ERRATA LIST

Robust and Adaptive Control with Aerospace Applications – 1st Edition, Springer, 2013
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Revised on 23 Jul 2013

Part I

Introduction

1) Page 10, change the top equation to:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ -\dot{h} \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

Chapter 1

1) Page 21, remove boxes from around the equations

$$\boxed{\begin{aligned} \dot{x} &= A_p x + B_p (C_c x_c + D_{c1} y_{\text{in}}) = A_p x + B_p C_c x_c + B_p D_{c1} y_{\text{in}} \\ y_{\text{out}} &= C_p x + D_p (C_c x_c + D_{c1} y_{\text{in}}) = C_p x + D_p C_c x_c + D_p D_{c1} y_{\text{in}} \end{aligned}}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} &= \underbrace{\begin{bmatrix} A_p & B_p C_c \\ 0_p & A_c \end{bmatrix}}_{A_{\text{Lo}}} \begin{bmatrix} x \\ x_c \end{bmatrix} + \underbrace{\begin{bmatrix} B_p D_{c1} \\ B_{c1} \end{bmatrix}}_{B_{\text{Lo}}} y_{\text{in}} \\ y_{\text{out}} &= \underbrace{\begin{bmatrix} C_p & D_p C_c \end{bmatrix}}_{C_{\text{Lo}}} \begin{bmatrix} x \\ x_c \end{bmatrix} + \underbrace{\begin{bmatrix} D_p D_{c1} \end{bmatrix}}_{D_{\text{Lo}}} y_{\text{in}} \end{aligned}} \quad (1.53)$$

E. Lavretsky and K.A. Wise, *Robust and Adaptive Control*, Advanced Textbooks in Control and Signal Processing, DOI 10.1007/978-1-4471-4396-3_15,

Chapter 2

1) Page 34, change

$$H^*(x,\nabla_x J^*,t) = x_1^4 + \frac{1}{4}\left(\frac{\partial J^*}{\partial x_2}\right)^2 + \frac{\partial J^*}{\partial x_1}x_2 - 2\frac{\partial J^*}{\partial x_2}x_1 - 3\frac{\partial J^*}{\partial x_2}x_2 - \frac{1}{2}\frac{\partial J^*}{\partial x_2} \quad (2.26)$$

to

$$H^*(x,\nabla_x J^*,t) = x_1^4 + \frac{1}{4}\left(\frac{\partial J^*}{\partial x_2}\right)^2 + \frac{\partial J^*}{\partial x_1}x_2 - 2\frac{\partial J^*}{\partial x_2}x_1 - 3\frac{\partial J^*}{\partial x_2}x_2 - \frac{1}{2}\left(\frac{\partial J^*}{\partial x_2}\right)^2$$

2) Page 34, change

$$\begin{aligned} -\frac{\partial J^*}{\partial t} &= H^*(x,\nabla_x J^*(x,t),t) \\ &= x_1^4 + \frac{1}{4}\left(\frac{\partial J^*}{\partial x_2}\right)^2 + \frac{\partial J^*}{\partial x_1}x_2 - \frac{\partial J^*}{\partial x_2}\left(2x_1 - 3x_2 - \frac{1}{2}\right) \quad (2.27) \end{aligned}$$

to

$$\begin{aligned} -\frac{\partial J^*}{\partial t} &= H^*(x,\nabla_x J^*(x,t),t) \\ &= x_1^4 - \frac{1}{4}\left(\frac{\partial J^*}{\partial x_2}\right)^2 + \frac{\partial J^*}{\partial x_1}x_2 - \frac{\partial J^*}{\partial x_2}(2x_1 - 3x_2) \end{aligned}$$

3) Page 36, change

$$-\frac{\partial J^*}{\partial t} = x^T Qx + \tfrac{1}{4}\frac{\partial J^*}{\partial x}BR^{-1}B^T\nabla_x J^* + \frac{\partial J^*}{\partial x}Ax - \tfrac{1}{2}\frac{\partial J^*}{\partial x}BR^{-1}B^T\nabla_x J^* \quad (2.34)$$

to

$$-\frac{\partial J^*}{\partial t} = x^T Qx - \tfrac{1}{4}\frac{\partial J^*}{\partial x}BR^{-1}B^T\nabla_x J^* + \frac{\partial J^*}{\partial x}Ax$$

