Spanish Heritage in Mathematics and **Mathematics Education**

L. Rico

On the occasion of the 12th ICME, the Spanish Committee of Mathematics Education decided to prepare a national presentation entitled Spanish Cultural Heritage.

The presentation takes the form of a series of posters, each of which has a special focus, showing relevant historical events identified according to time and institutions. As a whole, the posters outline a comprehensive historical trajectory devoted to the Hispanic Heritage.

The relevance of mathematics in the relations between Spain and America has remained unbroken since its beginning 520 years ago. Julio Rey Pastor emphasizes the importance and scope of this heritage for its scientific and technological use and its benefits since the discovery of America. Since then, throughout 520 years of continuous cultural cooperation, the mathematical background shared by Spain and the American Republics, people and countries, that have remained solid and permanent.

To present the Spanish Heritage in the ICME 12 of Seoul (Korea), from the Education Commission of the CEMAT (Spanish Mathematics Committee) have been prepared 27 posters, which set out key moments, characters and events in the history of mathematics.

The list of themes chosen is as follows:

- 1. Spanish Heritage in Mathematics and Mathematics Education.
- 2. Mathematics and Science in the Discovery of America.
- 3. The Founding of the First American Universities.
- 4. First Mathematical-Scientific Publication in the New World.
- 5. The House of Trade: Navigation, Cartography, and Astronomy.
- 6. The 16th-Century Mathematics Academy: Philip II, Siliceo, Juan de Herrera.
- 7. Science and Technology in the 16th Century.

University of Granada, Granada, Spain

L. Rico (\subseteq)

- 8. Mathematics in the Baroque Period in Spain.
- 9. Scientific Policy of the First Bourbons. The Jesuits and Mathematics.
- 10. Enlightenment Mathematics. The Reforms of Charles III.
- 11. José Celestino Mutis. An Enlightened Scientist in the New World.
- 12. Jorge Juan and Antonio de Ulloa. Meridian measurement in Quito.
- 13. Educational Reforms in Hispano-America, based on the 1812 Constitution.
- 14. The Compendium of Pure and Mixed Mathematics by D. José Mariano Vallejo.
- 15. 19th-Century Mathematics.
- 16. The Metric System in Textbooks in the Second Half of the 19th Century.
- 17. The Mathematicians of Scientific '98.
- 18. Andrés Manjón and the Ave María Schools at the end of the 19th Century.
- 19. The Spanish Republican Exile: the Mathematicians in America.
- 20. Researching Together: Return Journeys.
- 21. The Iberoamerican Mathematical Olympiad.
- 22. Research Centers.
- 23. Journals, research and collaboration in Mathematics Education.
- 24. ICME 8 Seville (Spain), July 1996.
- 25. Miguel de Guzmán (1936–2004) Academic, scientific, and educational legacy.
- 26. Mathematical Research in Ibero-America, Spain and Portugal.
- 27. Spanish Mathematics: the last 20 years.

The posters has been prepared by:

- M. de León and A. Timón, from the Institute of Mathematical Research (ICMAT).
- J. Peralta, from University Autonomous of Madrid.
- A. Maz; N. Adamuz; N. Jiménez-Fanjul; M. Torralbo and A. Carrillo de Albornoz, from the University of Cordoba.
- L. Rico; E. Castro-Rodríguez; J. A. Fernández-Plaza; M. Molina; M.C. Cañadas; J.F. Ruiz-Hidalgo; J.L. Lupiáñez; M. Picado; I. Segovia; I. Real and F. Ruiz, University of Granada.
- I. Gómez-Chacón; M. Castrillón and M. Gaspar, from the University Complutense of Madrid.
- M. Sierra and M.C. López, from University of Salamanca.
- B. Gómez, L. Puig and O. Monzó, University of Valencia.

