

## Chapter 5

# Mathematics and Bildung 1810 to 1850



Hans Niels Jahnke with a reaction by Michael N. Fried

**Abstract** Section 5.1 of this chapter is written by Hans Niels Jahnke on the basis of his presentation at ICME 13. Michael Fried was invited to react to this presentation at ICME 13 and elaborated his reaction as Sect. 5.2 of this chapter. Although the authors are only responsible for their respective parts, the parts belong together and are therefore published here as a joint chapter. The first part analyzes the role of mathematics within the ideas on education of the neo-humanist movement. It refers to the period of around 1800–1850 and concentrates on the thinking of W. von Humboldt and the two catchwords of ‘anti-utilitarianism’ and ‘self-directed activity’. From this general educational attitude resulted a certain preference for pure mathematics which had to be balanced against the daily needs of shopkeepers and workmen. A compromise on this issue was developed and implemented in the 1820s. Nevertheless, a strong emphasis on theoretical thinking, understanding and pure science remained for a long time the main stream of educational thinking. In the eyes of the neo-humanists this was not a denial of the demands of practical life, but the best way to meet them. In his reaction entitled “Bildung, Paideia, and some undergraduate programs manifesting them,” Michael N. Fried discusses how notions similar to that of Bildung are enshrined in the idea of paideia and the classical concept of the liberal arts. He shows that such ideas also work in modern times in the English speaking world by hinting at examples of prominent colleges in North America.

**Keywords** Anti-utilitarianism · Bildung · Everyday applications · Liberal arts · Paideia · Pure mathematics · Self-directed activity

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### **Preliminary Remark**

In this and the following Chap. 6 the authors use the two German terms *Bildung* and *Allgemeinbildung* since there is no adequate translation into English which reflects all the connotations they have taken on in German. The English words ‘education’ and ‘general education’ are only first approximations of their German counterparts. However, as Mogens Niss (Chap. 6) rightly remarks, there exist also in other countries and other cultures ideas similar to the ones enshrined in those German words.

A second difficulty concerns the distinction between the two terms of ‘*Bildung*’ and ‘*Allgemeinbildung*’. One can observe that as a rule authors of the 19th century used the term *Bildung* whereas in more recent times *Allgemeinbildung* became more common. In the background there are subtle differences of meaning. Whereas *Bildung* emphasizes the comprehensive intellectual, ethic and aesthetic cultivation of personalities, *Allgemeinbildung* refers more explicitly to the idea that such a cultivation should be made accessible to all human beings. With these different nuances the terms are used in for example (Klafki 2000) as well as in this chapter. But the reader should be careful. Already W. von Humboldt and other neo-humanist thinkers considered *Bildung* as a right to which all human beings should have access (see below), and, of course, also modern conceptions of *Allgemeinbildung* aim at the cultivation of personalities in all their dimensions.

This chapter and Chap. 6 discuss the relation of *Bildung* and mathematics in Germany at two periods of time separated by a time span of nearly two hundred years. The reader who is interested in the development of educational thinking and the teaching of mathematics in Germany during the period between Humboldt and PISA is referred to Chap. 1 of this book “Educational Research on Mathematics—a Short Survey of its Development in German Speaking Countries”. On the evolution of educational thinking in general she/he might also consult Blankertz (1982) and Klafki (2000).

## **5.1 Mathematics and *Bildung* at the Time of Humboldt<sup>1</sup>**

### *Systematizing Public Education*

The educational reforms in Prussia of the early 19th century which influenced the development in the other German states and which bear the name of W. von Humboldt (see below) were part of a longer process beginning in the second half of the 18th century to reorganize and systematize the proliferation of schools which had been created by local communities for different needs. In cities existed learned schools (“*gelehrte Schulen*”) teaching mainly Latin as the language of the educated people giving access to the knowledge of the time. The process of how these learned schools were transformed into the system of Prussian gymnasia has been analysed in detail by Jeismann (1974). Besides the learned schools there were also “*Bürgerschul-*

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<sup>1</sup>Many thanks go to Michael Fried for his linguistic advice and thoughtful remarks.

len” [“schools for citizens”] with a more practical orientation to which children of workmen and shopkeepers were sent. However, the great majority of the population lived in the country and children there attended, if at all, elementary schools which provided, frequently under very bad conditions, some basic reading, writing and reckoning. By means of several laws from 1717, 1763 and 1794 school education had become compulsory in Prussia. But data from 1816 say that only 60% of the children attended a school, and one must infer that this figure was not higher than 50% during the second half of the 18th century (Lundgreen 1980, Chap. 2; Leschinsky and Roeder 1976, 137; Jahnke 1990a, 6). Only during the 19th century the Prussian state succeeded in getting school education really compulsory, by educating teachers and creating new elementary schools. But the social situation of the lower classes was so bad that frequently the police had to be sent to poor people who preferred to have their children earning some money instead of attending school. Of course, child labour of up to twelve hours a day is not compatible with school attendance.

In the following we confine ourselves to the role of neo-humanism in this process of systematizing public education under the perspective of the teaching of mathematics. Neo-humanism favoured a revival of ancient Greek at learned schools (gymnasia) and also pleaded for a strong position of mathematics (Jahnke 1990a, 333 pp; Schubring 1987, 207). It was not a closed system of propositions, but a more or less vague collection of ideas and convictions which were developed and refined by different persons in different directions. Nevertheless, there was a common core which was condensed in the concept of ‘Bildung’ and proved heavily influential during the entire 19th century.

After 1810, as reaction of its military defeat against Napoleonic France, Prussia launched a number of social and political reforms whose core lay in restructuring and, in part, establishing a new educational system. To some people in Prussia it appeared reasonable to imitate the earlier educational reforms of revolutionary France which had proved so successful, as the military events had shown, and to give up the seemingly outmoded universities. However, the final decisions were just the other way round. Instead of closing universities and learned schools, Prussia made at first sight a conservative turn: it maintained universities and schools, but endowed them with a new understanding of education and research. This development was deeply rooted in a cultural and intellectual milieu to which we now turn.

### ***The Cultural and Intellectual Milieu Influencing the Emergence of the Neo-Humanist Movement***

We begin with considering the biographies of two men who during some years of their lives had been in close contact yet, nevertheless, were representatives of opposite currents of educational thinking, Johann Heinrich Campe (1746–1818) and Wilhelm von Humboldt (1767–1835), the latter being the most important representative of the neo-humanist movement.

