Chapter 8
Cascading Failures in Interdependent Economic Networks

Shlomo Havlin and Dror Y. Kenett

Abstract Throughout the past decade, there has been a significant advance in understanding the structure and function of networks, and mathematical models of networks are now widely used to describe a broad range of complex systems, such as socio-economic systems. However, the significant majority of methods have dealt almost exclusively with individual networks treated as isolated systems. In reality an individual network is often just one component in a much larger complex multi-level network (network of networks, NON). The NON framework provides critical new insights into the structure and function of real-world complex systems. One such insight is that NON system is significantly more vulnerable to shocks and damages, which has lead to the development of the theory of cascading failures in interdependent networks. Here we provide an overview of this theory, and one example of its application to economic systems.

8.1 Introduction

The growth of technology, globalization, and urbanization has caused world-wide human social and economic activities to become increasingly interdependent [1–13]. From the recent financial crisis it is clear that components of this complex system have become increasingly susceptible to collapse. Current models have been unable to predict instability, provide scenarios for future stability, or control or even mitigate systemic failure. Thus, there is a need of new ways of quantifying complex system vulnerabilities as well as new strategies for mitigating systemic damage and increasing system resiliency [14, 15]. Achieving this would also provide new insight into such key issues as financial contagion [16, 17] and systemic risk [18–20] and would provide a way of maintaining economic and financial stability in the future.

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Throughout the past decade, there has been a significant advance in understanding the structure and function of networks, and mathematical models of networks are now widely used to describe a broad range of complex systems, from techno-social systems to interactions amongst proteins [21–32]. However, the significant majority of methods have dealt almost exclusively with individual networks treated as isolated systems. In reality an individual network is often just one component in a much larger complex multi-level network (network of networks). As technology has advanced, the coupling between networks is becoming stronger and stronger. For example, there is a strong coupling between human mobility (which can be tracked by mobile networks) and transport networks. In these interdependent networks, the failures of nodes in one network will cause failures of dependent nodes in other networks, and vice-versa [33–41]. This process happens recursively, and leads to a cascade of failures in the network of networks system. As in physics, when only the individual particles were studied it was made possible to understand the properties of gas; however, when the transition was made to study the interactions between these particles, it was finally made possible to understand and describe liquids and solids, as well as the concept of phase transitions. Such a development in network science has led to a significant paradigm shift, which has opened the door to the understanding of a multitude of new features and phenomena (see schematic overview in Fig. 8.1). Here we will review the theory of cascading failures in interdependent networks, and present one application in economic networks.

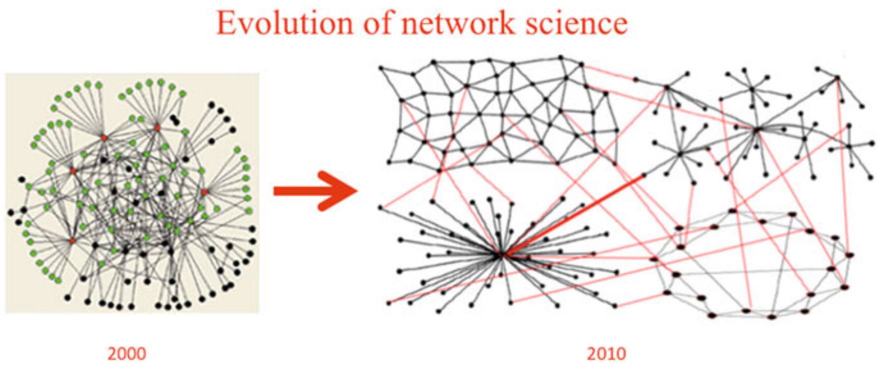

Fig. 8.1 Schematic representation of the scope of network science research from the beginning of the twenty first century, from focusing on the case of a single network, to the case of interconnected and interdependent networks. The *black links* represent connectivity links while the *red links* are dependency links. The concept of dependency links and the generalization of percolation theory to include such links was first introduced in Buldyrev et al. [33]

