Chapter 8
An Ethnomathematical Perspective on the Question of the Idea of Multiplication and Learning to Multiply: The Languages and Looks Involved

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8.1 Introduction

One of the most delicate tasks performed by students and teachers continues to be learning about the objects and relationships of mathematics in school. Apart from this discipline having been referred to as difficult to understand for centuries, students are divided between those who fear it and those who adore it.

This passion for numbers is nurtured early on, not only in the school environment, but it must also be encouraged and developed through the guidance of elders who already perform more complex operations. The elders demonstrate to the young people the need to elaborate mathematical reasoning for resolution of daily problems, to use it as an object of leisure through hobbies, and to interconnect it with different areas of knowledge such as art, the exact sciences, the humanities, and the social sciences.

Mathematics is present in our daily routines. This causes all of us to use it consciously or unconsciously for the accomplishment of our tasks.

In this sense, one of the mathematical concepts used by all of us involves multiplying quantities. It is present in almost all of our tasks and in the classroom full of students trying to learn and their worried teacher. However, most children and adults perform this activity easily out of the school context, although some of them have not attended school or even have not learned to develop some mental calculation techniques; nor do they comprehend the use of algorithms for multiplication.

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It is also worth emphasizing that thinking about the learning of mathematical concepts in the scope of educating the human being (a noun) to be (a verb) human requires a process in which mathematics is worked on as a whole—that is, by including and relating to it during the learning of content by the students. From the teacher, this demands an attitude of an educator involving himself or herself more and more in the ethnomathematics program, since, in education today, we believe that the process goes beyond teacher–student dialogue. It involves knowledge of mathematics, social relations, and the use of technology, which are present in all our actions, even though they are not available to all people of the world. This is one of the great challenges of the present century.

The computerization of societies, contact with technology through social networks, and the use of technology as a form of registration through the image, and also as a way of expressing what is thought, are parts of the universe of the young people of today. The act of teaching using new technologies or virtual environments is just another way of bringing the mathematics that exists in the world into their objects and relationships, packaged in a language that is easily accessible for the young.

The choice of appropriate language to extend the understanding of mathematical content makes the students more interested in learning something that matches the realities in which they exist. The use of art through symbols, icons, and photographs is getting closer and closer to young people.

In this context, we want to report here some mathematical ways and thoughts about this task, which involves the most basic as well as advanced calculations, but which are part of the daily life of some professionals and students.

The elaboration of the concept of multiplication happens naturally in the history of man. The elaboration of an algorithm to facilitate the solution of a problem was a human creation. However, these practical devices are not yet in the domain of the entire world population, although, in practice, everyone has a way to perform a necessary or requested calculation.

Perhaps this difference in looking at it causes a gap between practical and theoretical knowledge. Since the practice of some teachers is increasingly related to their classroom, this may make education decontextualized.

In this way, some groups of educators are increasingly placed in a macroperception between scientific knowledge and the cultural knowledge of the social groups in which teachers and students exist. According to gestalt theory, the perception of the whole context in which the individual exists gives him or her elements to better understand mathematical knowledge.

Another way of thinking about this involves the concept map of a website, where anyone can have access to all of the concepts involved and only perceive it in a linear way in which they relate to it. Weaving through the web in all dimensions will require the interaction of the individual in his or her learning.

8.2 Alternative Modes

8.2.1 Project Learning

Escola Municipal de Ensino Fundamental (EMEF) Desembargador Amorim Lima was initially inspired by the Escola da Ponte to promote a major change in its political pedagogical project. Today, with its own identity, this Brazilian school has a slightly more flexible political calendar, thus allowing newly proposed workshop activities to be added to the basic curriculum, which is composed of research and execution of scripts elaborated by the school.

These completed scripts work as miniprojects. They are integrated into the different areas of knowledge about the researched subject in the sense of giving autonomy to different ways of learning and discovering the world at the rate at which students develop their activities.

The school's physical plan has been remodeled to carry out all the activities provided for in the political pedagogical project. The main building consists of three floors. On the ground floor, there is the computer room (which is well equipped with microcomputers and the internet), the capoeira room, the literacy room for first-grade children, and a repository of the teaching materials available to teachers. In the courtyard, there is a stage. On its wall are the principles of the school, written by teachers and students together. At the end of the inner courtyard, there is the canteen and, at its side, the stairs that lead to the classrooms. Through a corridor next to the canteen are the library and the art room. Accessing the school by the service entrance, the first room is for the secretaries of the school, the rooms of the teachers, the classroom of the first grade, the room of the pedagogical coordination, the boardroom (where meetings with teachers are held), and finally, the bathrooms (Fig. 8.1).

