

A TEXTBOOK ON ORDINARY DIFFERENTIAL EQUATIONS
S. Ahmad & A. Ambrosetti

ERRATA

1. pag.30 and following: in the proof of Lemma 2.5.2: replace all the integrals

$$\int_{t_0}^t \cdots ds \quad \text{by} \quad \left| \int_{t_0}^t \cdots ds \right|$$

2. Ch.2, Sec.2.5, pg.31, after (2.11) up to the end of the proof, replace the text by:

Consider the series

$$(x_1(t) - x_0) + (x_2(t) - x_1(t)) + \dots + (x_k(t) - x_{k-1}(t)) + \dots$$

The uniform upper bound (2.11) and the fact that the numerical series $\sum \frac{M}{L} \frac{(b-a)^k L^k}{k!}$ converges to $\frac{M}{L} e^{L(b-a)}$ allows us to use the Weierstrass M-test to infer that $\sum (x_k(t) - x_{k-1}(t))$ is uniformly (and absolutely) convergent to a function $\psi(t)$ in $[a, b]$. Since

$$x_k(t) = x_0 + (x_1(t) - x_0) + \dots + (x_{k-1}(t) - x_{k-2}(t)) + (x_k(t) - x_{k-1}(t)),$$

then the uniform convergence of $\sum_{j=1}^{j=k} (x_j(t) - x_{j-1}(t))$ to $\psi(t)$ implies the uniform convergence of $x_k(t)$ to $x_0 + \psi(t)$. This completes the proof.

3. pag.32, line 4: replace

$$\dots \leq \int_{t_0}^t |f(s, x(s)) - f(s, y(s))| ds \leq L \int_{t_0}^t |x(s) - y(s)| ds$$

by

$$\dots \leq \left| \int_{t_0}^t |f(s, x(s)) - f(s, y(s))| ds \right| \leq L \left| \int_{t_0}^t |x(s) - y(s)| ds \right|$$

4. pag.38, second equation: replace

$$\frac{1}{x(\alpha - \beta x)} \quad \text{by} \quad \frac{1}{x(\alpha - \beta x)}$$

5. pag.38, Ch.3, Sec.3.1.1: replace the last 5 lines by
 from which we obtain

$$\left| \frac{x(t)}{\alpha - \beta x(t)} \right| = ke^{\alpha t}, \quad k = e^{\alpha c}.$$

This is the general solution in implicit form. To solve for x we recall that either (i) $\alpha - \beta x(t) > 0$ for all $t \geq 0$, or (ii) $\alpha - \beta x(t) < 0$ for all $t \geq 0$. In the case (i) we find

$$\frac{x(t)}{\alpha - \beta x(t)} = ke^{\alpha t} \quad \text{which yields} \quad x(t) = \frac{\alpha ke^{\alpha t}}{1 + \beta ke^{\alpha t}}.$$

In the case (ii) we find

$$-\frac{x(t)}{\alpha - \beta x(t)} = ke^{\alpha t} \quad \text{which yields} \quad x(t) = \frac{-\alpha ke^{\alpha t}}{1 - \beta ke^{\alpha t}}.$$

This shows....

6. pag.51, line -5: replace $\frac{e^x}{(x+1)^2}$ by $\frac{e^x}{x+1}$
7. pag.62, ex. n.1: replace $p, q \neq 0$ by $p \geq 0, q \geq 1$.
8. pag.62, ex. n.2: delete $t, x \geq 0$.
9. pag.63, ex. n.17: after "its singular points" add "when $p = 2, q = 1$ and $a, b, d \neq 0$."
10. pag.63, ex. n.27: replace "for any $f(y) \neq 0, g(y)$ " by "for any differentiable $f(y) \neq 0$ and any continuous $g(y)$ "
11. pag.97, C7: in the system replace $z'' = y - 2z$ by $z' = y - z$.
12. pag.108, ex. A1: replace $t^3 - 3t$ by $x_2 = t^3 - 3t$.
13. pag.108, ex. A11: replace $x'' + \frac{x}{t} + q(t)x = 0$ by $x'' + \frac{x'}{t} + q(t)x = 0$.
14. pag.109, ex. B3: replace $x'' + 8x + 16 = 0$ by $x'' + 8x' + 16x = 0$
15. pag.109, ex. B10: replace $k \geq 0$ by $k > 0$

16. pag.109, ex.B11: delete $x(0) = 0$
17. pag.109, ex.B14: replace "only one solution tends to a constant." by "only constant solutions tend to constants".
18. pag.110, ex.C14: replace "has to change sign in (a, b) " by "cannot be non-negative in (a, b) "
19. pag.111, ex. D4: replace $q(t) > 0$ by $p(t) > 0$
20. pag.121, ex. n.8: replace $-3x'$ by $-x'$
21. pag.153, C7: in the third equation replace $-2z$ by $-z$.
22. pag.153, D5: replace $-3xx' = -2$ by $-3x' = -2x$.
23. pag.170, ex. n.7: replace "all the solutions" by "all the nontrivial solutions".
24. pag.171, ex. n.9: in the second equation of the system replace $-x$ by $-2x$.
25. pag.172, ex. n.21: replace $x(0) = 1$ by $x(0) = 2$.
26. pag.185, line 4: replace $R - t_0 < t < t_0 + R$ by $t_0 - R_i < t < t_0 + R_i$.
27. pag.185, lines 4 and 5: replace $a(t_0)$ by $a_0(t_0)$
28. pag.185, line 8: replace "is the smallest of the" by "is at least as large as the smallest of the"
29. pag.185, line 5: after "singular point." add "The reader should notice that we have already used the term singular point with a different meaning, dealing with exact equations $Mdx + Ndy = 0$, see Section 3.2."
30. pag.187, lines 6 and 11, delete $1+$ (three times); line 12, replace $x(t) = c(1 + \sinh t)$ by $x(t) = c \sinh t$
31. pag.254, n.5: in the second equation replace x by $3x$
32. pag.255, n.13: replace $x'_2 = x_1 - x_2 + x_3$ by $x'_2 = -x_2 + x_3$.