The main objective of this work was to present the joint activity on mathematics and mathematics education, thought and written in Spanish, conducted by Spanish and American in more than 500 years of history and shared culture. We will stress the links established between Americans and Spaniards, as demonstrated by the information presented. We will underscore the scientific, technological, or cultural value of these events, their subsequent implications, and the social impact they produced in their time.

As there is a common language, a shared history and culture, there are ways of thinking and doing math based on that language, that culture and that history. This work aims show and claim the shared heritage in mathematics and mathematics education in this community. We have done a selection of the information presented in the posters and we will comment it here.

We have organized the posters considering five general comprehensive periods:

- 1. Discovery and colonization.
- 2. The Creole society.
- 3. The Century of Independence.
- 4. 20th Century: mutual assistance and help.
- 5. Current cooperation.

Summary of key moments and ideas of the above mentioned periods.

1. Mathematics and Science in the Discovery of America.

On October 12, 1492, a Spanish expedition commanded by Admiral Christopher Columbus arrived at the island of Guanahani and took possession of the land in the name of their Majesties Isabella of Castile and Ferdinand of Aragon. This act of Discovery is essential to the birth of historical modernity and of science. It marks the origin of a sociocultural community, the Ibero-American community, based on the unique relationship between Spaniards and Americans. Columbus's goal was to reach Asia, that say, the island of Cipangu (Japan), which was thought to be at the same latitude as the Canary Islands. Columbus was not trying to discover a new continent, but rather to "reach the East by sailing West." The information that Columbus used involved several significant errors and to understand them it was needed a big change of ideas. Toscanelli's map, reflects the ideas of many navigators and geographers of the period, describes the route that Columbus believed he had travelled. He thus believed that the distance from the Canary Islands to Japan was 800 leagues by west (4,500 km), when it was actually about 3,500 leagues (19,500 km). These data were sufficient grounds for undertaking his first and the subsequent voyages. When Columbus arrives in the Antilles, he was convinced that he has reached the western coast of Asia hence his naming these lands the West Indies (Fig. 1).

Institutions: the first Universities. Starting with their first years in the colonies, the Crown, the Church, and the religious orders intervene in the area of education to teach and train clergy, government officials, and the middle classes. Their knowledge was classified into study in the trivium (grammar, rhetoric, and logic) and the quadrivium (arithmetic, geometry, music, and astronomy). Founding the Universities and Colleges in America is a historical feat and cultural phenomenon of prime importance, particularly in the first half of the 16th century. The first universities were the universities of Santo Domingo, Lima and Mexico, that were respectively founded in 1538 (Santo Domingo) and in 1551 (Lima and Mexico).

MATHEMATICS AND SCIENCE IN THE DISCOVERY OF AMERICA

- Authors:
- Dr. Luis Rico. University of Granada. Irico@ugr.es Spanish ICMI Representative (CEMat)
- Da. Elena Castro- Rodríguez, University of Granada, elecr@correo.ugr.es



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The information that Columbus used involved several significant errors.

It assumed that the earth had a maximum circumference of 29,000 km., estimated according to Posidonius and the length of a terrestrial degree, which was based on Arabic, not the shorter Italian, mile. Posidonius calculated the earth's circumference as less than three fourths

of its real size, and this figure had been accepted as scientific truth since the time of Eratosthenes.

On the other hand, interpreting the information from the voyages of Marco Polo, Columbus estimated the width of Eurasia to be 225°, subtracting only 135° of ocean.

He thus believed that the distance from the Canary Islands to Japan was 800 leagues by west (4500 km), when it was actually about 3500 leagues (19,500 kilometers). But these data were sufficient grounds for undertaking his first and the subsequent voyages. When Columbus arrives in the Antilles, he is convinced that he has reached the western coast of Asia hence his naming these lands the West Indies.