Going up the stairs from the inner courtyard on the first floor, there is a room for English workshops, two rooms for project realization, and the hall—inspired by the Escola da Ponte, formed by three linked rooms—where there are students from second to fourth grades. This hall has three living spaces and six computers, which are used by students doing research. The second floor is laid out similarly to the first floor; the only difference is the occupants. The activities here are for students in grades 5–8. Throughout the school building, there are works by the students. The walls are well painted. The rooms and halls are clean and organized between one activity and another. Green curtains complement the spaces, provide protection from the sun, and brighten the environment. Around the building, there is an outdoor area with courts, a skating rink, a vegetable garden (planted together by teachers and students; the food produced is used to prepare lunch in the canteen), a white event tent (which was donated to the school), and a wooden tipi (created by Guarani natives from the village Morro da Saudade), which was part of an exchange with the school.

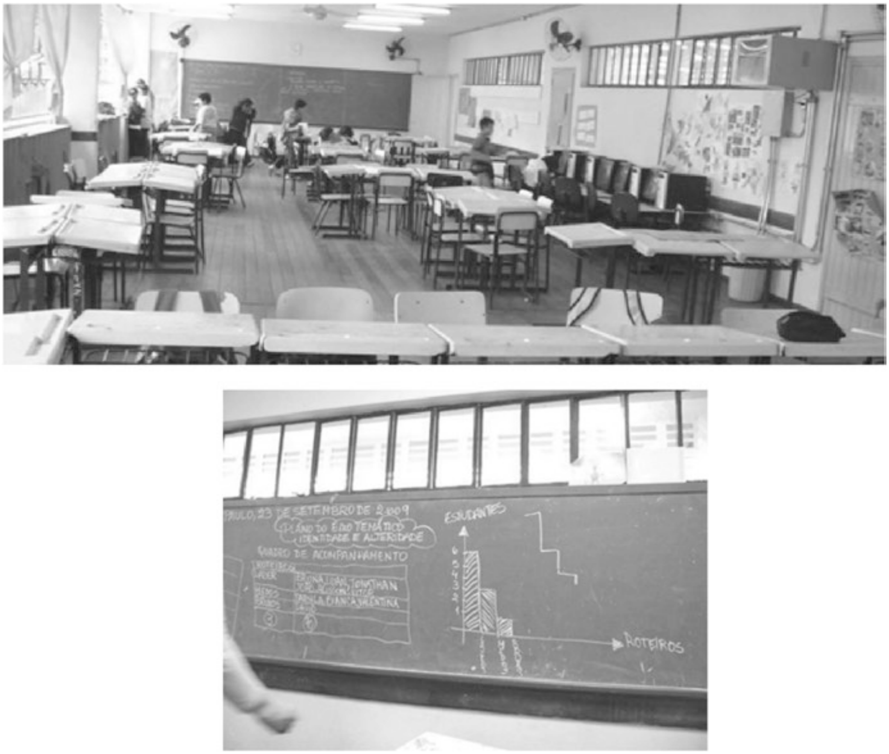


Fig. 8.1 The hall (*top*) and a blackboard (*bottom*) at Escola Municipal de Ensino Fundamental (EMEF) Desembargador Amorim Lima (2004)

As of February 2009, Prof. Dr. Geraldo Tadeu Souza introduced the fourth thematic axis: our world. In 2010, a fifth axis was added, so the work is organized as follows by the coordinating teachers of these axes:

1. *Alterity and identity*: This uses theatrical language, a resource that has been elaborated since 2007 with the help of a professor from the Faculty of Education (FE) at the University of São Paulo (USP), Brazil.
2. *Life*: This axis has been built through experiments and their documentation, under the scientific eye; for this, the school has spaces that allow this type of activity with groups.
3. *Our planet*: In this axis, the focus is on chemical experiments that are performed and recorded.
4. *Our world*: In this axis, art is predominant. Through activities that involve the construction of models and going to the cinema, the students produce reports in elaborate scripts.
5. *Literacy*: This axis was added to the project in 2010. It arose from the need for experience by practice and to give more support to the activities of documentation and communication through literacy. As described by D'Ambrosio (1999):