Johann Heinrich Campe (Fig. 5.1), born in a small village in the kingdom of Hannover, studied theology and linguistics in Helmstedt and Halle and worked for a time as a Pastor in Potsdam (cf. Blankertz 1982, 79 ff). In 1776, he became head of a reform school (a so-called Philanthropin) in Dessau and later founded another such school

in Hamburg. He was a leading figure of the German *enlightenment pedagogy*. Following the ideas of Rousseau these people wanted to inaugurate *new schools* and *new teaching*. Their basic motive was a fight against the antiquated customs of scholastic teaching. Schools should impart to their students *useful knowledge for their future life*. The method of teaching should no longer emphasize the written word, but show to students *real phenomena*. After 1783 Campe gave up practical pedagogical work and dedicated himself exclusively to literary activities. He wrote, on the one hand, theoretical works on pedagogy and linguistics and, on the other hand, as a sort of practical pedagogy, books addressed especially to children and young persons. From 1785 to 1792 he edited a huge work of 16 volumes describing education in Germany, the *Examination of the entire system of schools and education: by a society of practical educators* (“*Allgemeine Revision des gesamten Schul- und Erziehungswesens: von einer Gesellschaft praktischer Erzieher*”) (Campe 1785–1792). For a time Campe was one of the private tutors of Wilhelm von Humboldt with whom, in 1789, he travelled to Paris where they witnessed the early phases of the French revolution. Campe was one of the most progressive educational thinkers of late 18th century Germany: he carried out educational experiments and fought for freeing education from seemingly outmoded traditions and for introducing new useful subjects into teaching. As we have mentioned he was also an early and influential writer of young adult literature.

The most important figure of the neo-humanist movement was Wilhelm von Humboldt (Fig. 5.2), the elder brother of Alexander von Humboldt (cf. Scurla 1976). Humboldt’s father was a military man and had been elevated to nobility by the Prussian king Frederick II. The family was well-off so that Humboldt was financially independent. He received a careful private education and then studied in Göttingen philosophy, history and ancient languages. In 1801 he entered the service of the Prussian state as ambassador at the Vatican. In 1809 he was put in charge of the educational reforms in Prussia. He served in this position only for sixteen months, and then resigned because the Prussian king refused to establish a separate government department for education as Humboldt had claimed for. After 1819 he finally resigned from Prussian services altogether finding the spirit of the ongoing restoration at variance with his own perspective. From then on he worked exclusively as a private scholar on political philosophy and comparative studies on language and art and was very productive in these domains. Humboldt cultivated contacts with numerous intellectuals all over Europe.

Campe and Humboldt represented different generations, and Humboldt’s thinking on matters of education was to develop in a direction quite different from Campe’s. Nevertheless, they maintained friendly relations and shared a number of opinions, for example a more or less positive view of the revolutionary events in France. The reader should keep this in mind, since neo-humanist views on education as they emerged in the first decades of the 19th century were frequently exposed as a sharp and sometimes unfair and insulting critique of enlightenment pedagogy (see Niethammer 1808). In a way, the personal connection between Campe and Humboldt symbolizes the overarching relation between enlightenment and neo-humanism, the

**Fig. 5.1** J. H. Campe  
(1746–1818) (source  
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latter being so-to-speak a child of the former. And, as is frequently the case, children come to criticize their parents.

For understanding the general intellectual climate in Germany which was to become the breeding ground of educational philosophy after 1810 it is very illuminating to take a look at the small city of Jena in the 1790s. Between 1794 and 1800 quite a number of German intellectuals attended the university of Jena and listened to the lectures of philosopher J. G. Fichte (1762–1814). Fichte had publicly defended the French revolution in 1793. As a philosopher he started with an elaboration of Kant's philosophy and then conceived of a philosophical system of his own based on the idea of the *self-development of the individual*. He became *the* philosopher of the romantic movement in Germany. Later in 1808, after Prussia's military defeat at the hands of Napoleon, Fichte delivered a series of lectures in Berlin under the title *Speeches to the German Nation* in which he recommended a reshaping of education as a necessary requirement for a revival of Germany. He pointed explicitly to the Swiss pedagogue Johann Heinrich Pestalozzi (1746–1827) as a thinker who had developed decisive ideas for elementary education. This caused sort of a Pestalozzi hype in Germany, and the Prussian administration decided to send a number of teachers to Switzerland in order to learn about Pestalozzi's teaching methods.

**Fig. 5.2** W. von Humboldt (1767–1835) (*source* Wikipedia Public Domain, ©)



Back to the 1790s we name some thinkers who had come to the university of Jena in order to listen to Fichte. Among others there were

J. F. Herbart (1776–1841),  
 K. Chr. Fr. Krause (1781–1832),  
 J. F. Fries (1783–1843),  
 F. W. J. Schelling (1775–1854),  
 G. W. F. Hegel (1770–1831),  
 A. W. and F. Schlegel (1767–1845; 1772–1829)  
 Friedrich von Hardenberg (Novalis) (1772–1801).

In 1809 Herbart took over Kant's chair of philosophy at the university of Königsberg, and, by the middle of the 19th century, he was considered the most influential pedagogue of Germany. Krause, Fries and Schelling were widely known philosophers though representing quite different directions of philosophical thinking. Hegel took over the prestigious chair of philosophy at the university of Berlin in 1818 and remained there until his death. A. W. and F. Schlegel who were brothers were to become the most influential propagators of the romantic movement in Germany, and Novalis became *the* romantic writer. Intellectually, these people developed in quite different directions. Nevertheless, there was a common core of interest. They all observed the French revolution and they all wrote about mathematics. A detailed

exposition of the views of Herbart and Novalis on mathematics can be found in Jahnke (1990b, 1991).

From 1794 to 1796 also W. von Humboldt spent his time in Jena where he became close to Friedrich Schiller, who, like Fichte, was professor of philosophy (and history) at the university. Humboldt also entered personal relations with J. W. Goethe who lived in Weimar twenty kilometers distant from Jena and who functioned at that time as ministry of the grand duchy Sachsen-Weimar-Eisenach responsible for the university (Fig. 5.3).

Much earlier Goethe had brought Johann Gottfried Herder (1744–1803) to Weimar and Jena. Herder was a theologian and in 1776 he took over the position of a ‘Generalsuperintendent’ (superintendent general) in the grand duchy Sachsen-Weimar-Eisenach. In this function he presided the highest administrative body of the protestant church in the grand duchy (the ‘Oberkonsistorium’) and was among others responsible for the schools. He functioned in this position until his death in 1803. Herder was an important philosopher of history and language and a central figure in what later on became to be called ‘Deutsche Klassik’. In the German historiography of pedagogy, he is considered to be the “founder” of the German theory of Bildung (Horlacher 2004, 420). According to Herder “Bildung is a non-political concept that focuses on the individual’s process of inner self-development, unfolding, self-cultivation—in accordance with an organic concept of nature and natural development.” (l.c., 421) In forming and elaborating the concept of Bildung Herder relied among others on his reception of Shaftesbury and Rousseau, and thus the very concept of Bildung is rooted in the European enlightenment and in its origin was not an exclusively German idea (see Oelkers 1999).

