

Chapter 8

Classroom Studies—Sociological Perspectives



Uwe Gellert and Götz Krummheuer

Abstract The specific aspects of Classroom Studies, as a focus within the German Speaking Traditions in Mathematics Education Research, rest on the fundamental sociological orientation on mathematics lessons. Initiated by the works of Heinrich Bauersfeld, the first sociological perspective unfolds its power of description by reconstructing social processes regarding the negotiation of meaning and the social constitution of shared knowledge through collective argumentation in the daily practice of mathematic lessons. A second sociological perspective aims at the reconstruction of the conditions and the structure surrounding the construction of performance and success in mathematics lessons.

Keywords Access · Argumentation · Interpretative studies · Negotiation of meaning · Participation · Performance

8.1 Introduction

The specificity of classroom studies as a focus within the German-speaking traditions in mathematics education research is due to the fundamental sociological orientation of the view into mathematics teaching and learning. Two sociological points of view are distinguished: (a) the constituents and effects of learning in mathematics classroom activities; and (b) the effects and constituents of stratification in mathematics teaching and learning. We do not attempt to provide an inclusive overview of classroom studies in German-speaking countries, but rather point out the special nature of the sociology-based tradition.

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- (a) Initiated by the work of Heinrich Bauersfeld, the first sociological perspective unfolds a considerable descriptive power in the reconstruction of social processes of the negotiation of meaning and the social constitution of taken-as-shared knowledge. The *modus operandi* here is collective argumentation in the everyday practice of mathematics teaching and learning. With respect to the sociological reference to Symbolic Interactionism, Conversation Analysis and Ethnomethodology, a microsociology of the teaching and learning of mathematics in school is constructed. In the first section, Götz Krummheuer reports on “Interpretative Classroom Research: Origins, Insights, Developments”.
- (b) The second sociological perspective aims at reconstructing the conditions and structures of the construction of achievement and success in mathematics education. In this respect, the mechanisms of interaction and their (often indirect) social effects are the focus of attention. In doing this, the microsociology of the teaching and learning of mathematics in school is systematically related to meso-sociological and macro-sociological, institutional and societal structures. For this purpose, theories of the sociology of education serve as a central reference. In the second section, Uwe Gellert describes “Classroom Research as Part of the Social-Political Agenda”.

8.2 Interpretative Classroom Research: Origins, Insights, Developments

8.2.1 Introduction¹

The process of the development of an interactional theory of learning mathematics reaches back to the years between 1970 and 1980. In these years, the foundations were laid for the basic concepts for a theory of interaction of mathematics learning and teaching in the work group around Heinrich Bauersfeld at the IDM of the University of Bielefeld (Krummheuer and Voigt 1991). This theoretical approach was further expanded (Jungwirth and Krummheuer 2008) and in time efforts were undertaken to make these concepts and the typical way of thinking behind this approach known

¹As the sole author, I alone am responsible for this part of the paper. However, without the intensive work with other colleagues, the presented results would not be possible. I would like to especially mention:

- At the Purdue University in West-Lafayette, Indiana USA: Terry Wood, Erna Yackel und Paul Cobb, and in Germany,
- at the Institute for Didactic of Mathematics at the University of Bielefeld: Heinrich Bauersfeld and Jörg Voigt,
- at the Free University Berlin: Natascha Naujok, Birgit Brandt and Uwe Gellert, and
- at the Goethe University Frankfurt am Main: Marei Fetzer, Christof Schreiber, Marcus Schütte, Anne Fellmann, Anne Vogler, Ergi Acar Bayraktar, Melani Beck, as well as my colleagues Birgit Brandt, Helga Jungwirth and Rose Vogel.

to practicing teachers (Fellmann 2014; Fetzner and Krummheuer 2007; Krummheuer and Fetzner 2005). The theory development as well as the endeavour to make the results available to teachers were usually funded by grants, for example the German Science Foundation. Most of the projects deal with mathematics teaching in grade school. More recent projects involve also preschool as well as familial settings.

In the following the development of the theory (Sect. 8.2.2), its practical relevance (Sect. 8.2.3) and the practice of research (Sect. 8.2.4) are described. Due to space constraints, the methodological foundation of this approach is not outlined. This is unfortunate in so far as its name “interpretative classroom research” refers to its usual methodological classification. It should be mentioned that there are more approaches that apply interpretative methods. Usually, their theoretical perspective is not a genuine sociological one as in Steinbring’s epistemological work or in the research of the group around Nührenböcker (Schwarzkopf et al. 2018; Steinbring 2005).

8.2.2 Theory

The theoretical approach that is referred to here as interactionist is based on three basic assumptions:

1. The subject matter to be learned as well as the learning conditions that are necessary for its acquisition are situationally bound in interactive exchange between the participants in the process of the negotiation of meaning.
2. The constitutive social condition of the possibility of learning of a mathematical content, term or procedure is the participation in a process of collective argumentation concerning the content, terms or other procedures.
3. The indication of a successful process of learning of a pupil is the increased autonomous participation in such collective argumentation in the process of a current interaction and/or in the following interaction that is thematically imbedded in the actual situation.

In the following these three points are outlined in more detail.

8.2.2.1 Emergent Interactional Processes: Negotiation of Meaning and Conditions of Learning

Basically, this approach refers to symbolic interactionism and ethnomethodology. Below a twofold hypothesis is stated, whereby the first one represents the other reason why this approach is interpretative.

