Mathematical Literacy

Mogens Niss

Introduction

The actual design and implementation of the structure and organisation of the four TSG sessions was carried out by *Mogens Niss*, with the assistance of *GwiSoo Na*, *Eduardo Mancera*, and *Michèle Artigue*. Unfortunately, Eduardo Mancera was eventually unable to attend the Congress and the TSG.

This TSG was included for the first time in the history of the ICMEs. Hence, there was no established ICME tradition to build on concerning this topic. Moreover, generally speaking, the very notion of mathematical literacy is not well-defined, especially as several related concepts, such as numeracy, quantitative literacy, mathematical proficiency, and mathematical competencies, are in general use as well. Against this background it was decided to devote a fair proportion of the session time to coming to grips with the notions of mathematical literacy and its "relatives".

The presentations given in the four sessions of TSG 6 were partly commissioned papers, partly contributed ones. As is often the case with TSGs, the attendance to this TSG was not completely stable, but varied across the four sessions, the average attendance being about twenty participants per session.

The themes of the four sessions were chosen as a reflection of perceived intellectual and scholarly needs, and of the papers contributed by participants. The main theme of the opening session was the *Notions and interpretations of mathematical literacy*, whilst *The role and impact of mathematical literacy in national and*

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S.J. Cho (ed.), *The Proceedings of the 12th International Congress on Mathematical Education*, DOI 10.1007/978-3-319-12688-3_31

international studies was the main theme chosen for Session 2. Session 3 was primarily devoted to the theme *The role, use and implementation of mathematical literacy in educational systems and institutions.* The main theme of the fourth and final session was *Mathematical Literacy and teachers*. In order to ensure discussion of the presentations, each session was concluded by questions or comments, in Sessions 1 and 4 assisted by a round-table.

Major Points from the Four Sessions

Session 1: Tuesday, 10th July, 10:30–12:00

To set the stage for the work of TSG 6, *Mogens Niss* gave a 30-minute introduction focusing on the notions of mathematical literacy and its relatives. He began by observing that mathematics educators have always insisted that knowledge, skills and insights pertaining to elementary mathematics go far beyond facts, rules and procedures. This was already the case with the First International Mathematics Study (FIMS), conducted by the IEA and published in 1967, which spoke about five "cognitive behaviours". Later on, organisations such as NCTM and OECD-PISA, and several individual researchers, made an effort to identify aspects of the "add-ons" involved, suggesting various terms for the enterprise. The term Mathematical Literacy was used at least as early as in 1944, but the first attempt at a definition seems to have been made in the first OECD-PISA framework in 1999, with minor modifications in subsequent frameworks. Other related terms are numeracy, quantitative literacy, mathematical proficiency, and mathematical competence (competencies). Niss asked whether these terms are just different names for the same thing, or each term stands for something independent. He concluded that when it comes to mathematical literacy, numeracy and quantitative literacy, many people use them interchangeably, even though it is actually possible to attach distinct specific meanings to these terms. Given that people tend not to stick to definitions, he proposed to use Mathematical Literacy as the overarching term for the common underlying idea of promoting mathematical empowerment by making mathematics functional in extra-mathematical contexts. In contrast, the terms mathematical proficiency and mathematical competencies refer to a much wider spectrum of mathematical mastery, pertaining also to intra-mathematical contexts.

Next, two 15-minute presentations on aspects of the range and scope of mathematical literacy were given. In the first one, *Steve Thornton* (with John Hogan) (Australia), suggested to utilise a notion of "slow mathematics"—inspired by the notion of "slow food" in contrast to fast food—as a metaphor for quality mathematics education and for mathematical literacy. Slow mathematics is meant to capture what working mathematically is actually about, going against the "one-size-fits-all" idea typical of traditional curricula. Thornton proposed that working mathematically should be made *the* curriculum, whereas content should be of

secondary importance. He illustrated his ideas by two examples, one on sums occurring in the dice game of Yahtzee, and one on cyclones and tides. *Karen François* (Belgium) (co-author not present) discussed the relationship between mathematical literacy and statistical literacy from a theoretical perspective, based on a literature review. François concluded that whilst there are indeed clear similarities and links between mathematical and statistical literacy there are also significant differences (e.g. statistical literacy focuses on decision making under uncertainty), as reflected in the sociological fact that statistical literacy and statistics education have developed into independent notions and fields of study. In other words statistical literacy should not simply be perceived as a special sub-field of mathematical literacy.

