An "Optimal" k-Needle Placement Strategy Given an Approximate Initial Needle Position

Markus Kukuk[1,2]

[1] SIEMENS Corporate Research, Imaging & Visualization, Princeton, NJ, USA
[2] Stanford University, Department of Radiology, Stanford, CA, USA
kukuk@stanford.edu

Abstract. In this paper we address the problem of finding an "optimal" strategy for placing k biopsy needles, given a large number of possible initial needle positions. This problem arises for example in guided, endoscopic needle biopsies, where the position of the endoscope's tip is known with some error. We consider two variations of the problem: (1) Calculate the smallest set of needles[1], needed to guarantee a successful biopsy. (2) Given a number k, calculate k needles such that the probability of a successful biopsy is maximized. We formulate both problems in terms of two general, NP-hard optimization problems. Our solution to both problems is "optimal" with respect to the best approximative algorithm known for the respective NP-hard problem. For the latter problem there exists an approximative algorithm which requires virtually no implementation effort and is guaranteed to be within a factor of $1 - \frac{1}{e}$ of the exact solution. For both variations of the problem we are able to provide success probabilities for each needle to the physician. We have implemented the approximative algorithm for the second variation. The resulting probabilities show that our approach can provide valuable decision support for the physician in choosing how many needles to place and how to place them.

1 Introduction

A biopsy is a minimally invasive surgical procedure, often used in the diagnosis and staging of cancer patients. In general, the goal is to take a sample of the suspicious tissue (target) by placing a biopsy needle inside the target. Since the target is often not directly visible for the physician, numerous methods for guiding biopsies have been developed. Procedures that have attracted special attention in recent years include the biopsy of the prostate, breast, liver and lung.

In many cases it is common practise to take more than one tissue sample, in order to increase the probability of hitting the target. Instead of using a simple trial-and-error approach, biopsy strategies have been developed, among others for prostate cancer biopsies [1],[2]. The "k-Needle Placement Strategy" is

[1] We use "needle" short hand for the parameter vector that specifies the needle placement

R.E. Ellis and T.M. Peters (Eds.): MICCAI 2003, LNCS 2878, pp. 116–123, 2003.
© Springer-Verlag Berlin Heidelberg 2003

a biopsy protocol that specifies how to place k (biopsy) needles, such that the probability of success is maximized. The placement of a needle is specified by a suitable parameterization of its degrees of freedom, e.g. by two angles and an insertion depth.

In this paper we present an "optimal" k-needle placement strategy for a special class of biopsy problems, where the initial needle position is known approximately. A typical example for such a procedure is a "transbronchial needle aspiration biopsy" (TBNA) [3], [4]. Traditionally this biopsy is performed by maneuvering a bronchoscope to a suitable site within the tracheobronchial tree. Then the bronchoscopist inserts a needle through the bronchoscope and punctures the bronchial wall in order to hit the target behind. Methods to guide TBNAs are based on determining the position and orientation of the bronchoscope's tip. Solomon *et al.* (see [3]) uses position sensors attached to the tip, Mori *et al.* [5] analyzes the video images from a CCD camera inside the bronchoscope's tip to achieve a continuous tracking and our group [3] recently introduced a new approach to the problem by using a model of a flexible bronchoscope, to "predict" the position and orientation of its tip. All three approaches have in common that they determine the tip position approximately, due to sensor, video tracking or model inaccuracies.

The main contribution of this work is an algorithm that finds for a given set of possible initial needle positions, the smallest set of needles needed to guarantee a successful biopsy. Another contribution is an algorithm that maximizes the coverage of the possible initial positions for a given maximum number of k needles. The advantage of considering this variation of the problem is that there exists an approximative solution, which is easy to implement and is guaranteed to be within a factor $1 - \frac{1}{e}$ of the exact solution. We use our model of a flexible endoscope [4] together with our virtual endoscopy system to experimentally validate the approach.

1.1 Assumptions

The problem is based on the following three assumptions.