In the sequence of interaction during a process of teaching, a situational meaning of the content at stake is negotiated by acts of speech and accompanying actions and is based on processes of interpretation conducted by the participants. Thus, one can differentiate between the intended content as exemplarily presented in the teaching material and the

situationally emerging themes as they emerge by the process of negotiation (Krummheuer 1995; Krummheuer and Brandt 2001).

The cognitive processes of the pupils refer to these interactively produced and situationally bound themes and not to the provided mathematical content.

Through previous experiences in similar processes of negotiation, often the interpretations by the participants are routinized and standardized. Alluding to Goffman (1974) Krummheuer calls these ways of interpretation “framing”. These are structured individual processes of interpreting that through adjustment to previous processes of negotiation have found a certain routine in their recall as well as a certain standardization in their use (Krummheuer 1995, 2007).

On the interactive level, routinized processes of negotiation have been reconstructed above all in teacher guided situations. Voigt (1995) introduces the terms of “pattern of interaction” and “thematic procedures” for these phenomena. He understands here a specific, thematically focused rule in the process of interaction. It is essential for him that these patterns refer to the process of negotiation and thus contain a subject matter component. This distinguishes his definition from those which do not concern themselves with interaction structures that are content-bound, as for example (Mehan 1979) with his interaction pattern „initiation-reply-evaluation“ (p. 54). Furthermore, Voigt’s work refers to Bauersfeld’s concept of the „funnel pattern” (1980). In his later research this led him to speak of “thematic procedures”, a more exact terminology. In the Sect. 8.3.2.2 the concept of pattern of interaction will be mentioned again.

Beside this empirical research on teacher-guided instruction, several studies have dealt with interaction processes in group work of pupils in mathematics classes. Naujok (2000) reconstructs work phases of different types of cooperation, such as helping, collaborating and parallel forms of work. Lange (2013) was able to reconstruct further types of cooperation in a study on the processes of problem solving in mathematics group work in the secondary school. Krummheuer and Brandt (2001) differentiate in groups of pupils between “stable, collective work processes” and the “parallel handling of problems” (pp. 66ff).

In considering the mathematical aspects in the attempts at theory building, the following two characteristics have been more exactly examined in corresponding empirical studies:

- (a) the dependency on language
- (b) the specific inscriptionality of mathematics and mathematics education

Here a short discussion of these two points:

- (a) With reference to the dependency on language that distinguishes itself among others through a specificity of the subject matter, Bauersfeld (1995) speaks of “language games” in allusion to Wittgenstein (1963). Here Bauersfeld expresses the idea that the language in the mathematics teaching interaction is characterized by specific forms, ways of expression and means of dealing with each other that, taken together, build “a culture of a mathematical classroom” (ibid., p. 282).

A similar idea is expressed in the term „mathematics discourse“ (Moschkovich 2007; Sfard 2008).

Schütte (2009) states the following hypothesis about this classroom culture based on his socio-linguistic oriented analysis of mathematics teaching: In the introductory situations in mathematics classrooms the mathematics contents that are to be learned are often presented in oral everyday language, while the later required achievement tests in the class and written papers are conducted in a formal subject-matter language of mathematics and mathematics education which most of the pupils do not have at their disposal (p. 195). This everyday classroom practice follows an “implicit pedagogy” (p. 196) in which pupils learn the contents that are to be negotiated more on the basis of their out-of-school mathematics and language competencies rather than through a mathematics specific language game developed in the interaction of the classroom. These linguistic and the above-mentioned frame analyses both point to the qualitative difference in the use of language and the habits of interpretation between the teacher and the mathematically more competent pupils. Conversely, other pupils interpret with everyday language the happenings in the classroom by a way of framing that is not appropriately adapted to the treated mathematics contents. Krummheuer (1995) characterizes such phenomena as “framing differences”.

- (b) Besides the special mathematics language, other specific graphic signs and/or symbols are used in mathematics lessons that can be communicated only in a written or drawn form. These place special demands on the interactive negotiation of meaning in mathematics lessons. It should be remembered that also from a perspective of mathematics education, even more inscriptional elements flow into the teaching interaction in form of learning materials and visual aids. Fetzer (2003a, b) studies these specific content and didactically motivated inscriptions in mathematics classes. For her analyses, she refers to the model of a two-dimensionality framework of orality and literacy from Koch and Oesterreicher (Oesterreicher 1997). It is important to mention that the inscriptional elaborations of the pupils are a blending of their current mathematical ideas and their anticipated expectations about what effect their publication might have on the class. Hereby, a second level enters the processes of negotiation. Fetzer speaks here of a “double interactionism” of mathematics teaching processes (2003a, p. 86).

Schreiber (2004) adapts the semiotics approach of Peirce (1978) in order to theoretically encompass the inscription aspects of mathematics interaction. Thus, an alternative semiotic-oriented understanding of the inscriptional processes in mathematics teaching is developed in addition to Fetzer’s linguistics-based approach. Schreiber reconstructs semiotic processes that are characterized by a chain of signs and attributed meanings and which within these meanings again becomes signs in a negotiation by interactive turns. With reference to the construction of a term of Presmeg (2002) he speaks here of a “chaining” i.e. of “complex semiotic processes” (Schreiber 2010, p. 40). Further Schreiber takes up Peirce’s concept of “diagram” of such semiotic processes: Diagrams are inscriptions, which are constituted by a system of rules concerning their

generation, use and transformation (Bauersfeld and Seeger 2003; Dörfler 2006; Kadunz 2006; Morgan 1998). Thus, processes of thought do not (only) take place internally in the cognition of the learner; much more these processes can take place also in the external manipulation of diagrams, guided by certain rules and, at the same time, with “reduced speech”. Schreiber expands here an up to now mostly non-researched field at the juncture of semiotics and interactionism.

