# Modeling Human Control Strategies in Simulated RVD Tasks through the Time-Fuel Optimal Control Model 

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#### Abstract

Human performance modeling has become more and more popular in cognitive science recently. This paper applies a time-fuel optimal control model to model the human control strategies in a simplified RVD task. Preliminary comparisons had been made between the model performance and the performance of human operators. Results show that the model can model the performance of human operators and individual differences. Discussion reveals that the human control strategies in the simplified RVD task depend on a ratio of two time estimates. This finding can provide useful guide for the further cognitive modeling of the RVD tasks.


Keywords: Human Performance Modeling, RVD, Time-Fuel Optimal Control model.

## 1 Introduction

Manually controlled rendezvous and docking (RVD) is a challenging space task for astronauts. The operator performing RVD task observes the information displayed on the monitoring interface and manipulates the controllers to complete the manual RVD task. The RVD system includes two controllers in the chaser spacecraft: one translation controller, which controls the $\mathrm{X}, \mathrm{Y}$, and Z axes of the chaser's position, and one orientation controller, which controls the yaw, pitch, and roll of the chaser's attitude. The monitoring interface directly displays the Y-Z control plane, in which Y axis is the horizontal direction and Z axis is the vertical direction. X axis represents the distance between the chaser spacecraft and the target spacecraft. The distance information can be perceived by human operator through the alterable size of the target spacecraft image displayed on the monitoring interface. Initial conditions of the chaser's position and attitude can be configured in the RVD simulation system.

Recently, modeling human performance has become more and more popular in cognitive science since it can provide a flexible and economical way to evaluate
the design of human-machine interaction [1/3]. We are interested in modeling human performance in RVD tasks.

The present study investigates the human control strategies in the elimination of only the Y axis deviation between the chaser spacecraft and the target spacecraft, which is a basic and simplified RVD task. To complete this task, the operator needs to eliminate both the position deviation and velocity deviation in the approaching process, which means that the relative position deviation and the relative velocity deviation between the chaser spacecraft and the target spacecraft is approximately zero when the X -axis distance of the two spacecrafts is zero. The translation controller is used to accelerate and decelerate the chaser spacecraft. The operator must eliminate the Y axis deviation with minimal time cost and minimal fuel cost which is measured by the summation of the absolute value of the instantaneous acceleration.

We suppose that operators' performance, especially the performance of the well-trained operators, is close to the optimal control performance in the simplified RVD tasks. As a result, the time-fuel optimal control model, which aims to minimize the time cost and fuel cost simultaneously in a control task, is employed to model human control strategies in performing the simplified RVD task.

The goal is to use the model to better understand the cognitive processes associated with the performance, to support the future cognitive modeling of RVD tasks.

## 2 Model

The basic and simplified RVD task whose target is to eliminate the Y-axis deviation can be viewed as a time-fuel optimal control problem. The task is illustrated in Fig. 1. The chaser spacecraft of which the initial position deviation is $y_{0}$ and the initial velocity deviation is $V_{y 0}$ is represented by the circle. The target spacecraft which can be viewed as fixed is represented by the origin. The operator can use the translation controller to provide a constant instantaneous acceleration for the chaser spacecraft which can be positive or negative. The positive directions of the position deviation $y$, the velocity deviation $V_{y}$, and the acceleration are the same.

We assume that the initial time is zero and the constant acceleration is represented by $a$. The dynamical system of the simplified RVD task is a second-order system which is governed by

$$
\begin{equation*}
\dot{y}=V_{y}, \dot{V}_{y}=u \tag{1}
\end{equation*}
$$

where $u$ is a bounded scalar control variable:

$$
\begin{equation*}
-a \leq u \leq a \tag{2}
\end{equation*}
$$

Let $x_{1}=y, x_{2}=V_{y}$, and we get

$$
\begin{equation*}
\dot{x}_{1}=x_{2}, \dot{x}_{2}=u \tag{3}
\end{equation*}
$$



Fig. 1. The illustration of the simplified RVD task

Given $x_{1}(0)=y_{0}, x_{2}(0)=V_{y 0}$ and the free terminal time $t_{\mathrm{f}}$, the time-fuel optimal control problem is to find $u$ to minimize

$$
\begin{equation*}
J=\int_{0}^{t_{\mathrm{f}}}[\rho+|u(t)|] d t \tag{4}
\end{equation*}
$$

with specified terminal conditions

$$
\begin{equation*}
x_{1}\left(t_{\mathrm{f}}\right)=0, x_{2}\left(t_{\mathrm{f}}\right)=0 \tag{5}
\end{equation*}
$$

In (4), $\rho(>0)$ is the time weight parameter and larger $\rho$ represents smaller time cost. The former of $J$ denotes the weighted time cost while the latter of $J$ denotes the fuel cost.