1. There exists an initial position domain $P \subset \mathcal{M}$, which is a set of possible initial locations for the endoscope, before needle placement. The endoscope is assumed to be given by the model described in [4], namely by a sequence of links, interconnected by joints. Let $\tilde{\mathbf{p}} \in P$ denote the real, but unknown position of the endoscope after insertion. It is assumed that $\tilde{\mathbf{p}}$ does not change during needle placement.
2. There exists a target domain $T \subset \mathbb{R}^3$.
3. There exists a function $f : P \times T \to N$, which computes for a given $\mathbf{p} \in P$ and $\mathbf{t} \in T$ the necessary needle parameter $\mathbf{n} \in N$ to hit $\mathbf{t}$ from position $\mathbf{p}$. $N \subset \mathbb{R}^3$ is denoted as the "needle parameter domain". Function $f()$ represents a model of the endoscope's active tip deflection, as described in [3].
   There also exists a dual function $\bar{f} : P \times N \to \mathbb{R}^3$, which computes for a given position $\mathbf{p}$ and a needle parameter $\mathbf{n}$ the resulting position of the

needle tip. Given the same model of active tip deflection as mentioned above, the realization of $\bar{f}$ is straightforward.

Note that the codomain of $\bar{f}$ is $\mathbb{R}^3$ (and not T), because for $\mathbf{p} \neq \mathbf{q}$:

$$(\,\mathbf{n} = f(\mathbf{p},\,\mathbf{t})\,) \;\not\Rightarrow\; (\,\bar{f}(\mathbf{q},\,\mathbf{n}) \text{ is an element of } T\,), \quad \mathbf{p},\mathbf{q} \in P,\ \mathbf{t} \in T \tag{1}$$

2 An "Optimal" Strategy

The basic idea is to find needles that "cover" as many of the initial endoscope positions as possible. A needle "covers" an area, if for any endoscope within this area the needle in question hits the target. One goal is to solve the problem of minimizing the number of needles needed for a full coverage. The problem of finding the smallest set of needles that cover all initial positions is formulated as the problem of finding the "minimum set cover" in the needle parameter domain. This problem in turn, can be directly formulated as the "Set Covering Problem", a well known NP-hard optimization problem.

Another goal is to maximize the number of initial positions covered by a given number of k needles. This problem is formulated as the "Maximum k-Coverage Problem", likewise a NP-hard, general optimization problem.

2.1 Formulation as an Optimization Problem

An "optimal" k-needle placement strategy is a set $N_{\mathsf{opt}} \subset N$ of needle parameters, such that

$$\begin{array}{l} 1.\ P_{\mathsf{opt}} = P \quad \text{and} \\ 2.\ |N_{\mathsf{opt}}| = \text{minimal}\,, \end{array} \tag{2}$$

where $P_{\mathsf{opt}} = \{\mathbf{p} \in P \,|\, \bar{f}(\mathbf{p},\,\mathbf{n}) \text{ is an element of } T,\ \mathbf{n} \in N_{\mathsf{opt}}\}$. In other words, for all endoscope positions $\mathbf{p} \in P$ there exists a needle in N_{opt} which hits the target and no set smaller than N_{opt} guarantees the same.

The basic idea behind finding the smallest set of subsets in P, is to consider a "dual problem" in the needle parameter domain N. The problem is transformed into N by sampling P and calculating a "scan" of target T from the "perspective" of each sample. The dual problem is then to find a minimum number of points in N such that each scan covers at least one point. This set of points is equivalent to N_{opt}.

2.2 Transformation into the Needle Parameter Domain

To transform the problem into the needle parameter domain, the following definition is used:

Definition 1 ($S^T(\mathbf{p})$) *$S^T(\mathbf{p})$ denotes a "scan" of T from a given position $\mathbf{p} \in P$:*

$$S^T(\mathbf{p}) = f(\mathbf{p},\,T)\,.$$

$S^T(\mathbf{p}) \subset N$ is the set of all needle parameters needed to hit all $\mathbf{t} \in T$ from a fixed $\mathbf{p}$. Position $\mathbf{p}$ is called the "viewpoint" of the scan. □

