# Smooth Signed Distance Surface Reconstruction and Applications

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**Abstract.** We describe a new and simple variational formulation to reconstruct the surface geometry, topology, and color map of a 3D scene from a finite set of colored oriented points. Point clouds are nowadays obtained using a variety of techniques, including structured lighting systems, pasive multi-view stereo algorithms, and 3D laser scanning. In our formulation the implicit function is forced to be a smooth approximation of the signed distance function to the surface. The formulation allows for a number of different efficient discretizations, reduces to a finite dimensional least squares problem for all linearly parameterized families of functions, does not require the specification of boundary conditions, and it is particularly good at extrapolating missing and/or irregularly sampled data. The resulting algorithms are significantly simpler and easier to implement than alternative methods. In particular, our implementation based on a primal-graph octree-based hybrid finite element-finite difference discretization, and the Dual Marching Cubes isosurface extraction algorithm is very efficient, and produces high quality crack-free adaptive manifold polygon meshes. After the geometry and topology are reconstructed, the color information from the points is smoothly extrapolated to the surface by solving a second variational problem which also reduces to a finite dimensional least squares problem. The resulting method produces high quality polygon meshes with smooth color maps, which accurately approximate the source colored oriented points. An open source implementation of this method is available for download. We describe applications to digital archaeology, 3D forensics, and 3D broadcasting.

**Keywords:** surface reconstruction, multi-view stereo, geometry processing, digital archaeology, digital forensics.

## 1 Introduction

A new variational formulation was recently introduced [7,8] for the problem of reconstructing a watertight surface defined by an implicit equation $f(p) = 0$ from a finite set of oriented points $\{(p_1 n_1), \ldots, (p_N, n_N)\}$. Oriented point clouds are obtained from laser scanners, structured lighting systems, and multi-view

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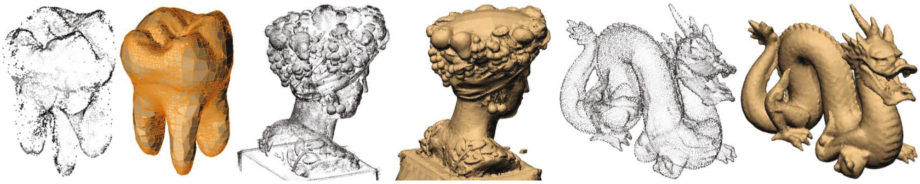

**Fig. 1.** We have developed a new method of reconstructing high resolution watertight surfaces from oriented points which is particularly good at extrapolating under-sampled areas, missing and irregularly sampled data [7]

stereo algorithms. This problem has received an immense amount of attention since the mid 80s. An extensive review of the literature and the state-of-the-art is provided in [7,8]. We have shown that this approach provides an interesting alternative to other popular methods, it is much simpler to implement, and performs particularly well on unevenly sampled data sets at comparable cost. The implicit surface $S = \{p : f(p) = 0\}$ is estimated by minimizing the energy function

$$E_1(f) = \frac{\alpha_0}{N}\sum_{i=0}^{N} f(p_i)^2 + \frac{\alpha_1}{N}\sum_{i=0}^{N} \|\nabla f(p_i) - n_i\|^2 + \frac{\alpha_2}{|V|}\int_V \|Hf(p)\|^2 dp \qquad (1)$$

where $\nabla f(p)$ is the gradient of the implicit function, $V$ is a bounding volume, $|V|$ is the measure of this volume, $\|Hf(p)\|^2$ is the Frobenius norm of the Hessian of the implicit function (sum of squares of second order partial derivatives), and $\alpha_0$, $\alpha_1$ and $\alpha_2$ are positive weights. In this formulation the first two terms of the energy function –the data terms– force the implicit function to approximate the signed distance function to the underlying surface, and the third term –the regularization term– forces the gradient of the function to be as close as possible to constant away from the data points. In our view this is a more natural regularization condition to impose, and it is responsible for the good behavior of the associated algorithms.

