Erratum: On the Functor ℓ^2

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The main purpose of this erratum is to correct a claim made in "On the functor ℓ^2" (Computation, Logic, Games, and Quantum Foundations, Lecture Notes in Computer Science Volume 7860, 2013, pp 107–121) in Lemma 5.9. Namely, positive operators on Hilbert space are not necessarily isomorphisms, but merely *bimorphisms*, *i.e.* both monic and epic; this is precisely the issue in 2.8. Here is the corrected version.

Lemma 5.9. *Positive operators on Hilbert spaces are bimorphisms.*

Proof. Let $p\colon H \to H$ be a positive operator in **Hilb**. If $p(x) = 0$ then certainly $\langle p(x) \,|\, x\rangle = 0$ which contradicts positivity. Hence $\ker(p) = 0$, and so p is monic.

To see that p is epic, suppose that $p \circ f = p \circ g$ for parallel morphisms f, g. Then $\langle p \circ (f - g)(x) \,|\, x\rangle = 0$ for all x. By positivity, For each x there is $p_x > 0$ such that $p \circ (f - g)x = p_x \cdot (f - g)(x)$. Hence $\langle (f - g)(x) \,|\, x\rangle = 0$ for all x, that is, $f = g$ and p is epic. □

Definition 5.10 then needs to be adapted accordingly: a functor $F\colon \mathbf{C} \to \mathbf{D}$ is essentially full when for each morphism g in $\mathbf{D}$ there exist f in $\mathbf{C}$ and bimorphisms u, v in $\mathbf{C}$ such that $g = v \circ Ff \circ u$

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