Probabilistic Congruence for Semistochastic Generative Processes

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Abstract. We propose an SOS transition rule format for the generative model of probabilistic processes. Transition rules are partitioned in several strata, giving rise to an ordering relation analogous to those introduced by Ulidowski and Phillips for classic process algebras. Our rule format guarantees that probabilistic bisimulation is a congruence w.r.t. process algebra operations. Moreover, our rule format guarantees that process algebra operations preserve semistochasticity of processes, i.e. the property that the sum of the probability of the moves of any process is either 0 or 1. Finally, we show that most of operations of the probabilistic process algebras studied in the literature are captured by our format, which, therefore, has practical applications.

1 Introduction

Probabilistic process algebras have been introduced in the literature (see, among the others, [2, 3, 8, 9, 10, 11, 13]) to develop techniques dealing with both functional and non-functional aspects of system behavior, such as performance and reliability. *Probabilistic transition systems* (PTSs, for short), which extend classic labeled transition systems by some mechanism to represent the probabilistic choice, have been employed as a basic semantic model of probabilistic processes. In order to abstract away from irrelevant information on the way that processes compute, several notions of behavioral *equivalence* and *preorder* have been considered. *Probabilistic bisimulation* relates two processes iff they have the same probabilistic branching structure. In the process algebras of [2, 3, 8, 9, 10, 11, 13]), probabilistic bisimulation is a *congruence* w.r.t. all operations, which is an important property to fit it into an axiomatic framework.

Usually, PTSs are defined by means of a *structural operational semantics* [14, 15] (SOS, for short) consisting of a set of *transition rules* of the form $\frac{\text{premises}}{\text{conclusion}}$, which, intuitively, determine how probabilistic moves of processes can be inferred by probabilistic moves of other processes. A set of syntactical constraints on the transition rules is called a *transition rule format* [16]. In the area of classic (i.e., non-probabilistic) process algebras, rule formats have been widely employed to fix results holding for classes of process algebras. For instance, several rule formats proposed in the literature ensure that a given behavioral equivalence is a congruence (for a survey see [1]). Other rules formats ensure that a given property of security is preserved by process algebra operations [17, 18].

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An interesting issue is to develop rule formats for probabilistic process algebras. To take a step in this direction, we propose a rule format for process algebras respecting the *generative* model of probabilistic processes [11], which requires that a single probability distribution is ascribed to all moves of any process. Such a generative model differs w.r.t. the *reactive* model of probabilistic processes, which requires that the kind of action of any process is chosen nondeterministically, and that, for any action and any process, a probability distribution is ascribed to the moves of that process labeled with that action.

Our format admits transition rules of the following form:

$$\frac{\{x_i \xrightarrow{a_i, p_i} y_i \mid i \in I\} \cup \{x_j \xrightarrow{A_j, p'_j} \mid j \in J\} \cup \{x_h \xrightarrow{B_h} \mid h \in H\}}{f(\overrightarrow{x}) \xrightarrow{a, \frac{\prod_{i \in I} p_i}{\prod_{j \in J}(1-p'_j)} \cdot w_\rho} t}$$

Hence, our format extends the classic *de Simone format* [16] with probability (i.e., a probability value p appears in transition labels), premises $x_j \xrightarrow{A_j, p'_j}$ meaning that the argument j of f performs actions in the set A_j with total probability p'_j, and premises $x_h \xrightarrow{B_h}$ meaning that the argument h of f performs at least one action in the set B_h. Then, to give a semantics to a given process algebra, we require that the transition rules are partitioned in n *strata* $\mathcal{R}_1, \ldots, \mathcal{R}_n$, for some $n \in \mathbb{N}$. The interpretation is that the moves of a given process t can be inferred from rules in $\mathcal{R}_i$ only if no move of t can be inferred from rules in $\mathcal{R}_j$, for any $j < i$. Hence, the partitioning gives rise to an ordering relation between transition rules analogous to those introduced for classic process algebras in [19].

We prove that process algebra operations captured by our format preserve *semistochasticity* of processes, i.e. the property that the sum of the probability of the moves of any process is either 0 or 1. This is a central issue in the theory of probabilistic processes, since semistochasticity is required by most of authors, such as [3, 5, 8], which concentrate on so called *semistochastic languages* [11].

Then, we prove that probabilistic bisimulation is a congruence w.r.t. all operations captured by our format.

To show that our format has practical applications, we prove that it captures most of operations of the probabilistic process algebras proposed in the literature.

Finally, we prove that our format can be enriched by *double testing* as in *GSOS format* [7], and by *look ahead* as in *tyft/tyxt format* [12]. We discuss also the possibility to admit *predicates*, as in formats *path* [4] and *panth* [20].

We discuss the related work [6], where a very preliminary rule format for the reactive model of probabilistic processes is introduced.

2 Background

Let us begin with recalling the model of probabilistic transition systems.

For any set S, let $\mathcal{M}(S)$ denote the collection of multisets over S.

Definition 1. *A* probabilistic transition system *(PTS, for short) is a triple* $(\mathcal{S}, Act, T)$*, where* $\mathcal{S}$ *is a set of* states*,* Act *is a set of* actions*, and* $T \in \mathcal{M}(\mathcal{S} \times$

$Act \times (0,1] \times \mathcal{S})$ *is a multiset of* transitions *such that, for all states* $s \in \mathcal{S}$, $\sum\{\!| p \,|\, \exists a \in Act, s' \in S : (s,a,p,s') \in T |\!\} \in [0,1]$.

Def. 1 respects the *generative* (or *full*) model of probabilistic processes [11], where a single probability distribution is ascribed to all moves of any process. On the contrary, we recall that the *reactive* model admits that the kind of action is chosen nondeterministically, i.e. the multiset T satisfies the following property: for all states $s \in \mathcal{S}$ and actions $a \in Act$, $\sum\{\!| p \,|\, \exists s' \in \mathcal{S} : (s,a,p,s') \in T |\!\} \in [0,1]$.

Definition 2. *A state* $s \in \mathcal{S}$ *is* semistochastic *iff* $\sum\{\!| p \,|\, \exists a \in Act, s' \in S : (s,a,p,s') \in T |\!\} \in \{0,1\}$. *If this sum is 1 then s is* stochastic. *A PTS is semistochastic iff all its states are semistochastic.*

As in [3, 5, 8], we concentrate on semistochastic PTSs, which are the semantic model of the so called *semistochastic languages* [11].