**Fig. 5.3** Meeting in Jena. Schiller, W. and A. von Humboldt, Goethe (left to right) (source Wikipedia Public Domain, ©)



It was at this time and in these circles that a general outlook on science emerged which can be called “*cultural foundation of science*” (Jahnke 1993a, 266 pp) and which was influential in Germany all over the 19th century. The term is intended to convey the idea that science is pursued not so much for the sake of technical or commercial applications but as a human effort to foster a comprehensive *understanding* of nature and of culture. By way of this, science contributed to the development of a certain awareness of life and of notions by which a human being may gain a better understanding of himself and his aims, and to the cultivation (*Bildung*) of the individual. Wilhelm Dilthey has described the spirit of these times by the apt phrase that the interpretation of the world out of itself became the watchword of all free minds (Dilthey 1905, 211). Just as philosophy and art generate interpretations of reality, science was seen as part of an interpretive effort as well, and mathematics was an important component of this undertaking.

Beyond the special views the persons mentioned above held on philosophy and the world in general it was an important fact that they lived in a permanent intellectual discourse, often quite controversial. They were not just a collection of students, poets and philosophers, but people joined together by intense intellectual and emotional relations. The romanticists Novalis and brothers Schlegel created the artificial word “*symphilosophieren*” (joint philosophizing) as a designation of this mixture of personal company and intellectual exchange. Many people know from their own experience that this can be a great, even decisive influence in the course of their lives.

### ***Essentials of the Neo-Humanist Educational Reforms***

As mentioned above, with W. von Humboldt as the person responsible for the educational system in Prussia, far-reaching reforms were launched. Before entering into details of this reform we sketch Humboldt’s ideas on *Bildung*, school teaching and education following the succinct account by Blankertz (1982; see also part 2 of this chapter).

Humboldt’s theory of *Bildung* was not at all confined to schools (Blankertz 1982, 101 pp). Rather, he saw *Bildung* as the way of a human being to himself and, therefore, as an infinite task which will never end. This amounted to a consequent siding with the individual and against any subordination of the individual to the needs of society. According to Humboldt individuality is the capacity of a human to transform the outer world into his own inner being. This constitutes a dialectical relationship between the individual and the world. On the one hand any human being considers the world by way of her/his specific, individual perspective (in analogy to Kant’s concept of ‘*a priori*’). On the other hand, this individual perspective emerges from being active in the world. In German at that time this capacity or perspective was frequently called ‘*Form*’ from which derived the concept of ‘*formale Bildung*’. According to Humboldt the key component in this dialectical relationship between an individual and the world is language, any language constitutes a certain world-view. Thus Humboldt did not conceive of individuals as isolated beings living in their own world, but as social beings not subordinated to the ‘*needs of society*’ but contributing to society.

Blankertz (1982, 119 pp) condenses Humboldt’s views on education into four principles. First of all, general education takes precedence over professional educa-



tion. Since ‘Bildung’ should be thought of in terms of the individual and its self-development, public schools under the governance of the state should provide the best opportunities such that the individual can develop and strengthen her/his intellectual, aesthetic and emotional capabilities.

“General teaching at schools aims at the human being on the whole, namely ... at the main functions of his personality.” [“Der allgemeine Schulunterricht geht auf den Menschen überhaupt, und zwar ... auf die Hauptfunktionen seines Wesens.”] (l.c., 188/9, transl. by HNJ)

From this principle Humboldt derived that public schools should provide **historical, linguistic and mathematical subjects**.

What students needed for a profession should be acquired separately after general education had been completed. Training for special professions should take place in special schools.

The second principle follows from the first. According to Humboldt there can be *only one general education*. Thus, there should be a system of ‘comprehensive schools’. As Humboldt wrote:

“Thus, this entire instruction has to rest on only one and the same foundation. For originally, the lowest day labourer and the most delicately educated [person] have to be equally tuned in their mind so that the former should not be brutish and below human dignity and the latter should not be sentimental, airy-fairy, crank and below human strength. ... in this way having learnt Greek could be as little useless to the carpenter as making tables to the scholars.” [“Dieser gesamte Unterricht kennt daher auch nur Ein und dasselbe Fundament. Denn der gemeinste Tagelöhner, und der am feinsten Ausgebildete muss in seinem Gemüth ursprünglich gleich gestimmt werden, wenn jener nicht unter der Menschenwürde roh, und dieser nicht unter der Menschenkraft sentimental, chimärisch und verschroben werden soll. ... Auch Griechisch gelernt zu haben, könnte auf diese Weise dem Tischler ebenso wenig unnütz sein, als Tisch zu machen dem Gelehrten”] (l.c., 189, transl. by HNJ)

Of course, it was not Humboldt’s idea that every child should learn ancient Greek nor did he think that everybody should learn to make tables. But he had the vision that there should be a common foundation in education for everybody. This was an extraordinarily political and at the same time theoretical statement: the right and the aims of Bildung and education are a *matter of human dignity* and not of serving the needs of society. Stated in modern words: *Bildung and education are a human right!*

Third, the state has to take care of schools, but at the same time the influence of the state on schools has to be restricted—a somewhat paradoxical idea. Fourth, Humboldt, according to Blankertz, believed that by way of Bildung students will become (also politically) self-determined persons.

Essentials of the educational reforms inaugurated by Humboldt can be seen in three crucial junctures.

- (1) In 1810 Prussia founded a new university in its capital Berlin which was to inaugurate a new understanding of the spirit and the way of functioning of a university in entire Germany. The slogan of a “*unity of research and teaching*” implied the idea that students and professors should closely cooperate. Instead of outsourcing research into academies as had been frequently done in earlier

times seminars and laboratories were founded as university institutions in which such cooperation could take place. It is easy to see behind this the romantic idea of continual open discussion and exchange of ideas which Humboldt and so many others had experienced in their youthful days at the university of Jena. Humboldt's 'definition' of a university expressed this idea in a perfect way:

"In fact, attending courses is only secondary, the essential thing is that between school and the entrance to life the young man dedicates a number of years exclusively to scientific reflection at a place which brings together many persons, teachers and learners." ["Das Collegienhören selbst ist eigentlich nur zufällig; das wesentlich Nothwendige ist, dass der junge Mann zwischen der Schule und dem Eintritt ins Leben eine Anzahl von Jahren ausschließend dem wissenschaftlichen Nachdenken an einem Orte widme, der Viele, Lehrer und Lernende in sich vereinigt."] (von Humboldt 1809, p. 171, transl. by HNJ)

Of course, it was not by chance that philosopher Fichte was elected as first rector of the university of Berlin, though he never actually served in this position owing to his sudden death in 1814.

- (2) In 1810 an 'Edict Concerning the Introduction of a General (university) Examination of Future Teachers' was issued. The edict concerned future teachers at gymnasium, and among others defined the subjects which should/could be studied in order to become teacher. Among these subjects mathematics had a prominent place, and universities had to offer courses in mathematics and to create the necessary professorships. Thus, for the first time, mathematics became a full-fledged university discipline rather than a mere component of the elementary *studium generale*, as had been the practice until then.
- (3) In 1812 the Prussian government issued the 'Edict Concerning the Students Entering the Universities'. According to this edict entrance to a university was no longer a matter of feudal privileges but required passing the 'Abitur'. The edict regulated which subjects a student had to study successfully at gymnasium in order to be admitted to university. Without going into details it can be said that compared with 18th century Latin Schools two disciplines were given new emphasis. As mentioned above, these were *ancient Greek* and *mathematics*. Ancient Greek was part of the special ideology of neo-humanism, but how can we explain the strengthening of mathematics?

