

A Generalised-Mutual-Information-Based Oracle for Hierarchical Radiosity

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Abstract. One of the main problems in the radiosity method is how to discretise a scene into mesh elements that allow us to accurately represent illumination. In this paper we present a new refinement criterion for hierarchical radiosity based on the continuous and discrete generalised mutual information measures between two patches or elements. These measures, derived from the generalised entropy of Harvda-Charvát-Tsallis, express the information transfer within a scene. The results obtained improve on the ones based on kernel smoothness and Shannon mutual information.

1 Introduction

The radiosity method solves the problem of illumination in an environment with diffuse surfaces by using a finite element approach [1]. The scene discretisation has to represent the illumination accurately by trying to avoid unnecessary subdivisions that would increase the computation time. A good meshing strategy will balance the requirements of accuracy and computational cost.

In the hierarchical radiosity algorithms [2] the mesh is generated adaptively: when the constant radiosity assumption on a patch is not valid for the radiosity received from another patch, the refinement algorithm will subdivide it in a set of subpatches or elements. A refinement criterion, called *oracle*, informs us if a subdivision of the surfaces is needed, bearing in mind that the cost of the oracle should remain acceptable. In [3,4], the difficulty in obtaining a precise solution for the scene radiosity has been related to the degree of dependence between all the elements of the adaptive mesh. This dependence has been quantified by the mutual information, which is a measure of the information transfer in a scene.

In this paper, a new oracle based on the *generalised mutual information* [5], derived from the generalised entropy of Harvda-Charvát-Tsallis [6], is introduced. This oracle is obtained from the difference between the continuous and discrete generalised mutual information between two elements of the adaptive mesh and expresses the loss of information transfer between two patches due to the discretisation. The results obtained show that this oracle improves on the kernel smoothness-based [7] and the mutual information-based [8,9] ones, confirming the usefulness of the information-theoretic approach in dealing with the radiosity problem.

2 Preliminaries

2.1 Radiosity

The radiosity method uses a finite element approach, discretising the diffuse environment into patches and considering the radiosities, emissivities and reflectances constant over them. With these assumptions, the discrete radiosity equation [1] is given by

$$B_i = E_i + \rho_i \sum_{j \in S} F_{ij} B_j, \quad (1)$$

where S is the set of patches of the scene, B_i , E_i , and ρ_i , are respectively the radiosity, emissivity, and reflectance of patch i , B_j is the radiosity of patch j , and F_{ij} is the patch-to-patch form factor, defined by

$$F_{ij} = \frac{1}{A_i} \int_{S_i} \int_{S_j} F(x, y) dA_y dA_x, \quad (2)$$

where A_i is the area of patch i , S_i and S_j are, respectively, the surfaces of patches i and j , $F(x, y)$ is the point-to-point form factor between $x \in S_i$ and $y \in S_j$, and dA_x and dA_y are, respectively, the differential areas at points x and y . Using Monte Carlo computation with area-to-area sampling, F_{ij} can be calculated:

$$F_{ij} \approx A_j \frac{1}{|S_{i \times j}|} \sum_{(x, y) \in S_{i \times j}} F(x, y), \quad (3)$$

where the computation accuracy depends on the number of random segments between i and j ($|S_{i \times j}|$).

To solve the system (1), a hierarchical refinement algorithm is used. The efficiency of this algorithm depends on the election of a good refinement criterion. Many refinement oracles have been proposed in the literature (see [10] for details). For comparison purposes, we review here the oracle based on kernel smoothness (KS), proposed by Gortler et al. [7] in order to drive hierarchical refinement with higher-order approximations. When applied to constant approximations, this refinement criterion is given by

$$\rho_i \max\{F_{ij}^{\max} - F_{ij}^{\text{avg}}, F_{ij}^{\text{avg}} - F_{ij}^{\min}\} A_j B_j < \epsilon, \quad (4)$$

where $F_{ij}^{\max} = \max\{F(x, y) \mid x \in S_i, y \in S_j\}$ and $F_{ij}^{\min} = \min\{F(x, y) \mid x \in S_i, y \in S_j\}$ are, respectively, the maximum and minimum radiosity kernel values estimated by taking the maximum and minimum value computed between pairs of random points on both elements, and $F_{ij}^{\text{avg}} = F_{ij}/A_j$ is the average radiosity kernel value.