We write $s \xrightarrow{a,p} s'$ to denote that $(s,a,p,s') \in T$, and we call s and s' *source* and *target* of the transition, respectively. For a set of actions $A \subseteq Act$, we write $s \xrightarrow{A,p}$ to denote that $\sum\{\!| q \,|\, \exists a \in A, s' \in \mathcal{S} : s \xrightarrow{a,q} s' |\!\} = p$. If this multiset is empty, then we write $s \xrightarrow{A,0}$. Finally, we write $s \xrightarrow{A}$ to denote that there is at least one transition (s,a,p,s') in T with $a \in A$, for some p and s'.

Before defining probabilistic bisimulation, we need some definitions.

For an equivalence relation $\mathcal{R}$ over $\mathcal{S}$, we write $\mathcal{S}/\mathcal{R}$ to denote the set of equivalence classes induced by $\mathcal{R}$.

Definition 3. $\mu : \mathcal{S} \times Act \times 2^{\mathcal{S}} \to [0,1]$ *is the function given by:* $\forall s \in \mathcal{S}$, $\forall a \in Act$, $\forall S \subseteq \mathcal{S}$

$$\mu(s,a,S) = \sum\{\!| p \,|\, s \xrightarrow{a,p} s' \text{ and } s' \in S |\!\}$$

Definition 4. *An equivalence relation* $\mathcal{R} \subseteq \mathcal{S} \times \mathcal{S}$ *is a* probabilistic bisimulation *if* $(s_1,s_2) \in \mathcal{R}$ *implies:* $\forall S \in \mathcal{S}/\mathcal{R}$, $\forall a \in Act$,

$$\mu(s_1,a,S) = \mu(s_2,a,S)$$

The union of all probabilistic bisimulation is, in turn, a probabilistic bisimulation. We denote it by $\approx$, and we write $s_1 \approx s_2$ for $(s_1,s_2) \in \approx$.

Let us recall now the notions of signature and term over a signature.

A *signature* is a set Σ of *operation symbols* together with an *arity* mapping that assigns a natural $ar(f)$ to every $f \in \Sigma$. If $ar(f)$ is 0, f is called a *constant.*

For a set of *variables* $\mathtt{Var}$, ranged over by $x, y, \ldots$, the set of (*open*) *terms* $\mathtt{T}(\Sigma, \mathtt{Var})$ over Σ and $\mathtt{Var}$, ranged over by $s, t, \ldots$, is the least set such that: 1) each variable $x \in \mathtt{Var}$ is a term; 2) $f(t_1, \ldots, t_{ar(f)})$ is a term whenever $f \in \Sigma$ and $t_1, \ldots, t_{ar(f)}$ are terms. *Closed terms* are terms that do not contain variables.

A *substitution* is a mapping $\sigma : \mathtt{Var} \to \mathtt{T}(\Sigma, \mathtt{Var})$. With $\sigma(t)$ we denote the term obtained by replacing all occurrences of variables x in term t by $\sigma(x)$.

The abstract syntax of probabilistic process description languages is usually given by a signature Σ, whose closed terms are called *probabilistic processes.* The semantics is usually given by a PTS, where states are probabilistic processes.

3 Definitions

In this section we introduce the notions of PB transition rule and PB transition system specification (PB stays for probabilistic bisimulation).

Definition 5. *For any operation* $f \in \Sigma$ *and tuple* $\overrightarrow{x} = x_1, \ldots, x_{ar(f)}$ *of variables, a* PB transition rule ρ *is of the form*

$$\frac{\{x_i \xrightarrow{a_i, p_i} y_i \mid i \in I\} \cup \{x_j \xrightarrow{A_j, p'_j} \mid j \in J\} \cup \{x_h \xrightarrow{B_h} \mid h \in H\}}{f(\overrightarrow{x}) \xrightarrow{a, \frac{\prod_{i \in I} p_i}{\prod_{j \in J}(1-p'_j)} \cdot w_\rho} t}$$

where:

1. I*,* J*,* H *are subsets of* $\{1, \ldots, ar(f)\}$ *such that* $J \subseteq I$*;*
2. $a_i \in Act$ *for* $i \in I$*,* $A_j \subseteq Act$ *for* $j \in J$*,* $B_h \subseteq Act$ *for* $h \in H$*,* $a \in Act$*;*
3. *for all* $i \in I$ *and* $j \in J$ *such that* $i = j$*, it holds that* $a_i \notin A_j$*;*
4. p_i *is a variable with range* $(0, 1]$ *for* $i \in I$*,* p'_j *is a variable with range* $[0, 1)$ *for* $j \in J$*;*
5. t *is a term over* Σ *and* $\overrightarrow{x} \cup \{y_i \mid i \in I\}$*;*
6. w_ρ *is the* weight *of* ρ *and satisfies* $0 < w_\rho \leq 1$*.*

Transitions $\{x_i \xrightarrow{a_i, p_i} y_i | i \in I\}$ are the *active premises*; variables $\{x_i | i \in I\}$ are the *active variables*; transitions $\{x_j \xrightarrow{A_j, p'_j} | j \in J\}$ are the *unneeded premises*; transitions $\{x_h \xrightarrow{B_h} | h \in H\}$ are the *unquantified premises*; transition $f(\overrightarrow{x}) \xrightarrow{a, \frac{\prod_{i \in I} p_i}{\prod_{j \in J}(1-p'_j)} \cdot w_\rho} t$ is the *conclusion*; $f(\overrightarrow{x})$ is the *source*; t is the *target* of ρ.

Given terms $\overrightarrow{t}$, values $\{q_i \mid i \in I\}$ in $(0, 1]$, and values $\{q'_j \mid j \in J\}$ in $[0, 1)$, Def. 5 says that term $f(\overrightarrow{t})$ has the move $f(\overrightarrow{t}) \xrightarrow{a, q} t[\overrightarrow{t}/\overrightarrow{x}][\overrightarrow{s}/\overrightarrow{y}]$, with $q = \frac{\prod_{i \in I} q_i}{\prod_{j \in J}(1-q'_j)} \cdot w_\rho$, provided that t_i has the move $t_i \xrightarrow{a_i, q_i} s_i$, for all $i \in I$, the sum of the probability of the moves of t_j with label in A_j is q'_j, for all $j \in J$, and t_h has at least one move with label in B_h, for all $h \in H$.