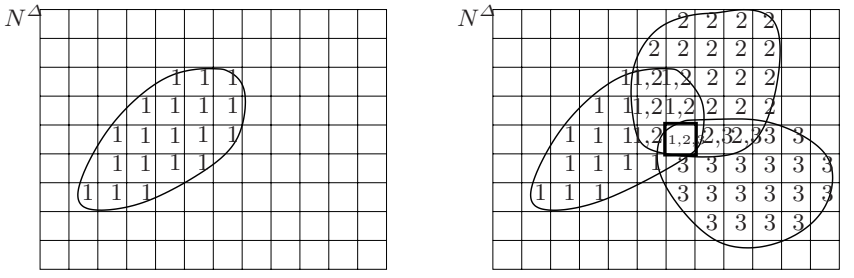

Fig. 1. Left: A scan $S^{T^{\Delta}}(\mathbf{p}_1)$ in N^{Δ}. Each cell shows its set of viewpoints V_i (subscript of $\mathbf{p}_1$ only). Right: Three scans from viewpoint $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$. Set V_i of cell i gives the indices of the viewpoints, whose scan cover cell i. Only one cell (boxed) is covered by all three scans.

Let target T be discretized into T^{Δ}, which consists of voxels or cells of side length Δ_T. A discretization of T also requires a discretization of N in the sense that two needle parameters which map a position $\mathbf{p} \in P$ into the same voxel of T^{Δ}, can be regarded as one needle parameter.

Definition 2 (N^{Δ}) *The needle parameter domain N is discretized into cells. The centers of all cells represent the discretized needle parameter domain N^{Δ}. Cell size Δ_N is derived from the cell size Δ_T in T^{Δ}. Let $d()$ be the euclidian distance:*

$$\Delta_N = d(\mathbf{n}_1, \mathbf{n}_2) \rightarrow \max \quad \textit{such that for a } \mathbf{p} \in P : d(\bar{f}(\mathbf{p}, \mathbf{n}_1), \bar{f}(\mathbf{p}, \mathbf{n}_2)) \leq \Delta_T$$

□

In the following the transition is made from $S^T(\mathbf{p}) \subset N$ to $S^{T^{\Delta}}(\mathbf{p}) \subset N^{\Delta}$, where $S^{T^{\Delta}}(\mathbf{p})$ is the scan of T^{Δ} from viewpoint $\mathbf{p} \in P$. The idea is to "round" each $\mathbf{n} \in S^{T^{\Delta}}(\mathbf{p})$ to the center of the cell it falls in. If one or more $\mathbf{n}$ fall into the same cell, we say the cell is "covered" by the scan. Consequently, it is sufficient to store for each cell the viewpoint $\mathbf{p}$ of the scan, which "covers" the cell. This yields the following:

Definition 3 ($\mathbf{n}_i^c$ and V_i) *Each cell of N^{Δ} stores two pieces of information:*

1. $\mathbf{n}_i^c \in N^{\Delta}$ *the center of cell i ,*
2. $V_i \subseteq P$ *the set of viewpoints of cell i*

The "center of cell i" is the needle parameter in the center of cell i. Set V_i is the set of viewpoints of all scans that cover cell i. □

Figure 1 (left) shows N^{Δ} divided into cells and a scan $S^{T^{\Delta}}(\mathbf{p}_1)$. For each cell the set of viewpoints V_i is given. The set is either $\{1\}$ (subscript of viewpoint $\mathbf{p}_1$) if the cell is covered by the scan or the empty set $\{\}$, if the cell is not covered.

To transform the problem from P to N^{Δ}, P is sampled and a scan $S^{T^{\Delta}}(\mathbf{p}_i)$ is calculated for each $\mathbf{p}_i \in P$. Figure 1 (right) shows an example for three samples

$\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$. Each cell's set of viewpoints V_i is given. Note that one cell (boxed) is covered by all three scans. With $\mathbf{n}_i^c$ the center of this cell, this can be interpreted as:

$$\bar{f}(\mathbf{p}_1, \mathbf{n}_i^c) \in T^\Delta \ \wedge \ \bar{f}(\mathbf{p}_2, \mathbf{n}_i^c) \in T^\Delta \ \wedge \ \bar{f}(\mathbf{p}_3, \mathbf{n}_i^c) \in T^\Delta \tag{3}$$

In other words, only needle parameter $\mathbf{n}_i^c$ is needed to map all three positions into the target. Positions $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ are members of the same subset, induced by $\mathbf{n}_i^c$. The goal of dividing P into a minimum number of subsets can now be formulated as the problem of selecting a minimum number of cells in N^Δ, such that each scan covers at least one selected cell. This problem is reduced to the following "classic" optimization problem.