2.2 HCT Entropy and Generalised Mutual Information

In 1967, Harvda and Charvát [6] introduced a new generalised definition of entropy. In 1988, Tsallis [11] used this entropy in order to generalise the Boltzmann-Gibbs entropy in statistical mechanics.

Definition 1. *The Harvda-Charvát-Tsallis entropy (HCT entropy) of a discrete random variable X , with $|X| = n$ and p_X as its probability distribution, is defined by*

$$H_\alpha(X) = k \frac{1 - \sum_{i=1}^n p_i^\alpha}{\alpha - 1}, \tag{5}$$

where k is a positive constant (by default $k = 1$) and $\alpha \in \mathbb{R} \setminus \{1\}$ is called entropic index.

This entropy recovers the Shannon discrete entropy when $\alpha \rightarrow 1$, $H_1(X) \equiv -\sum_{i=1}^n p_i \ln p_i$, and fulfils good properties such as non-negativity and concavity.

On the other hand, Taneja [5] and Tsallis [12] introduced the generalised mutual information.

Definition 2. *The generalised mutual information between two discrete random variables (X, Y) is defined by*

$$I_\alpha(X, Y) = \frac{1}{1 - \alpha} \left(1 - \sum_{i=1}^n \sum_{j=1}^m \frac{p_{ij}^\alpha}{p_i^{\alpha-1} q_j^{\alpha-1}} \right), \tag{6}$$

where $|X| = n$, $|Y| = m$, p_X and q_Y are the marginal probability distributions, and p_{XY} is the joint probability distribution between X and Y .

The transition of $I_\alpha(X, Y)$ to the continuous generalised mutual information is straightforward. Using entropies, an alternative form is given by

$$I_\alpha(X, Y) = H_\alpha(X) + H_\alpha(Y) - (1 - \alpha)H_\alpha(X)H_\alpha(Y) - H_\alpha(X, Y). \tag{7}$$

Shannon mutual information (MI) is obtained when $\alpha \rightarrow 1$. Some alternative ways for the generalised mutual information can be seen in [13].

3 Generalised Mutual Information-Based Oracle

We will see below how the generalised mutual information can be used to build a refinement oracle within a hierarchical radiosity algorithm. Our strategy will be based on the estimate of the discretisation error from the difference between the continuous and discrete generalised mutual information (6) between two elements of the adaptive mesh. The discretisation error based on Shannon mutual information was introduced by Feixas et al. [8] and applied to hierarchical radiosity with good results.

In the context of a discrete scene information channel [4], the marginal probabilities are given by $p_X = q_Y = \{a_i\}$ (i.e., the distribution of the relative area of patches: $\frac{A_i}{A_T}$, where A_T is the total area of scene) and the joint probability is given by $p_{XY} = \{a_i F_{ij}\}$. Then,

Definition 3. *The discrete generalised mutual information of a scene is given by*

$$I_\alpha = \frac{1}{1 - \alpha} \left(1 - \sum_{i \in S} \sum_{j \in S} \frac{a_i^\alpha F_{ij}^\alpha}{a_i^{\alpha-1} a_j^{\alpha-1}} \right) = \sum_{i \in S} \sum_{j \in S} \tau_\alpha(a_i F_{ij}, a_i a_j), \tag{8}$$

where, using $1 = \sum_{i \in S} \sum_{j \in S} a_i a_j$ and $\tau_\alpha(p, q) = \frac{1}{1-\alpha} \frac{q^\alpha - p^\alpha}{q^{\alpha-1}}$, the last equality is obtained.

This measure quantifies the discrete information transfer in a discretised scene. The term $\tau_\alpha(a_i F_{ij}, a_i a_j)$ can be considered as an element of the generalised mutual information matrix I_α , representing the information transfer between patches i and j .