The session ended with a round-table in which *Nitsa Movshovitz-Hadar* (Israel), the speakers and the members of the audience discussed the range and scope of the concept of mathematical literacy. Movshovitz-Hadar made the point that mathematical literacy should encompass insights into the reality of current mathematical developments and described a project in Israel in which secondary school students were exposed to contemporary "mathematical snapshots" once every two weeks.

Session 2: July 11th Wednesday, 10:30–12:00

The session opened by a 30-minute invited presentation by *Ross Turner* (Australia) (in charge of implementing the mathematics part of the OECD-PISA study for several cycles). After considering the genesis and meanings of the notions of mathematical and scientific literacy, numeracy and quantitative literacy in various reports, Turner zoomed in on the ways in which the notion of mathematical literacy was developed in different PISA cycles, right from the beginning. The concept of mathematical literacy in PISA has always given rise to some tension within the group of participating countries. The key tension can be phrased as one between seeing mathematics as a superset, having mathematical literacy is a smaller part, or seeing mathematical literacy as the overarching domain, with mathematics as a subset. The tension is both a conceptual one, reflected in the ways in which different versions of the PISA framework draws upon mathematical competencies and overarching content areas ("big ideas"), and a political one, reflected in the fear voiced in some quarters, that PISA, by focusing on contextualised mathematics, would not provide an adequate coverage of school mathematics curricula, as only relatively low level mathematics seems to be needed to solve PISA problems. Nevertheless, several PISA items could be solved by a tiny minority of students only. Another problem is that the word "literacy" does not exist in many languages, making translation difficult. These tensions gave rise to a strong pressure on the OECD, and then on those in charge of PISA, to change the focus of PISA 2012 towards a more traditional view of mathematics as being constituted by well-known content areas, without directly forbidding the use of the term mathematical literacy. Turner concluded by mentioning the promising work done by some of the PISA

mathematics experts on the impact of mathematical competencies on the intrinsic demands of PISA items, and on these demands as predictors of observed item difficulty.

In the first of three 15-minute presentations, *Jeff Evans* (UK) offered a comparative analysis of the definition of numeracy in PIAAC (Project for International Assessment of Adult Competencies) and the definition of mathematical literacy in PISA, 2006. He found the PISA definition somewhat broader and more "humanistic" than that in PIAAC. Finally, Evans pointed to the criticism, raised by some, of the unidimensionality of the performance levels in both surveys.

Next, *Kees Hoogland* (The Netherlands) reported on a randomized, controlled, comparative study of 38,000 Dutch students solving image-rich, respectively word-based, numeracy problems. The aim of the study was to test the hypothesis that replacing word problems with image-rich problems would have a significant positive effect on students' result, and even more so with vocational students. In the study, 24 pairs of mathematically equivalent numeracy problems were constructed such that each pair contained a language-rich version and an image-rich version of the "same" problem. Each student was randomly given 12 problems of each type. The study was found to provide a fair degree of confirmation of the hypothesis stated.

The final presentation was given by *Yukihiko Namikawa* (Japan), who described a national project in Japan which first focused on scientific literacy and then moved on to mathematical literacy, focusing on citizenship. A key part of this project was the publication "Mathematical Literacy for All Japanese", containing chapters on the nature of mathematics, on the central objects and concepts of mathematics, on mathematical methods and mathematical competencies, on mathematical topics, and, finally, on the relationship of mathematics with humanity and science. Following a report published in 2008 by the Central Council for Education, a new comprehensive, national standards curriculum emphasising mathematical literacy for all is being phased in, challenging the education of teachers at all levels.

Session 3: Friday 13th July, 15:00–16:30

This session contained a variety of short presentations. *John Hogan* (with Steve Thornton) (Australia), after having proposed to define "being mathematically literate" as more or less the same as "being numerate", went on to suggest that this cannot be developed or observed in the mathematics classroom alone, it has to go across the curriculum. To illustrate how this can be pursued, Hogan briefly outlined some settings in the arts, English, health and physical education and science, corresponding to early, middle and later years, respectively. He finally sketched a numeracy framework developed for diagnostic, analytic and practical purposes.

Yelena Baishanski (USA, with co-author not present) spoke about achieving literacy through articulated reasoning in remedial mathematics courses for US community college students (i.e. La Guardia CC, New York). The project involved

activities on simple applied arithmetico-algebraic problems, arising out of "current compelling issues" meant to be engaging and meaningful to students, on which they can develop and practice their own skills in reasoning and written communication about reasoning, so as to develop confidence in their own powers of deduction.