2.3 "Set Covering Problem" and "Maximum k-Coverage Problem"

The "Set Covering Problem" or short SCP is a well known NP-hard combinatorial optimization problem, which can be formulated as:

Set Covering Problem (SCP): A finite set U of elements and a class $\mathcal{S}$ of subsets of U is given. Let S_i denote the i-th subset in $\mathcal{S}$.
The task is to select subsets S_i, such that every element in U belongs to at least one S_i. A selection $\mathcal{W} \subseteq \mathcal{S}$ with this property is called a set cover of U with respect to $\mathcal{S}$.
The optimization problem is to find a set cover $\mathcal{W}$ of minimum cardinality:

$$\mathsf{SCP}(U, \mathcal{S}) = \{\mathcal{W} \mid \mathcal{W} \text{ is a set cover of } U \text{ of minimum cardinality}\}. \tag{4}$$

The SCP is a subject of numerous publications in the operations research and mathematical literature. Many applications of the set covering problem to real-world problems, such as resource allocation and scheduling have been described. Exact solutions for modestly sized problems using a dual heuristic, have been reported by Fisher and Kedia [6]. For large problems, approximative schemes have been suggested by Beasley [7].

An interesting variation of the SCP is the "Maximum k-Coverage Problem" (kCP).

Maximum k-Coverage Problem (kCP): A set U and a class of subsets $\mathcal{S}$ is given, as in the SCP, as well as an integer k. Each element $u \in U$ has an associated weight $w(u)$.
The optimization problem is to select k subsets S_i from $\mathcal{S}$, such that the weight of the elements in $\bigcup_{i=1}^{k} S_i$ is maximized. □

Hochbaum and Pathria [8] have shown that the greedy approach to this NP-hard problem, which selects at each stage the subset that gives maximum improvement, is guaranteed to be within a factor of $1 - (1 - \frac{1}{k})^k > 1 - \frac{1}{e}$ of the optimal solution.

2.4 Formulation of the Problem as an SCP and kCP

The connection between our problem and the SCP can be established as follows: Let P^{Δ} be a set of samples of P, $V_i \subseteq P^{\Delta}$ the set of viewpoints of cell i and $\mathcal{W}$ an arbitrary minimal set cover:

$$\mathcal{W} \in \mathsf{SCP}(U, \mathcal{S}), \text{ where } U = P^{\Delta}, \;\; \mathcal{S} = \{V_1, V_2, \ldots, V_{|N^{\Delta}|}\} \tag{5}$$

Let $\mathbf{n}_i^c \in N^{\Delta}$ be the needle parameter in the center of cell i. Then an "optimal" k-Needle placement strategy is given by:

$$N_{\mathsf{opt}} = \{\mathbf{n}_i^c \,|\, V_i \in \mathcal{W}\}\,. \tag{6}$$

The $P_{\mathsf{opt}} = P$ condition of Equation 2 follows from the SCP condition that every element in U belongs to at least one selected subset S_i. The '$|N_{\mathsf{opt}}|$ = minimal' condition follows from the minimization of the set covers' cardinality.
For example, given the situation shown in Figure 1 (right), $U = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$, $\mathcal{S} = \{\{\}, \{\mathbf{p}_1\}, \{\mathbf{p}_2\}, \{\mathbf{p}_3\}, \{\mathbf{p}_1, \mathbf{p}_2\}, \{\mathbf{p}_2, \mathbf{p}_3\}, \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}\}$, $\mathcal{W} = \{\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}\}$ and $N_{\mathsf{opt}} = \{\mathbf{n}_i^c\}$, where i is the boxed cell. With this formulation, a subset of P, induced by a $\mathbf{n}_i^c \in N_{\mathsf{opt}}$ is given by V_i. It is important to note that the quality of solution N_{opt} depends on the sample density of P^{Δ}.

The connection between our problem and the kCP follows directly from the above theorem, with the weight function given by: $w(u) = 1$, for all $u \in U$. This weight function favors cells that are covered by many scans, since the kCP maximizes the sum of the weights of all elements of all selected subsets.