– LITERACY: It is the ability to process written and spoken information, which includes reading, writing, calculation, dialogue, eclogue, the internet in daily life [COMMUNICATION INSTRUMENTS].
– MATHERACY: It is the capacity to interpret and analyze signs and codes, to propose and use models in daily life, to elaborate abstractions about representations of the real [INTELLECTUAL INSTRUMENTS].
– TECHNOCRACY: It is the ability to use and combine simple or complex instruments, including the body itself, evaluating its possibilities and limitations and its suitability to diverse needs and situations [MATERIAL INSTRUMENTS].

It is important to emphasize that this proposal, as a whole, seeks to build knowledge based on the freedom to comprehend the world and to educate for responsibility and freedom.

At this pedagogical moment, the traditional teacher is replaced by a tutor teacher responsible for 20 students, arranged into groups of five. The purpose is to organize the research work to be done in the resolution of the scripts aside from assisting classmates to overcome some doubts.

8.2.2 Thinking of Multiplication Through Research Scripts

Although the mathematical content is not perfectly integrated into the projects in this school, in general, the questions cover all areas of knowledge. It is possible to think of the following topics for work:

Theme: Bees (second and third grades).
Questions
1. How are the bees arranged in the hive?
2. How are the combs arranged in the hive?
3. When observing the work of bees, can we say that there is some kind of organization and hierarchy in this environment?
4. Draw a piece of honeycomb on a 10 cm × 15 cm wooden frame. Describe some ways to organize mathematical thinking to count all the honeycomb cells.
5. How can we count the bees that are in the combs? Exemplify ways of counting by drawing it. Does knowing the number of cells help calculate the number of bees?
6. What are the most common types of bees? Where do they come from?
7. Would there be another geometric figure that could be used by the bees to build a honeycomb? Which one do you consider the best? Does the hexagon suit the honey bees?

This example makes it possible to explore with groups not only the ways of counting the number of cells but also the beginning of the organization of mathematical thinking by introducing some notions to begin the discussion of the concept of multiplication. So, if we are able to think of groups of 2 cells, we can count how many there are and carry out multiplication. Likewise, as we increase the group size to 3, 4, and so on, we will reach a maximum group where the children by themselves will understand that it leads to the need to create an algorithm.

As we can see in the schematic drawings, when we count by 2s, there is a large group to be marked.

When you choose groups of 3, the number starts to decrease, and so on. Sometimes the group of students themselves suggests choosing groups of 10 so that the counting becomes much easier.

However, when we choose groups larger than 10, it is easier to visualize the groups, but the counting begins to be difficult mentally. At this point, one may suggest or even begin to question the need to use a process or a way of performing calculations for numbers in this situation, so next we show how the multiplication algorithm works in order to facilitate the counting to be performed (Fig. 8.2).

Evaluation and self-evaluation: Elaboration of the activities and later complementation of the activities start in the classroom. At the end of each activity, it is recommended that students have a prearranged space to write their self-assessment of the knowledge presented. In this way, the student will say in his or her own words what he or she has learned that day, what still needs to be learned, and the curiosities or doubts that the lesson has generated, in order to research them later with the help of an older person, the internet, or the teacher.

It is important to emphasize that it is not only the work of observation, analysis, control, and evaluation that the tutor performs. At the same time, he or she also initiates the construction of deep links bound both to the learning of scripts and to the personal and collective responsibilities of the group.

At this school, it is possible to perceive the affection and the concern of the teachers for the students. Besides recognizing each one by name and always looking directly in their eyes when talking and explaining the activities and content to them, the teacher guides the groups during analysis and evaluation of the results. Perhaps this is one of the most humanitarian facets of education, which puts an end to indiscipline and allows students to respect themselves as well as their teachers and all who contribute to the administration of the school.