To compute I_α , the Monte Carlo area-to-area sampling (3) is used, obtaining for each pair of elements

$$\begin{aligned}
 I_{\alpha_{ij}} &= \tau_\alpha(a_i F_{ij}, a_i a_j) = \frac{1}{1-\alpha} \frac{a_i^\alpha a_j^\alpha - a_i^\alpha F_{ij}^\alpha}{a_i^{\alpha-1} a_j^{\alpha-1}} \\
 &\approx \frac{1}{1-\alpha} \frac{A_i}{A_T} \frac{A_j}{A_T} \left(1 - A_T^\alpha \left(\frac{1}{|\mathcal{S}_{i \times j}|} \sum_{(x,y) \in \mathcal{S}_{i \times j}} F(x,y) \right)^\alpha \right). \tag{9}
 \end{aligned}$$

The information transfer between two patches can be obtained more accurately using the continuous generalised mutual information between them. From the discrete form (8) and using the pdfs $p(x) = q(y) = \frac{1}{A_T}$ and $p(x,y) = \frac{1}{A_T} F(x,y)$, we define

Definition 4. *The continuous generalised mutual information of a scene is given by*

$$I_\alpha^c = \int_S \int_S \tau_\alpha \left(\frac{1}{A_T} F(x,y), \frac{1}{A_T^2} \right) dA_y dA_x. \tag{10}$$

This represents the continuous information transfer in a scene. We can split the integration domain and for two surface elements i and j we have

$$I_{\alpha_{ij}}^c = \int_{S_i} \int_{S_j} \tau_\alpha \left(\frac{1}{A_T} F(x,y), \frac{1}{A_T^2} \right) dA_y dA_x \tag{11}$$

that, analogously to the discrete case, expresses the information transfer between two patches.

Both continuous expressions, (10) and (11), can be solved by Monte Carlo integration. Taking again area-to-area sampling (i.e., pdf $\frac{1}{A_i A_j}$), the last expression (11) can be approximated by

$$\begin{aligned}
 I_{\alpha_{ij}}^c &\approx A_i A_j \frac{1}{|\mathcal{S}_{i \times j}|} \sum_{(x,y) \in \mathcal{S}_{i \times j}} \tau_\alpha \left(\frac{1}{A_T} F(x,y), \frac{1}{A_T^2} \right) \\
 &= \frac{1}{1-\alpha} \frac{A_i}{A_T} \frac{A_j}{A_T} \left(1 - A_T^\alpha \left(\frac{1}{|\mathcal{S}_{i \times j}|} \sum_{(x,y) \in \mathcal{S}_{i \times j}} F(x,y)^\alpha \right) \right). \tag{12}
 \end{aligned}$$

Now, we define

Definition 5. *The generalised discretisation error of a scene is given by*

$$\Delta_\alpha = I_\alpha^c - I_\alpha = \sum_{i \in S} \sum_{j \in S} \Delta_{\alpha_{ij}}, \quad (13)$$

where $\Delta_{\alpha_{ij}} = I_{\alpha_{ij}}^c - I_{\alpha_{ij}}$.

While Δ_α expresses the *loss of information transfer* in a scene due to the discretisation, the term $\Delta_{\alpha_{ij}}$ gives us this loss between two elements i and j . This difference is interpreted as the benefit to be gained by refining and can be used as the base of the new oracle.

From (13), using (9) and (12), we obtain

$$\Delta_{\alpha_{ij}} \approx A_i A_j A_T^{\alpha-2} \frac{1}{1-\alpha} \delta_{\alpha_{ij}}, \quad (14)$$

where

$$\delta_{\alpha_{ij}} = \left(\frac{1}{|\mathcal{S}_{i \times j}|} \sum_{(x,y) \in \mathcal{S}_{i \times j}} F(x,y) \right)^\alpha - \frac{1}{|\mathcal{S}_{i \times j}|} \sum_{(x,y) \in \mathcal{S}_{i \times j}} F(x,y)^\alpha. \quad (15)$$

Accordingly to the radiosity equation (1) and in analogy to classic oracles, like KS, we consider the oracle structure $\rho_i \sigma B_j < \epsilon$, where σ is the geometric kernel [14]. Now, we propose to take the generalised discretisation error between two patches as the kernel ($\sigma = \Delta_{\alpha_{ij}}$) for the new oracle based on generalised mutual information (GMI_α). To simplify the expression of this oracle, we multiply the inequality by the scene constant $A_T^{2-\alpha}(1-\alpha)$.

