Flexible risers:

- *They are useful as production risers.*
- *Flow lines.*

Top Tensioned Risers:

- *These are required to ensure stability.*
- *They connect the seabed through a stress joint.*
- *In addition, these risers will also have keel joint, which is located at keel level of the platform.*
- *As these risers are highly flexible in terms of its cross section, they undergo large deformation.*

5. Describe the steps involved in the identification of explosion damage.
   One of the common methods to estimate the consequences of explosion damage is TNT equivalence method. The steps involved are as follows:

   - *Calculation of total mass of the fluid involved.*
   - *Calculation of explosion energy.*
   - *Computation of TNT equivalence.*

   $$m_{\mathrm{TNT}} = \eta \frac{m\Delta H_c}{E}$$

   - *Calculation of scaled distance.*

   $$Z_e = \frac{r}{\sqrt[3]{m_{\mathrm{TNT}}}}$$

   - *Calculation of peak overpressure.*

6. Mention the basic problems in the offshore platforms that make fire protection very difficult.

- *Offshore platforms generally have congested layout of topside module.*
- *They operate in a very confined and compact space.*
- *Offshore production equipment is not laid horizontally but laid vertically.*
- *They impose extensive weight and space limitation for fire protection equipment to be provided on board.*
- *Usage of seawater for fire protection may lead to corrosion.*
- *Pure water to be used for fire protection system in offshore platforms.*

7. Compute the front wall loading due to blast waves traveling horizontally on the building module of offshore platform. Length = 15 m, breadth = 30 m, height = 5 m, and the blast wave approaches the building in the direction parallel to the length of the building. The peak side-on overpressure is 50 kPa for the duration of 0.06 s.

   (i) *Shock wave parameters*:

      a. *shock wave velocity,* $u = 345\ (1 + 0.0083P_{so})^{0.5}$ = 410.39 m/s.
      b. *length of the pressure wave* $= u \cdot t_d$ = 24.623 m.
      c. *peak dynamic wind pressure,* $q_s = 0.0032(P_{so})^2$ = 8 kPa.

   (ii) *Front wall loading*:
   Reflected overpressure, $C_r = 2 + 0.0073P_{so}$ = 2.365
   $P_r = C_r P_{so}$ = 118.25 kPa.
   Clearing distance is the least of B_H = 5 m *and* $B_W/2$ = 5 m.
   S = 5 m
   Reflected overpressure clearing time, $t_c = 3S/u$ = 0.0366.
   Drag coefficient = 1.0
   Stagnation pressure, $P_s = P_{so} + C_d q_s$ = 58 kPa.
   Front wall impulse, $I_w = 0.5(P_r - P_s)t_c + 0.5P_s t_d$ = 2.84 kPa s
   Effective duration, $t_e = \frac{2I_w}{P_r}$ = 0.048 s.

8. Write a case study on the piper alpha accident and deepwater horizon accident.
   Piper alpha accident:

   - *July 6, 1998, in the North Sea.*
   - *Fire and explosion occurred.*
   - *The consequences are highly severe. 167 people died and financial loss of about 3.4 billion US dollars in 1998.*
   - *Accident occurred due to human error during operation/maintenance and faulty design of the platform.*

   Deepwater horizon:

   - *It is a semisubmersible MODU.*
   - *Accident occurred on April 20, 2010, in Gulf of Mexico.*
   - *It caused severe human and environmental impact.*
   - *Fire and explosion occurred which leads to capsizing of the vessel and oil spill.*
   - *Reason for the explosion is faulty design.*