

10. Some theorems concerning deducibility.

Only for some of these propositions we indicate a proof. (Cf. also Hermes [4].)

(10.1) $\vdash \alpha\alpha$.

Use (P_x) where not Free $x \alpha$. Cf. (6.1).

(10.2) If $\vdash \Sigma\alpha$ then $\vdash \Sigma'\alpha$, if each member of Σ is a member of Σ' .

Use $(K_0), (K_1)$ and (10.1).

(10.3) If $\vdash \Sigma_1\alpha$ and $\vdash \Sigma_2\alpha\beta$ then $\vdash \Sigma_{12}\beta$ (syllogism).

Use (10.1), (W), (E).

(10.4) $\vdash =tt$.

(10.5) $\vdash =t_1t_2 =t_2t_1$.

(10.6) $\vdash =t_1t_2 =t_2t_3 =t_1t_3$.

(10.7) $\vdash =s_1t_1 \dots =s_r t_r =fs_1 \dots s_r ft_1 \dots t_r$.

(10.8) $\vdash =st s t$ and $\vdash =st t s$.

Use (10.1), the second rule for identity, the rule of substitution and (10.2).

(10.9) If $\vdash =\alpha\beta$ then $\vdash =\epsilon x\alpha\epsilon x\beta$.

Use (10.8) and (E_x) .