The kCP is an interesting variation for two reasons: Firstly, the greedy approach is easy to implement, by simply selecting at each stage the cell with the highest cardinality of V_i and subsequently updating all V_i. Secondly, as shown by Hochbaum *et al.* [8] for small k, a greedily constructed solution is within an acceptable factor from the exact solution. For $k < 3$ the factor is > 0.7.

3 Experiment

We have implemented the greedy approach to the kCP using Matlab. Figures 2(a)-(l) depict the needle parameter domain N^{Δ}. The grid indicates the cells in N^{Δ}. The cell size is given by $5^{\circ} \times 5^{\circ} \times 2\,\mathrm{mm}$. Each scan of target T^{Δ} is depicted as a transparent (alpha blending) convex hull. Figure (a) shows the scans from all viewpoints in P. In Figure (b) the cell of maximum coverage was determined, all scans covering this cell were removed and a "1" was drawn in the center of that cell. Given the remaining scans, the cell of maximum coverage was determined, all scans covering this cell were removed and a "2" was drawn in the center of that cell. The result is shown in Figure (c). This procedure was repeated until 99.9% of the scans were removed (Figure (l)). Each number i drawn in N^{Δ} represents three alignment parameters α_i (twist), β_i (tip angle), and d_i (needle length), needed to manoeuvre the needle tip into the target. The percentage of scans removed in each step corresponds to the probability of success for the respective biopsy needle placed. This probability and the cumulative