8.2 Overview of Cascading Failure Processes in Interdependent Networks

The theory for cascading failures in interdependent networks was introduced in [33, 34, 36, 37, 42, 43], and we review it shortly in this section. In order to model interdependent networks, consider two networks, A and B, in which the functionality of a node in network A is dependent upon the functionality of one or more nodes in network B (see Fig. 8.2), and vice-versa: the functionality of a node in network B is dependent upon the functionality of one or more nodes in network A. The networks can be interconnected in several ways. In the most general case we specify a number of links that arbitrarily connect pairs of nodes across networks A and B. The direction of a link specifies the dependency of the nodes it connects, i.e., link $A_i \rightarrow B_j$ provides a critical resource from node A_i to node B_j. If node A_i stops functioning due to attack or failure, node B_j stops functioning as well but not vice-versa. Analogously, link $B_i \rightarrow A_j$ provides a critical resource from node B_i to node A_j.

To study the robustness of interdependent networks systems, we begin by removing a fraction $1-p$ of network A nodes and all the A-edges connected to these nodes. As an outcome, all the nodes in network B that are connected to the removed A-nodes by $A \rightarrow B$ links are also removed since they depend on the removed nodes in network A. Their B edges are also removed. Further, the removed B nodes will cause the removal of additional nodes in network A which are connected to the removed B-nodes by $B \rightarrow A$ links. As a result, a cascade of failures that eliminates virtually all nodes in both networks can occur. As nodes and edges are removed, each network breaks up into connected components (clusters). The clusters in network A (connected by A-edges) and the clusters in network B (connected by B-edges) are different since the networks are each connected differently. If one assumes that small clusters (whose size is below certain threshold) become non-functional, this may invoke a recursive process of failures that we now formally describe.

The insight based on percolation theory is that when the network is fragmented the nodes belonging to the giant component connecting a finite fraction of the network are still functional, but the nodes that are part of the remaining small

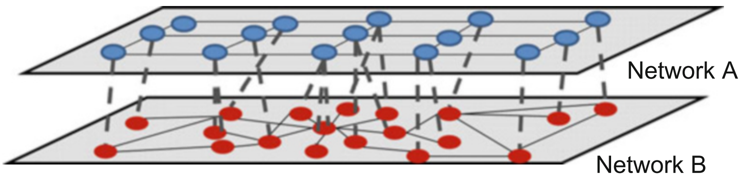

Fig. 8.2 Example of two interdependent networks. Nodes in network B (e.g. communications network) are dependent on nodes in network A (e.g. power grid) for power; nodes in network A are dependent on network B for control information

clusters become non-functional. Thus in interdependent networks only the giant mutually-connected cluster is of interest. Unlike clusters in regular percolation whose size distribution is a power law with a p-dependent cutoff, at the final stage of the cascading failure process just described only a large number of small mutual clusters and one giant mutual cluster are evident. This is the case because the probability that two nodes that are connected by an A-link and their corresponding two nodes are also connected by a B-link scales as $1/N_B$, where N_B is the number of nodes in network B. So the centrality of the giant mutually-connected cluster emerges naturally and the mutual giant component plays a prominent role in the functioning of interdependent networks. When it exists, the networks preserve their functionality, and when it does not exist, the networks split into fragments so small they cannot function on their own. In Fig. 8.3 we present a schematic representation of an example of a tree-like network of networks, composed of five networks. The cascading failure process is applied by removing $1-p$ nodes, and calculating the size of the mutual giant component, P_∞. We present (Fig. 8.2) a comparison between P_∞ of $n = 1, 2, 5$ networks, and show that the network of networks system is more vulnerable to cascading failures. Finally, we show (Fig. 8.2) the analytical