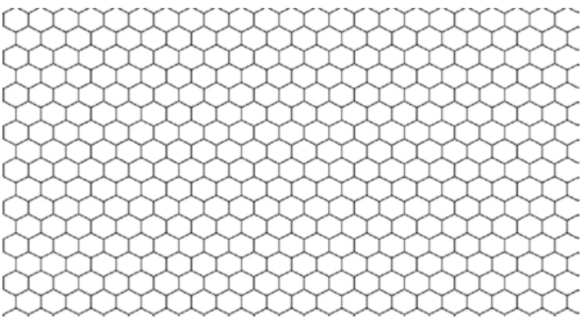


Fig. 8.2 Honeycomb

8.3 Multiplication: Tables with Polygons

In general, mathematical educators are very concerned with learning and teaching operations, especially with the concept of multiplication. According to Isoda and Olfos (2009a, b, pp. 46–55), in *Guías para la Enseñanza de la Matemática*, the concept of multiplication is approached by showing variations of how to represent a quantity that is repeated several times, going through concrete examples until it is abstracted for the calculation that one wishes to make.

In the Waldorf Schools of São Paulo, the concept of multiplication is built together with the geometry of flat figures through the elaboration of mathematical thought in conjunction with string figures created on a wooden table with nails (geoplan) and strings. These are designed especially for each child to be able to manipulate and understand the concept (Fig. 8.3).

Tag: Mathematical knowledge of the product of two natural numbers.
Topic: Learn and teach the product of two natural numbers.
Duration: For each multiplication table, it is necessary for the teacher to work through the mathematical content with the students. It is hoped that five lessons will be enough for each of them; however, this will depend on the knowledge of the group.
Content to be developed: Multiplication tables for numbers 1 to 12, polygons, relation of similarity, and direction.
Objectives: To give support for learning of multiplication tables, polygons, and similarity.
Methodology adopted: This class should be used after the teacher has already talked with the students and has exemplified and worked on individual activities on the concept of multiplication with an experimental class. This is followed by discussion in groups, formed by two or three students, for the elaboration of a geoplan, where the figures that geometrically represent the multiplication tables will be constructed. It is expected that from such discussion, geometric forms will emerge that will help the student to remember the results that are found.
Teaching resources: White board, pencil suitable for use on the board, kraft paper, piece of wood 25 cm × 25 cm, nails, string, hammer, thick wool, notebook, pencil, crayons.

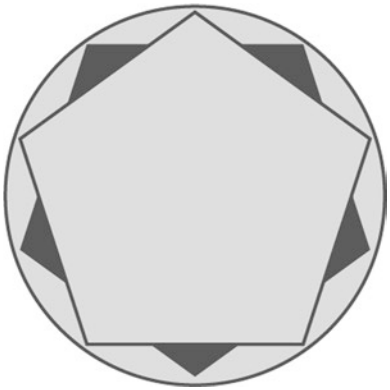


Fig. 8.3 Image of geoplan (Table 2)

Description of activities: 10 nails are hammered with equal spaces between them on the circle. With a string or woolen ball, we begin the multiplication table and with each number found, we take a turn on the nail. For the table of 2, $2 \times 1 = 2$, so we mark position 2; $2 \times 2 = 4$, so we mark position 4. This goes up to $2 \times 6 = 12$, so we take one turn and two more. In the same way, $2 \times 7 = 14$, so we reach position 4. We continue until $2 \times 10 = 20$, which falls in the tenth position. In this way, the figure formed is a pentagon, which is associated with the results of this table.

We consider this idea relevant because it works with the results of the multiplication tables associated with the number they represent and gives an opportunity to work with the geometric figures.

8.4 Multiplication using Art and Technology

Leonardo da Vinci, a Renaissance artist and scientist, developed interfaces between art and science, not only to transform painting into a faithful representation of the world around us, through the use of perspective and nature as templates, but also to invent machines to change some things in nature. In this way, he conceptualized many machines, such as flying machines and machines to move large volumes of land or to defend the nobles for whom he worked at that time. He explored, studied, designed, and reproduced, on canvas, the geology of soils and the structure of various plants and flowers. In his records and drawings of the human body, we can analyze his explanations and clear observations of how the human being is constituted, from the development of the fetus until birth.

In this context, bringing the figure of Leonardo into the classroom attracts the attention of young and old alike, both for the artistic value of his works of art and for the concepts explored in his inventions that inspire the desire to understand everything around us.

Sabba (2004) studied the works and the life of Leonardo, which show the integration between scientific knowledge and art, besides listing categories that could help the development of the human being as a whole by valuing emotions and sensations through categories such as corporality (*corporalità*) and sensations (*sensazione*) using art and drawings. These categories are already developed in schools such as the Waldorf Schools and the Sakura School in Tsukuba municipality, according to Isoda and Olfos (2009a, b, p. 55).