4) Page 36, change

$$u^*(x,\,t) = -\tfrac{1}{2}R^{-1}B^T\frac{\partial J^*(x,\,t)}{\partial x} = -\underbrace{R^{-1}B^TP(t)}_{K(t)}x = -K(t)x \qquad (2.39)$$

to

$$u^*(x,t) = -\frac{1}{2}R^{-1}B^T\nabla_x J^* = -\underbrace{R^{-1}B^TP(t)}_{K(t)}x = -K(t)x$$

5) Page 46, change

$$\det[sI - H] = \phi_{cl}(s)\phi_{cl}(-s) \qquad (2.81)$$

The asymptotic properties we desire to explore are those associated with the migration of these eigenvalues, as the numerical values in the LQR penalty matrices Q and R are varied. We can examine these eigenvalues (roots of $\phi_{cl}(s)$) through the polynomial formed by expanding the $\det[sI - H]$. We begin with some elementary row and column operations on H. First, we multiply the first row of H by $-Q(sI - A)H^{-1}$ and add it to the second row. This yields

to

$$-Q(sI - A)^{-1}$$

Chapter 3

1) Page 57, change

$$\dot{z} = \tilde{A}z + \tilde{B}\mu z = \begin{bmatrix} e \\ \dot{x} \end{bmatrix}, \mu = \dot{u}$$
$$\tilde{A} = \begin{bmatrix} 0 & C_c \\ 0 & A \end{bmatrix}, \tilde{B} = \begin{bmatrix} D_c \\ B \end{bmatrix} \qquad (3.24)$$

to (need comma and space)

$$\dot{z} = \widetilde{A}z + \widetilde{B}\mu, \quad z = \begin{bmatrix} e \\ \dot{x} \end{bmatrix}, \mu = \dot{u}$$
$$\widetilde{A} = \begin{bmatrix} 0 & C_c \\ 0 & A \end{bmatrix}, \widetilde{B} = \begin{bmatrix} D_c \\ B \end{bmatrix}$$

2) Page 57, change

$$\dot{z} = \tilde{A}z + \tilde{B}\mu z = \begin{bmatrix} e \\ \xi \end{bmatrix}, \xi = \ddot{x} - \omega^2 x, \mu = \ddot{u} - \omega^2 u,$$
$$\tilde{A} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2 & 0 & C_c \\ 0 & 0 & A \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0 \\ D_c \\ B \end{bmatrix} \tag{3.25}$$

to

$$\dot{z} = \widetilde{A}z + \widetilde{B}\mu, \quad z = \begin{bmatrix} e \\ \xi \end{bmatrix}, \xi = \ddot{x} - \omega^2 x, \mu = \ddot{u} - \omega^2 u,$$
$$\widetilde{A} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2 & 0 & C_c \\ 0 & 0 & A \end{bmatrix}, \widetilde{B} = \begin{bmatrix} 0 \\ D_c \\ B \end{bmatrix}$$

3) Page 59, change

$$P\tilde{A} + \tilde{A}^T P - P\tilde{B}R^{-1}\tilde{B}^T P + Q = 0 \tag{3.36}$$

The resulting steady-state $n_u \times (n_r + n_x)$-dimensional feedback controller gain matrix is

to

$$n_u \times (pn_r + n_x)$$

4) Page 60, change

$$u = -K_x x + \sum_{i=1}^{p} s^{-i} \left(a_i \left(\overset{(p-i)}{u} + K_x \overset{(p-i)}{x} \right) - K_i \overset{(p-i)}{e} \right) \tag{3.40}$$

to

$$u = -K_x x + \sum_{i=1}^{p} s^{-i} (a_i (u + K_x x) - K_i e)$$

5) Page 60, change

$$\dot{z} = \left(\tilde{A} - \tilde{B}K_c\right)z + Fr \tag{3.41}$$

$$\text{where} \quad F = \left[\, -I_{n_u \times n_u} \quad 0_{n_u \times n_x} \,\right]^T .$$

to

$$\dot{x}_e = \left(\widetilde{A} - \widetilde{B}K_c\right)x_e + Fr$$

where the extended state x_e is (3.18) integrated p-times, and $F = \left[\, -I_{n_u \times n_u} \quad 0_{n_u \times n_x} \,\right]^T$.