Definition 6. *The hierarchical radiosity oracle based on the generalised mutual information is defined by*

$$\rho_i A_i A_j \delta_{\alpha_{ij}} B_j < \epsilon. \quad (16)$$

4 Results

In this section, the GMI_α oracle is compared with the KS and MI ones. Other comparisons, with a more extended analysis, can be found in [14]. All oracles have been implemented on top of the hierarchical Monte Carlo radiosity method.

In Fig. 1 we show the results obtained for the KS (a) and GMI_α oracles with their Gouraud shaded solutions and meshes. In the GMI_α case, we show the results obtained with the entropic indexes 1 (b) (i.e., note that $\text{GMI}_1 = \text{MI}$) and 0.5 (c). For the sake of comparison, adaptive meshes of identical size have been generated with the same cost for the power distribution: around 19,000 patches and 2,684,000 rays, respectively. To estimate the form factor, the number of random lines has been fixed to 10.

In Table 1, we show the Root Mean Square Error (RMSE) and Peak Signal Noise Ratio (PSNR) measures for KS and GMI_α (for 5 different entropic indexes) oracles for the test scene. These measures have been computed with respect to the corresponding converged image, obtained with a path-tracing algorithm

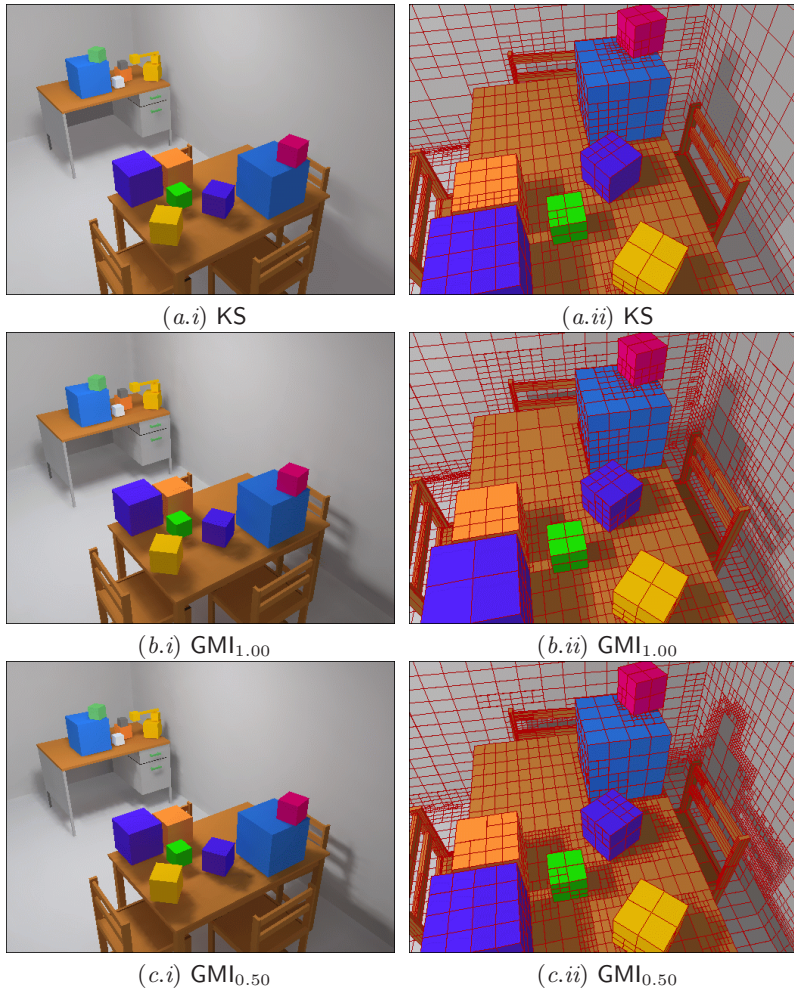


Fig. 1. (a) KS and GMI_α (entropic indexes (b) 1 and (c) 0.5) oracles. By columns, (i) Gouraud shaded solution of view₁ and (ii) mesh of view₂ are shown.

with 1,024 samples per pixel in a stratified way. For each measure, we consider a uniform weight for every colour channel ($RMSE_a$ and $PSNR_a$) and a perceptual one ($RMSE_p$ and $PSNR_p$) in accordance with the sRGB system.