Notice that the conclusion is triggered by both active and unquantified premises, and does not require unneeded premises, meaning that p'_j could be 0 for some $j \in J$. Unneeded premises are used to compute the probability of the conclusion. More precisely, they permit normalization of probability, which, as we will see in next sections, is needed in several operations of process algebras, such as restriction and priority. The probability of the conclusion depends on the weight of ρ and on $\frac{\prod_{i \in I} p_i}{\prod_{j \in J}(1-p'_j)}$, which is the conditional probability that all x_i perform a_i under the assumption that all x_j are not allowed to perform actions in A_j. Unquantified premises do not contribute in computing the probability of the conclusion. They are "necessary conditions" for the application of ρ.

Definition 6. *A* PB transition system specification *(PB TSS, for short) is formed by a set* $\mathcal{R}$ *of PB transition rules such that:*

1. *$\mathcal{R}$ is partitioned into n* strata *$\mathcal{R}_1, \ldots, \mathcal{R}_n$, for some $n \in \mathbb{N}$;*
2. *for each stratum $\mathcal{R}_u$, operation f and tuple of variables $\overrightarrow{x} = x_1, \ldots, x_{ar(f)}$ s.t. $\mathcal{R}_u$ has at least one PB transition rule with source $f(\overrightarrow{x})$, it holds that:*
   (a) All PB transition rules with source $f(\overrightarrow{x})$ in stratum $\mathcal{R}_u$ have the same set of unquantified premises $\{x_h \xrightarrow{B_h} \mid h \in H\}$;
   (b) All PB transition rules with source $f(\overrightarrow{x})$ in stratum $\mathcal{R}_u$ have the same set of unneeded premises $\{x_j \xrightarrow{A_j, p'_j} \mid j \in J\}$;
   (c) All PB transition rules with source $f(\overrightarrow{x})$ in stratum $\mathcal{R}_u$ have the same set of active variables $\{x_i \mid i \in I\}$;
   (d) Given actions $\{a'_i \mid i \in I\}$ such that $a'_i \notin A_j$ for all indexes i and j with $i = j$ and $x_j \xrightarrow{A_j, p'_j}$ an unneeded premise, then there is at least one PB transition rule with source $f(\overrightarrow{x})$ in $\mathcal{R}_u$ with active premises $\{x_i \xrightarrow{a'_i, p_i} y_i \mid i \in I\}$;
   (e) Given the PB transition rules $\rho_1, \ldots, \rho_m$ in $\mathcal{R}_u$ with source $f(\overrightarrow{x})$ having the same active premises, their weights satisfy $w_{\rho_1} + \cdots + w_{\rho_m} = 1$.

The meaning of clause 1 is that the rules in stratum $\mathcal{R}_u$ can be applied only if no rule in strata $\mathcal{R}_1, \ldots, \mathcal{R}_{u-1}$ can be applied (see Def. 7 below).

Let us take any $f \in \Sigma$. Clause 2a implies that unquantified premises trigger either all rules with source $f(\overrightarrow{x})$ in $\mathcal{R}_u$, or none of them. In the first case, we can prove that clauses 2b–2e ensure that, given semistochastic processes $\overrightarrow{t}$, then the sum of the probability of the moves of $f(\overrightarrow{t})$ that are derivable by the rules in $\mathcal{R}_u$ is either 0 or 1. Let us distinguish two cases. In the first case, some t_i with $i \in I$ is not stochastic. Since it is semistochastic, t_i has no move. Hence, since clause 2c implies that a move of t_i is needed to infer a move of $f(\overrightarrow{t})$, no move of $f(\overrightarrow{t})$ can be derived from the rules in stratum $\mathcal{R}_u$, and, therefore, the sum of the probability of the moves of $f(\overrightarrow{t})$ derivable from $\mathcal{R}_u$ is 0. In the second case, all t_i with $i \in I$ are stochastic. Let us assume that, for all $j \in J$, q'_j is the probability such that $t_j \xrightarrow{A_j, q'_j}$. Value $\prod_{j \in J}(1 - q'_j)$ is the probability that each t_j does not perform any action in A_j. All combinations of arbitrary moves $\{t_i \xrightarrow{a_i, q_i} t'_i \mid i \in I\}$, with $a_i \in Act$ for each $i \in I$, fall into two categories:

- Some a_i is in A_j for the index $j = i$. Clause 3 of Def. 5 ensures that no move of $f(\overrightarrow{t})$ is inferred by rules in $\mathcal{R}_u$ from moves $\{t_i \xrightarrow{a_i, q_i} t'_i \mid i \in I\}$.
- No a_i is such that $a_i \in A_j$ for any index $j = i$. Since t_i is semistochastic, this implies $q'_j \neq 1$ for all $j \in J$. By clause 2d of Def. 6 there exist rules $\rho_1, \ldots, \rho_m$ with source $f(\overrightarrow{x})$ in $\mathcal{R}_u$, for some $m \in \mathbb{N}$, with active premises $\{x_i \xrightarrow{a_i, p_i} y_i \mid i \in I\}$. Hence, $f(\overrightarrow{t})$ has m moves with probabilities $w_{\rho_1} \cdot \frac{\prod_{i \in I} q_i}{\prod_{j \in J}(1 - q'_j)}, \ldots, w_{\rho_m} \cdot \frac{\prod_{i \in I} q_i}{\prod_{j \in J}(1 - q'_j)}$. Notice that these probabilities are well defined, since $q'_j \neq 1$ for all $j \in J$. Now, since $w_{\rho_1} + \cdots + w_{\rho_m} = 1$ by clause 2e of Def. 6, the sum of these probabilities is $\frac{\prod_{i \in I} q_i}{\prod_{j \in J}(1 - q'_j)}$.

Since we have assumed that all $\overrightarrow{t}$ are stochastic, and that for all $j \in J$, q'_j is the probability of $t_j \xrightarrow{A_j, q'_j}$, the overall probabilities of the combinations of moves $\{t_i \xrightarrow{a_i, q_i} t'_i \mid i \in I\}$ falling in the second category is $\prod_{j \in J}(1-q'_j)$. Hence, if $q'_j = 1$ for some $j \in J$, $f(\overrightarrow{t})$ has no move and the sum of the probability of the moves of $f(\overrightarrow{t})$ derivable from $\mathcal{R}_u$ is 0. Otherwise, if $q'_j \neq 1$ for all $j \in J$, the sum of the probability of the moves of $f(\overrightarrow{t})$ derivable from $\mathcal{R}_u$ is $\frac{\prod_{j \in J}(1-q'_j)}{\prod_{j \in J}(1-q'_j)} = 1$.