A good example of working in the classroom in this sense is the area of mathematics represented by the features that make up Leonardo's work *Vitruvian Man*. This presents a man inside a square and a circle, such that it is inscribed in the figures, besides having the body marked by proportions that make the human being so harmonious in our eyes.

Figure 8.4 shows the relationships that exist in our body—how it is possible to construct the height of the individual by knowing the size of his arm span and how there are symmetries between the parts in a certain direction.

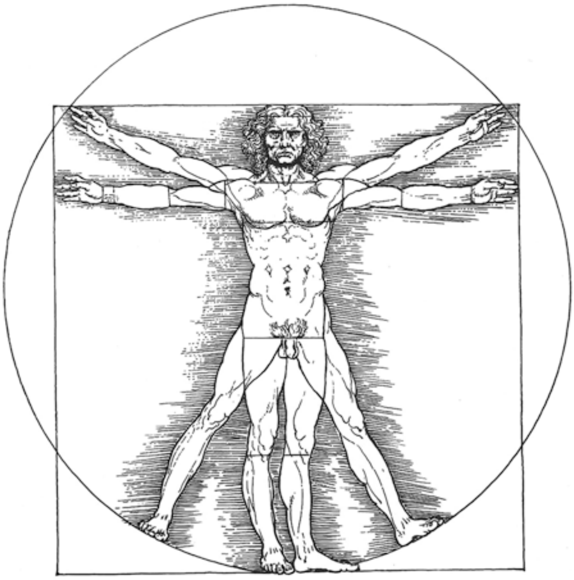


Fig. 8.4 Leonardo da Vinci's *Vitruvian Man*

An activity to be developed would involve pieces of kraft paper, the size of which would be slightly larger than the height of the student. In this way, a young man would lie on top of the sheet while another one would draw his outline with a marker pen. In this drawing, we would mark the student with his feet together and with his arms stretched out, without moving the upper body. Then, his outline is drawn in another position with his feet and arms spread half open, just like the position shown in Leonardo's *Vitruvian Man*.

With the drawings at hand, we would ask them to draw the proportions that exist in the body and, after that, a square and a circle around the body. First, we would observe the ideas of students on how they would perform the activity, and then explore the geometric relationships that would involve the square, the circle, and also where the square is inscribed or circumscribed.

When the teacher decides to use this drawing, some students may feel uncomfortable; thus, the activity may not achieve its purpose. In order to avoid this, we can substitute a different approach that children always use that would also produce the same effect. For example, using a cell phone for taking pictures, instead of drawings, does not pose any problems for young students; on the contrary, this procedure shows them how mathematics is part of our universe and how it is possible to work with mathematical content within the world of information technology in which the students exist.

Perhaps a teacher who was born before the 1990s may feel uncomfortable using this approach. However, the students will feel confident in photographing their friends in the requested poses and can still work with a ruler and compass to perform the calculations. It is possible to work out the proportions that exist on the face, as well as the relations between the body parts. For example, the foot is the size of the forearm, and the perimeter of a closed hand also corresponds to this measure; these are among many such relationships that exist in the human body.

Tag: Mathematical knowledge of division and products, construction of a square and circumference.
Theme: Learning and teaching of division and multiplication of integers.
Duration: It is expected that two lessons will be enough for the student to work through the mathematical content, with the help of the teacher in the second class; however, the teacher will decide, depending on the knowledge of the group.
Content to be developed: Relation of similarity, construction of the square and the circumference.
Objectives: To give support for learning of multiplication tables, polygons, and similarity.
Methodology adopted: This lesson should be used after the teacher has already talked with the students about Leonardo da Vinci and decided with them what methodology to use. Such activity is expected to show how the proportions construct the whole and make the symmetries of the figure more beautiful.
Teaching resources: Kraft paper roll, marker pen, whiteboard, pen suitable for use on the picture, notebook, pencil, colored pencil, mobile phone, computer, printer.

8.4.1 Multiplication Using the Calculator

Students are still prohibited by some teachers from using a calculator in tests and school activities, which at the end of their training entails a lack of knowledge of how to use and exploit such a tool that is necessary to facilitate the daily routines of many engineers, administrators, vendors, and others.