8.2.2.2 The Condition of the Possibility of Learning: Collective Argumentation in Formats

The second premise of the interactionist approach says that the constitutive condition for the possibility of learning of mathematics is the participation in a collective argumentation (see above). In Sect. 8.3.2.1 the term argumentation is dealt with. Then in Sect. 8.3.2.2 the learning theoretical content of this premise is explained.

The Concept of Argumentation

The mathematics discourse among mathematicians is distinguished by a specific “accounting practice” (Garfinkel 1967, p. 1) that is determined by strictly logical argumentation as in the mathematical proof as a special form of argumentation. We also speak of mathematics discourse referring to interactions in mathematics classes. Here, the accounting practice might look different, especially if one thinks of grade school, kindergarten and preschool. In general, by an argumentation “a” a process of negotiation is accomplished in such a way, that “a” supports an utterance “b” so that the participants agree with the correctness of “b” (Kopperschmidt 1989). Usually an argument consists of several utterances that assume various functions. Some of them take over the function of summarizing these statements in the current situation that are unequivocally accepted by the members of a group. Toulmin (1969) speaks here of “data”. The general idea of an argumentation is that one can refer back from the current utterance (b) to other undoubted statements (a), the data. Then (b) appears as a “conclusion”. If necessary, such an inference from (a) to (b) has to be legitimated. Toulmin categorizes such comments as “warrants” (ibid., p. 98). Other utterances have the function of referring to the acceptability of such warrants. Toulmin (1969) calls them “backings”. They represent undoubtable basic convictions.

According to Toulmin (1969), “analytic” or “substantial” types of argumentation can be accomplished. To the analytic type of argumentation belongs deduction and thereby the mathematical proof. The information of the backing is transferred to the conclusion. An argument is called “substantial” when the backing does not contain the complete information that is transferred to the conclusion. In them a convincing

soundness between statements, references to other statements and/or modification of statements is established (*ibid.*, p. 125).²

Toulmin (1969) emphasizes in his writing that the term argumentation should not be reserved only for analytic argumentations. Human undertakings are in a much broader sense argumentative, i.e. grounded on rationality. If one accepts Toulmin's differentiation between types of argumentation, one can expect to find (mostly) substantial argumentation already in the mathematics discourse in the preschool and grade school. Exemplarily, two types of substantial argumentation might illustrate this approach: the narrative argumentation (a) and the diagrammatic argumentation (b).

- (a) Empirically, it is possible to reconstruct a narrative (substantial) argumentation with reference to early learning process of mathematics above all in arithmetical problem solving situations among preschool and grade school children (Krummheuer 1997). The here asserted narrativity is seen in the typical patterned sequences of action in the interaction. In such an argumentation, the claimed solution of a mathematics problem is placed in relationship to a familiar solving routine, like counting that has been accomplished before. The participation skills in such a "narrative discourse" (Tomasello 2003, p. 244) are grounded on the mastery of processes of basic language acquisition and interactive competences, that usually are developed at the end of the process of language acquisition, that is in the fifth to sixth year (*ibid.*, pp. 266ff).
- (b) Typical for mathematics discourse is, as has already been mentioned, the common use of inscriptions. Such inscriptions can be the conventional presentation of numbers in the decimal system, illustrations that are didactically motivated and much more. In view of the mathematics learning processes in preschool and grade school lessons, it can also involve presentations with concrete materials as with wooden blocks, wooden beads, a ten-frame etc. (Krummheuer 2009; Latour and Woolgar 1986; Roth and Mc Ginn 1998; Schreiber 2010). Such notes, sketches, drawings, presentations based on materials etc. are here of interest when they are used for the demonstration, clarification, backing and consolidation in an oral(-vocal) interaction process. The intentional manipulation of diagrams can also assume the function of an argumentation. Van Oers (1997) shows that such inscriptions like diagrams demonstrate not only objects but also interpretations of the situation for the child that is drawing. As such, diagrams as well as the process of their construction are generally connected with verbal talk in a process of a negotiation.

Krummheuer (2013) reconstructs that in the childhood development of mathematics argumentation the relationship between diagrammatic argumentation and narrative argumentation is complex. While at preschool age apparently both types of argumentation are relatively unrelated to each other, narrative types of narration seem to dominate in first grade, at least in the content area of arithmetic. Hereby not necessarily

²Among others, Toulmin's functional analysis of argumentation is also used by Schwarzkopf (2000) and Meyer (2015).

mathematically elaborate arguments are developed. These narrative argumentations are often bound to counting strategies but express furthermore an adequate level of language development (see above Sect. 8.3.2.1 and Tomasello 2003).

