Mathematical Knowledge for Teaching at the Secondary Level

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Overview

TSG 24 at ICME-12 aimed to especially examine current scholarship and research on mathematical knowledge for teaching at the secondary level by collecting, comparing and discussing research experiences in this area, through the following three questions: What mathematical knowledge is needed for teaching at secondary level? What are the status quo of knowing and using mathematical knowledge for teaching at secondary level? How should we move forward (or what we have done) towards better equipped with mathematical knowledge for teaching at secondary level? In ICME 12, TSG 24 gathered 23 oral presentations from Canada, China, Finland, France, India, Ireland, Korea, Norway, South Africa, Spain, Sweden, and Turkey. They were presented in terms of four subtopics.

Organizers Co-chairs: Aihui Peng (China), Hikma Smida (Tunisia); Team Members: Hakan Sollervall (Sweden), Dongwon Kim (Korea), Karin Brodie (South Africa); Liaison IPC Member: Mercy Kazima (Malawi).

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Theoretical Perspective and Conceptual Framework for Mathematical Knowledge for Teaching at Secondary Level

The first presentation entitled "secondary school teachers' mathematical problem-solving knowledge for teaching" was presented by Olive Chapman (Canada). The study identified the nature of mathematical problem-solving knowledge for teaching and how this knowledge could support students' development of proficiency in problem-solving, which has significant implications for teacher education. In particular, the author discussed what should teachers know to teach for problem-solving proficiency and what knowledge should teachers hold to help students to become proficient in problem solving. These questions were addressed from a theoretical perspective and from a study that investigated secondary school teachers' knowledge in terms of their conceptions and teaching of problem solving in relation to contextual problems.

The second presentation "the ladder of knowledge: A model of knowledge for second level mathematics teachers" was presented by Niamh O'Meara (Ireland). In this study, the authors developed a new model of knowledge to meet the needs of curricula with a strong focus on mathematical applications.

The third presentation "competence in didactic analysis in the pre-service training of secondary school mathematics teachers in Spain" was presented by Vincent Font (Spain). The study illustrated how one of the components of the broad competence in didactic analysis (identifying potential improvements to be implemented in future classes) was developed within the context of the University of Barcelona.

The fourth presentation "coordinating theories to analyze the relationship between teachers' actions and teachers' knowledge—a presentation of a methodological approach" was presented by Erika Stadler (Sweden). The study presented a tentative methodological framework to analyze what kind of mathematical knowledge for teaching, MKT, novice mathematics teachers use when teaching. The main idea of the framework is to coordinate three different theoretical frameworks, which provide a methodological tool for analyzing the relationship between teachers' teaching actions and mathematical knowledge.

The fifth presentation "the structure of knowledge of teaching of student teachers on the topic of distance formula" was presented by Lin Ding (China). The presentation provided a new approach of interpreting knowledge of teaching (KOT) of secondary mathematics student teachers by examining its structure (i.e. mathematics, student and pedagogy). A brief analysis on two examples regarding the structure of KOT was provided in order to illustrate how this approach works.

The sixth presentation "A pre-analysis of the creation of teacher's resources for developing instruction in basic logic in French high schools" was presented by Zoe Mesnil (France). The author presented studies on the role of logic in mathematics education in order to show how it can help students to improve their skills in

language and expression. Through the analysis of curricula and textbooks, the study presented an overview of the process of didactic transposition for teaching the concepts of logic.

Pre-service Mathematics Teachers' Knowledge

This subtopic consists of five presentations from USA, Ireland and Turkey. The first presentation "secondary teacher candidates' mathematical knowledge for teaching as demonstrated in their portfolios" was presented by Hari Koirala (USA). Their study focused on prospective secondary school teachers' mathematical knowledge and their ability to demonstrate how their learning of mathematics from their university courses applies to the teaching of secondary school mathematics.

The second presentation "Chinese and US pre-service mathematics teachers' knowledge for teaching algebra with a focus on representational flexibility" was presented by Rongjin Huang (USA). Their study examined Chinese and U.S prospective middle grade teachers' knowledge of algebra for teaching with a focus on representational flexibility. It was found that the Chinese participants not only demonstrated sound knowledge needed for teaching the concept of function, but also had the flexibility in using representations appropriately. In contrast, the U.S. counterparts showed their weakness of using these concepts to solve problems and using appropriate representations.

The third presentation "whose fault is it anyway? The truth about the mathematical knowledge of prospective secondary school teachers and the role of mathematics teacher educators" was presented by Miriam Liston (Ireland). The author presented an empirical research study which aims to contribute to the understanding of prospective secondary level mathematics teachers' mathematical knowledge for teaching. The findings suggest that prospective mathematics teachers may not have sufficient subject matter knowledge to alter their teaching strategies and ultimately teach for understanding.

The fourth presentation is "pre-service secondary school mathematics teachers' specialized content knowledge of complex numbers" presented by Fatma Aslan (Turkey). The author reported the findings of a study of pre-service secondary school mathematics teachers' learning of complex numbers during a content course. According to the author's findings, participants were able to build connections between their mathematical understanding as teachers with their teaching practice and students' mathematical ideas.

The fifth presentation "a comparative analysis of the content knowledge for secondary pre-service mathematics teachers" was presented by Wei Sun (USA). In his study, it focused on the knowledge that the secondary pre-service teachers gain during their study in the teacher education program. Two mathematics teacher preparation programs were examined, one from China and the other from the US,

with the intent to shed light on this important issue and help mathematics educations understand mathematics teacher education from a broader (international) perspective.

In-service Mathematics Teachers' Knowledge

The first presentation "seeing mathematics through processes and actions: investigating teachers' mathematical knowledge and secondary school classroom opportunities for students" was presented by Rose Mary Zbiek (USA). The study described the processes and actions approach. The authors proposed a more general way to characterize MKT than is typically used.