Various discretizations are possible with the problem reducing to the solution of a sparse linear system of equations. The resulting algorithms are simple and easy to implement, and produce results of quality comparable with state-of-the-art algorithms, if not better, particularly when the point cloud constitutes an uneven sampling of the subjacent surface. We have shown an efficient implementation based on a primal-graph octree-based hybrid finite element-finite difference discretization, and the Dual Marching Cubes isosurface extraction algorithm, which produces high quality crack-free adaptive manifold polygon meshes, as shown in Figure 5. A reference implementation of this algorithm, distributed as Open Source, can be downloaded from [8]. Leveraging this software, and targeting archaeology and forensics applications, we have build a processing pipeline for multi-view stereo surface reconstruction. This software, which is also distributed in Open Source, can be downloaded from [12]. The pipeline consists of three previously proposed methods that together reconstruct a complete 3D model from a collection of

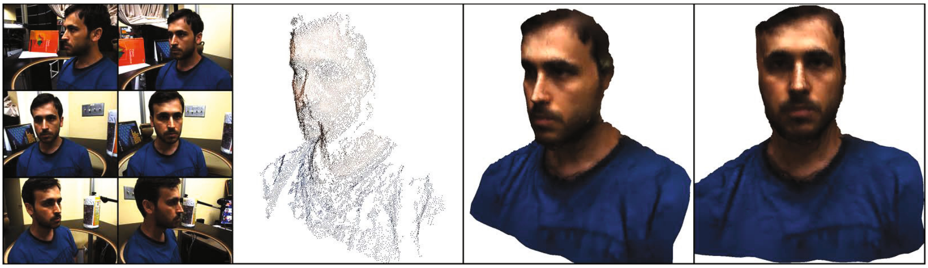

**Fig. 2.** Surface reconstructed from a colored oriented point cloud generated by the a multi-view stereo algorithm. This is an application to real-time view interpolation for face-to-face teleconferencing.

images taken from different camera viewpoints. The first step is to recover a set of camera parameters and 3D locations for keypoints in each image using Bundler [2], a method proposed to perform structure from motion (SfM) on unordered image collections. The second step is to generate a dense point cloud using PMVS [5], a patch-based multiview stereo method. The last step is to reconstruct the surface using our Smooth Signed Distance surface reconstruction method [7,8]. A precursor to this formulation, the VFIso algorithm introduced in [1], was used in our prior work on shape from depth discontinuities [3,4]. However, the VFIso implementation, based on finite differences on a regular voxel grid discretization, does not scale up gracefully.

## 2 Multi-view Stereo Surface Reconstruction

The problem of reconstructing colored 3D models from multiple images captured by inexpensive consumer level digital cameras has a wide range of applications in industry, entertainment, human-computer interaction, surveillance, navigation, archaeology, forensics, medicine, sports, architecture, and many other fields. Popular Multi-View Stereo algorithms such as [5,6] produce a dense colored oriented point cloud $\{(p_1, n_1, c_1), \ldots, (p_N, n_N, c_N)\}$ from a collection of images.

Originally targeting the needs of the REVEAL Digital Archaeology project [11,12], we have recently extended the SSD Surface Reconstruction method described above to produce colored surfaces. In our implementation the colored surface is represented as a polygon mesh with colors per vertex, which are then continuously interpolated within the faces by the rendering engine. The implicit surface color is represented as a color map function $c(p) = (r(p), g(p), b(p))$ defined on the bounding volume $V$. The color field is estimated independently of the geometry, by minimizing the following energy function

$$E_2(c) = \frac{\beta_0}{N} \sum_{i=0}^{N} \|c(p_i) - c_i\|^2 + \frac{\beta_2}{|V|} \int_V \|Dc(p)\|^2 dp \qquad (2)$$

where $Dc(p)$ is the Jacobian of the color map function, and $\beta_0$ and $\beta_1$ are positive constants.