Observe in the view₁, obtained with GMI_α (Fig. 1.i.b-c), the finer details of the shadow cast on the wall by the chair on the right-hand side and also the better-defined shadow on the chair on the left-hand side and the one cast by the desk. In view₂ (Fig. 1.ii) we can also see how our oracle outperforms the KS, especially in the much more defined shadow of the chair on the right. Note the superior quality mesh created by our oracle.

Table 1. The RMSE and PSNR measures of the KS and GMI_α oracles applied to the test scene of Fig. 1, where the KS and $GMI_{\alpha \in \{0.5, 1\}}$ results are shown. The oracles have been evaluated with 10 random lines between each two elements.

oracle	view ₁				view ₂			
	RMSE _a	RMSE _p	PSNR _a	PSNR _p	RMSE _a	RMSE _p	PSNR _a	PSNR _p
KS	13.791	13.128	25.339	25.767	15.167	14.354	24.513	24.991
$GMI_{1.50}$	11.889	11.280	26.628	27.084	13.046	12.473	25.821	26.211
$GMI_{1.25}$	10.872	10.173	27.405	27.982	11.903	11.279	26.618	27.086
$GMI_{1.00}$	9.998	9.232	28.133	28.825	10.438	9.709	27.758	28.387
$GMI_{0.75}$	9.555	8.786	28.526	29.254	10.010	9.257	28.122	28.801
$GMI_{0.50}$	9.370	8.568	28.696	29.473	9.548	8.740	28.533	29.300

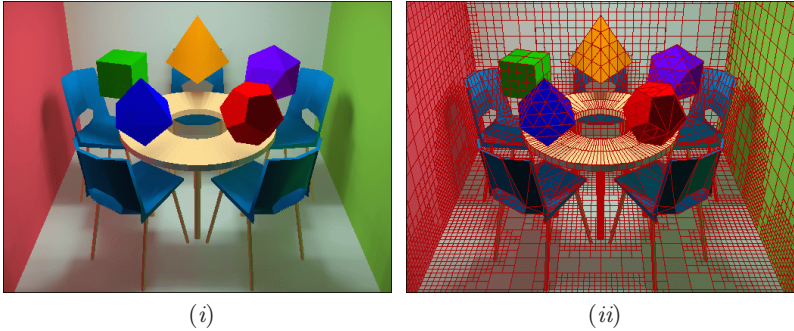


Fig. 2. $GMI_{0.50}$ oracle: (i) Gouraud shadow solution and (ii) mesh are shown

Table 2. The RMSE and PSNR measures of the GMI_α oracle applied to the scene of Fig. 2, where the $GMI_{0.5}$ result is shown. The oracle has been evaluated with 10 random lines between each two elements.

oracle	RMSE _a	RMSE _p	PSNR _a	PSNR _p
$GMI_{1.50}$	16.529	15.530	23.766	24.307
$GMI_{1.25}$	15.199	14.145	24.494	25.119
$GMI_{1.00}$	14.958	13.844	24.633	25.306
$GMI_{0.75}$	14.802	13.683	24.724	25.407
$GMI_{0.50}$	14.679	13.573	24.797	25.477

In general, the improvement obtained with the GMI_α oracle is significant. Moreover, its behaviour denotes a tendency to improve towards subextensive entropic indexes ($\alpha < 1$). To observe this tendency, another test scene is shown in Fig. 2 for an entropic index 0.5. Its corresponding RMSE and PSNR measures are presented in Table 2. The meshes are made up of 10,000 patches with 9,268,000 rays to distribute the power and we have kept 10 random lines to evaluate the oracle between elements.

5 Conclusions

We have presented a new generalised-mutual-information-based oracle for hierarchical radiosity, calculated from the difference between the continuous and discrete generalised mutual information between two elements of the adaptive mesh. This measure expresses the loss of information transfer between two patches due to the discretisation. The objective of the new oracle is to reduce the loss of information, obtaining an optimum mesh. The results achieved improve on the classic methods significantly, being better even than the version based on the Shannon mutual information. In all the tests performed, the best behaviour is obtained with subextensive indexes.

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