We can now formalize how PTSs are generated by PB TSSs.

Definition 7. *Assume a PB TSS with strata* $\mathcal{R}_1, \dots, \mathcal{R}_n$.

1. *A transition* $t \xrightarrow{a,q} s$ *is* provable from stratum $\mathcal{R}_u$ *iff there is a closed substitution instance* $\dfrac{\{t_i \xrightarrow{a_i, q_i} s_i \mid i \in I\} \cup \{t_j \xrightarrow{A_j, q'_j} \mid j \in J\} \cup \{t_h \xrightarrow{B_h} \mid h \in H\}}{t \xrightarrow{a,q} s}$ *of a PB transition rule in* $\mathcal{R}_u$ *such that:*
   (a) *for all* $i \in I$, $t_i \xrightarrow{a_i, q_i} s_i$ *is a transition provable from the TSS;*
   (b) *for all* $j \in J$, $q'_j = \sum\{\!|q| \exists a \in A_j, s' : t_j \xrightarrow{a,q} s' \text{ is provable from the TSS}|\!\}$;
   (c) *for all* $h \in H$, *at least one transition* $t_h \xrightarrow{a, q_h} u_h$ *with* $a \in B_h$ *is provable from the TSS, for some* q_h *and* u_h;
2. *A transition* $t \xrightarrow{a,q} s$ *is* provable from the TSS *if it is provable from some stratum* $\mathcal{R}_u$ *and no transition with source* t *is provable from strata* $\mathcal{R}_1, \dots, \mathcal{R}_{u-1}$.

Moves of terms are proved inductively w.r.t. their structure. In fact, first of all we can prove moves of constants from strata $\mathcal{R}_1, \dots, \mathcal{R}_n$ and, then, we can prove moves of constants from the TSS. This is possible since PB transition rules having a constant as source have no premise. Then, after moves of terms $\overrightarrow{t}$ have been proved from the TSS, we can prove moves of $f(\overrightarrow{t})$ from $\mathcal{R}_1, \dots, \mathcal{R}_n$ and, then, we can prove moves of $f(\overrightarrow{t})$ from the TSS.

Let us recall that, according to the classical definition (see, e.g., [12]), a (non-probabilistic) transition $t \xrightarrow{a} t'$ is provable from a given TSS iff there exists a well-founded, upwardly branching tree whose nodes are labeled by closed transitions, whose leaves have empty label, whose root is labeled by $t \xrightarrow{a} t'$, and, whenever K is the (possibly empty) set of labels of the nodes directly above a node labeled by β, then K/β is a closed substitution instance of a transition rule in the TSS.

We need a more complicated definition since our rules have the unneeded premises and the unquantified premises that are not "pure" transitions. Hence, we cannot construct the branching tree that is considered in the classical definition. Moreover, as in [19], we have to take into account that there is an ordering relation between the transition rules, given by the partitioning in n strata.

Definition 8. *The* PTS induced by a PB TSS *is the PTS having as transitions the transitions that are provable from the TSS.*

4 Examples

In this section we show that most of operations offered by the probabilistic process algebras proposed in the literature can be expressed by our PB TSSs.

Example 1 (Constants). Stratum $\mathcal{R}_1$ contains the following rule, for all $a \in Act$:

$$\frac{}{a \xrightarrow{a,1} 0}$$

Term a performs action a, and, then, it behaves as the idle process 0.

Let us show now that we can express the probabilistic sum of [2, 3, 8, 9, 11].

Example 2 (Probabilistic sum). Let $0 < p < 1$. Stratum $\mathcal{R}_1$ contains the following rules, for all $a_1, a_2 \in Act$, where p and $1 - p$ are their weights:

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1 \quad x_2 \xrightarrow{a_2,p_2} y_2}{x_1 +^p x_2 \xrightarrow{a_1,p_1 \cdot p_2 \cdot p} y_1} \qquad \frac{x_1 \xrightarrow{a_1,p_1} y_1 \quad x_2 \xrightarrow{a_2,p_2} y_2}{x_1 +^p x_2 \xrightarrow{a_2,p_1 \cdot p_2 \cdot (1-p)} y_2}$$

Stratum $\mathcal{R}_2$ contains the following rule, for all $a_1 \in Act$:

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1}{x_1 +^p x_2 \xrightarrow{a_1,p_1} y_1}$$

Stratum $\mathcal{R}_3$ contains the following rule, for all $a_2 \in Act$:

$$\frac{x_2 \xrightarrow{a_2,p_2} y_2}{x_1 +^p x_2 \xrightarrow{a_2,p_2} y_2}$$

Let us take term $t_1 +^p t_2$. Index p means that, when both t_1 and t_2 can move, t_1 moves with probability p, and t_2 moves with probability $1-p$. Rules in $\mathcal{R}_1$ (with weights p and $1 - p$) are applied when both t_1 and t_2 are stochastic; rules in $\mathcal{R}_2$ (with weight 1) are applied when only t_1 is stochastic; rules in $\mathcal{R}_3$ (with weight 1) are applied when only t_2 is stochastic. In the first case, since t_2 (resp. t_1) is stochastic and the sum of the probability of its moves is 1, from $t_1 \xrightarrow{a_1,p_1} t_1'$ (resp. $t_2 \xrightarrow{a_2,p_2} t_2'$) we infer moves of $t_1 +^p t_2$ labeled a_1 (resp. a_2) with total probability $p_1 \cdot p$ (resp. $p_2 \cdot (1 - p)$). In the other two cases, from $t_1 \xrightarrow{a_1,p_1} t_1'$ (resp. $t_2 \xrightarrow{a_2,p_2} t_2'$), we infer $t_1 +^p t_2 \xrightarrow{a_1,p_1} t_1'$ (resp. $t_1 +^p t_2 \xrightarrow{a_2,p_2} t_2'$).

Let us consider now the interleaving operation of [3].