Application in a Theory of Learning Mathematics: The Term “Format of Argumentation”

As with the reference to Tomasello above, it has been inferred that the acquisition of the mother tongue represents something like a paradigmatic example for any kind of learning- and developmental process. For the successful development of the acquisition of the mother tongue the necessity of a “Language Acquisition Support System” (LASS) is postulated, by which, in combination with the genetically inherited Language Acquisition Device (LAD), it is possible for the child to acquire its mother tongue (Bruner 1983, p. 19). Empirically, Bruner reconstructs specifically structured patterns of interaction which he calls “formats”. Formats are similar, structured patterns of interaction in which over time a switch takes place in the roles, allowing and demonstrating the child’s increased autonomy in its language performance (*ibid.*, p. 39).

Besides these considerations that have to do more with the structure of an interaction process that makes learning possible. Miller (1987) is concerned with the question which thematic aspects of an interactive negotiation process can be seen as the condition for the possibility of learning. In an almost seemingly inevitable logical conclusion, he emphasizes that this can only be in the form of the “collective argumentation”. Collective argumentation represents thereby a specific form of negotiation that allows for the possibility of learning. In such a process of meaning-making the participating individuals cannot experience a systematic transcendence of their own possibilities for the construction of meaning. Furthermore, it also influences the triggered cognitive processes in two respects:

1. The experienced genesis of a collective argumentation functions as an orientation for the cognitive restructuring of the individual. The orientation is more effective, the more rational and plausible these argumentations are for the individual since it then can anticipate more easily the further development of this argumentation.
2. The negotiated collective argumentation allows also a function of convergence between the different definitions of the situation by the participants so that the developing individual definitions show a better fit to the results of the commonly negotiated meaning. Also here, it is again the mentioned rationality by means of the collective argumentation that allows the fitting of the new individual constructions of meaning (for both points see Krummheuer 2007).

Both of these aspects of the socially constituted conditions for the possibility of learning (format and collective argumentation) are brought together by Krummheuer in the concept of the “format of argumentation” (*ibid.*). These formats are to be understood as specific, argumentatively shaped thematic procedures supporting the learning of mathematics (see above Sect. 8.3.1).

8.2.2.3 Indicator for Successful Learning: The Increasing Degree of Autonomy in Participation

Under an interactive perspective learning is not conceptualized exclusively as an inner, cognitive process, but as a process that concomitantly takes place internally in the individual in the sense of a cognitive reconstruction, as well as in the interaction processes in which the individual participates. Cobb and Bauersfeld (1995) argue that these two constituents are complementary to each other and should be treated in analogy to Heisenberg's uncertainty principle: "When the focus is on the individual, the social fades into the background, and vice versa" (p. 8).

The play that is difficult to grasp between individual and social constituents can be described as follows: if the participation on a collective argumentation functions for the mathematical thinking as orientation and convergence, then the success of learning is expressed in an increasingly good fit of the individual definition of the results of the interactively negotiated meaning. On the level of interaction this fit appears as an increase in the activities of the learners in an established format of argumentation throughout several situations of interaction. The fit of the actions and interpretations of the individual can be empirically reconstructed as an increasingly autonomous adoption of steps of action within such a format. Learning can be described as an "improvement" of participation. Sfard (2008) suggests substituting the concept of learning of "learning-as-acquisition" with one of "learning-as-participation" (p. 92).

Lave and Wenger (1991) characterize the beginning of such a process of acquisition as "legitimate peripheral participation (*ibid.*, p. 35). A learning process can then be described on the interactive level as the way from legitimate peripheral participation to full participation (*ibid.*, p. 37). Krummheuer (2011) argues that the typical polyadic interaction during teaching requires a further differentiation of this terminology. Thus, the legitimate peripheral participation can be filled by different recipient statuses. It can, for example, be an accepted listener who is spoken to by full participants or just a listener who is not spoken to. Furthermore, the legitimate peripheral participation can also be understood as a first step toward participation by which the learner attempts to copy partial steps and parts of the vocal remarks of the full participants.

From the interactionist perspective, mathematical learning and the development of mathematical thinking—as mentioned—can be understood as an increasing degree of autonomy in participation on formats of argumentation. This leads to the two following questions:

1. At the interactive level: in which way does the interaction system make it possible for the participants, who in the beginning are still in the status of the legitimate peripheral participation, to adopt a growing degree of participation?
2. At the individual level: in which way does the individual make use of his opportunity to change his status of participation offered to him by the interaction system?

Both questions are interdependent. Brandt (2004) introduces for this the term "Partizipationsspielraum", which can be translated into English as "leeway of participation". It describes under which particular emerging conditions in the interaction

a person can shape his participation. Such conditions can be limited so that, for example, no other offer of participation exists other than imitating the teacher's actions. If these conditions are more open, then the participants have the chance to construct their participation in different statuses. If it is possible to reconstruct stability and regularity within different situations of interaction with the same participants, as is the case, for example in school classes, then it is possible to describe participatory types of the individual. Brandt summarizes these types with the term of "profiles of participation" ("Partizipationsprofil", *ibid.*, p. 147).

8.2.2.4 Conditions of the Possibility of Mathematics Learning Processes

Krummheuer and Brandt (2001) developed a multi-dimensional model that describes components for the reconstruction of the conditions of the possibility of learning mathematics in social settings. The above text mentions this model and further discusses new research results. The conditions of the possibility of mathematics learning are dependent on:

- the quality of the development of the theme
- the characteristic of the practice of rationalization (explicitness of the arguments as in Toulmin, emergence of a format of argumentation) and
- the flexibility of the leeway of participation (active and receptive participation).