Columbus's subsequent voyages demonstrated the crucial importance of the new lands and cultures discovered for the construction and grounding of new knowledge. The exploration of the New World established a contrast between reality and speculative ancient doctrines on the size and shape of the earth's geography. This stimulated the spirit of free investigation and empirical contrast of hypotheses. Geography, cartography, cosmography, navigation, geodesy, and the physics of the globe ceased to be practical techniques and began their careers as physical-mathematical sciences.



The problem of geographical longitudes also had to be solved to respond to the need for nautical maps. The demand for greater precision led to the emergence of new techniques and instruments for measurement, which required a considerable advance in science, particularly mathematics.

Although Columbus made four voyages, explored the islands of Guadalupe and Puerto Rico, reached the mouth of the Orinoco River, navigated the islands of Chacachacare and Margarita, Tobago and Granada, and landed in region of Panama, he died without knowing that he had discovered the "New World" for Europeans. In 1507, the German cartographer Waldseemiller published a geography book that included a map. The book represented the new lands and included the tales of a Florentine navigator, Amerigo Vespucci, who stated that the discoveries were not Asian lands, but a new continent. In Vespucci's honor, the 1507 map called the new lands and merica.



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Fig. 1 Mathematics and science in the discovery of America

2. The Creole Society.

To talk about the Creole society we have fixed our attention in the following ideas:

- Scientific Policy of the First Bourbons. The Jesuits and Mathematics.
- Enlightenment Mathematics. The Reforms of Charles III.
- José Celestino Mutis. An Enlightened Scientist in the New World.
- Jorge Juan and Antonio de Ulloa. Meridian measurement in Quito.

At the beginning of the 18th century, the Jesuits assumed responsibility for educating the nobility, through Seminaries for Nobility, which began in Madrid. This institution's model of teaching spreads to Barcelona, Valencia, Gerona, and other cities. Based on its model, new centers are founded throughout the 18th century. In Mexico, the Royal Academy of San Carlos of the Noble Arts of New Spain is founded. These centers also trained the American elites. The Seminaries of Nobility become one of the most important centers of teaching and research in America. The Jesuits authored fertile textbooks in mathematics.

José Celestino Mutis y Bosio (Cadiz 1732; Bogotá 1808) Botanist, doctor, astronomer, and mathematician. Mutis developed important scientific work on American soil. He gave the inaugural speech for the Chair of Mathematics at the College of Our Lady of the Rosary in Santa Fe de Bogotá. There, he held the positions of rector and director. He determined the coordinates of Santa Fe de Bogotá, observed an eclipse of a satellite of Jupiter, and was one of the observers of the Transit of Venus on June, 1769.

Jorge Juan y Antonio de Ulloa. They participated in the expedition from 1735 to 1744 to measure an arc of 1° of latitude near the equator and one near the pole, to determine the lemon or orange shape of the earth.

The Royal Academy of Sciences decided to undertake the task of obtaining precise data from two meridian positions at two locations on Earth: Lapland (North Pole) and the Viceroyalty of Peru (the equator). To do this, two expeditions were organized. If the measurements obtained by both expeditions were the same, the Earth was sphere-shaped. If the measurement was greater at the pole, there was flattening at the poles.

If the polar measurement was smaller, the French were right and the lemon shape shall be the model. To carry out the expedition to the cities of Quito and Cuenca, located today in the Republic of Ecuador (Fig. 2).

- 3. The Century of independence. We have fixed our attention on the following subjects.
- Educational Reforms in Hispano-America, based on the 1812 Constitution.
- The Compendium of Pure and Mixed Mathematics by D. José Mariano Vallejo.
- 19th-Century Mathematics.
- The Metric System in Textbooks in the Second Half of the 19th Century.
- The Mathematicians of Scientific '98.

Jorge Juan and Antonio de Ulloa. Meridian measurement in Quito

Authors: Dr. B. Gómez, Dr. L. Puig, D. O. Monzó. University of Valencia

THE EARTH IS NOT ROUND

The 17th century saw the height of the polemic concerning the shape and size of the Earth.