In the next presentation, *Jenna Tague* (USA, with co-authors not present), dealt with two linked topics: the so-called STEM (Science, Technology, Engineering and Mathematics education) reform in the USA and a related development project at Ohio State University, reconceptualising engineering courses by focusing on mathematical literacy. More specifically, Tague proposed to devise a mathematical literacy framework within a "STEM for engineering students" context, taking inspiration from the Danish KOM project on mathematical competencies.

Based on the observation that many interpretations of mathematical literacy give a crucial role to mathematical modelling, *Abolfazi Refiepour Garabi* (Iran) presented two related empirical studies, one of Iranian mathematics textbooks and one of teachers' views about application and modelling problems in their classrooms. Mathematical modelling and applications were introduced in Iranian textbooks in 2008/2009. Comparing with Australian textbooks, the author finds that measured by the number of real world modelling problems, these textbooks tend to have a larger emphasis on mathematical literacy than do Iranian textbooks. Iranian mathematics teachers experience difficulties in using applications and modelling problems in their classrooms, especially because they don't have access to adequate sources for modelling tasks.

The final presentation was given by *Luis Rico Romero* (Spain, with co-authors not present). He presented a study in progress on Spanish in-service secondary teachers' assessment of mathematical competences. In the Spanish curriculum of 2006, the notion of competence, including mathematical competence, is given a key role at all educational levels, and also in the related system of performance indicators. The study focuses on teachers' understanding of, and intended methods with regard to, competency assessment in mathematics. The components of a workshop on this topic for teachers were outlined.

Session 4: Saturday 14th July, 10:30–12:00

In the first presentation, *Cigdem Arslan* (with *Günes Yavuz*), Turkey, reported on a research study on the mathematical literacy self-efficacy of prospective mathematics teacher students (PTs) in different programmes in a Turkish university. The study was conducted by way of a 25 item questionnaire, where each item was to be answered in a five-point Likert scale format. The study found that PTs indicate an above-medium level of mathematical literacy self-efficacy, and that there were no significant differences between their mean scores with respect to their year in university, between male and female students, or with respect to their choice of programme.

The presentation by *Lyn Webb* (South Africa, with co-authors not present) was based on the fact that mathematical literacy was introduced in South Africa as a mathematics option for prospective teachers as an alternative to "usual" mathematics. This led to the establishment of mathematical literacy programmes at some higher education institutions. The degrees obtained are rather different from those of traditional mathematics programmes. Two mathematics programmes are offered at two universities in KwaZulu Natal. After comparing the programmes with respect to their overall design, Webb concluded from the study that a balanced mix of types of knowledge, particularly disciplinary, pedagogical, practical and situational learning, is essential for teacher training qualification, and that content knowledge is not sufficient.

The final presentation was delivered jointly by *Dave Tout* (Australia) and *Iddo Gal* (Israel). They set out by contrasting internal views of educational goals (learning the trade of the discipline) with external views of educational outcomes focusing on real-world functional demands ("literacies"/"competencies"). Different surveys of students (e.g. TIMSS, PISA) and adults (e.g. ALL, PIAAC) have been conducted to shape educational policies and to design interventions. Mathematical literacy and numeracy are of the same nature, but mathematical literacy sits (mainly) in student and school contexts and numeracy in adult world contexts. This is reflected in PIAAC's definition of numeracy, focusing on the mathematical demands of a range of situations in adult life and on associated facets of numerate behaviour. The presentation went on to highlight various results from PIAAC and other adult numeracy surveys, and concluded by calling attention to three kinds of challenges to mathematical literacy/numeracy: Conceptual challenges ("what is it?"), educational challenges ("how can we develop it?"), and systemic challenges ("where is it (to be) located?").

This conclusion provided a handy lead-on to the final part of the session, a combined round-table and discussion amongst participants. Members of the round-table, moderated by *Mogens Niss*, were *GwiSoo Na*, *Yukihiko Namikawa* and *Ross Turner*. The round-table and the audience focused on important points for future work on mathematical literacy, such as examining the relationship between mathematical literacy and mathematical knowledge and skills, and finding ways to develop teaching and learning of mathematical literacy so as to ensure that all students (and adults) get something out of their mathematical education of subjective and objective value.