With this model, more and less optimal conditions of the possibility of learning can be described:

For an improvement of the social conditions of the possibility of learning mathematics, it is necessary that a development of themes is made possible not within inflexible interaction patterns, but that, step by step, the learner can assume flexible roles with varying degrees of originality and responsibility until finally evolving his own responsibility for the production of a complete mathematical argumentation. The process of argumentation that develops should not "flatten", but include the production of argumentative components that are concerned with deeper insights in the legitimacy of a negotiated conclusion of given statements about the claim that is to be justified. This is of great significance for learning through active doing, as well as through recipient participation, for example, in legitimate peripheral participation. If characteristics of structuring of this description are evident, then we call the interaction process a "condensed course of interaction". In the opposite case, we speak of a "smooth course of interaction" (Krummheuer 2007).

The smooth course of interaction is characterized by a minimal amount of energy and conflict potential and refers to passages of interaction that are almost without friction and conflict (see Bauersfeld 2000). The possibilities for learning that emerge here, however, cannot be seen as optimal. The condensed course of interaction presents an improvement of these possibilities. Their processes of interaction are rather filled with more crisis and friction. Formats of argumentation portray an interactive "crisis management" by which, through the production of a certain

interactively structured collective argumentation, the interaction is foreseeable thus making a cognitive adaptation possible (Krummheuer 2007).

8.2.2.5 Summary

In the above the results of an empirically based development of theories has been presented. They were successively developed in a series of research projects. In the following these results are once more summarized in a short overview.

Holding to the premises of interactionism, the mentioned empirical studies enabled the following insights:

Interactive processes of negotiation represent the starting point for the development of mathematically specific language games, on which the participating pupils become increasingly competent. In these processes of negotiation, the situationally-bound thematic development as well as the situationally-bound conditions of the possibility of learning mathematics are accomplished. This condition is the participation in collective argumentation. At least in mathematics lessons in grade school, these arguments are generally of a substantial nature.

- The processes of negotiation are characterized by differences in framing and qualitative differences in the use of language among the participants. These differences can be understood as a constitutive condition for the possibility of learning mathematics.
- Interaction in mathematics classes is characterized by content-relevant patterns of interaction. Teaching outside such patterns is not feasible in the long term.
- Patterns that are supportive of learning are formats of argumentation. In them the differences of framings are constructively taken up and the autonomous acts of the pupils are supported with argumentative means. The learners take part in a process of development from legitimate peripheral participation to a full participation.

Favourable conditions for the learning of mathematic in teaching situations are interaction processes that are characterized by a condensed course of interaction. They are, first of all, characterized by a collective argumentation in which explicit warrants and if necessary also backings of an argument are used. Furthermore, the pupils act in the demanding status of participation, which are seen as intermediary stages toward full participation.

These participatory intermediate stages are filled more or less successfully by the pupils depending on the leeway of participation that emerges and on the profile of participation that was established for them over several situations.

Courses of condensed interaction emerge from a course of smooth interaction and usually fade back in time into a smoother interaction. At least in the empirical studies on which this article is based, permanent courses of condensed interaction could not be constructed as processes of negotiation in mathematics classrooms. Such everyday situations seem not to be characterized by a continual improvement of the conditions of learning. Possibly this expectation is too high and unrealistic for the everyday mathematics classroom. Often an implicit pedagogy is in effect in

everyday teaching in which, for example, the differences of frames in the class are not explicitly spoken of and those pupils have an advantage who have formal language competencies from their familial background.

To summarize, the discussed approach focuses on the situationally-bound aspects of mathematics learning. It is concerned with the theoretical exploration of social situations in which mathematics learning processes are initiated, supported and consolidated. The perspective is on the here and now where persons are interacting with each other. The empirical analyses are directed at the reconstruction of such interactively accomplished products such as the negotiation of meaning, argumentation and the structure of interaction etc. In the terms of Schütz and Luckmann (1979), it has to do here with the emerging “everyday worlds” in mathematics classrooms and the conditions of learning that are produced in them.

8.2.3 *Practice Relevance*

The interactionist theory approach allows the relationship of everyday teaching to the changing of teaching to define in a specific way. “To teach” and “changing teaching” can be understood as two different realizations of classroom interaction along these dimensions. In order to improve mathematics teaching, condensed courses of interaction should be recognized more often and with more certainty. In either case of classroom interaction, everyday teaching is characterized by the fact that one makes decisions under pressure that should maintain the flow of interaction. This is not easy—one should not hesitate too frequently, but make decisions with the other participants, and take some matters to be certainly given. Such assumptions in everyday mathematics lessons, for example, are the use of patterns of interaction processes with a less elaborated accounting practice in the sense of a smooth course of interaction.

It is this everyday practice that should be changed. This modified practice would again become everyday but naturally another, in which the course of condensed interaction can not only take place more often but also be methodically better developed. One can understand this development as a cycle: from being everyday to a modified everyday practice with a phase of change in between (Gellert 2003). This cycle can reoccur several times.

With reference to the possibilities of modifying this everyday practice, one can ask how a teacher can learn to accomplish her own perceptions of change in this everyday setting and to critically reflect on it. A teacher will be more successful in these matters when she assumes more autonomy in her own decisions and actions. An increase in autonomy means the teacher does not attempt to exactly copy suggestions made by other teachers or researchers, but that she can be enabled to develop and experiment on her own alternative visions about the condensed course of interaction. Gellert (2003) distinguishes here between a conception of education that aims at directions about action and those that enable the (future) teachers to develop their

own approaches for alternative action (p. 139). These alternative actions should aim at the production of a condensed course of interaction.