### ***Cultural Meaning of Mathematics***

The new strong position of mathematics in school teaching and at universities was partially due to the model of the educational reforms in France. The Prussian government was impressed by the success of French military and economy, and viewed them in terms of the new French education. Nevertheless, the German view on mathematics was markedly different from the French and stressed the *cultural meaning* of mathematics in agreement with what we have called the cultural foundation of science. A sense of this view can be gained from a sample from Humboldt's writings. It should be said that these quotations come from opinions which Humboldt had written for the Prussian administration in an official capacity. Thus, in his time

they were not known to a broader public. Nevertheless, they express the spirit of the time in a perfect way.

According to Humboldt, education was to ensure

“that understanding, knowledge and intellectual creativity become appealing not because of external circumstances, but because of their internal precision, harmony and beauty. It is primarily mathematics that must be used for this purpose, starting with the very first exercises of the faculty of thinking.” [“...dass das Verstehen, Wissen und geistige Schaffen nicht durch äussere Umstände, sondern durch seine innere Präcision, Harmonie und Schönheit Reiz gewinnt. Dazu und zur Vorübung des Kopfes zur reinen Wissenschaft muss vorzüglich die Mathematik und zwar von den ersten Uebungen des Denkvermögens an gebraucht werden.”] (von Humboldt 1810, p. 261, transl. by HNJ)

Mathematics is seen as the most important subject “from the first exercises of the faculty of thinking” to provide to pupils a feeling for the fascination of intellectual creativity, and this is possible because of its “internal precision, harmony and beauty”. In Humboldt’s thinking precision and beauty, rigour and aesthetics entered a close relationship.

At another place, Humboldt expressed himself against the tendency

“... of digressing from the possibility of future scientific activity and considering only everyday life .... Why, for example, should mathematics be taught according to Wirth, and not according to Euclides, Lorenz or another rigorous mathematician? Any suitable mind, and most are suitable, is able to exercise mathematical rigour, even without extensive education; and if, because of a lack of specialized schools, it is considered necessary to integrate more applications into general education, this can be done particularly toward the end of schooling. However, the pure should be left pure. Even in the field of numbers, I do not favour too many applications to Carolins, Ducats, and the like.” [“...sich selbst von der Möglichkeit künftiger Wissenschaft zu entfernen, und aufs naheliegende Leben zu denken. Warum soll z. B. Mathematik nach Wirth und nicht nach Euclides, Lorenz oder einem andern strengen Mathematiker gelehrt werden? Mathematischer Strenge ist jeder an sich dazu geeignete Kopf, und die meisten sind es, auch ohne vielseitige Bildung fähig, und will man in Ermangelung von Specialschulen aus Noth mehr Anwendungen in den allgemeinen Unterricht mischen, so kann man es gegen das Ende besonders tun. Nur das Reine lasse man rein. Selbst bei den Zahlverhältnissen liebe ich nicht zu häufige Anwendungen auf Carolinen, Ducaten und so fort.”] (von Humboldt 1809/1964, p. 194, transl. by HNJ)

This is a core quotation for the whole bundle of ideas we are talking about. Firstly, there is a strong anti-utilitarian sentiment. Humboldt is against “considering only everyday life” as a benchmark for determining the subjects of education. At the end of the quotation, he repeats again that he does not favour “too many applications to Carolins, Ducats and the like”. [Carolins and Ducats were common monetary units of the time.] He also emphasizes two times the importance of mathematical rigour—a remarkable statement. We remark that Humboldt did not plead for cancelling “Carolins, Ducats and the like” from teaching, but that he did not want too many applications to them. Thus, the role of “Carolins, Ducats and the like” in teaching elementary arithmetic is a question of the right measure and has to be balanced with the requirement of developing rigorous thinking.

No wonder then that

„... any knowledge, any skill which does not elevate the faculty of thinking and imagination by a complete insight into the rigorously enumerated reasons or by elevation to a universal intuition (like the mathematical or aesthetical) [is] dead and sterile.” [“ist ... jede Kenntnis, jede Fertigkeit, die nicht durch vollständige Einsicht der streng aufgezählten Gründe, oder durch Erhebung zu einer allgemeingültigen Anschauung (wie die mathematische und ästhetische) die Denk- und Einbildungskraft, und durch beide das Gemüth erhöht, todt und unfruchtbar.”] (l.c., 188, transl. by HNJ)

In what follows we shall enlarge upon two points of view expressed in Humboldt's texts, namely his “anti-utilitarianism” and his “preference for pure mathematics”.

The latter term needs some qualification. As we have seen, Humboldt did not use the term “pure mathematics”, but spoke of “rigour” and “rigorous mathematician”. Nevertheless, it seems rather clear what he had in mind. Treating “Carolins, Ducats and the like” in teaching, should not obscure the conceptual and logical relations within mathematics. On the contrary, clarifying these relations should be the central concern of teaching. Since the term “reine Wissenschaft” (“pure science”) was frequently used at the time and for lack of a better word for expressing this idea we shall use the phrase “preference for pure mathematics” knowing the difficulties of the distinction between “pure” and “applied” mathematics.

### *Anti-utilitarianism and Self-directed Activity*

To enlarge on this preference for pure mathematics we consider elementary education. Looking upon arithmetic under the perspective of the needs of society it might seem to be most important to future workmen and shopkeepers that they be able to calculate fluently with money and the elementary magnitudes of length, area, volume and weight. This was even more so in a politically fragmented country like Germany with numerous different systems of money and different units of measurement. To carpenters and shopkeepers it was a matter of survival to know these different units and magnitudes and to master the resulting calculations. Thus, it seemed to be most important to children becoming acquainted with “Carolins, Ducats, and the like”, and in fact training faculties in the domains of measurement and money had been the core of arithmetic teaching at traditional 18th century elementary and higher schools.

Now the neo-humanist ideas were radically the other way round. These people thought that it is most important to young children to become first and foremost acquainted with the very concept of number and with number relations independent of magnitudes. Children should become fluent and flexible in calculating with abstract numbers. The handling of money and magnitudes was considered secondary and an easy exercise once children had learnt to operate with abstract numbers. These ideas were completely in line with Pestalozzi's approach to elementary education, and this explains why Pestalozzi became such a prominent person in German elementary education. And of course, this is also the modern view on the teaching of arithmetic. Surely, money and simple magnitudes are part of the teaching of elementary arithmetic but also to us it seems more important that children understand the concept of number and number relations in themselves.

