# Mathematical Problem Solving

**Manuel Santos-Trigo and Zahra Gooya**

## Introduction

The program was designed to set up to organize, structure, and discuss the academic agenda of mathematical problem solving and its developments. The program included an open invitation to the mathematics education community to contribute and reflect on research and practicing issues that involve: (a) Addressing the origin, characterization, and foundation of mathematical problem solving, (b) discussing problem solving frameworks used to support research and curricula reforms in mathematical problem solving; (c) analyzing local and international research programs in mathematical problem solving; (d) discussing curriculum proposals that support the development of mathematical problem solving; (e) analyzing different ways to assess mathematical problem solving performances; (f) discussing the role played by the use of different digital tools in students' development of mathematical problem solving proficiency; (g) addressing programs that foster learners' development of problem solving approaches beyond school; and (h) identifying future developments of the field.

The international problem solving community responded to the invitation and sent more than 30 proposals, of those 18 were selected for presentation during the

**Organizers** Co-chairs: Manuel Santos-Trigo (Mexico), Zahra Gooya (Iran); Team Members: Jiang Chunlian (China), Mangoo Park (Korea), Dindyal Jaguthsing, Singapore; Liaison IPC Member: Yuriko Baldin (Brazil).

M. Santos-Trigo (✉)
Mathematics Education Department, Centre for Research and Advanced Studies, Cinvestav-IPN, Tsukuba 07360, Mexico
e-mail: msantos@cinvestav.mx

Z. Gooya
Shahid Beheshty University, Tehran, Iran
e-mail: zahra.gooya@yahoo.com



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sessions, and 10 were assigned to the poster session. In this report, we inform about the subjects and themes that authors addressed in their written proposals, and the results and discussions that emerged during the authors' oral presentations held during the development of the sessions at the ICME conference. A pdf file that includes all authors' contributions can be retrieved from: http://www.matedu.cinvestav.mx/~santos/icme12/ICME12TSG15book.pdf.

## An Overview

The authors' contributions addressed and discussed several issues that were identified in the open invitation letter they received and was available through the congress web-page. Here, we highlight common issues addressed in the contributions that include mathematical reflections on what problem solving entails, the variety of studies and methodologies used to frame research studies, the range of participants in those studies that involves elementary, secondary, high school students, in-service and practicing teachers, and university students, and a variety of theories used to support and develop problem solving research.

(a) Two contributions reviewed issues regarding what types of problems are relevant to discuss with students, and the importance for instructors to create an instructional environment in which students can actively be engaged in problem solving experiences. One example used to illustrate problem solving strategies and conjectures that emerged during the solution process was a variant of a task discussed by Polya (1954, pp. 43–52): *Into how many parts is space divided by 5 planes?* The discussion became important to identify ways to formulate and pursue conjectures in which a set of heuristics appears important during the entire solution process. The same theme "heuristic methods" is also addressed in another contribution to discuss examples where students have an opportunity to rely on strategies such as pattern recognition, working backwards, guessing and testing, looking for simpler problems, etc. to solve tasks set in different contexts. Both contributions offer ways to analyze tasks that can be useful to construct instructional paths to foster students' mathematical problem solving experiences.

(b) Eight contributions recognized the importance for learners to work on small groups to discuss and defend their ideas, listen to others, and communicate results. Two contributions emphasized students' social interactions as a way to enhance cognitive experiences. One proposes a teaching module to guide university students to comprehend and develop conceptual knowledge associated with a first differential equation course. In general, authors used a bricolage perspective that relies on several conceptual frameworks to support the study; another contribution builds up a local conceptual framework to guide practicing elementary teachers to develop problem-solving experiences through social interactions.

(c) Four contributions rely on statistical analyses to compare students' problem solving performances. For example, three studies emphasize the use of pre and post-tests to analyze and compare groups of students' problem solving achievements as a result of receiving differential problem solving instruction. For instance, one group explicitly addressed the importance of using analogical thinking in their approaches versus a group that followed a regular teaching approach. Other studies relied on the use of Case Study methodology in which the participants' problem solving behaviors are analyzed in detail. It is common in this process or use of task-based interviews, groups or class videos, or a combination of qualitative tools to gather data and to foster the development of problem solving approaches. In general, a tendency in six contributions was to rely on both the use of quantitative and qualitative tools to analyze learners' problem solving behaviours.

(d) It was observed that five contributions have explicitly relied on frameworks that extend problem-solving approaches such as models-and-modeling perspectives. The analyses of problem solving performance of students that consistently have shown high achievement in international assessments was also addressed in seven of the contributions. For example, a study focused on analyzing the extent to which some Korean students epistemological beliefs about mathematics are related to their problem solving behaviours. Similarly, another study analyzes how a problem-based learning (PBL) was implemented in China.

It must be noted that the use of mathematical competitions to promote learners' development of problem solving skills has been encouraged in different countries. For example, one study analyzes how a web-based mathematical problem competition became important for 13–14 years to engage in problem solving experiences that go beyond those that appear in regular classroom contexts. Yet, another contribution analyses how a set of didactic techniques based on the problem centred Japanese tradition is implemented in Swedish. In this particular study the author relies on the use of Anthropological Theory of Didactics which is a framework commonly used in the French mathematics education tradition.

(e) Problem solving activities also play an important role in teachers professional development programs and the education of prospective teachers. A contribution focuses on fostering both prospective and practicing teachers' competence to pose, formulate, and pursue questions or problems. The framework that authors used to support the problem posing experiences involves epistemic, cognitive, and mediation analysis of tasks and learners interaction and is called an Onto-Semiotic approach. Likewise, the implementation of problem solving activities has taken different directions and aims. For instance, one contribution emphasizes the second Polyas' proposed stage of problem solving "designing a plan or planning the solution" to improve colleges students abilities to solve arithmetic problems.

## Remarks and Future Directions

Learning, constructing, or developing mathematical knowledge via problem solving activities continues to be an important goal in curriculum proposals and a central theme in research programs around the world. However, a salient feature of the group contributions is that there are multiple ways and a variety of interpretations of what a problem solving approach to learn mathematics entails, and ways to frame and implement curriculum proposals. To analyze and reflect on common aspects around problem solving approaches we must construct and activate an international community that continuously shares research programs and discusses problem-solving developments. This community must include active researchers whose academic agenda involves both theoretical and practicing themes in problem solving. And teachers who show clear interest in implementing problem solving approaches in their classrooms are key elements since they look for ideas to consistently frame their practices around problem solving activities. In particular, teachers' discussions focus on demanding actions and directions that will help reduce efficiently a long list of contents and to concentrate on problem solving activities to study key concepts deeply. What fundamental mathematical ideas and processes should be central in curriculum proposals that promote problem-solving approaches?

Another teachers' interest is to address the role of students' international assessments (PISA, TIMMS) in problem solving approaches. That is, to discuss the extent to which the mathematics and ways of reasoning involved in those international assessments is consistent with problem solving approaches. Another important issue that emerged during the group session is the role played by the use of different forms of digital technology in fostering learners' development of mathematical problem solving experiences. It was recognized that there is little information on the type of mathematical reasoning that students construct as a result of using several technologies, and how that reasoning expand or complement paper and pencil approaches. There was a consensus that it is urgent to include in the research and practicing agenda the extent to which theoretical and conceptual frameworks used in problem solving needs to be adjusted in order to explain and foster the students' development of mathematical learning in problem solving scenarios that enhance the systematic use of digital technology.



## Reference

Polya, G. (1954*). Induction and analogy in mathematics education. Volume 1 of mathematics and plausible reasoning*. NJ: Princeton University Press.