The ability of the autonomous development and trial of such alternative teaching is most likely encouraged when the participants are able to interpret interaction in classroom episodes in new alternative ways by means of an improved perception. Inevitably, the possibility of the alternative structuring is based on the ability of interpreting in divergent ways. As a motto, it could be phrased: “changing through interpreting” (Fetzer and Krummheuer 2007).

In the teaching situation, the teacher generally acts rationally based on her definition of the situation. If we wish that she acts differently, this too should be a rational undertaking. This can arise more often when she is able to perceive aspects of interaction that lie outside the horizon of her definition of the current situation. In shaping a more developed competency of interpretation lies the foundation for any possibility of changing everyday teaching. On this basis then new assignments, new access to contents of teaching, new contents on the whole or even completely new cultures of lessons with the chance of a permanently effective change of teaching can be introduced. In teacher education as in further education seminars of currently practicing teachers, an essential element is the initiation of a practice of joint interpretation of classroom episodes in the sense of the above goal. On given classroom documents or on participants’ own documented teaching episodes, interpretations of these scenes of everyday teaching are generated together based on the above described theoretical approach. Hereby innovative teaching concepts can be already demonstrated in these documents (Krummheuer 2008).

Exactly in the sense of interpretative classroom research, it is not attempted to conduct quantitative studies on effectiveness. It rather tries to reconstruct teachers’ interpretations of classroom interaction. Fellmann (2014) reconstructs in a qualitative comparative study the professional self-concepts of the three common professional groups (students in the school of education, practice teachers in their preparatory seminars and practicing teachers), who attempt to use an innovative concept of teaching of cooperative learning in mathematics classes in grade school. She further generated a wealth of alternative interpretations in practice organized by the author of commonly interpreting episodes of their realized practice. Fellmann could assign different interpretations of the participants to certain professional roles (pp. 184ff). These types allow an evaluation of the scope or the generalization of innovative approaches and demands in different professional groups.

8.2.4 Summary—A Reflection on Research Practice

An empirically grounded theory of mathematics classroom interaction is presented. It rests on many separate empirical research projects in which due to the necessity of the reduction of complexity in the implementation of each single project, it was necessary to concentrate in each project only on certain facets. Likewise, in the course of over 30 years of research, certain aspects could be stated more precisely

or were enriched. Especially in the broad theory complex concerning the format of argumentation new theoretical approaches were integrated with reference to the works of Bruner and Toulmin.

All in all, an approach involving an interactionist theory of mathematics learning in the context of school classrooms has been described with the goal of the development of an empirically rich theory of middle range. It places a clear emphasis on the effectiveness of collective argumentation for the learning of mathematics and relies on the idea that the most “rational” of all sciences, namely mathematics, is learned through the participation of pupils in argumentative processes of negotiation about the rationality of their mathematics actions in the classroom.

8.3 Classroom Research as Part of the Social-Political Agenda

8.3.1 Introduction

This chapter sketches the accumulating body of classroom research in Germany that explicitly relates the interpretations of classroom interaction to the macro-sociological or institutional conditions of its production. It aims at complementing Krummheuer’s description of the development of an interpretative theory of learning mathematics in the preceding chapter.³

Interpretative studies of interaction in German primary and secondary mathematics classrooms (Bauersfeld 1978; Krummheuer 1997; Voigt 1984) have focussed, and continue to focus, on the reconstruction of interaction patterns and formats of argumentation. These are seen as the social building blocks for mathematical activity in the classroom. The results of interactionist research, taken together, yield a rather coherent picture of mathematics classroom micro-cultures. In a nutshell, interactionist studies regard communication in the mathematics classroom as “a process of mutual adaptation wherein individuals negotiate meanings by continually modifying their interpretations” (Cobb and Bauersfeld 1995, p. 8). The ‘social’ in interactionist studies of mathematics classroom micro-culture is firmly located in the interpersonal space of those who interact. This space is considered a contingent sphere in which mathematical meaning emerges as the product of processes of negotiation. Methodologically, the reconstructivist paradigm, to which most studies subscribe, refers to the textual data of classroom interaction (in which videotape is considered as textual data, too) as the inescapable ground for all interpretation. What cannot be evidenced by the data, cannot form part of the interpretation. As an example, the social or cultural background of individual students involved in a collaborative activity is only taken into consideration once the collaborating students refer explicitly

³I am indebted to Götz Krummheuer as my mentor when introducing me to interpretative analyses of classroom interaction years ago.

to their social or cultural backgrounds. The students' 'social condition' is irrelevant for the interactionist's analysis unless the participants of the interaction refer to it.

This paradigmatic decision of excluding everything invisible in the interaction from the interpretative analysis is essentially facilitating reconstructions of the emergence of meaning in the classroom. A different sociological perspective considers schools and classrooms not only as places in which learning occurs (qualification), but also as institutional loci in which further societal functions of schooling—allocation, integration, and cultural reproduction—need to be pursued parallel to qualification (cf. Fend 2006). At stake in mathematical activities in the classroom is not only the development of students' knowledge and skills, but also the creation of hierarchies of achievement in mathematics, of differential access to valued forms of mathematics, and of familiarisation with work ethic and norms of compartment. Mathematics, as a school subject, seems to play a particularly important role in this respect. It has been called a 'gate-keeper' for students' career options, and, as Robert Veal (1999, p. 206) has argued from a sociolinguistic perspective, "mathematics is a discipline whose discursive construction through language seems to be unusually closely aligned to the regulative discourse of the classroom and the macro-regulative discourse of the ordering of space and time in the school".