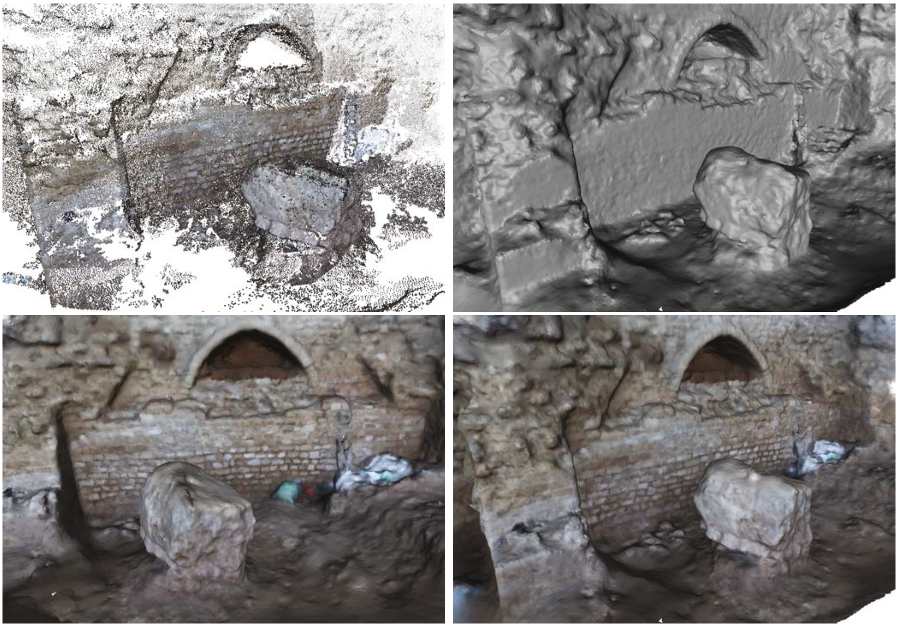

**Fig. 3.** Reconstruction of the side of a castle model: The input point cloud (top-left), Surface reconstructed by the proposed algorithm(top-right), Two views from the surface and color map reconstructed by the proposed algorithm (bottom)

In our octree-based implementation, after the geometry is reconstructed, a piecewise constant discretization with one RGB color value per octree cell is used to reduce the estimation of surface colors to a least square problem on the octree dual graph. This problem reduces to the solution of another (Laplacian) sparse linear system. Even a simple cascading multi-grid gradient descent algorithm (such as the Jacobi method) converges very fast to the solution. Figures 2, 3, 6, and 7 show applications to view interpolation for face-to-face teleconferencing, forensic 3D reconstruction of footprints where we have shown that this method is competitive with laser scanners [6], reconstruction of architecture from outdoors photography, and interesting problems in Archaeology.

Very often objects have relatively simple geometry but very detailed textures. In these cases it is convenient for a polygonal approximation scheme to represent the surface color data not as a color vector per vertex, face, or corner, but by using a coarsely tessellated polygon mesh and texture mapping. A tradeoff needs to be reached to determine where in between the two extremes the produced surface will be. Surfaces reconstructed from colored point clouds produced by multi-view stereo algorithms such as [5,6] usually result in polygon meshes of much lower resolution than can be derived from the input images, because the geometry is usually reconstructed from decimated images due to memory constraints. Using the process described above to estimate mesh color also results in degraded resolution, since the polygon mesh resolution is the limiting factor. One approach is to detect in the original images regions which may require high