Example 3 (Interleaving). Let $0 < p < 1$. Stratum $\mathcal{R}_1$ contains the following rules, for all $a_1, a_2 \in Act$, where p and $1 - p$ are their weights:

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1 \quad x_2 \xrightarrow{a_2,p_2} y_2}{x_1 \|^p x_2 \xrightarrow{a_1,p_1 \cdot p_2 \cdot p} y_1 \|^p x_2} \qquad \frac{x_1 \xrightarrow{a_1,p_1} y_1 \quad x_2 \xrightarrow{a_2,p_2} y_2}{x_1 \|^p x_2 \xrightarrow{a_2,p_1 \cdot p_2 \cdot (1-p)} x_1 \|^p y_2}$$

Stratum $\mathcal{R}_2$ contains the following rules, for all $a_1 \in \mathit{Act}$:

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1}{x_1 \parallel^p x_2 \xrightarrow{a_1,p_1} y_1 \parallel^p x_2}$$

Stratum $\mathcal{R}_3$ contains the following rules, for all $a_2 \in \mathit{Act}$:

$$\frac{x_2 \xrightarrow{a_2,p_2} y_2}{x_1 \parallel^p x_2 \xrightarrow{a_2,p_2} x_1 \parallel^p y_2}$$

As in Ex. 2, given a term $t_1 \parallel^p t_2$, index p means that, when both t_1 and t_2 can move, t_1 moves with probability p, and t_2 moves with probability $1 - p$.

Let us consider now the synchronous product of PCCS [10, 11].

Example 4 (Synchronous product). Stratum $\mathcal{R}_1$ contains the following rules, for all $a_1, a_2 \in \mathit{Act}$:

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1 \quad x_2 \xrightarrow{a_2,p_2} y_2}{x_1 \parallel x_2 \xrightarrow{a_1 \times a_2, p_1 \cdot p_2} y_1 \parallel y_2}$$

Here, at each computation step, term $t_1 \parallel t_2$ can move only by combining an action of t_1 and an action of t_2. Actions are composed by means of operator $\times$.

Let us consider now the probabilistic version of CCS parallel composition [3].

Example 5 (Interleaving plus synchronization). Let $0 < p, q < 1$. Stratum $\mathcal{R}_1$ contains the following rules, for all $a_1, a_2 \in \mathit{Act}$ such that $a_2 \neq \overline{a_1}$:

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1 \quad x_2 \xrightarrow{a_2,p_2} y_2}{x_1 \parallel^p_q x_2 \xrightarrow{a_1, p_1 \cdot p_2 \cdot p} y_1 \parallel^p_q x_2} \qquad \frac{x_1 \xrightarrow{a_1,p_1} y_1 \quad x_2 \xrightarrow{a_2,p_2} y_2}{x_1 \parallel^p_q x_2 \xrightarrow{a_2, p_1 \cdot p_2 \cdot (1-p)} x_1 \parallel^p_q y_2}$$

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1 \quad x_2 \xrightarrow{\overline{a_1},p_2} y_2}{x_1 \parallel^p_q x_2 \xrightarrow{a_1, p_1 \cdot p_2 \cdot p \cdot (1-q)} y_1 \parallel^p_q x_2} \qquad \frac{x_1 \xrightarrow{a_1,p_1} y_1 \quad x_2 \xrightarrow{\overline{a_1},p_2} y_2}{x_1 \parallel^p_q x_2 \xrightarrow{\overline{a_1}, p_1 \cdot p_2 \cdot (1-p) \cdot (1-q)} x_1 \parallel^p_q y_2}$$

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1 \quad x_2 \xrightarrow{\overline{a_1},p_2} y_2}{x_1 \parallel^p_q x_2 \xrightarrow{\tau, p_1 \cdot p_2 \cdot q} y_1 \parallel^p_q y_2}$$

Stratum $\mathcal{R}_2$ contains the following rules, for all $a_1 \in \mathit{Act}$:

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1}{x_1 \parallel^p_q x_2 \xrightarrow{a_1,p_1} y_1 \parallel^p_q x_2}$$

Stratum $\mathcal{R}_3$ contains the following rules, for all $a_2 \in \mathit{Act}$:

$$\frac{x_2 \xrightarrow{a_2,p_2} y_2}{x_1 \parallel^p_q x_2 \xrightarrow{a_2,p_2} x_1 \parallel^p_q y_2}$$

Let us take $t_1 \|_q^p t_2$. When t_1 and t_2 intend to perform actions a_1 and a_2 with $a_2 \neq \overline{a_1}$, t_1 moves with probability p and t_2 moves with probability $1-p$, as in the case of interleaving operator of Ex. 3. When t_1 and t_2 intend to perform actions a_1 and $\overline{a_1}$, either they synchronize with probability q, thus producing action τ, or they do not synchronize with probability $1-q$. In this second case, t_1 moves with probability $p\cdot(1-q)$, and t_2 moves with probability $(1-p)\cdot(1-q)$.

Let us consider now the operation of sequential composition of terms of [3].

Example 6 (Sequencing). Stratum $\mathcal{R}_1$ contains the following rules, for $a_1 \in Act$:

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1}{x_1 \cdot x_2 \xrightarrow{a_1,p_1} y_1 \cdot x_2}$$

Stratum $\mathcal{R}_2$ contains the following transition rules, for all $a_2 \in Act$:

$$\frac{x_2 \xrightarrow{a_2,p_2} y_2}{x_1 \cdot x_2 \xrightarrow{a_2,p_2} y_2}$$

Let us take $t_1 \cdot t_2$. If t_1 moves, then rules in $\mathcal{R}_1$ can be applied and $t_1 \cdot t_2$ moves as t_1, else, if t_2 moves, rules in $\mathcal{R}_2$ can be applied and $t_1 \cdot t_2$ moves as t_2.

Let us consider now the restriction operation of [2, 8, 9, 11]. This is the first example in which we employ unneeded premises.

Example 7 (Restriction). Let $A \subseteq Act$. Stratum $\mathcal{R}_1$ contains the following rules, for all $a_1 \in Act \setminus A$:

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1 \quad x_1 \xrightarrow{A,p}}{x_1 \backslash A \xrightarrow{a_1,\frac{p_1}{1-p}} y_1 \backslash A}$$

Term $t_1 \backslash A$ behaves as t_1, but it cannot perform actions in A. Let us assume that the sum of the probability of the moves of t_1 with label in A is q, i.e. $t_1 \xrightarrow{A,q}$. If $q = 1$, then no move of $t_1 \backslash A$ can be inferred by the rules in $\mathcal{R}_1$. Hence, $t_1 \backslash A$ has no move and it is semistochastic. If t_1 has a move $t_1 \xrightarrow{a_1,q_1} t_1'$, with $a_1 \notin A$, then $t_1 \backslash A$ has the same move, but with probability $\frac{q_1}{1-q}$, which is the conditional probability that t_1 has the move $t_1 \xrightarrow{a_1,q_1} t_1'$ under the assumption that t_1 is not allowed to perform actions in A. Hence, the sum of the probability of the moves of $t_1 \backslash A$ is $\frac{1-q}{1-q} = 1$, and $t_1 \backslash A$ is stochastic.