From such a point of view, issues such as the distribution of knowledge, access and the students' resources are crucial ingredients to the forms the interaction in the mathematics classrooms may take. It is of interest, for instance, how mathematics instruction deals "with the correspondence between the hierarchy of social groups and their differential power external to the school and the hierarchies of knowledge, possibility and value within" it (Gellert 2008, p. 216). The interplay of the macro-regulative social order and the micro-dynamics of classroom interaction come to the fore in research on mathematics classroom interaction that does not confine its sociological perspective to the micro view.

Based on the extensive experience of micro-sociological interpretative studies of interaction in mathematics classrooms in Germany and, particularly, on the accomplishments in terms of methodological elaboration, several researchers have engaged in extending the analytical framework, with the purpose of getting hold of a variety of macro-sociological or institutional factors. A selection of their work is presented in this chapter in order to illustrate how classroom research as part of the social-political agenda has developed in a close relation to the traditional interactionist research that Krummehuer has reported in the preceding chapter. Of course, the presentation cannot discuss the social issues in-depth, here. Although this presentation is organised chronologically, only some of the examples build explicitly on each other while for others it is more difficult to trace interconnections and development.⁴

⁴Some of the research referenced in this section has been published as a book or a Ph.D. thesis in German language and might thus be difficult to access. Anyhow, summaries and selected results of the research in English language can easily be found in international journals and as chapters in edited volumes (e.g. in Gellert et al. 2018).

8.3.2 Research Issues

Studies, which interpretatively analyse classroom interaction and go beyond the reconstruction of the emergence of mathematical meaning, have dealt with a variety of issues. This variety falls into three categories. First, the dimension of the students' sex and the gendered formation of mathematical identities has received some attention. Second, the influence of national traditions on classroom interaction has been illuminated through international comparative analysis. Third, the construction of differential performance in mathematics classrooms has been traced to issues of language diversity, openly selective school systems, and to the hidden structures of pedagogic discourse on several levels. It might be interesting not only to consider which issues attention has been paid to. The absence of categories of, for instance, race and ethnicity in research on interaction in mathematics classrooms in Germany might be a fact to be observed.

8.3.2.1 The Dimension 'Sex' in Mathematics Classrooms

The heading of this subsection refers to Helga Jungwirth's (1991) research on the ways in which the sex of the students frames their interaction with the mathematics teacher. In this study, Jungwirth reconstructs differences in the patterns of interaction between teachers and male and female students.

The findings show that the interaction is not coloured by the sex of the participants over many periods. Phases however, in which the basic structure of the interaction is altered, are arranged differently according to the sex of the participating students. This is due to the fact that certain routines of the teachers are modified according to the sex of the students they interact with and the fact that girls and boys have specific ways of acting of their own. In summary boys are more familiar with those methods which enable students to participate successfully in the mathematics classroom. Therefore boys appear to be more competent in mathematics than girls. (p. 87)

Jungwirth concludes that the interaction in the mathematics classroom contributes to the formation of gendered mathematical identities.

8.3.2.2 National Traditions Framing Interaction in the Mathematics Classroom

Christine Knipping (2003) combines the micro-sociological with an international comparative perspective. She reports on proving and application processes in classroom practices in Germany and France, focusing on the Pythagorean theorem as the mathematical content. The comparison of the two national contexts points out two differing curricular tendencies. While, for instance, the observed mathematical lessons in Germany emphasise the application of the Pythagorean theorem in calculations of roofs, distance estimation, etc., the French lessons focus on the activity of proving in geometry. The research exemplifies in which ways the teaching of

mathematics is not independent from context and culture. The students learn different mathematical knowledge and experience different practices of using this knowledge. Knipping concludes that it is clear and surprising at the same time how much the analysed French and German teaching situations differ.

Although not basing her analyses exclusively on an interactionist methodology, Eva Jablonka's (2004) comparative study of classroom interaction in Germany, Hong Kong and the United States takes up the accomplishments and methodology of German interpretative studies on mathematics education and integrates them into a 'complementary accounts methodology' as the frame of the Learner's Perspective Study. The Learner's Perspective Study context is the context in which her classroom data had been generated. Her methodology combines a zooming into the patterns of interaction at micro-level and a zooming-out to see the structuration of classroom practice. By looking at structures in (national) diversity, the study contributes to our understanding of the practice of learning and teaching mathematics in schools at a very general level. From the international comparative perspective and based on the theoretical foundation of Jablonka's analyses, the learning of mathematics appears primarily as an initiation into the practice of school mathematics. However, as Jablonka points to, "student and teacher practices only have meaning with relation to the organisational forms in which they exist" (p. 6).