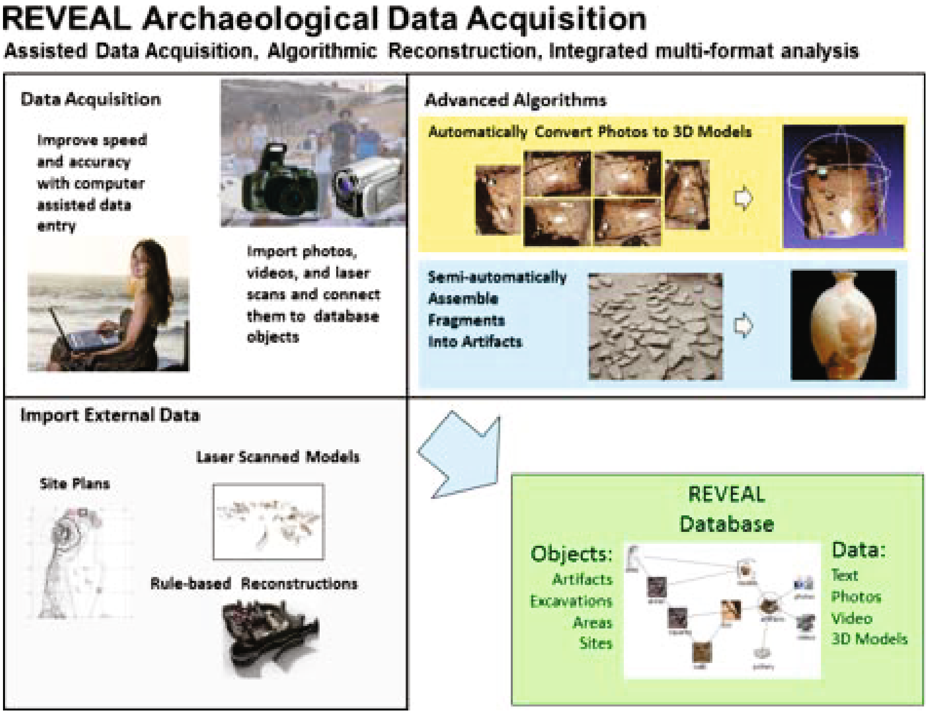


**Fig. 4.** The REVEAL Digital Archaeology System components

resolution and then modify the reconstruction process to refine the geometry of those regions alone, as will be discussed in a later section. Another possibility is to go back to the original high resolution images, and using the coarse reconstructed geometry and visibility information from the camera poses and surface geometry, create a texture atlas composed of non-overlapping charts cut from the original source images. If the main purpose of reconstructing the surface is visualization, then this approach is most efficient when the models to be created have low geometric complexity, but are highly textured. We are currently working on an extension of our surface reconstruction software which will create textured polygon mesh models.

## 3 Application to Digital Archaeology

REVEAL (Reconstruction and Exploratory Visualization: Engineering meets ArchaeoLogy) [11,12] is a four year NSF-funded project promoting paradigm shifts in archaeology. This is a project to create an environment for acquiring and presenting archaeological data in a way that streamlines the excavation process and supports and enhances the experts understanding of the data. REVEAL leverages three aspects of the technology: using vision algorithms to speed up or replace measurement and documentation tasks, using computer automation to speed up