The solution of the problem is as follows [4]:

$$
u^{*}=\left\{\begin{array}{rl}
+a, \text { while }\left(x_{1}, x_{2}\right) & \in R_{3}  \tag{6}\\
-a, \text { while }\left(x_{1}, x_{2}\right) & \in R_{1} \\
0, \text { while }\left(x_{1}, x_{2}\right) & \in R_{2} \cup R_{4}
\end{array} .\right.
$$

where

$$
\left\{\begin{array}{l}
R_{1}=\left\{\left(x_{1}, x_{2}\right): x_{1} \geq-\frac{1}{2 a} x_{2}\left|x_{2}\right|, x_{1}>-\frac{\rho+4}{2 \rho a} x_{2}\left|x_{2}\right|\right\}  \tag{7}\\
R_{2}=\left\{\left(x_{1}, x_{2}\right): x_{1}<-\frac{1}{2 a} x_{2}\left|x_{2}\right|, x_{1} \geq-\frac{\rho+4}{2 \rho a} x_{2}\left|x_{2}\right|\right\} \\
R_{3}=\left\{\left(x_{1}, x_{2}\right): x_{1} \leq-\frac{1}{2 a} x_{2}\left|x_{2}\right|, x_{1}<-\frac{\rho+4}{2 \rho a} x_{2}\left|x_{2}\right|\right\} \\
R_{4}=\left\{\left(x_{1}, x_{2}\right): x_{1}>-\frac{1}{2 a} x_{2}\left|x_{2}\right|, x_{1} \leq-\frac{\rho+4}{2 \rho a} x_{2}\left|x_{2}\right|\right\}
\end{array} .\right.
$$

and

$$
\left\{\begin{array}{l}
r_{+}=\left\{\left(x_{1}, x_{2}\right): x_{1}=\frac{1}{2 a} x_{2}^{2}, x_{2} \leq 0\right\}  \tag{8}\\
r_{-}=\left\{\left(x_{1}, x_{2}\right): x_{1}=-\frac{1}{2 a} x_{2}^{2}, x_{2} \geq 0\right\} \\
\beta_{+0}=\left\{\left(x_{1}, x_{2}\right): x_{1}=-\frac{\rho+4}{2 a} x_{2}^{2}, x_{2} \geq 0\right\} \\
\beta_{-0}=\left\{\left(x_{1}, x_{2}\right): x_{1}=\frac{\rho+4}{2 \rho a} x_{2}^{2}, x_{2} \leq 0\right\}
\end{array} .\right.
$$

Figure 2 shows the state space trajectories and switching curves for the optimal control example. We implement the time-fuel optimal control model using MATLAB software.


Fig. 2. The state space trajectories and switching curves for the optimal control example

## 3 Model Validation

Three initial RVD task conditions are set and four participants, including two experts and two less-skilled persons, each performed six trials of RVD tasks in the RVD simulation system under the same initial conditions. The three initial RVD task conditions are: (1) position deviation is 2.2 m , and velocity deviation is positive $0.1 \mathrm{~m} / \mathrm{s}$, which means that the chaser spacecraft is flying away the target spacecraft in Y axis direction; (2) position deviation is 2 m , and velocity deviation is $0 \mathrm{~m} / \mathrm{s} ;(3)$ position deviation is 4 m , and velocity deviation is $0 \mathrm{~m} / \mathrm{s}$.

It should be noted that all the three conditions are just about Y axis and the initial Z-axis deviation and attitude deviation are both zero. The initial X -axis position deviation is 20 m , and the X -axis velocity deviation is always negative $0.2 \mathrm{~m} / \mathrm{s}$, which means that the chaser spacecraft is moving close to the target spacecraft at a constant velocity in X -axis direction.

For each participant and each initial condition, we implemented the simulation using the time-fuel optimal control model. The model had to adjust the time weight parameter $\rho$ to better fit the human data.

Preliminary comparisons have been made between the model performance and the performance of human operators.