We believe that many activities can be done in order to facilitate conscious learning of the algorithms that we use to perform operations and also knowledge of the resources available to facilitate day-to-day calculations.

For example, the teacher usually asks students to solve problems that offer two numbers to multiply and find the result. Actually, in that case, the calculator solves everything and there is no room for many interactions.

However, if the educator offers the result of the product and one of the numbers involved in multiplication, the student will have to find the other number, which will lead to an inverted table exercise. For example, to solve the problem proposed in Fig. 8.5, without using the division algorithm, we must investigate the results of the three-dimensional table to deduce that it is a 3-digit number.

In this way, the student should think about which number, when multiplied by 3, will have a 2 in the units place. They will remember that if the number is 4, the result is 12. One group of 10 is carried over to the tens place, and 2 is in the units place. Again, remembering the results of the table of number 3, they will look for a number in the tens place such that, when multiplied by 3 and with the addition of 1 to the product, will give 2. This number is 7. Using the same analogy, the student would find the number 6. Therefore, the number sought is 674 (Fig. 8.6).

Still on this subject, after teaching the algorithm on multiplying 2-digit numbers or 3-digit numbers, the teacher could propose exercises, as shown in the Fig. 8.6, suggesting some of the numbers to help the others search Fig. 8.6.

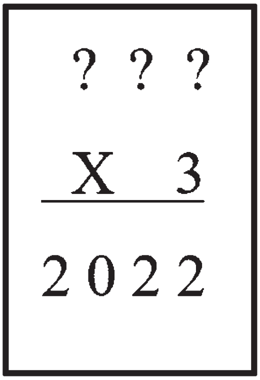


Fig. 8.5 Multiplication algorithm

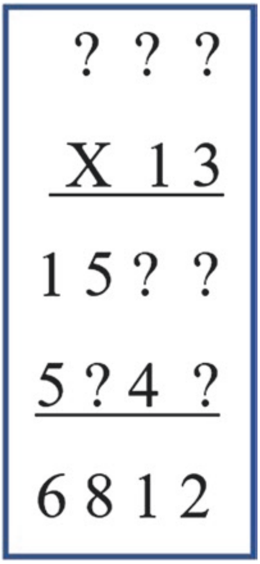


Fig. 8.6 ??? × 13 = 6812, finish the count

In the same way, the activities would allow an investigation on the part of the student, allowing him or her to think of several methods to solve a single problem as well as reminding him or her of the importance of the positional numeration that the numerical system possesses.

Tag: Mathematical knowledge of the product of natural numbers by units and by numbers greater than 10.
Theme: Learn and teach the product of natural numbers.
Duration: It is expected that each algorithm model will be worked on in three classes after the students have been familiarized with it; however, the teacher will decide, depending on the knowledge of the group.
Content to be developed: Multiplication of natural numbers, multiplication tables.

Objectives: To give support for learning of multiplication tables and algorithms for multiplication, developing logical reasoning.
Methodology adopted: This class should be taught after the teacher has explained each of the algorithms.
Teaching resources: Whiteboard, pen suitable for use on the picture, notebook, pencil, colored pencils.

In all activities—both those described here and those that the teacher has already incorporated into his or her classroom practice—it is possible to ask the student to make a self-assessment by writing a few lines about their improvement, presenting their ideas in front of the class, and requesting them to write down the questions or the important points learned that day. We believe that honest self-assessment on the part of the student—and, likewise, reading and adaptation of the teacher's practices—will aid in the success of teaching and learning outcomes.

8.5 Some More Ideas About Learning and Teaching of Mathematical Knowledge

Although the learning and teaching of mathematics are generally decontextualized from the reality of the students and teachers for different reasons, it is important to observe that there is a worldwide increase in the number of both teachers and students seeking ethnomathematics as a way to improve mathematical education and also to increase their self-confidence in the learning of educational content.

The ethnomathematics program allows integration of everyday knowledge to the scientific knowledge presented in schools and in unification the knowledge, as the human being is directly responsible for comprehension of their knowledge as well as for interfering in the environment around them and in reality, through the use of this knowledge. It is important for the teacher to contextualize each new topic in order to articulate this new knowledge in the context of the knowledge has already been acquired.