Newton, using his law of universal gravitation, obtained an ellipsoid of revolution for a homogeneous mass in uniform rotation. Based on this result, there were two possibilities for the Earth: In England, scientists argued that the ellipsoid of revolution was flattened at the poles (in the shape of an orange), whereas the French argued that it was elongated at the poles (lemon-shaped).



The method chosen to resolve this question consisted of measuring an arc of 1° of latitude near the equator and one near the pole. The Royal Academy of Sciences decided to undertake the task of obtaining precise data from two meridian positions at two locations on Earth: Lapland (North Pole) and the Viceroyalty of Peru (the equator). To do this, two expeditions were organized. If the measurements obtained by both expeditions were the same, the Earth was sphere-shaped. If the measurement was greater at the pole, there was flattening at the poles, and the English were right. If the polar measurement was smaller, the French were right. The procedure was based on the correspondence of angular and linear values



The procedure was based on the correspondence of angular and linear values. It can be explained as follows: Take two pairs of points (A. B) and (A', B') which define arcs of the same meridian. For the same angular value be between their zeniths, if the Earth is spherical, their lengths will be the same. If the Earth is an ellipsoid with flattening at the poles, as Newton proposed, the angular value b" would remain the same; since the radius of curvature of the circumference is greater at the pole than at the equator, the length of the arc at the pole would be greater than the length of the arc at the equator.

When the two measurements were made, it was confirmed that the Earth was shaped like an orange, as Newton had imagined, as an ellipsoid of revolution flattened at the poles, and not lemon-shaped as Cassini had suggested.

EXPEDITION TO THE VICEROYALTY OF PERU (1735-1744)

To carry out the expedition to the Vicerovalty of Peru, specifically, to the cities of Quito and Cuenca, located today in the Republic of Ecuador, King Louis XV of France had to ask permission of Philip V of Spain, since these territories belonged to the Spanish Crown

Spain granted permission, but on the condition that two Spanish scientists, experts in mathematics and science, participated in all of the operations.

Jorge Juan and Antonio de Ulioa were given this commission. They were 21 and 19 years old, respectively. Jorge Juan was to be the mathematician and Antonio de Ulioa the naturalist. The French commission was composed of Louis Godin, Pierre Bouguer, and Charles Marie de La Condamine. (At the end of their mission, Godin remained as Chair Professor of Mathematics and Cosmography at the University

To achieve their goal of obtaining the length of one degree of an arc of the meridian, they took measurements in two stages. One was geographic, measuring by triangulation the distance from north to south at a position close to the Cuenca meridian, which spanned a length of 78 leagues. The results of the measurement from Peru were more precise, with a possible error of 0.04% (approximately 22

toesas). (The toesa was the unit of measure of length that

had been used in France since the time of Charlemagne.

Later, it was replaced by the toesa of Peru.)



To perform the measurements for the astronomical phase, barometers, plumb lines, thermometers, and an octant were used. The octant was used in navigation to determine the location of a ship by the position of the stars. To use the octant, one places the index arm toward the Great Bear and consults the scale on the arc of the instrument's frame to measure the angle that the horizon forms with the heavenly bodies. The octant has a mirror that enables one to visualize and measure the angle between these objects. The octant was also used to measure the earthly latitude of a position, that is, the degrees it is elevated above the horizon. This problem was solved by measuring the height of the Sun as it passed over the

After the geometric measurement of this portion of the arc of the meridian by triangulation, the next sten was to measure the size of the angle of the arc triangulated, using astronomical procedures. From these measurements, it would be possible to know the value of the degree near the equator, which required measuring the latitude at each extreme of the triangulation.







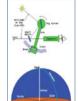




meridian of the location during the day (at its highest point) and by observing the North Star in the northern hemisphere or the Southern Cross in the southern. The line marked by the plumb line is called the vertical and forms an angle of 90° with the plane of the horizon. It intersects the celestial sphere at a point called the zenith. By extending this line from north to south (or from south to north-it doesn't matter), one obtains the meridian of the position, as this line passes through the northern point of the horizon, the zenith, and the southern point, dividing the celestial sphere into eastern and western quadrants.