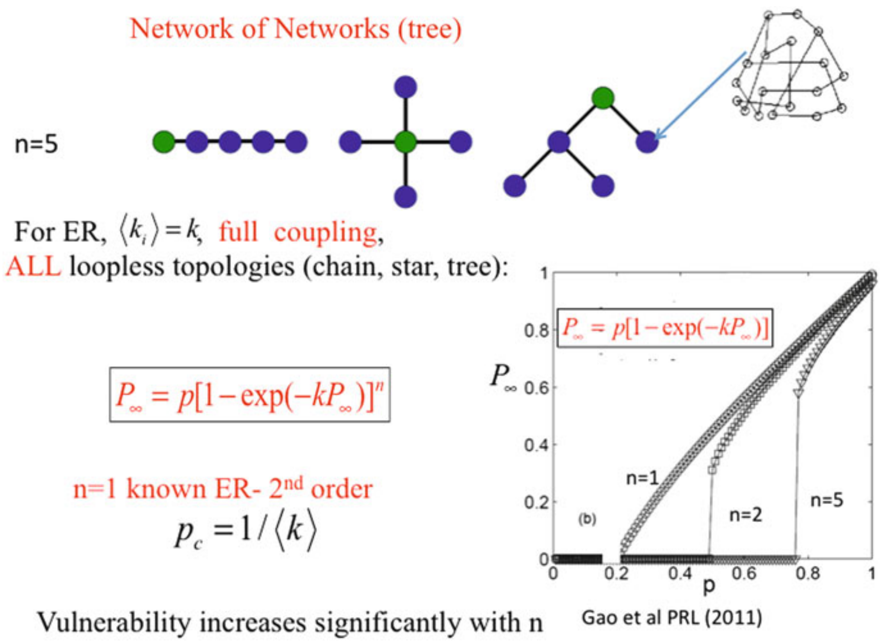

Fig. 8.3 Schematic representation of an example of a tree-like network of networks, composed of 5 networks. The cascading failure process is applied by removing a fraction $1 - p$ nodes, and calculating the size of the mutual giant component, P_∞. We present a comparison between P_∞ of $n = 1, 2, 5$ networks, and show that the network of networks system is more vulnerable to cascading failures. Finally, we show the analytical relationship between P_∞, n, k and p, which for the case of one network collapses to the well known ER formalism

relationship between P_∞, n, k and p, which for the case of a single network ($n = 1$) collapses to the well known ER formalism [24].

8.3 Cascading Failures in Economic Networks

Network science has greatly evolved in the twenty first century, and is currently a leading scientific field in the description of complex systems, which affects every aspect of our daily life [2, 22–25]. Network theory provides the means to model the functional structure of different spheres of interest, and thus, understanding more accurately the functioning of the network of relationships between the actors of the system, its dynamics and the scope or degree of influence. In addition, it measures systemic qualities, e.g., the robustness of the system to specific scenarios, or the impact of policy on system actions. The advantage offered by the network science approach is that instead of assuming the behavior of the agents of the system, it rises empirically from the relationships that they really hold; hence, the resulting structures are not biased by theoretical perspectives or normative approaches imposed 'by the eye of the researcher'. On the contrary, the modeling by network theory could validate behavioral assumptions by economic theories. Network theory can be of interest to various edges of the financial world: the description of systemic structure, analysis and evaluation of contagion effects, resilience of the financial system, flow of information, and the study of different policy and regulation scenarios, to name a few [44–57]. Once the network structure and topology is uncovered, it is possible to test many features of the economic system. One critical issue is the resilience of economic and financial systems to shock scenarios, which is commonly investigated using stress tests [58–61]. Cascading failure processes can be applied to study the stability of economic and financial systems, and uncover global and local vulnerabilities to the system. Here, we review a recent application of the theory of cascading failures in interdependent economic systems to quantify and rank the economic influence of specific industries and countries, which was recently introduced by Li et al. [51].

Li et al [51] have examined the interdependent nature of economies between and within 14 countries and the rest of the world (ROW), using input-output table [62] during the period 1995–2011. The economic activity in each country is divided into 35 industrial classifications. Each cell in the table shows the output composition of each industry to all other 525 industries and its final demand and export to the rest of the world (see [63]). From the IO table, an output network is constructed using the 525 industries as nodes and the output product values as weighted links based on the input-output table. The goal of this work is to introduce a methodology for quantifying the importance of a given industry in a given country to global economic stability with respect to other industries in countries that are related to this industry. The authors use the theory of cascading failures in interdependent networks to gain valuable information on the local and global influence on global stability of different economic industries.