33. pag.255, n.19 in the second equation replace $-3x'$ by $-3x$.

34. pag.255, ex. n.20 replace the system by

$$\begin{cases} x'' - 2y' = ax + 3y \\ y'' + 2x' = 3x + ay \end{cases}$$

and " $a < 3$ " by " $|a| < 3$ ".

35. pag.256, ex. n.27 and 28: in the first equation of the system, replace $-y$ by y .

36. pag.257, replace ex. n.35 by

35. Show that $x'' + x + e^{-t}x = 0$ has oscillatory solutions.

37. pag.276, ex. n.8: replace " $x'(b) = 1$ " by " $x'(b) = 0$ ".

38. pag.276, ex. n.11: add "where $k \neq 0$ ".

39. pag.289, ex. n.17: after the equation, replace the given answer by
"The singular points are $(0, 0)$ and $(\frac{b^2}{ad}, -\frac{b^3}{ad^2})$."

40. pag.289, ex. n.33: add \pm before the fraction.

41. pag.290, ex. n.1 and 2: replace "a lipschitzian first derivative" by "a continuous first derivative".

42. pag.290, ex. n.37: replace the given solution by $x(t) = \frac{1}{4}t^2$

43. pag.291, ex. A11: replace $W(7) = \frac{49}{6}$ by $W(7) = 6$

44. pag.291, ex. B8: replace by "One root of the characteristic equation is positive (if $\beta > 0$) or null (if $\beta = 0$)"

45. pag.291, ex. B17: replace $a - b < 0$ by $a^2 - b < 0$

46. pag.293, ex. n.15: replace the given solution by $x(t) = \frac{e^{-bt}}{a-b} - \frac{e^{-at}}{a-b}$.

47. pag.293, ex. n.20: replace "has at least one negative root $m = -1$ " by "has the negative root $m = -1$ ".

48. pag.293, ex. n.26: replace $x(t) = 2 + 2t - 2e^t + t^2$ by $x(t) = -1 + \frac{1}{2}e^t + \frac{1}{2}e^{-t} - \frac{1}{2}t^2$.

49. pag.294, ex. B5: replace $\frac{7}{3}e^t$ by $2e^t$ and $-\frac{2}{3}e^t$ by $-\frac{1}{2}e^t$.
50. pag.294, ex. B8: replace the given solution by $x = c_1e^{2t} + c_2e^{3t}$, $y = c_1e^{2t}$.
51. pag.294, ex. B11: replace the given solution by $x = 3c_1e^{2t} + c_2e^{-2t}$, $y = c_1e^{2t} - c_2e^{-2t}$.
52. pag.294, ex. C7: replace $\frac{5}{6}$ by $\frac{1}{3}$.
53. pag.294, ex. D2: replace the given solution by $x = \frac{3}{2}t^2 + \int y(t)dt$, where $y = e^{-\frac{1}{2}t^2} (c_1 + 3 \int t^2 e^{\frac{1}{2}t^2} dt)$.
54. pag.294, ex. D6: replace the given solution by $x = c_1t - e^{c_1}$, $y = t - \frac{1}{2}c_1t^2 + c_2$ as well as $x = t \ln t - t + c'_1$, $y = t - \frac{1}{2}t^2 \ln t + \frac{1}{4}t^2 + c'_2$.
55. pag.295, ex. n.3: replace $\alpha = \beta$ by $\alpha = -\beta$.
56. pag.296, ex. n.3: replace the given solution by

$$x(t) = t + \sum_{n \geq 1} \frac{t^{3n+1}}{(3n+1)3n(3n-2)(3n-3) \cdots 4 \cdot 3} + \frac{t^2}{2} + \sum_{n \geq 1} \frac{t^{3n+2}}{(3n+2)(3n+1)(3n-1)(3n-2) \cdots 5 \cdot 4 \cdot 2}$$

57. pag.297, ex. n.8: replace a_0 by 0:
58. pag.298, ex. n.38: replace the given solution by $x = (4 - \lambda_1)A_1e^{\lambda_1 t} + (4 - \lambda_2)A_2e^{\lambda_2 t}$, $y = A_1e^{\lambda_1 t} + A_2e^{\lambda_2 t}$, where $A_i = \frac{\lambda_i - 2}{2\lambda_i + 2}$, $\lambda_{1,2} = -1 \pm \sqrt{8}$.
59. pag.299: starting from n.11, all the numbers of the exercises have to be diminished by 1.
60. pag.300, ex. n.11: replace by

$$G(t, s) = \begin{cases} \frac{-1}{k \sinh(k)} \cdot \sinh(kt) \cdot \sinh[k(s-1)], & \text{if } t \in [0, s] \\ \frac{-1}{k \sinh(k)} \cdot \sinh(ks) \cdot \sinh[k(t-1)], & \text{if } t \in [s, 1] \end{cases}$$