In his Historical Account of the Journey to South America (1748), Ulloa told of the many difficulties and hardship hackbon by the bountey to sodorn-when or 124-bit, whose told on the many but mission and hardships that the expedition had to endure. The process of triangular measurement was not simple, given the extreme climatological and physical conditions under which they had to record data, and defects in the instruments required the expedition to repeat its calculations numerous









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Fig. 2 Jorge Juan and Antonio de Ulloa. Meridian measurement in Quito

Conception of education. Spanish resistance to Napoleon's invasion in 1808 led to the formation of the Courts of Cadiz, which developed the Constitution of 1812, oriented ideologically to exalt and safeguard individual liberty.

The Constitution's ninth title, dedicated to education, articulated the liberal idea of education, defending the idea of general, uniform education for all citizens and the need to form a Council for Public Education. This Council prepared a report that can be considered the most representative document on liberal ideology in matters of education. The report was published in Cadiz on September 9, 1813, under the following title: Report of the Council created by the Regency to propose the means for proceeding to regulate the various branches of Public Education.

It was established that "education must be universal, uniform, public, and free, and it must enjoy liberty." This Report formed the basis and origin of the educational reforms put into effect throughout Hispano-America after the revolutions that led to the independence of the Spanish colonies.

The 19th century is a turbulent period in the history of Spain. It begins with the invasions of Napoleon's armies in 1808 and ends with the Spanish-American War in 1898, known as the *disaster of '98*. The beginning of the 19th century brings the independence of the former colonies in America, giving rise to the new American republics. Spain loses its status as world power. The 19th century ends with the loss of Cuba, Puerto Rico, and the Philippines and the defeat of the Spanish fleet in Santiago, Cuba. Spain concludes its political presence in America. Spain is aware of its cultural and educational decline, a feeling aggravated by the loss of its colonies. It is thought that the military defeat was caused in good part by the country's scientific and technical backwardness: *the crisis of '98* (Fig. 3).

4. 20th Century: mutual assistance and help.

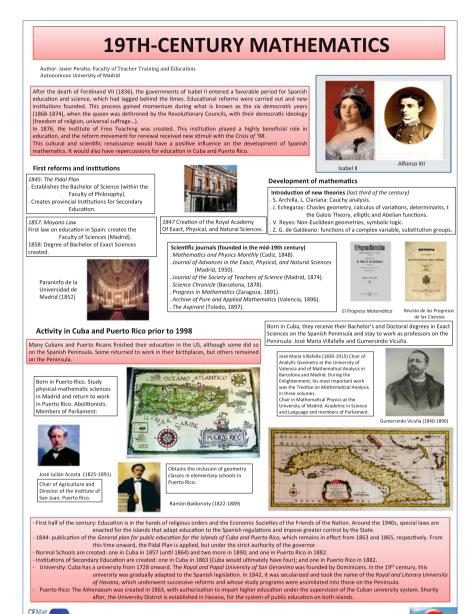
Where we are? Who are the leaders? We choose five points to reflect about our common work during the 20th Century.

- Andrés Manjón and the Ave María Schools at the end of the 19th Century.
- The Spanish Republican Exile: the Mathematicians in America.
- Researching Together: Return Journeys.
- The Iberoamerican Mathematical Olympiad.
- ICME 8 Seville (Spain), July 1996. Current research and cooperation (Fig. 4).

5. Current cooperation.

To describe this point we selected the following reflections:

- Research Centers.
- Journals, research and collaboration in Mathematics Education.
- Miguel de Guzmán Ozámiz (1936–2004) Academic, scientific, and educational legacy.
- Mathematical Research in Ibero-America, Spain and Portugal.
- Spanish Mathematics: the last 20 years (Fig. 5).