In order to identify and rank the influence of industries in the stability of this global network, the authors perform a cascading failure tolerance analysis [33, 51]. The model can be described as follows. Suppose industry A fails, other industries can no longer sell their products to industry A and thus they lose that revenue. The revenue of each industry is reduced by a fraction p', which for each industry is defined as the revenue reduction caused by the failure of industry A divided by that industry's total revenue. The tolerance fraction ϕ is the threshold above which an industry fails. This occurs when reduced revenue fraction p' is larger than tolerance fraction ϕ. Here we assume that (i) ϕ is the same for all industries and that every industry fails when its $p' > \phi$ and (ii) the failure of an industry in country A does not reduce the revenue of the other industries in the same country A because they are able to quickly adjust to the change. The methodology can be schematically illustrated as follows (see Fig. 8.4): In step 1, industry A in country i fails. This causes other industries in other countries to fail if their $p' > \phi$. Assume that in step 2 industries B, C, and D fail. The failure of these industries in step 2 will reduce other industries' revenue and cause more industries including those in country i to have a reduced fraction p'. Thus in step 3 there is an increased number of industries whose $p' > \phi$. Eventually the system reaches a steady state in which no more industries

Fig. 8.4 Schematic representation of each step in the cascading failure propagation in the world economic network (**a**⟶**b**⟶**c**⟶**d**). We present an example of two countries, where *circle* nodes represent country 1, and *triangles* represent country 2. Both countries have the same industries, and the *arrow* between two nodes points in the direction of money flow. The different subpanels demonstrate the cascade of the damage, after an initial failure in electrical equipment industry in Country 1 (*circle*) which causes a failure of electrical equipment industry in Country 2, which cascades into other industries. After [51]

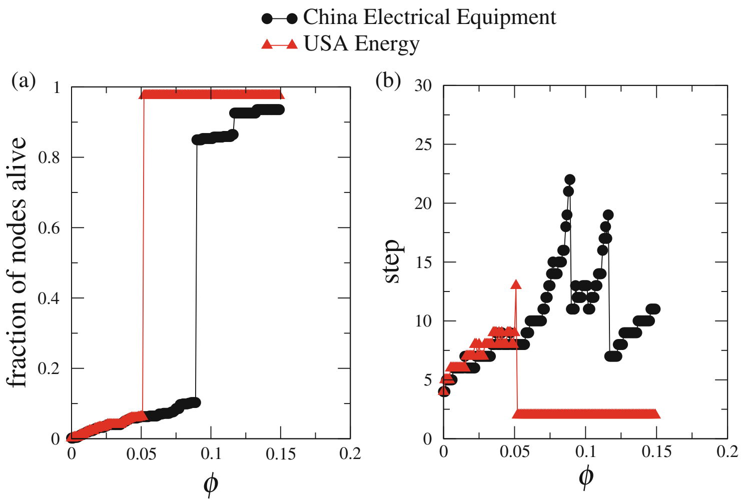

Fig. 8.5 Typical examples of industry tolerance threshold ϕ_c. (**a**)(*left*) the *black curve* shows the fraction of surviving industries as a function of tolerance threshold for the case when the electrical equipment industry in China fails in year 2009 and the *red curve* represents the case of the failure of the energy industry in 2009 in the USA. (**b**)(*right*) Number of failure steps as a function of p. The total number of steps is the number of cascades it takes for the network to reach a steady state after certain initial failure. After [51]

fail. The surviving industries will all have a reduced revenue fraction that is smaller than the tolerance fraction, i.e., $p' \leq \phi$.

