Summary

We call a commutative ring extension $A \subset R$ Prüfer, if A is an R-Prüfer ring in the sense of Griffin (Can. J. Math. 26 (1974)). These ring extensions relate to Manis valuations in much the same way as Prüfer domains relate to Krull valuations. In the Introduction we tried to explain why Prüfer extensions and Manis valuations deserve attention from a geometric viewpoint. In Chapter I we develop a basic theory of Prüfer extensions and give some examples. Then in Chapter II we explicate the multiplicative ideal theory related to Prüfer extensions. Finally, in Chapter III we take a closer look at Manis valuations. We single out the all-important subclass of PM-valuations (= Prüfer-Manis valuations) and study relations between PM-valuations and other Manis valuations.

Prüfer extensions may be viewed as families of PM-valuations. This viewpoint will dominate Part II of the book.

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