Let us consider now the operator of priority. This is the first example in which we employ unquantified premises.

Example 8 (Priority of a over b). Let $a, b \in Act$. Stratum $\mathcal{R}_1$ contains the following rules, for all $a_1 \in Act \setminus \{b\}$:

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1 \quad x_1 \xrightarrow{\{b\},p} \quad x_1 \xrightarrow{\{a\}}}{\vartheta_b^a(x_1) \xrightarrow{a_1,\frac{p_1}{1-p}} \vartheta_b^a(y_1)}$$

Stratum $\mathcal{R}_2$ contains the following rules, for all $a_1 \in Act$:

$$\frac{x_1 \xrightarrow{a_1,p_1} y_1}{\vartheta_b^a(x_1) \xrightarrow{a_1,p_1} \vartheta_b^a(y_1)}$$

Term $\vartheta_b^a(t_1)$ behaves as t_1, but it can perform action b only if it cannot perform a. Rules in $\mathcal{R}_1$ are applied only if t_1 can perform a. In this case, if the sum of the probability of the moves of t_1 labeled b is q (i.e. $t_1 \xrightarrow{\{b\},q}$), then, from any move $t_1 \xrightarrow{a_1,q_1} t_1'$ with $a_1 \neq b$, we infer a move of $\vartheta_b^a(t_1)$ with label a_1 and probability $\frac{q_1}{1-q}$, which is the conditional probability that t_1 has the move $t_1 \xrightarrow{a_1,q_1} t_1'$ under the assumption that t_1 is not allowed to perform b. So, the sum of the probability of the moves of $\vartheta_b^a(t_1)$ is $\frac{1-q}{1-q} = 1$, and $\vartheta_b^a(t_1)$ is stochastic. Rules in $\mathcal{R}_2$ can be applied only if t_1 cannot perform a. In this case, $\vartheta_b^a(t_1)$ behaves as t_1.

5 Results

Theorem 1. *The PTS induced by any PB TSS is semistochastic.*

Proof. We have to prove that, given an arbitrary term t, the sum of the probability of the moves of t is either 0 or 1. This property follows by two facts: 1) The moves of t can be derived only by the rules that are in one stratum $\mathcal{R}_u$; 2) the sum of the probability of the moves of t derivable by the rules in any stratum $\mathcal{R}_u$ is either 0 or 1, as we have proved in the previous section. □

Theorem 2. *The probabilistic bisimulation induced by any PB TSS is a congruence.*

Proof. Let R be the least equivalence relation over PTS states such that:

1. $s\,R\,t$ whenever $s \approx t$;
2. $f(\overrightarrow{s})\,R\,f(\overrightarrow{t})$ whenever $s_1\,R\,t_1, \ldots, s_{ar(f)}\,R\,t_{ar(f)}$.

Lemma 1. *Given a term u over variables $\overrightarrow{x} = x_1, \ldots, x_n$ and tuples of terms $\overrightarrow{s} = s_1, \ldots, s_n$ and $\overrightarrow{t} = t_1, \ldots, t_n$, if $s_i\,R\,t_i$ holds for all $1 \leq i \leq n$, then $u[\overrightarrow{t}/\overrightarrow{x}]\,R\,u[\overrightarrow{s}/\overrightarrow{x}]$.*

To prove the thesis, it suffices to prove that, for arbitrary terms s and t, $s\,R\,t$ implies $s \approx t$. In fact, by the two clauses of the definition of R, this property implies that R and $\approx$ coincide and that $\approx$ is a congruence.

Let us reason by induction over the definition of R. The base case where $s\,R\,t$ is due to $s \approx t$ is immediate. Let us concentrate on the inductive step, where $s \equiv f(\overrightarrow{s})$, $t \equiv f(\overrightarrow{t})$, and $s\,R\,t$ is due to $s_1\,R\,t_1, \ldots, s_{ar(f)}\,R\,t_{ar(f)}$. We can assume, by the inductive hypothesis, that $s_1 \approx t_1, \ldots, s_{ar(f)} \approx t_{ar(f)}$.

We have to prove that, for any value $0 < q \leq 1$, action $a \in Act$ and equivalence class $S \in \mathcal{S}/R$, $\mu(f(\overrightarrow{s}), a, S) = q$ iff $\mu(f(\overrightarrow{t}), a, S) = q$. We prove that $\mu(f(\overrightarrow{s}), a, S) = q$ implies $\mu(f(\overrightarrow{t}), a, S) = q$; the converse is analogous.

Since $\mu(f(\overrightarrow{s}), a, S) = q$, it holds that in some stratum $\mathcal{R}_u$ of the TSS, and for some $k \in \mathbb{N}$, there exist PB transition rules $\rho_1, \ldots, \rho_k$ such that:

1. for all $1 \leq l \leq k$, from rule ρ_l we infer m_l transitions $f(\overrightarrow{s}) \xrightarrow{a,q_{l,1}} u_{l,1}, \ldots, f(\overrightarrow{s}) \xrightarrow{a,q_{l,m_l}} u_{l,m_l}$, for some $m_l \in \mathbb{N}$;
2. $\sum_{1 \leq l \leq k} \sum_{1 \leq i \leq m_l} q_{l,i} = q$;
3. for all $1 \leq l \leq k$, $u_{l,1}, \ldots, u_{l,m_l} \in S$,

and, moreover, no move of $f(\overrightarrow{s})$ is derived from rules in $\mathcal{R}_1, \ldots, \mathcal{R}_{u-1}$.