Humboldt's disdain of “Carolins, Ducats, and the like” echoes the prominent pedagogue F. A. W. Diesterweg (1790–1866) who wrote:

„Arithmetic lessons are no stores and no fairs, no stock market and no courtroom.”  
 [Die Rechenstunde ist kein Materialladen und kein Jahrmarkt, keine Börse und keine Gerichtsstube.] (Diesterweg 1859, quoted according to: von Sallwürk 1899, 479, transl. by HNJ)

Diesterweg also made it clear that the refusal of the utilitarian point of view is closely connected with another important idea. As we have seen neo-humanist pedagogues considered as the foremost aim of school teaching the raising of autonomous personalities, and, therefore, another central catchword of the neo-humanist reforms was “Selbsttätigkeit” (“self-directed activity”). As Diesterweg put it:

“Teach in such a way that the self-directed activity of the pupil is developed as far as possible.”  
 (“Unterrichte so, dass überall die Selbsttätigkeit des Schülers möglichst ausgebildet werde.”  
 (Diesterweg 1844, III/IV, transl. by HNJ)

Diesterweg favoured especially mental arithmetic, since it is in this domain that children can develop individual and flexible strategies of calculation.

“However, in mental arithmetic there is much more freedom for individual activity, decision and discretion. For this reason mentally agile children love mental arithmetic so much. They like treating a task in multiple ways and in their own manner .... Therefore, by way of exercises in mental arithmetic [the teacher] should strive for the unleashing and liberation of the young mind by as many different solution methods as possible.” [„Beim Kopfrechnen dagegen. herrscht viel mehr Freiheit, welche eigene Bewegung, Auswahl und Belieben zulässt. Darum lieben geistig bewegliche Kinder so sehr das Kopfrechnen. Es gefällt ihnen, eine Aufgabe in mannigfacher Art, auf ihre Weise zu behandeln .... Darum strebe man ja an den Kopfrechenaufgaben durch möglichst mannigfache Auflösungsweisen die Entfesselung und Befreiung des jugendlichen Geistes an.”] (l.c., IX, transl. by HNJ)

Thus, mental arithmetic should not be a matter of mere memorizing, but a matter of thinking. Pupils were expected to develop their own clever strategies of calculation. This in fact is again a completely modern idea.

For his approach to calculating and mental arithmetic Diesterweg coined the artificial word “Denkrechnen” as a compound of “thinking” and “calculating”. The idea that mental arithmetic should be a matter of thinking and further the development of self-directed activity was supported by many educators and teacher trainers for elementary schools and was a hallmark of the Pestalozzi school in Germany. We name Wilhelm Harnisch (1787–1864), Ernst Tillich (1780–1807), W. von Türk (1774–1846), Peter Kawerau (1789–1844), Ernst Hentschel (1804–1875), Carl Gotthilf Ehrlich (1776–1857) (see Jänicke 1877; Radatz and Schipper 1983, 31 pp; Biermann 2010, 95 pp).

Diesterweg was a teacher trainer for elementary schools and the quotations above refer to elementary education, but a similar and even stronger rejection of a utilitarian point of view was also true for gymnasium and higher education in general. It is plausible that this entire approach to education was very much in line with the general scientific culture of the time which stressed the notion of *understanding*. Educators considered it more important that students understand something than to merely train them in seemingly useful skills.

The “imperative of understanding” provides also another reason why neo-humanists wanted to exclude preparation for special professions from the state-driven public schools. Humboldt explained:

In regard to skills in application “it is very often necessary to confine oneself to results which are not understood from their reasons, since the skill must be there, and time or talent for insight are missing. Such is the case with unscientific surgeons, many factory owners etc.” [Im Hinblick auf Fertigkeiten zur Anwendung “muss man sich sehr oft auf in ihren Gründen unverstandene Resultate beschränken, weil die Fertigkeit da seyn muss, und Zeit oder Talent zur Einsicht fehlt. So bei unwissenschaftlichen Chirurgen, vielen Fabrikanten u.s.f.” (l.c., 188, transl. by HNJ)

The knowledge necessary for special professions implied too many elements which cannot be understood from their reasons and are only recipes contrary to the principle of understanding as the core of teaching and learning.

### ***Pure Mathematics and Every Day Applications: A Compromise***

As we have seen, in Humboldt’s and his followers’ eyes emphasis should be put on pure mathematics. However, to understand this adequately, one has to distinguish between two different types of applications. One type consisted of those elementary calculations mentioned above which are needed in everyday life and in the daily affairs of shopkeepers and workmen. We call these the *everyday applications*. A completely different matter was the application of mathematics in astronomy, physics and other mathematized sciences. There is no common term for these applications; we will call them *theoretical applications*. Whereas the neo-humanist-idealist pedagogues intended to reduce the amount of everyday applications in teaching, they favoured theoretical applications. In fact, taken as a mathematical theory mechanics is simply mathematics. In the first syllabus for gymnasium of 1816 which never became obligatory mechanics (kinematics, statics) was included in the mathematics lessons of the upper grades of gymnasium. However, when trying to implement this syllabus at some schools this proved to be unfeasible, and very soon the mechanics lessons were cancelled from the mathematics plan.

The discussion on whether the everyday applications should at all be included in the mathematics lessons at gymnasium lasted until the end of the 1820s. The intention to break with 18th century traditions and to introduce exclusively “scientific” mathematics that is mathematics beyond these practical needs into gymnasium met severe problems with parents and students. As a rule, a gymnasium was attended by a student population with quite mixed interests. It served for pupils who were intending to take the university entry qualification and to continue with university studies as well as for children who were going to become shopkeepers and workmen. The latter left school after grade seven or eight and were called “Frühabgänger” (they were “early leaving”). Their parents expected their children to be trained in elementary arithmetic and the everyday applications and, thus, resisted against a radical change of mathematics teaching. In the 1820s it became clear that a compromise was necessary, and finally, the government decided that elementary calculations with numbers including the everyday applications became subjects in the first two grades of gymnasium whereas the course on “scientific mathematics” started only in the

third grade (see Jahnke 1990a, 377, 388 pp). The distinction between “Calculating” (“Rechnen”) and (“scientific”) Mathematics” remained common for a long time, in fact well into the 1960s until it finally was overcome by the New Math Movement.

Since the 1830s, the mathematical syllabus at gymnasium comprised in the two lowest grades a course on elementary calculating after which, in the higher grades, the course in scientific mathematics followed. Standard topics of the latter were until the 1870s Euclidean elementary geometry, trigonometry, spherical trigonometry and a strand of arithmetic, algebra and sort of elementary analysis. The latter comprised some simple infinite series, but no differential and integral calculus.