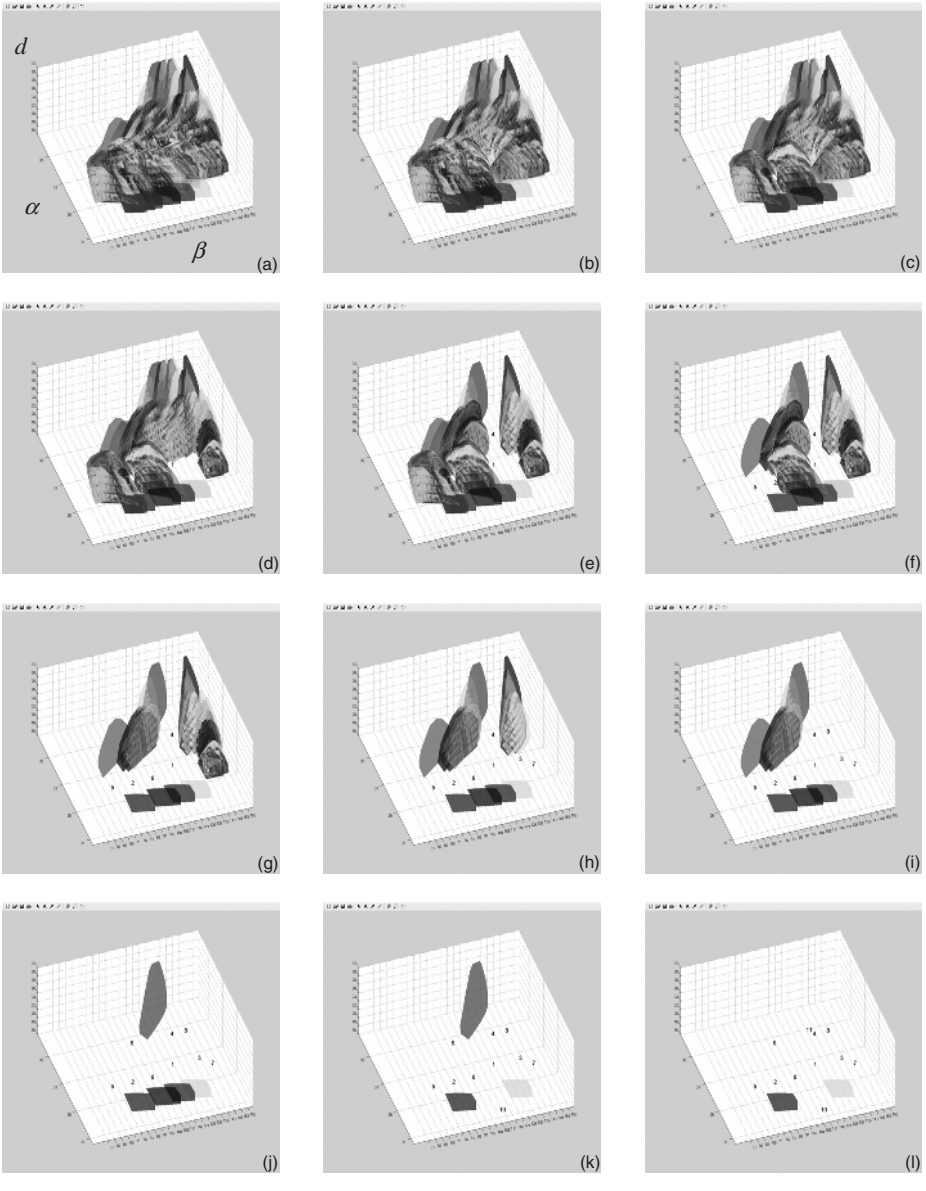

Fig. 2. Calculating a strategy for placing $k = 6$ needles, using the greedy implementation of the "Maximum k-Coverage Problem". (a) Scans from 283 viewpoints depicted in the needle parameter domain. (b)-(l) Iteratively selecting the cell of highest coverage and removing the corresponding scans. (b)-(g) The centers of the first six selected cells represent the sought needle parameters. Percentages of needles 1-6 are given in the table.

probability was calculated for needle $i = 1 \ldots 6$ as $\frac{|V_i|}{|P^{\Delta}|=283}$. The result is shown in Figure (b)-(g) and in the following table: The table shows that two needles

Needle	1	2	3	4	5	6
Probability [%]	42	28	16	4	4	3
Sum	42	71	86	90	94	97

cover 71% and three needles 86% of all initial positions. The table represents a valuable decision support tool for the bronchoscopist. Depending on the concrete condition of the patient, he/she can decide whether or not a third or even a fourth needle is advisable. Based on this table, a third needle gives a considerable improvement of 16%, whereas the improvement of a fourth, fifth or sixth needle is negligible ($\leq 4\%$). Computation time for the experiment was about two minutes.

4 Conclusion

We have presented an "optimal" strategy for placing k biopsy needles given a large number of possible initial needle positions. Beside the actual needle parameters, we provide a table to the physician, which contains a probability of success for each needle. By placing the needles in order of decreasing probability, the physician can decide after each needle, whether the gain in the overall probability of success by employing the next needle outweighs the risk to the patient. Overall, our approach can provide valuable decision support to the physician regarding how many needles to place and how to place them.

References

1. D. Shen et al. A Statistical Atlas of Prostate Cancer for Optimal Biopsy. In *MICCAI*, volume 2208 of *LNCS*, pages 416–424. Springer, 2001.
2. J. Zeng et al. Distribution of Prostate Cancer for Optimized Biopsy Protocols. In *MICCAI*, volume 1935 of *LNCS*, pages 287–296. Springer, 2000.
3. Markus Kukuk et al. TBNA-protocols - Guiding TransBronchial Needle Aspirations Without a Computer in the Operating Room. In *MICCAI '01 - LNCS 2208*, pages 997–1006. Springer.
4. Markus Kukuk and Bernhard Geiger. A Real-Time Deformable Model for Flexible Instruments Inserted Into Tubular Structures. In *MICCAI '02 - LNCS 2489*, pages 330–338. Springer.
5. Kensaku Mori et al. A Method for Tracking the Camera Motion of Real Endoscope by Epipolar Geometry Analysis and Virtual Endoscopy System. In *MICCAI*, volume 2208 of *LNCS*, pages 1–8. Springer, 2001.
6. M. L. Fisher and P. Kedia. Optimal solution of set covering/partitioning problems using dual heuristics. In *Management Science-36*, pages 674–688, 1990.
7. J. E. Beasley. A Genetic Algorithm for the Set Covering Problem. In *European Journal of Operational Research*, pages 392–404, 1994.
8. D. S. Hochbaum and A. Pathria. Analysis of the Greedy Approach in Problems of Maximum k-Coverage. In *Naval Research - 45*, pages 615–627, 1998.