8.3.2.3 The Construction of Differential Performance in Classroom Interaction

The differences in the mathematical performance of students in school are usually explained on the basis of different cognitive dispositions, which are mediated by individual performance motivation. From a micro-sociological perspective, however, Uwe Gellert and Anna-Marietha Hümmer (now A.-M. Vogler) (2008) see these differences as intersubjective constructions, thus differences in performance are intersubjective constructions too. These constructions and the mechanisms of their production are the subject of Gellert and Hümmer's study. The theoretical framework is provided by Bernstein's theory of pedagogic discourse, with its emphasis of the regulative principles of classroom interaction. The data of the study is made of video recordings from mathematic lessons at the very start of the fifth school year. The micro-sociological focus is on whether and how teachers clarify to their pupils what they have to achieve to be seen as high performers in mathematics. The study shows that performance expectations are not only related to mathematical abilities. Instead, it concludes, making disparities of performance explicit is linked to a process of coding and decoding of the subject-specific expectations to student participation in the mathematical activities in the classroom.

Marcus Schütte (2009) examines the linguistic organisation of mathematics teaching in primary school. He reconstructs this as a form of implicit pedagogy. The teacher in the lessons observed primarily provides a learning environment to the students, and pursues in the course of the lesson how the students develop their individual skills and talents. Such form of teaching, argues Schütte, can be considered a patho-

logical form of progressive education. It basically follows the idea that the students can access mathematical meaning simply because of their abilities brought to the classroom, and that the underlying relations of content and language arise by itself.

The discourse in the analysed lessons shows a predominant use of informal everyday language on the side of the teacher. With footing in the work of Bernstein, Cummins, and Halliday, Schütte considers this discourse as characterised by implicitness and contextuality. However, formal language skills, which are not detailed in this everyday discourse, are highly relevant within performance assessments. These formal language skills for understanding and using a kind of vertical discourse are not made accessible in the observed lessons. It can be assumed that for many children the classroom is the only possible space where they can acquire formal language skills. When teachers rely on everyday discourse only in mathematics lessons, the effect on students' language and knowledge acquisition can be detrimental. When this happens, the system of the school fails to provide access particularly for the non-privileged learners. Structures of privilege and discrimination are then reproduced.

The German school system can be characterised as overtly selective. Traditionally, there is a separation of those with 'intellectual' and those with 'practical talent' at the age of 10. Although the school system has passed substantial changes during the last years, by which its selective character has been downsized, one still finds schools in which the least successful (in terms of academic achievement during the primary school years) students gather. Hauke Straehler-Pohl and Uwe Gellert (2015) are interested in what kind of mathematics was made accessible to the students in these schools and in which form. The focus is on the interaction between teachers and students as well as on the effects that these interactions may have on students' identity formation as learners of mathematics. The study reconstructs how the students become mathematically disempowered by communication patterns that heavily prioritise conformance with the regulative discourse over engagement in the instructional discourse. The mathematics to be transmitted is ridiculously simple, so simple that any classification of students as 'good' or 'bad' could only be built along the students' compliance with the criteria of the regulative discourse. Accordingly, those students that try to resist the low academic expectations they face come quickly into conflict with the teachers and, in several cases, they abandon school at all. Straehler-Pohl and Gellert conclude that these pathological moments of pedagogy reveal the true structural condition of how mathematics is taught and learned within the overtly selective school system in Germany.

Nina Bohlmann's (2016) study develops a praxeological model of pedagogic practice which focuses on explication processes in mathematics classrooms. The theoretical ground is provided mainly by Bernstein's theory of pedagogic discourse, especially the concept 'pedagogic device'. The theoretical frame leads to a problematisation of implicitness in the structuring of pedagogic discourse. By analysing the structural specifics of mathematics classroom interaction, consequences for the explication of structures of mathematics classroom interaction are derived. The key assumption is that the implicit structuring of schooling is constitutive of the relationship between performance in school and the social background of students, and operates in a stratifying way. The study investigates by means of analyses of inter-

action how mathematics teachers attempt to make the structuring of schooling more explicit. It thus reconstructs explication processes in mathematics classrooms. It shows how certain aspects of pedagogic practice—that usually stay implicit—can indeed be explicated. In doing so, differences and commonalities in the setting of priorities and in the realisation of the intended explication processes are made visible.

The results reinforce the claim that explication processes, when systemically, consequentially and consistently implemented, can be seen as an approach to the postulation of equal opportunities in education and by education.

8.3.3 Conclusion

When researchers investigate classroom (inter-)activities from sociological perspectives, their particular sociological perspectives entail decisions on what of the social matter is foregrounded and what is kept in the background. It should be mentioned that these decisions are gradual and not dyadic, e.g. in terms of the micro and the macro, and that research studies may relate to several research branches (see, as an example, the positioning of Schütte (2009) in Krummheuer's and Gellert's discussions, in this section). Despite this, an important difference among sociological perspectives on interaction in mathematics classes should be pointed.

Interactionist studies of mathematics classroom activity usually reconstruct the emergence of meaning in negotiations between teachers and students or among students. The mathematics at stake appears as contingent and underdetermined by the social setting in which the negotiation takes place. From this perspective, the mathematics classroom is like a neutral platform on which learning happens. The character of this platform is deliberately blanked out.

Those researchers who acknowledge classroom interaction as the very locus where meaning is constructed, but who conceive the platform not as flat but as a strong relief folded by the tectonic forces of macro-social structures, trace the faulting visible in the construction of meaning in the mathematics classroom back to the structural conditions under which the teaching and learning of mathematics is organised. Needless to say, this is an ambitious expedition into an only roughly charted continent.

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