The *compromise* in regard to everyday applications made it possible that gymnasia in Prussia until the 1850s functioned as sort of *comprehensive schools* (see Müller 1977). With its mixed population Gymnasium took care for 7% of the respective male age groups at the beginning as well as at the end of the period. Beyond that, local circumstances made many modifications necessary, and the administration was flexible in doing this. When a city was too small and not enough children wanted to finish gymnasium schools were created covering only the first seven or eight grades. They were called “Höhere Bürgerschulen” (“Higher schools of citizens”). All in all Prussia established a school system consisting of elementary schools („Volksschulen”) for all children, and of secondary schools (“Gymnasium”) in order to prepare students for university studies, civil service careers, outstanding positions in commerce, or industry. On a level between “Volksschule” and “Gymnasium” so-called “Höhere Bürgerschulen” (“Higher schools of citizens”) emerged which educated for practical skills in craft and commerce. In the 1860s a separate system of schools with a stronger emphasis on science and mathematics (“Realschulen”) *in addition and parallel to gymnasium* was established. To avoid terminological confusion: These 19th century “Realschulen,” like the gymnasia, extended from grade 5–13, whereas in today’s Germany the term “Realschule” refers to schools extending only from grade 5–10.

### ***An Engineer’s Expert Opinion: Pure Mathematics Is the Best One Can Do for Applied Mathematics***

That pure mathematics rather than everyday applications should be the chief focus of mathematics teaching, was not only Humboldt’s opinion, but it was also that of most teachers and educators, including professional mathematicians. A good example was *August Leopold Crelle* (1780–1855). Crelle is well-known as the founder of the prestigious *Journal für die reine und angewandte Mathematik*. He worked for 20 years as an engineer in the Prussian administration, was engaged in the building of railways and advisor of the ministry of education for the teaching of mathematics. In a remarkable preface to a textbook on number theory (1845) he gave a justification why he thought pure mathematics so important in education and also the best one could do for applied mathematics. According to Crelle, the use of mathematics could be divided into two categories, one being its use in immediate applications, and the other, its use for training one’s ability to think. Upon weighing these two purposes against one another, Crelle argued that with most practical problems it is difficult to apply mathematics because they are too complicated. Frequently, the application of mathematics would even lead to serious mistakes and errors, because people rely on

mathematical rigour and certainty without considering the complexity of the conditions. Quite indubitable, however, was according to Crelle the use of mathematics to exercise the ability of thinking, and this in turn was then the indispensable condition for the direct application of mathematics (Fig. 5.4).

Only after a mathematical *spirit* has been awakened by assiduously exercising *judgment by means of mathematics* (without regard for applications), and only then, one may quite boldly count on the uses of mathematics in applications. Mere *knowledge* in mathematics, intended for *applications* ... is not sufficient for appropriate applications, but the guiding principle must be the mathematical *spirit*, the mathematical *way of thinking*. Only he who tackles applications *on this basis* will err less easily, for he will first of all examine *what* mathematics can properly achieve, and where and how the tool can be usefully applied .... Hence it is *quite right* that mathematics be exercised as much as possible in schools ... at first *without* any consideration of applications in common life. (Crelle 1845, pp. IX–X, transl. by HNJ).

With the notion of judgment, Crelle referred to Kant who in the *Critique of Pure Reason* had explained that a rule does not say by itself to which cases it can be applied, and that, therefore, the application of a rule requires faculty of judgment. Later, Kant elaborated this in his *Critique of Judgement*.

Considering Crelle's argument from a modern perspective we understand his point very well. To use 20th century terminology, we can say that modelling is an interplay of forming hypotheses, drawing consequences from them and evaluating the hypotheses in light of their consequences. It is exactly the latter process of evaluation which is aptly described by the concept of 'judgement'. A core component of judgement in applying mathematics is, of course, understanding the deductive interplay between hypotheses and its consequences, and by this we are back to pure mathematics. To say it shortly: proof and modelling are inseparably connected. It is no wonder, then, that in most documents on the aims of mathematics education all over the 19th century the term 'faculty of judgement' played a prominent role.

**Fig. 5.4** Commemorative plaque for A. L. Crelle (source Wikipedia Public Domain, ©)





Crelle and other mathematicians of the time were convinced that for the sake of applications we have to cultivate pure mathematics. This was in line with the general conviction of scientists and philosophers of the time that the cultivation of pure science would not lead to a contemplative stance. Philosopher Johann Gottlieb Fichte favoured an approach to education involving pure thinking because the ability to think abstractly was, in his opinion, the decisive precondition for imagining alternatives, and of being able to develop a new design for the future, thereby creating an ethically acceptable world. According to Fichte: “*That ability to independently design images which are by no means copies of reality but suitable to become ideals for it would be the first principle from which the cultivation of the species by means of the new education would have to proceed.*” (Fichte 1808, 31/2). This same idea is echoed by famous philologist August Böckh who in a speech as Rector of the University of Berlin said that not those people are suitable for the “higher service” who can adapt themselves to the given circumstances, but those who are ready to change them. These people have to be educated by pure science (Boeckh 1853, 97). Thus, the ability to think in terms of hypothetical alternatives was seen as a decisive condition of “aptness for the future” (“Zukunftsfähigkeit”). Changing to modern times, the motto of the university of Duisburg-Essen reads in German: “In Möglichkeiten statt in Grenzen denken.” [“Think in terms of possibilities, instead of limits.”] This exactly was the core of all these opinions.

### ***Final Remarks***

- a. Neo-humanist pedagogy defined Bildung as the comprehensive intellectual and emotional self-development of the individual and refused subordinating education at public schools by recourse to requirements of professional life and of needs of society. In this radical anti-utilitarianism neo-humanist pedagogy was unique, and this justifies continuing to use the German word ‘Bildung’ instead of the English ‘General education’.
- b. There was no noticeable difference between the number of weekly hours devoted to the teaching of mathematics at learned schools and “Bürgerschulen” of the 18th century and the respective number at a Prussian gymnasium in, say, the year 1837 (Jahnke 1990a, 335 pp). But the *character of mathematics* which was provided by school teaching in the 18th and 19th centuries had *deeply changed*. At elementary schools there were more and more teachers who followed the ideas of what was called the Pestalozzi school with its conception of “Denkrechnen” (“calculating by thinking”). At gymnasium only in the 19th century mathematics in a scientific sense was introduced. This was the great historical achievement of the neo-humanist pedagogy in regard to the teaching of mathematics in Germany.
- c. The strong emphasis of the neo-humanists on theoretical thinking, understanding and pure science was in their eyes not a denial of the demands of practical life, but the best way to meet them. In their eyes theoretical thinking is a necessary condition for change. Thus, to educate young people in theoretical thinking is the best way to make them “apt for the future”. Considering the dynamic technological development of Germany in the second half of the 19th century one might be inclined to agree.

- d. Even today, it is a critical question as to whether utility and the relatedness of a subject to everyday life is a sufficient argument for including it into the school curriculum. Among experts, it is clear that deeper conceptions than utility must play a part in our deliberations about what mathematics should be taught and how it should be taught. Nevertheless, pressure from the public (policy, economy, media) to reduce school teaching to everyday life is persistent and strong. It is the duty of pedagogical experts to clarify ‘*Bildung*’ in its proper sense and persuade the public of its cogency. The conviction that the creation and development of intellectual worlds by the individual is a fundamental qualification for becoming “apt for the future” was a strong motive for neo-humanist pedagogical thinking as well as it should be a guideline for modern approaches.