Figure 3 displays the position deviation and the velocity deviation during the simplified RVD task with initial condition 1 for both the model and for an expert. Figure 4 displays the position deviation and the velocity deviation during the simplified RVD task with initial condition 1 for both the model and for a less-skilled participant. The illustrations in Figure 3 and Figure 4 are intended to show that, the performance produced by the model is qualitatively similar to the performance produced by human participants under the initial condition 1.


Fig. 3. The performance of one of experts in the simplified RVD task with initial condition 1. Left: position deviation. Right: velocity deviation. Solid line: model. Dashdot line: human operator.


Fig. 4. The performance of one of less-skilled participants in the simplified RVD task with initial condition 1. Left: position deviation. Right: velocity deviation. Solid line: model. Dash-dot line: human operator.

Besides, we can find that the time cost of experts is smaller than that of lessskilled participants some difference between experts and less-skilled participants. In other words, less-skilled participants were 'hurry' to complete the RVD task while experts were 'calm'.

Table 1 shows the Pearson's linear correlation coefficients of the position deviation. Table 2 shows the Pearson's linear correlation coefficients of the velocity deviation. The correlation analysis had been implemented through cutting the

Table 1. The Pearson's linear correlation coefficients of the position deviation

| Initial condition | Expert 1 Expert 2 Less-skilled 1 Less-skilled 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.9962 | 0.9896 | 0.9946 | 0.9919 |
| 2 | 0.9999 | 0.9987 | 0.9970 | 0.9933 |
| 3 | 0.9932 | 0.9990 | 0.9992 | 0.9985 |

Note: all $p$ values are less than 0.0001 .

Table 2. The Pearson's linear correlation coefficients of the velocity deviation

| Initial condition | Expert 1 |  |  |  | Expert 2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Less-skilled 1 | Less-skilled 2 |  |  |  |  |
| 1 | 0.9402 | 0.9256 | 0.9372 | 0.9233 |  |
| 2 | 0.7880 | 0.7038 | 0.8573 | 0.5971 |  |
| 3 | 0.8896 | 0.8751 | 0.5120 | 0.7675 |  |

Note: all $p$ values are less than 0.0001 .
Table 3. The time weight parameters ( $\rho$ in (4)) of all participants and initial conditions

| Initial condition | Expert | Expert | Less-skilled | Less-skilled 2 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.1 | 0.1 | 0.5 | 0.5 |
| 2 | 0.1 | 0.05 | 0.5 | 0.5 |
| 3 | 0.2 | 0.13 | 0.5 | 0.75 |

human data to the length of the model data. From the two tables, we can conclude that the model can fit the human data very well.

However, there are still some obvious differences between the model data and the human data. We can divide the whole process to three steps. Step 1 is to accelerate toward the origin. Step 2 is to move toward the origin at the constant velocity. Step 3 is to decelerate toward the origin. From the velocity deviation in Fig. 3 and Fig. 3, we can easily find that Step 1 of the human is longer than


Fig. 5. The mean fuel cost of the model and participants. Dashed line: the model. Solid line: experts. Dotted line: less-skilled participants.
that of the model and Step 2 of the human is shorter than that of the model. Besides, the model is much more stable than the human in Step 3. The model achieved the excellent performance. Table 3 shows the time weight parameters ( $\rho$ in (4)) of all participants and initial conditions. The time weight parameters ( $\rho$ in (4)) of experts are larger than that of less-skilled participants significantly ( $p<0.001$ ). Remember that larger $\rho$ represents smaller time cost. So the time cost of experts is smaller than that of less-skilled participants. This has been verified by Fig. 3 and Fig. 4.

Figure 5 displays the mean fuel cost of the model and participants. The fuel cost of the model is much smaller than participants. Compared with less-skilled participants, the fuel cost of experts is much smaller.

In summary, the time-fuel optimal control model can model the performance of the human operators in the simplified RVD tasks and the time weight parameter ( $\rho$ in (4)) in the model can account for individual differences.

## 4 Discussion

Our goal is to investigate the human control strategies in the simplified RVD tasks. First, let us review the strategies of the time-fuel optimal model. In (6), the value of the optimal control $\left(u^{*}\right)$ depends on which state region does the current state, that is $\left(x_{1}, x_{2}\right)$, locate in. And the state regions in (7) are determined by four parabolas in (8). As the state regions in Fig. 2 is centrosymmetry, we just consider the right state plane $\left(x_{1}>0\right)$ which include two parabolas that are $r_{+}$ and $\beta_{-0}$. The state $x_{2}$ has two cases that are $x_{2}<0$ and $x_{2} \geq 0$.