In the sense of articulating mathematical knowledge as a whole, the gestalt theory opens up a new vision. Thus, this theory applied in teaching shows how important macrovisualization of the object under study is, as well as visualization of its parts. It raises an important question in showing that the sum of the parts is different from the interaction of the parts. The whole—that is, the totality that gestalt refers to—can be understood as the articulation of various mathematical theories or examples which are sometimes presented without apparent connections but have the same theoretical basis. We call attention here mainly to the use of gestalt in mathematics.

For example, a student should imagine the object or the problem in question as a photograph. It is noted that the focus—the attention—of seeing the picture is not in the details but in the overall aspect that it conveys. The photograph of the whole is important, as is the visualization of the parts and their interaction.

The presentation of the "photograph" gives the students a sense of location—where I am and where I should go—in order to acquire/construct knowledge.

This indication is sometimes neglected mainly in mathematics and contributes to its decontextualization, since the students end up learning mathematics (the whole) as a list of rules and topics (the parts) to be assimilated and reproduced in a test without understanding that it is part of a greater whole. The same thing happens in the area of Portuguese language when a student writes an essay in which their use of periods is grammatically correct although there is no coherence between them; the writing will not make much sense even if its parts are perfectly correct.

For better understanding of the interaction of the parts, it is worth remembering the idea of a movie. Analyzing a movie frame by frame does not show the movement and dynamism that the movie provides unless the frames are shown at a certain speed. In the same way, the articulation of mathematical content provides the idea of other mathematics.

An example of development in the practice of this vision would be the study of relations involving the volume of a cylinder, a sphere, and two cones, all with the same height, as shown in Fig. 8.7.

Figure 8.8 shows the articulation between the volumes (V) contained in half of a sphere, a cone, and half of a cylinder, all with height R.

$V_{\text{sphere}} = (2/3)\ V_{\text{cylinder}}$

$V_{\text{cylinder}} = 3V_{\text{cone}}$

$V_{\text{sphere}} = 2V_{\text{cone}}$

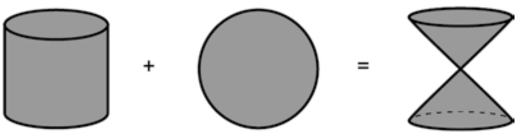
This shows the proportion between the volumes.

In general, the teaching of geometry in elementary school is based on a narrative that starts from simple elements—points, lines, and planes—in search of constitution of an image as the whole photograph.

The fundamental message that gestalt theory suggests is that the perception of the photograph gives rise to an interest in the points that constitute it. Beyond all that, the interaction between the points and the photograph can be only minimally understood if we limit it to a one-way direction that leads from the points to the photograph.

From what has been considered up to now, in all activities—both those described here and those that the teacher incorporates into his or her class practice—it is possible to ask students to make a self-evaluation by writing a few lines about their improvement, presenting their ideas in front of the class, and requesting them to write down the questions or the important points learned that day. It is important for students to learn not only to multiply and understand mathematics as techniques that relate numbers, points, and lines, but also to use this knowledge to solve practical problems in life.

Fig. 8.7 Geometry



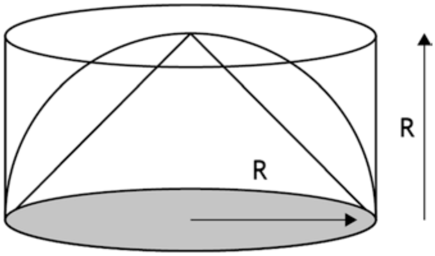


Fig. 8.8 Articulation between volumes

Foot notes

1. Geraldo Tadeu Souza is a professor at the Federal University of São Carlos (UFSCAR), Sorocaba Campus, and a doctor in linguistics from the Faculdade de Filosofia, Letras e Ciências Humanas (FFLCH) at USP.
2. Hernández (1998) explains that in a work project, the integrative aspect of knowledge construction violates the model of traditional education, so the transmission of knowledge compartmentalized and chosen by the teacher takes a much broader and dynamic context due to the interaction of meanings, resulting in active production of meanings and knowledge—that is, "a variety of actions of understanding that show an interpretation of the theme, and, at the same time, an advance on it." (Hernández, 2000, p. 184). The author further emphasizes that the project is not a methodology but a way of reflecting on the school and its function, which will present differences in each context.
3. EMEF Desembargador Amorim Lima is a public school in the city of São Paulo, SP, Brazil. Escola da Ponte is a Portuguese public school in Vila das Aves in the District of Porto, Portugal.

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