Let us consider any $1 \leq l \leq k$. Transition rule ρ_l has the form

$$\frac{\{x_i \xrightarrow{a_i,p_i} y_i \,|\, i \in I\} \cup \{x_j \xrightarrow{A_j,p'_j} \,|\, j \in J\} \cup \{x_h \xrightarrow{B_h} \,|\, h \in H\}}{f(\overrightarrow{x}) \xrightarrow{a,\frac{\prod_{i \in I} p_i}{\prod_{j \in J}(1-p'_j)} \cdot w_{\rho_l}} t}$$

Since $f(\overrightarrow{s}) \xrightarrow{a,q_{l,1}} u_{l,1}, \ldots, f(\overrightarrow{s}) \xrightarrow{a,q_{l,m_l}} u_{l,m_l}$ are derived from ρ_l, it holds that:

1. for all $i \in I$, there are states S_i s.t. $\mu(s_i, a_i, S_i) = q_i$, for some $0 < q_i \leq 1$;
2. for all $j \in J$, $s_j \xrightarrow{A_j,q'_j}$, for some $0 \leq q'_j < 1$;
3. for all $h \in H$, $s_h \xrightarrow{B_h}$;
4. $q_{l,1} + \cdots + q_{l,m_l} = w_{\rho_l} \cdot \frac{\prod_{i \in I} q_i}{\prod_{j \in J}(1-q'_j)}$.

By the inductive hypothesis, it follows that:

1. for all $i \in I$, there is a set of states S'_i such that $\mu(t_i, a_i, S'_i) = q_i$ and, for all $s' \in S'_i$,there is some state $s \in S_i$ such that $s \, R \, s'$;
2. for all $j \in J$, $t_j \xrightarrow{A_j,q'_j}$;
3. for all $h \in H$, $t_h \xrightarrow{B_h}$.

Hence, by applying ρ_l, we infer n_l moves $f(\overrightarrow{t}) \xrightarrow{a,q'_{l,1}} v_1, \ldots f(\overrightarrow{t}) \xrightarrow{a,q'_{l,n_l}} v_{n_l}$, for some $n_l \in \mathbb{N}$, where:

1. $v_1, \ldots, v_{n_l} \in S$, by Lemma 1 and the fact that for all $s' \in S'_i$ there is some state $s \in S_i$ such that $s \, R \, s'$;
2. $q'_{l,1} + \cdots + q'_{l,n_l} = q_{l,1} + \cdots + q_{l,m_l}$.

Since these arguments hold for all $1 \leq l \leq k$, it follows that by $\rho_1, \ldots, \rho_k$ we derive $\mu(f(\overrightarrow{t}), a, S) = q$, which implies the thesis. It remains to prove that we can apply $\rho_1, \ldots, \rho_k$, i.e. no move of $f(\overrightarrow{t})$ can be derived by any rule in any stratum $\mathcal{R}_v$ with $v < u$. This follows by the fact that no move of $f(\overrightarrow{s})$ can be derived by any rule in these strata, and that $s_i \approx t_i$ for $1 \leq i \leq ar(f)$. □

6 Extensions

The PB transition rules of Def. 5 extend the rules matching the *de Simone format* [16] with probability, unneeded premises and unquantified premises. Here we show how we can add to our rules some features offered by other formats proposed in the literature of non probabilistic process algebras.

The *GSOS format* [7] admits *negative premises* of the form $x_i \stackrel{a_i}{\not\longrightarrow}$ in rules with source $f(\overrightarrow{x})$, meaning that the i^{th} argument of f does not perform any action labeled a_i. In [19] a result is proved which assesses that negative premises can be simulated by suitable ordering relations between rules. Since the partitioning in strata of Def. 6 introduces ordering relations between PB transition rules that are less general than those used in [19], it would be interesting to extend Def. 6 to capture all the ordering relations of [19].

The GSOS format admits also *double testing*. Namely, rules with source $f(\overrightarrow{x})$ can have two (or more) premises $x_i \xrightarrow{a_{i_1}} y_{i_1}$ and $x_i \xrightarrow{a_{i_2}} y_{i_2}$ with the same variable x_i in the left side. Let us show how we can add double testing to our rules.

Definition 9. *A* PB transition rule with double testing ρ *is of the form*

$$\frac{\{x_i \xrightarrow{a_{i_l},p_{i_l}} y_{i_l} \mid i \in I, l \in I_i\} \cup \{x_j \xrightarrow{A_j,p'_j} \mid j \in J\} \cup \{x_h \xrightarrow{B_h} \mid h \in H\}}{f(\overrightarrow{x}) \xrightarrow{a,\frac{\prod_{i\in I}\sum_{l\in I_i} p_{i_l}}{\prod_{j\in J}(1-p'_j)}\cdot w_\rho} t}$$

where:

1. *clauses 1-6 of Def. 5 are respected;*
2. *for all $i \in I$, it holds that $a_{i_l} \neq a_{i_{l'}}$ for all $l, l' \in I_i$ such that $l \neq l'$;*
3. *for all $i \in I$ and $l \in I_i$, if $|I_i| > 1$ then there is an $h = i$ such that $a_{i_l} \in B_h$.*

Definition 10. *A* PB TSS with double testing *is defined as in Def. 6, except that clause 2d is replaced by the following clause:*

– *Given actions $\{a'_i \mid i \in I\}$ such that $a'_i \notin A_j$ for all indexes i and j with $i = j$ and $x_j \xrightarrow{A_j,p'_j}$ an unneeded premise, then there at least one PB transition rule with source $f(\overrightarrow{x})$ in $\mathcal{R}_u$* containing *the active premises $\{x_i \xrightarrow{a'_i,p_i} y_i \mid i \in I\}$.*

To explain clause 2 in Def. 9, let us take the following rule, which violates it:

$$\frac{x_1 \xrightarrow{a,p_1} y_1 \quad x_1 \xrightarrow{a,p_2} y_2}{f(x_1) \xrightarrow{b,p_1+p_2} 0}$$

Let t be the PCCS term $a{\cdot}0$, which has the move $t \xrightarrow{a,1} 0$. It holds that $f(t) \xrightarrow{b,2} 0$, and, therefore, $f(t)$ is not semistochastic. The problem is that the probability of the same move of t is summed twice when computing the probability of the move of $f(t)$. Clause 2 in Def. 9 prevents this problem, since different moves of the same argument of f can appear as premises only if they have different labels.