Case 1: $x_{2}<0$. Rewrite $r_{+}$and $\beta_{-0}$ as follows

$$
\begin{align*}
r_{+} & =\left\{\left(x_{1}, x_{2}\right): a \frac{x_{1}}{x_{2}^{2}}=\frac{2}{2}, x_{2}<0\right\} .  \tag{9}\\
\beta_{-0} & =\left\{\left(x_{1}, x_{2}\right): a \frac{x_{1}}{x_{2}^{2}}=\frac{\rho+4}{2 \rho}, x_{2}<0\right\} . \tag{10}
\end{align*}
$$

Then replace $x_{1}$ with $y$ and $x_{2}$ with $V_{y}$ in $a \frac{x_{1}}{x_{2}^{2}}$

$$
\begin{equation*}
a \frac{x_{1}}{x_{2}^{2}}=a \frac{y}{V_{y}^{2}}=\frac{\frac{y}{V_{y}}}{\frac{V_{y}}{a}}=\frac{T_{\mathrm{reach}}}{T_{\mathrm{dec}}} \tag{11}
\end{equation*}
$$

where $T_{\text {reach }}$ is the time to reach to the origin at the current velocity and $T_{\text {dec }}$ is the time to decelerate the current velocity to zero at the constant acceleration $a$.

The strategies of the model depend on the result of the comparison among $\frac{T_{\text {reach }}}{T_{\text {dec }}}, \frac{1}{2}, \frac{\rho+4}{2 \rho}$. If $\frac{T_{\text {reach }}}{T_{\text {dec }}}>\frac{\rho+4}{2 \rho}$, the optimal control is $-a$ which means accelerating toward the origin. If $\frac{1}{2}<\frac{T_{\text {reach }}}{T_{\text {dec }}} \leq \frac{\rho+4}{2 \rho}$, the optimal control is 0 which means moving constantly toward the origin. If $\frac{T_{\text {reach }}}{T_{\text {dec }}}<\frac{1}{2}$, the optimal control is $+a$ which means decelerating toward the origin.

Case 2: $x_{2} \geq 0$. We can also explain this case with (11). $x_{2}=V_{y} \geq 0$ means that the objcet is not moving or moving away from the origin. So, the $T_{\text {reach }}$ is much bigger than $T_{\mathrm{dec}}$. The optimal control should be $-a$ which is the same with (6).

Overall, the strategy of the model is determined by $T_{\text {reach }} / T_{\text {dec }}$.
As human performance in simplified RVD tasks is similar to that of the optimal control model, we deduce that the control strategies of the human operators also depend on the time ratio $T_{\text {reach }} / T_{\text {dec }}$ which had been verified in the interviews of the expert operators. But for the human, $T_{\text {reach }} / T_{\text {dec }}$ is the time estimates other than the precise time calculations for the model. $T_{\text {reach }}$ is the estimated time for the chaser to reach the target at the current velocity, and $T_{\text {dec }}$ is the estimated time for the chaser to decelerate the current velocity to zero at a constant acceleration.

The differences between the performance of the human operators and the performance of the optimal control model can be explained by the biased estimates of $T_{\text {reach }} / T_{\text {dec }}$ of the human operators in the different docking phases.

In the docking phase when the chaser is relatively far away from the target in Y axis direction, the estimated time ratio of the human operators is larger than that of the optimal control model, so the acceleration time duration of the human operators is longer. In the docking phase when the chaser is relatively close to the target, the estimated time ratio of human is smaller than that of the optimal control model, so the deceleration moment is earlier.

In addition, the time estimate ability is different between different people. Consequently, the time weight parameters of experts are larger than that of less-skilled participants.

## 5 Conclusions

Present study suggests that although the time estimate of the human operators is biased, human control strategies in simulated RVD tasks can still be modeled through the time-fuel optimal control model. The control strategies of the human operators in the simplified RVD tasks depend on the time estimate ratio $T_{\text {reach }} / T_{\text {dec }}$. This finding will not only support RVD training and selection, but also provide useful guide for the cognitive modeling of the RVD tasks. Future study will focus on the control strategies in more realistic and complex RVD tasks and the implementation of control strategies in a cognitive architecture such as ACT-R [5].

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