## 5.2 *Bildung* and Paideia and Their Presence in Some Undergraduate Programs (Reaction by Michael N. Fried)

On first sight, it may seem strange at the very least and, perhaps, even inappropriate that an English speaker, born in America and teaching at a university in Israel’s Negev Desert, a person far removed from the community of German mathematics educators, should presume to say anything about the concept of *Bildung*. Yet, in a way, a non-German speaking about *Bildung* is completely in line with Humboldt’s own ideas about the concept, which were directed towards an object at once individual and international. So, though a German word, the idea of *Bildung* should not be restricted to Germans, just as *mathematics*, a Greek word, should not be restricted to Greeks, but rather to human beings as far as they are human beings. *Bildung*, in this sense, is akin to the idea of liberal education, which was the subject of my own ICME invited lecture, for these “liberal arts” are directed towards human beings possessing *libertas*, the freedom to live as fully a human life as possible, as opposed to slaves prevented from fulfilling their human potential. Indeed, *Bildung* and liberal education are closely, even profoundly, related.

In the brief remarks below, with no claim of originality, I would like simply bring out the connection between *Bildung* and the classical liberal arts. Moreover, I would like to give a few examples of liberal arts colleges which, while they do not necessarily manifest that *other* Humboldtian ideal of research and teaching, nevertheless capture something of the deeper sense of *Bildung* as the formation of full human beings: thus they are also liberal arts colleges in the deeper sense of the liberal arts themselves. Significantly too, these college see mathematics as part of liberal education, and two, in particular, see mathematics as an *essential* part.

### *Humboldt’s Theory of Human Bildung*

Wilhelm von Humboldt’s (1767–1835) early fragment “Theorie der Bildung des Menschen” (1793) gives some idea of what Humboldt had in mind when he spoke

of *Bildung*, and therefore, how he conceived what education could achieve and what good it could do. In this early piece one can discern his pointed attention to wholeness—both in what we study and, more importantly, in our own being. In this connection, he says that in the narrowness and fragmentation standing opposed to that kind of integrity which he has his eye on,

...lies one of the preeminent reasons for the frequent, not, unjustified, complaint that knowledge remains idle and the cultivation of the mind unfruitful, that a great deal is achieved around us, but only little unimproved within us... (Humboldt 1960, pp. 234–235, Trans. Horton-Krüger, p. 58).

This sounds familiar. We are often asked to be more connected to the world, in the sense of being more interested in material progress, but we are also often reminded that material progress fails to match what we might call spiritual progress. We hear *both* the call to be practical and the call to be more spiritual or more moral. Then, as now, this incompatibility was expressed as a “complaint,” a sense of genuine dissatisfaction. However, the *source* of the problem, our lack of wholeness or fullness, it seems to me, is still not seen clearly today, even after Humboldt.

It is that picture of a human being engaged in a whole world, that is to say, not every detail in the world, not our love of information, but the world *as* something whole and our own wholeness within it, it is that which Humboldt wants us to understand. Although he leaned towards pure studies, specifically in mathematics (see Jahnke 1993b, p. 418), he would not have us completely retreat into a sort of *vita contemplativa* and ignore an active life in the world—and therefore, in the case of mathematical studies, he would not have us ignore the mathematics which is of use in daily life. He wants us, in fact, very much to look towards the external world around us, but he also wants to press the point that the ground for this should be, in a slightly paradoxical way, our own *inner* integrity. Thus he writes:

What man needs most, therefore, is simply an object that makes possible the interplay between his receptivity and his self-activity. But if this object is to suffice to occupy his whole being in its full strength and unity, it must be the ultimate object, the world, or at least (for only this is in fact correct) be regarded as such. Man seeks unity only to escape from dissipating and confusing diversity. In order not to become lost in infinity, empty and unfruitful, he creates a single circle, visible at a glance from any point. In order to attach the image of the ultimate goal to every step forward he takes, he seeks to transform scattered knowledge and action into a closed system, mere scholarship into scholarly *Bildung*, merely restless endeavor into judicious activity (Humboldt 1960, p. 238, translation by Horton-Krüger 2000, p. 60)

The word “picture” seems to me the right one, for it too is something grasped as a unity. For this reason, I do not take it as accidental that the word *Bildung* has a relation to *Bild* or picture. Indeed, I confess, as I read this fragment by Humboldt, a youthful piece to be sure, I begin to see in my mind—and begin to understand better—that iconic image of romanticism, the well-known painting by Caspar David Friedrich, *Wanderer above the Sea of Fog (Der Wanderer über dem Nebelmeer)* (1818), the original can be seen in the Kunsthalle Museum in Hamburg, where ICME-13 was held). It is not the pejorative view of the romantic lost in the clouds, for, note, the wanderer’s feet are firmly placed on the solid rock of a mountain. Moreover, we too

**Fig. 5.5** C. D. Friedrich, Wanderer above the sea of fog (source Wikipedia Public Domain, ©)



are standing on that mountain. Yet the landscape is open and undefined—that is to say, the solidness of the rocks do not crush or confine us, but free us. And “us” and “we” are the right words, for we are standing on the same rock, looking in the same direction—that of course is one reason why we only see the back of the man: the picture is not about him, but about us (Fig. 5.5).

### *Humanistic Education*

Now, a somewhat different picture is one by Botticelli, showing a young man introduced to the liberal arts—seven sisters—the core of his education. You will notice in the painting that there are eight, not seven, women. The eighth is seen holding a snake—she is Prudentia, or, to use the well-known Greek term, Phronesis: she is not one of the liberal arts, but she is the spirit of the full life the young man will live, reflecting, making decisions, and growing in wisdom (on the matter of animals, the one with the scorpion is logic, who possesses the power of stinging arguments) (Fig. 5.6).

The liberal arts formed a complete system—and included both arts of communication, i.e., constructions with words—grammar, logic, and rhetoric—and arts of things or of the world—the mathematical arts of arithmetic, music, geometry, and astronomy (in the Botticelli painting, three of the latter can be clearly seen: to the right of Prudentia, we have first geometry holding a right angle, then astronomy with

**Fig. 5.6** Botticelli, liberal arts—seven sisters (*source* Wikipedia Public Domain, ©)



an armillary sphere, and then music with her “positive organ”; arithmetic is likely the woman ushering in the young man). In late antiquity, these seven liberal arts were often completely identified with a Greek word, *Paideia*, a kind of general education, the word one hears in the word *Encyclopaedia*, literally, a “course of general education.” The truth of the matter though is that *Paideia*, like *Bildung*, is tremendously difficult to translate, for, like *Bildung*, it takes in education, culture, upbringing, to name a few. The great classical scholar Werner Jaeger (1945) required three thick volumes to explain the word *Paideia*, which is also the name of that three volume work.