The second presentation "what is pre-service and in-service Teachers' MKT in concept of vector" presented by Hyunkyoung Yoon (Korea) was to investigate the mathematical knowledge for teaching (MKT) of pre-service and in-service mathematics teachers on the concept of vector. 80 pre-service and 124 in-service mathematics teachers were asked to perform three questions based on MKT's subdomain. The results show that pre-service teachers have stronger common content knowledge. On the other hand, in-service teachers have stronger specialized content knowledge, knowledge of content and teaching.

The third presentation "pedagogical knowledge, pedagogical content knowledge, and content knowledge for teaching mathematics: how do they shape teaching practices?" was presented by Hee-Jeong Kim (USA). This empirical study offered a case of a proficient middle school mathematics teacher, well known as a highly skilled teacher in her district, and explored the teacher's decision-making in different teaching contexts. The author discussed what the contributions of different kinds of knowledge were and implied how we can support teachers with regard to knowledge for better mathematics teaching.

The fourth presentation "hypothetical teaching trajectories (HTT): analysing contingency events in secondary mathematics teachers' practice" was presented by Jordi Deulofeu (Spain). This paper showed through the work done by a future secondary mathematics teacher called Gabriel in his initial training at the university, how analyzing HTT can serve a double role: giving information about the prospective teacher's mathematical knowledge and helping to validate an instrument that serves teachers to reflect on their own mathematical knowledge in practice.

The fifth presentation "developing craft knowledge in mathematics teaching" was presented by Inger Nergaard (Norway). Her study focused on teachers' opportunities to develop craft knowledge through their engagement with students. Using video recordings of mathematics lessons and following up conversation with the teachers, two episodes of teaching were considered. In the first episode the teacher appears to close down opportunities for discussion of the unanticipated situations that arose and thus she denied herself opportunities to learn from the situation, while the second episode concerned a teacher who invites students into her teaching and thus enable further development of existing knowledge.

The sixth presentation "understanding teachers' knowledge of and responses to students' mathematical thinking" was presented by Shikha Takker (India). She reported a case study which aimed at understanding teachers' knowledge about students' mathematical thinking in situ. Teacher's response to students' mathematical thinking was characterized based on classroom observations, task-based interviews, complemented with the anticipation and reflection of students' responses to 'proportion' problems. It was found that such a framework helps in creation of conflict in the teacher and is a potential source of teacher reflection.

Methodology Issues on Mathematics Teachers' Knowledge

The first presentation "using scenarios validated as measures to explore subject matter knowledge (SMK) in an interview setting" was presented by Sitti Patahuddin (South Africa). The presentation focused on one scenario adapted from LMT (e.g. from the Learning Mathematics for Teaching—LMT—project) in order to explore how teachers interviewed engage with each of the responses offered.

The second presentation "instruments for improving teachers' use of artifacts for the learning of mathematics" was presented by Håkan Sollervall (Sweden). The author argued that teachers' mathematical knowledge has to include instruments for controlling how the artifact used become involved when students engage in solving mathematical tasks. The authors proposed to meet this demand by coordinating the matching notions of affordances (planning) and objects of activity (evaluation). They briefly illustrated how these notions can be used as analytical instruments in a fashion that connects to what teachers already do in their daily work.

The third presentation "exploring the influence of teachers' use of representation on students' learning of mathematics" was presented by Emmanuel Bofah (Finland). The aim of the study was to examine how teachers' use of different mathematics representations, in the domain of functions, affects students' behavior in the process of doing and learning mathematics.

The fourth presentation "consensuating the best profile of a mathematics teacher in the transition to secondary school; a discussion of experts using the Delphi method" was presented by Sainza Fernandez (Spain). This on-going investigation, embedded in a larger project that targets primary-secondary transition in mathematics, explored the knowledge of a group of expert mathematics teachers and experts involved in teachers' education using the Delphi method. The results arisen point at secondary teachers as more responsible for the success or failure of the process and their sensitivity as professionals of mathematics education as particularly determinant.

The fifth presentation is "a case study on the status quo of the development of Tibetan mathematics teacher's pedagogical content knowledge (PCK) in Lasha" (China). A case study was used to analyze the development of Tibetan mathematics teacher's PCK in secondary school in Lasha.

The sixth presentation "The project: collaborating to advance secondary teachers' mathematics proficiency for teaching" was presented by Pier Junior Clark (USA). Using the *Provisional Framework for Proficiency in Teaching Mathematics* as a guideline, the author examined the changes in the secondary teachers' mathematics proficiency and efficacy for teaching data analysis and statistics over a year-long professional development project.

Summary

TSG24 included presentations from many points of view:

Conceptual frameworks for mathematical knowledge for teaching at secondary level, e.g., what is the nature of mathematical knowledge for teaching at secondary level? What mathematical knowledge needs to know and how to use it from an advanced perspective for a secondary school teacher? What are the approaches, from the practice point of view, that could support teachers developing their mathematical knowledge that they need to know and know how to use it?

Empirical researches that aim to contribute our understanding of what mathematical knowledge is needed or how it is assessed in different scenarios, e.g., teachers' mathematical knowledge for teaching in specific activities, teachers' mathematical knowledge for teaching in specific domain, teachers' mathematical knowledge for teaching in special situations, such as information and communication technology environment, innovative and creative approaches of developing mathematical knowledge and the instruments for assessing these approaches specifically.

Empirical researches to explore relationships between teachers' learning of teaching (both pre-service and in-service) and students' learning of mathematics, e.g., the effect of mathematics knowledge for teaching on student achievement, the innovative and creative approaches of developing the effect of mathematics knowledge for teaching on students' learning and achievement.