To explain clause 3 in Def. 9, let us take the following rules, and note that the first one violates it:

$$\frac{x_1 \xrightarrow{a,p_1} y_1 \quad x_1 \xrightarrow{b,p_2} y_2}{f(x_1) \xrightarrow{d,p_1+p_2} 0} \qquad \frac{x_1 \xrightarrow{c,p_1} y_1}{f(x_1) \xrightarrow{e,p_1} 0}$$

Let t be the PCCS term $a \cdot 0 +^{\frac{1}{2}} c \cdot 0$, which has the moves $t \xrightarrow{a,\frac{1}{2}} 0$ and $t \xrightarrow{c,\frac{1}{2}} 0$. It holds that $f(t) \xrightarrow{e,\frac{1}{2}} 0$ is the only move of $f(t)$, which, therefore, is not semistochastic. The problem is that the probability of the move of t labeled a does not contribute in computing the probability of any move of $f(t)$, since t has no move labeled b and the premise $x_1 \xrightarrow{a,p_1} y_1$ appears only in the rule where there is also the premise $x_1 \xrightarrow{b,p_2} y_2$. Clause 3 in Def. 9 prevents this problem, since premises $x_1 \xrightarrow{a,p_1} y_1$ and $x_1 \xrightarrow{b,p_2} y_2$ are admitted only in rules that are in strata where all rules have an unquantified premise $x_1 \xrightarrow{B}$ with $a, b \in B$.

Finally, notice that the new clause of Def. 10 requires that at least one rule in $\mathcal{R}_u$ *contains* the premises $\{x_i \xrightarrow{a'_i,p_i} y_i \mid i \in I\}$, whereas the corresponding clause in Def. 6 requires that at least one rule in $\mathcal{R}_u$ *has exactly* the premises $\{x_i \xrightarrow{a'_i,p_i} y_i \mid i \in I\}$. The new clause allows double testing.

Theorem 3. *The PTS induced by any PB TSS with double testing is semistochastic. The probabilistic bisimulation induced by any PB TSS with double testing is a congruence.*

The *tyxt/tyft format* [12] admits *look ahead*. Namely, transition rules with source $f(\overrightarrow{x})$ can have premises $x_i \xrightarrow{a_i} y_i$ and $y_i \xrightarrow{b_i} z_i$, with the same variable y_i appearing in the right side of the first premise and in the left side of the second premise. Let us show how we can add look ahead to our PB TSSs.

Definition 11. *A* PB transition rule with look ahead ρ *is of the form*

$$\frac{\{x_i \xrightarrow{a_i,p_i} y_i | i \in I\} \cup \{y_i \xrightarrow{b_i,r_i} z_i | i \in I'\} \cup \{x_j \xrightarrow{A_j,p'_j} | j \in J\} \cup \{x_h \xrightarrow{B_h} | h \in H\}}{f(\overrightarrow{x}) \xrightarrow{a,\frac{\prod_{i\in I\setminus I'} p_i \cdot \prod_{i \in I'} p_i \cdot r_i}{\prod_{j\in J}(1-p'_j)} \cdot w_\rho} t}$$

where:

1. *clauses 1-6 of Def. 5 are respected;*
2. $I' \subseteq I$.

Also variables y_i with $i \in I'$ are called active variables.

Definition 12. *A* PB TSS with look ahead *is defined as in Def. 6, except that clauses 2c and 2d are replaced by the following clauses:*

1. *All PB transition rules with source $f(\overrightarrow{x})$ in stratum $\mathcal{R}_u$ have the same set of active variables $\{x_i \mid i \in I\} \cup \{y_i \mid i \in I'\}$;*
2. *Given actions $\{a'_i \mid i \in I\}$ such that $a'_i \notin A_j$ for all indexes i and j with $i = j$ and $x_j \xrightarrow{A_j, p'_j}$ an unneeded premise, and actions b'_i for all indexes $i \in I'$, then there is at least one PB transition rule with source $f(\overrightarrow{x})$ in $\mathcal{R}_u$ with active premises $\{x_i \xrightarrow{a'_i, p_i} y_i \mid i \in I\} \cup \{y_i \xrightarrow{b'_i, r_i} z_i \mid i \in I'\}$.*

The new clauses in Deff. 11–12 extend clauses in Deff. 5–6 to take into account that two consecutive moves of x_i are considered for all $i \in I'$.

Theorem 4. *The PTS induced by any PB TSS with look ahead is semistochastic. The probabilistic bisimulation induced by any PB TSS with look ahead is a congruence.*

Definitions of PB transition rule and PB TSS admitting both double testing and look ahead could be given immediately. By combining results of Thm. 3 and Thm. 4 we infer that the PB TSSs so obtained would induce semistochastic PTSs and probabilistic bisimulations being congruences.

Both *path format* [4] and *panth format* [20] admit *predicates*, i.e. transitions of the form $t\,P$, meaning that term t satisfies some property expressed by P. Since predicates have nothing to do with probability, they can be added to PB transitions rules and PB TSSs, without affecting results in Thm. 1 and Thm. 2.

7 Related and Future Work

In this paper we have proposed a rule format for probabilistic process algebras. We believe that our format has four main merits: 1) probabilistic bisimulation is a congruence w.r.t. process algebra operations respecting the format; 2) semistochasticity is preserved by process algebra operations respecting the format; 3) the main operations offered by the probabilistic process algebras studied in the literature are captured by the format, which, therefore, has practical applications; 4) features offered by known rule formats proposed for classic process algebras, such as look ahead and double testing, are offered by the format.

Now, let us recall that in [6] a rule format for probabilistic process algebras has been already proposed. The first difference between our paper and [6] is that we consider the generative model of probabilistic processes, whereas [6] considers the reactive model. Then, our definition of TSS requires some conditions (i.e. clauses 2c–2e in Def. 6) that guarantee semistochasticity. In [6] no syntactic constraint on transition rules guarantees semistochasticity of reactive processes, i.e. the property that the sum of the probability of the moves of any process *for the same label* is either 0 or 1. Hence, in [6] semistochasticity is not ensured by the format. In [6] neither unquantified premises nor unneeded premises nor stratification are considered. We need these features to express operations requiring redistribution of probability, such as restriction (see Ex. 7) and priority (see Ex.

8). In the reactive model restriction and priority do not require redistribution of probability, and, therefore, they can be expressed with the format in [6]. Problems in [6] arise in other operations requiring redistribution of probability, such as the relabeling operation $t[f]$, where $f : Act \longrightarrow Act$ is a relabeling functions.

Our results can be extended in several directions. We aim to develop a rule format for the reactive model of probabilistic processes that guarantees results analogous to those obtained in the present paper, i.e. bisimulation being a congruence, operations preserving semistochasticity, expressiveness. Moreover, we aim to develop rule formats for other behavioral equivalences, such as probabilistic weak bisimulation [5], and probabilistic testing equivalence [21]. Finally, we aim to develop rule formats guaranteeing that security properties for probabilistic processes, such as those defined in [2], are respected by process algebra operations, on the same line followed in [17, 18] for classic process algebras.

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