But one can say this about it:

- (1) *paideia* was rooted in the literature and thought of one’s tradition—here the translation, “culture,” is apt
- (2) It was meant to be carried throughout life, so that...
- (3) It was very much an expression of being a human in the fullest sense of the word, thus the Latin translation of *paideia* as *humanitas*.

I might mention as an aside that Werner Jaeger dreamed of a “third humanism,” where the second humanism happened to be that implicit in the aspirations of Humboldt’s *Bildung* (see, for example, Östling 2015, p. 209) But this is a complicated story....<sup>2</sup>

In any case, the three central characteristics of *Paideia* listed above seem truly very close to what Humboldt himself may have had in mind when he referred to *Bildung*.

<sup>2</sup>Marrou (1982) saw that Jaeger’s particular admiration of Spartan education gave voice to fascist and Nazi ideologies (see p. 23). Whether this is a fair assessment can be debated. Jaeger, it might be noted, fled Germany in 1936 and came to the United States, where he spent the rest of his life.

Indeed the difficulty of translating both words—*Bildung* and *Paideia*—may have much to do with their mutual attempts to grasp an almost unlimited whole.

### *Some Undergraduate Colleges*

My original task in this note was to reflect on these Humboldtian and classical ideas in the context of Israeli school mathematics or the Israeli educational system generally. But I could find, in fact, very little of that desire for a whole human being and a whole experience of human life—at least not explicitly, even though I could see that pieces were there. Nor, however, could I find this in other places I am familiar with. In almost every instance, I found far more pronounced that very tendency towards material progress without true inner human progress that Humboldt had hoped to steer us away from.

Where I do see something like an active embracing of *Bildung* or the liberal arts in the sense of *Paideia* is in two or three undergraduate colleges in North America (I am certain there must be similar islands of the liberal arts in other places as well). These include:

Quest University in British Columbia, Canada

St. Mary's College in Moraga, California, USA

St. John's College in Annapolis, Maryland and Santa Fe, New Mexico, USA.

All three have one degree—Quest a degree in the Arts and Science; and St. Mary's and St. John's a Liberal Arts degree. Quest has some choice in courses, while St. John's and St. Mary's (which is to a large degree modeled after St. John's, though St. Mary's has a religious orientation whereas St. John's is completely non-sectarian) have a set curriculum. Since time is short, I will only say a few words about St. John's.<sup>3</sup>

St. John's is based on a set of books studied in common by all students and very much at the center of the western tradition (there is a graduate program at the college which looks closely at the great books of Eastern traditions as well). The books thus form the foundation of a community in which both the students and faculty (called tutors rather than professors since they are not supposed to be professing something) share a common set of ideas—sometimes conflicting ideas, sometimes radically so—which forms, in turn, a foundation for continuous conversation. It is

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<sup>3</sup>Just for the record though, here is how Quest describes its own program on its website (<http://www.questu.ca/arts-and-sciences-degree.html>):

As the world further transitions from the industrial era into the information age, our education needs to keep pace. Technical, job-oriented training is no longer sufficient to handle the complex and shifting challenges of the 21st century.

Enter the liberal arts and sciences.

Consisting of arts, humanities, mathematics, and social, life and natural sciences, the liberal arts and sciences form the basis of a well-rounded education. They allow one to develop critical thinking, intellectual breadth, and an informed, worldly perspective—they allow one to really *think*.

telling in this regard that in all the classes students are seated around a large single table with their tutor sitting among them.

The books studied come from poetry and philosophy as well as from science, history, and mathematics. All students study mathematics. The questions and ideas in these books are considered complementary parts of a whole, which means not so much a single message as a single intellectual endeavor. The singleness of the endeavor—the continual reflection, reasoning, and discussion about the world and one’s place in it—implies that although mathematical studies may have a technical side (and students do learn some mathematical techniques) they can never be viewed as simply technical, only means to some other narrow end: they must always be viewed in terms of the greater endeavor. Therefore, it is typical—and expected—that a discussion of one set of books will come into play with another, possibly because the rock their own authors stand upon is common: a lively discussion, for example, of Plato’s *Meno* might well erupt as students try to make sense of Book X of Euclid’s *Elements*.

The list of mathematical books that students read in the “mathematics tutorial” has changed over the years, but certain works are quite stable in the program. These include Euclid’s *Elements* (all thirteen books) and Nichomachus’s *Arithmetic* in the first year; the first three books of Apollonius’s *Conics* and Descartes’s *La Géométrie* in the second year; Newton’s *Principia* and Dedekind’s *Continuity and Irrational Numbers* in the third year; and Lobachevski’s *Theory of Parallels* in the fourth year. In all cases, the theorems proven in the books or the problems solved are presented at the board by the students, and fully discussed.

Newton’s *Principia* of course could also be considered a reading in physics (and study read other works from physics in their laboratory tutorials), but in the mathematics tutorial a distinction between mathematics and physics or astronomy is not always made. Thus, in the second year the mathematics tutorial also includes Ptolemy’s *Almagest*; the third year includes readings from Galileo, and the fourth year Einstein’s two 1905 papers on special relativity and Minkowski’s 1908 lecture, “Space and Time.” The fact that the distinction between physics, astronomy, and mathematics is not always made itself a question for the students to consider, leading them to frequent discussions about the nature of mathematics.

From the emphasis on “great books,” one might get the false impression that St. John’s is based on western dogmas, partly because of the word “great.” Yet, nothing could be further from the truth. The goal of the program, in mathematics no less than in literature or philosophy, is to try to uncover and get behind the basic assumptions in the books being read. In this way, however, it *is* in line with one modern philosophical position, though one meant to keep questions alive, namely, Husserl’s idea that philosophy has the role of breaking through layers of conceptual “sedimentation” (see Husserl 1989, pp. 168 ff). Thus, Jacob Klein, who had a decisive role in shaping the St. John’s program and who was student of Husserl, writes:

The passing on of sciences, arts, and skills, especially of intellectual ones, cannot quite avoid the danger of blurring the original understanding on which those disciplines are based. The terms which embody that understanding, the indispensable terms of the art, of the *technē* in question, the “technical” terms, acquire gradually a life of their own, severed from the

original insights. In the process of perpetuating the art those insights tend to approach the status of sediments, that is, of something understood derivatively and in a matter-of-course fashion.... Liberal education has to counteract this process of sedimentation... (Klein 1985, p. 263)

And specifically about mathematics, he emphasizes:

It is necessary...to study mathematics, always bearing in mind that this studying has to be reflective and cannot be satisfied with a sedimented understanding of mathematical relationships. (p. 266)

Whether St. John's, or any of these schools, actually achieve the *Bildung* Humboldt aimed for or the *Paideia* the classical thinkers aimed for, I am not sure. It is clear, however, that they are turned towards these directions. And that may be the best we can hope for.

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