15. Completeness of restricted term calculus.

We want to show the second part of theorem 9.1 which has to do with restricted term calculus. What we have said in section 9, C and D, mutatis mutandis, also applies to restricted term calculus. According to (7.1) we obtain $\mathfrak{M} \vdash_1 \alpha$ from $\mathfrak{M} \vdash \alpha$. Hence we only have to show the analogue of the Theorem of satisfiability (13.1), where of course we have to replace the notion of consistency (which in (13.1) is related to the term calculus) by the analogous relation of consistency$_1$ (which is related to the restricted term calculus). The construction in section 13 also holds for restricted term calculus. It gives an interpretation, which satisfies $\mathfrak{M}$, over a semantical basis $\mathfrak{B} = \langle \omega, \pi, \mathcal{F}, \mathfrak{i}, \mathfrak{n}, \mathfrak{t}, \mathfrak{a} \rangle$, where in our case we only have to show that π has not more than one element. This can be shown as follows: In analogy to (13.8) we have

(15.1) If $\vdash_1 \alpha_1 \dots \alpha_n \alpha$ and $\alpha_1, \dots, \alpha_n \in \mathfrak{M}^*$, then $\alpha \in \mathfrak{M}^*$.

In analogy to (13.12) and (13.13) we get

(15.2) $\bar{s} = \bar{t}$ iff $=st \in \mathfrak{M}^*$.

π is defined as in (13.16):

(15.3) $\bar{s} \in \pi$ iff $s \in \mathfrak{M}^*$.

Let be $\bar{s} \in \pi$ and $\bar{t} \in \pi$. We have to show that $\bar{s} = \bar{t}$. With the help of the rule of restriction (R_x^y) and the rule of substitution we obtain

(15.4) $\vdash_1 s\ t\ =st$.

Since we have $\bar{s} \in \pi$ and $\bar{t} \in \pi$ we have $s \in \mathfrak{M}^*$ and $t \in \mathfrak{M}^*$ by (15.3). Now by (15.1) and (15.4) we get $=st \in \mathfrak{M}^*$, i.e. $\bar{s} = \bar{t}$ by (15.2).