Figure 8.5 shows an example of the failures of electric equipment industry in China and the energy industry in the US for the 2009 WIOT and shows the fraction of the largest cluster of connected industries as a function of the tolerance fraction ϕ after the Chinese electric equipment industry becomes malfunction and is removed from the network due to a large shock to the industry. The shock could result from different causes, such as natural environmental disaster, government policy changes, insufficient financial capability. The removal of China electric equipment industry will cause revenue reduction in other industries because China electric equipment industry is not able to buy products and provide money to other industries. When ϕ is small, the industries are fragile and sensitive to the revenue reduction, causing most of the industries fail, and the number of the surviving industries is very small. When ϕ is large, the industries can tolerate large revenue reduction and are more robust when revenue decreases. The number of the surviving industries tends to increase abruptly at a certain $\phi = \phi_c$ value as ϕ increases. Figure 8.5b shows the number of steps that elapse before a stable state is reached as a function of tolerance fraction ϕ after removing the Chinese electric equipment industry or the US energy industry. The number of steps reaches a peak when ϕ approaches criticality ϕ_c [64].

Finally, Li et al. [51] use the cascading failure methodology to rank the economic importance of individual countries, and track how it evolves in time. Figure 8.6 (left)

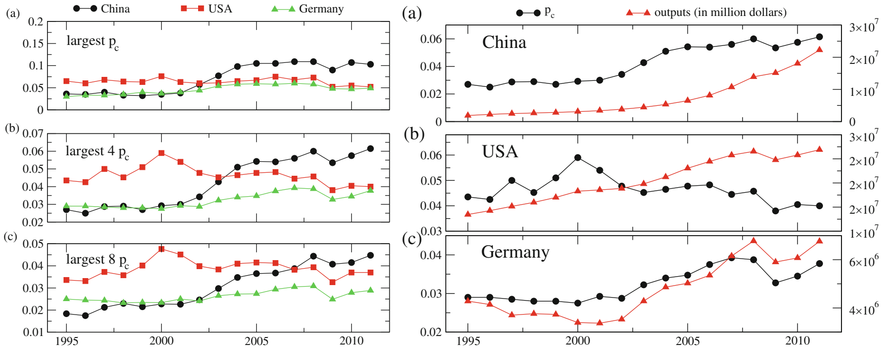

Fig. 8.6 (*left*) Tolerance ϕ_c changes of China, the USA and Germany for 17 years: *top*—the largest tolerance ϕ_c; *middle*—the average of 4 largest ϕ_c; and *bottom*—the average of 8 largest p_c in each country. These results show that the economic importance of China is increasing, while that of the USA is decreasing. (*right*) Tolerance ϕ_c of China, the USA and Germany comparing to the total product output value. For each country, the ϕ_c is an average of the largest four industry ϕ_c of this country (*black circles*). The product output (*red triangle*) value is the money flow a country supplies to the rest of the countries, which also indicates its impact to foreign countries. After [51]

shows the average of ϕ_c of country for the 17-year period investigated, for the case of China, USA and Germany: top—the largest tolerance ϕ_c; middle—the average of four largest industries ϕ_c; and bottom—the average of 8 largest ϕ_c in each country. The results of Li et al [51] present how the economic importance of China relative to that of the USA shows a consistent increase from year to year, illustrating how the economic power structure in the world's economy has been changing during time. Finally, to further validate these results, the total product output (see Fig. 8.6 (right), red triangles) and average tolerance ϕ_c (see Fig. 8.6 (right), black circles) for China, USA, and Germany, as a function of time. The product output value is the total money flow a country supplies to the other countries plus value added in the products, which also indicates its total trade impact on foreign countries.

8.4 Summary

In summary, this paper presents a review of the recently-introduced mathematical framework of for cascading failures in a Network of Networks (NON), particularly in economic NON. In interacting networks, when a node in one network fails it usually causes dependent nodes in other networks to fail which, in turn, may cause further damage in the first network and result in a cascade of failures with catastrophic consequences. This analytical framework enables to follow the dynamic process of the cascading failures step-by-step and to derive steady state solutions [65–67]. This formalism provides critical new information on the resilience and vulnerabilities of real world complex systems, such as economic and financial systems. In economics, some key applications include new stress test tools, such as those presented by Li et al. [51] and Levy-Carciente et al. [61]. Furthermore, these developed tools can be used to introduce intervention strategies in order to manage and mitigate once a cascade of failures is set off in the system (see for example [68]).

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