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Fig. 3 19th-century mathematics

Spanish Heritage

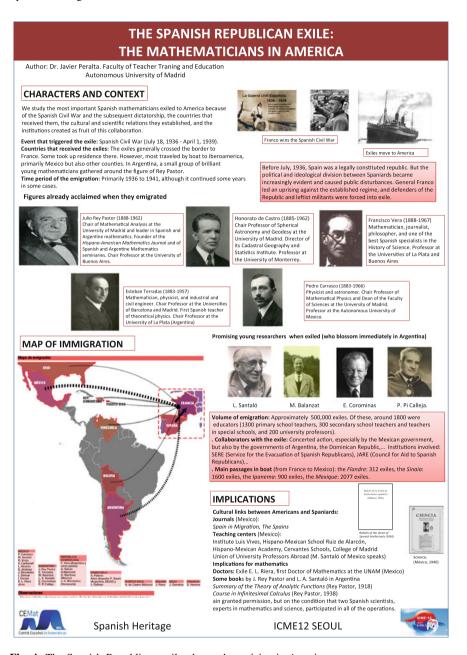


Fig. 4 The Spanish Republican exile: the mathematician in America

JOURNALS, RESEARCH AND COLLABORATION IN MATHEMATICS EDUCATION

Alexander Maz-Machado, Noelia Jiménez-Fanjul University of Córdoba (Spain)

Iberoamerican countries have a wide variety of journals for both research and popularization of Mathematics Education where Spanish, Portuguese, and Iberoamerican researchers currently publish.

Quadran

Doctoral Programs
In training researchers
in Spanish-speaking
countries, the
CINVESTAV of Mexico,
the University of Granada,

International Congresses

To collect and disseminate bibliographic

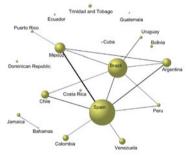
Information about scientific publications produced in Iberoamerican countries was created a network of bibliographical institutions that work collaboratively: LATINDEX is the product of this cooperation,

Two Iberoamerican journals in the area are indexed in the JCR: the Bolema-Mathematics Education Bulletin-Boletim de Educacao Matematica (BOLEMA) and the Journal of Latin American Research on Mathematics Education-Revista Latinoamericana de Investigación en Matemática Educativa-Relime (RELIME).

Ti**NUMPEROS** Iberoamerica as de International collaboration was strengthened in 2005, with

Educación Matemática by the creation of the Iberoamerican Federation of Societies for Mathematics Education.

Members include Societies from Argentina, Brazil, Chile, Columbia, Spain, Peru, Portugal, and Uruguay. This Society publishes the journal *Unión*.



Red de colaboración en artículos de Educación Matemática indexados en SCOPUS

If we examine the scientific productivity in the 770 articles indexed in SCOPUS as authored by Spanish or Iberoamerican researchers, we find systematic collaboration. This collaboration generates a network that revolves around Spain, Brazil, and Mexico.

A site for meeting and collaboration between Spain and Iberoamerica. Among the most significant are: the Latin American Meetings for Educational Mathematics ([RELME]), the Inter American Conference of Mathematics Education [CIAEM]), the Iberoamerican Congress of Mathematics Education [CIBEM]) and, in Spain, the Symposium for the Spanish Society for Mathematics Education (Simposio de la Sociedad Española de Educación Matemática [SEIEM])

and the Autonomous University of Barcelona are leading in number and quality of PhDs conducted.



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Matematica Educación Internet

Fig. 5 Journals, research and collaboration in mathematic education

The goal of the presentation was to show and underline the relevance of the cultural and scientific cooperation in mathematics and mathematics education between Spain and the American Republics over the last 500 years. The presentation seeks to publicize this common mathematical heritage by emphasizing its importance and the far-reaching influence these relationships have had and continue to have for science, technology, and education in our countries.

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