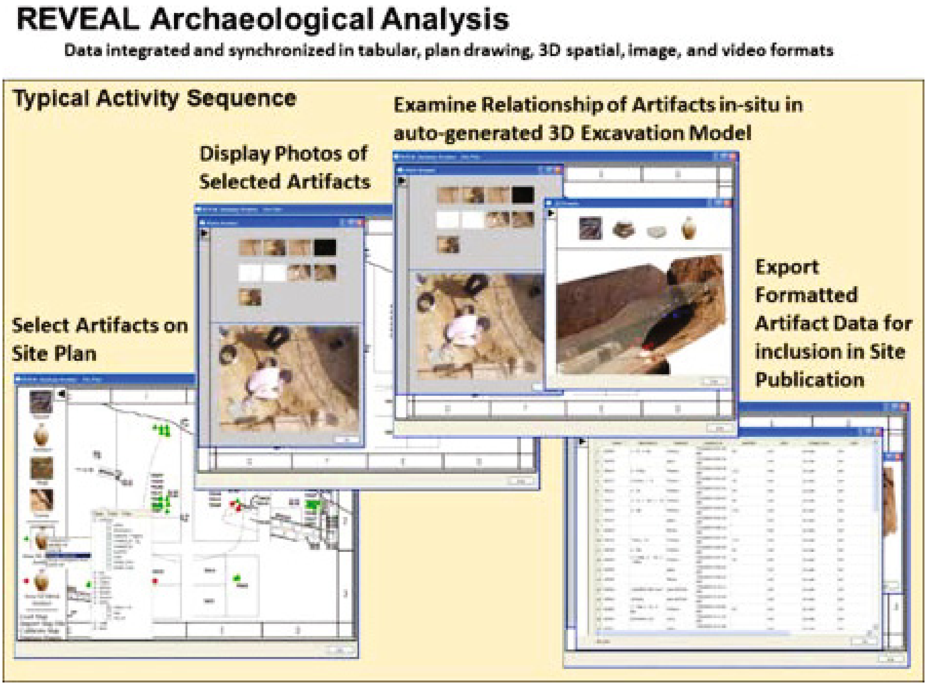


**Fig. 5.** The REVEAL Analyze Software System

data entry tasks, using integrated 2D and 3D media to enhance data comprehension. Figure 6 illustrates a particular digital archaeology application of surface reconstruction from multi-view stereo data which requires surface reconstruction at high resolution, but only at selective places. In this case the ultimate goal is to be able to perform character recognition from the reconstructed geometry. Archeology experts are able to differentiate cuneiform writing styles from different authors. Sufficient information to perform these recognition tasks may not be present in low resolution 3D reconstructions. However, in practice reconstructing the geometry at the required resolution with the algorithms described above may result in polygon meshes too large to be processed. As a result we are currently developing adaptive methods, which based on processing the geometry of low resolution reconstructions as well as the source images, regions in the 3D bounding volume which require reconstruction at high resolution will be determined. Then we will modify our algorithms to refine the octree only where it is needed. We envision a user-driven adaptive recognition system where a coarse reconstruction is first produced by the system and shown to the user along with the source images. Using the camera pose information, we will establish correspondences between the images and the reconstructed geometry within the interactive system. Then the user will be able to select the region of interest by painting or drawing on the images and/or coarse geometry and the system will refine the selected areas at interactive rate. Also, because of the tree structure, reconstructed details which are no longer

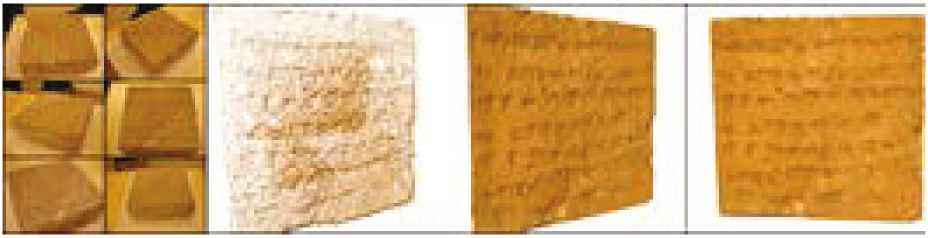

**Fig. 6.** Surface reconstructed from colored oriented point cloud generated by the PMVS multi-view stereo algorithm [5,6]. In this example we are exploring an application of the proposed techniques to Archaeology: the goal is to reconstruct the 3D geometry of a cuneiform tablet at sufficient resolution to be able perform 3D OCR.

of interest can be removed by cutting deep branches of the octree, and the surface re-estimated in real time.

## 4 Application to Forensics

Footwear impressions recovered from crime scenes are important to corroborate or refute hypotheses, or to narrow down the number of suspects. The long-standing standard used to obtain 3D models of 3D footwear impressions is casting. This method is slowly being replaced non-invasive techniques such as 3D laser scanning. In [10] we present an alternative method based on multi-view stereo data, which yields 3D models as accurate as those produced by 3D laser scanners, but at a much lower cost. We evaluated the results comparing our reconstructed 3D models with the ones acquired by 3D scanning, and we also examine the advantages and drawbacks of each method. Our solution relies on the pipeline developed for the REVEAL system mentioned above to reconstruct 3D surfaces using only digital photographs taken from the footwear print at the crime scene. In this work, we presented a pipeline to recover footwear impression from crime scenes based on a well known technique in Computer Vision, multi-view stereo, which has not been consider or analyzed for this kind of application

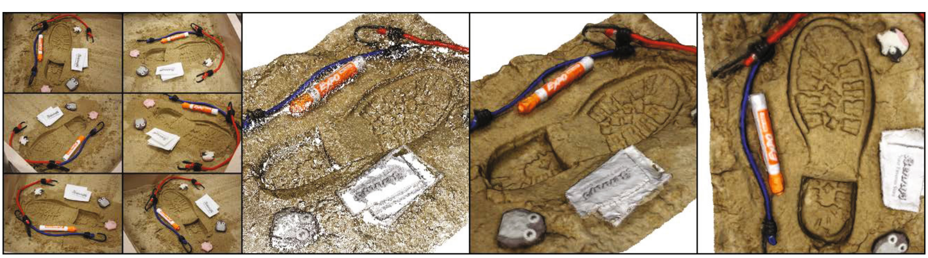

**Fig. 7.** Surface reconstructed from colored oriented point cloud generated by the PMVS algorithm [5,6]. This is an application to the forensic reconstruction of 3D shoeprints.

in the literature until now. Despite the simplicity for set up and acquisition, the reconstructed surfaces proved to be comparable with those produced by a 3D scanning system, a high-end technology used in practice, providing accurate 3D models of the shoe prints. A digital camera is the only equipment required to recover the evidence, which makes the process convenient and fast.

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