6) Page 63, change the following figure

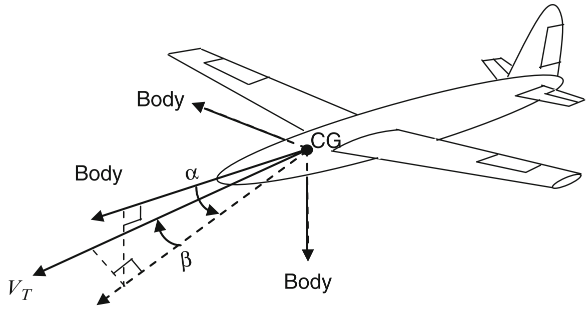
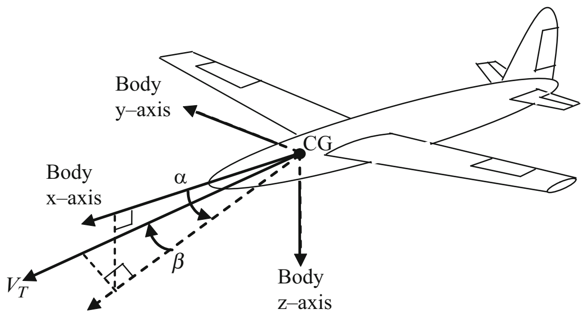


Fig. 3.3 Unmanned aircraft

to the same figure as in Figure 1.3, page 8.



7) Page 63, Change

$$A_z = -V\dot{\gamma} = VZ_\alpha\alpha + VZ_\delta\delta \tag{3.48}$$

to

$$A_z = -V\dot{\gamma} = Z_\alpha\alpha + Z_\delta\delta$$

8) Page 63, change

$$\begin{aligned}\dot{A}_z &= Z_\alpha A_z + VZ_\alpha q + VZ_\delta\dot{\delta}_e \\ \dot{q} &= \frac{M_\alpha}{VZ_\alpha}A_z + M_q q + \left(M_\delta - \frac{M_\alpha Z_\delta}{Z_\alpha}\right)\delta_e\end{aligned} \tag{3.49}$$

to

$$\begin{aligned}\dot{A}_z &= \frac{Z_\alpha}{V}A_z + Z_\alpha q + Z_\delta\dot{\delta}_e \\ \dot{q} &= \frac{M_\alpha}{Z_\alpha}A_z + M_q q + \left(M_\delta - \frac{M_\alpha Z_\delta}{Z_\alpha}\right)\delta_e\end{aligned}$$

9) Page 64, change

$$\begin{bmatrix}\dot{A}_z \\ \dot{q} \\ \dot{\delta}_\mathrm{e} \\ \ddot{\delta}_\mathrm{e}\end{bmatrix} = \begin{bmatrix} Z_\alpha & VZ_\alpha & 0 & VZ_\delta \\ M_\alpha/VZ_\alpha & M_q & \left(M_\delta - \frac{M_\alpha Z_\delta}{Z_\alpha}\right) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_\mathrm{a}^2 & -2\zeta_\mathrm{a}\omega_\mathrm{a}\end{bmatrix}\begin{bmatrix}A_z \\ q \\ \delta_e \\ \dot{\delta}_e\end{bmatrix} + \begin{bmatrix}0 \\ 0 \\ 0 \\ \omega_a^2\end{bmatrix}\delta_c \tag{3.51}$$

to

$$\begin{bmatrix}\dot{A}_z \\ \dot{q} \\ \dot{\delta}_e \\ \ddot{\delta}_e\end{bmatrix} = \begin{bmatrix} \frac{Z_\alpha}{V} & Z_\alpha & 0 & Z_\delta \\ M_\alpha/Z_\alpha & M_q & \left(M_\delta - \frac{M_\alpha Z_\delta}{Z_\alpha}\right) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_a^2 & -2\zeta_a\omega_a\end{bmatrix}\begin{bmatrix}A_z \\ q \\ \delta_e \\ \dot{\delta}_e\end{bmatrix} + \begin{bmatrix}0 \\ 0 \\ 0 \\ \omega_a^2\end{bmatrix}\delta_c$$

10) Page 64, change

$$\begin{bmatrix} \dot{e} \\ \ddot{A}_z \\ \ddot{q} \\ \ddot{\delta}_e \\ \delta_e \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & Z_\alpha & \mathrm{V}Z_\alpha & 0 & \mathrm{V}Z_\delta \\ 0 & M_\alpha/\mathrm{V}Z_\alpha & M_q & \left(M_\delta - \frac{M_\alpha Z_\delta}{Z_\alpha}\right) & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -{\omega_a}^2 & -2\zeta_a\omega_a \end{bmatrix} \begin{bmatrix} e \\ \dot{A}_z \\ \dot{q} \\ \dot{\delta}_e \\ \ddot{\delta}_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ {\omega_a}^2 \end{bmatrix} \dot{\delta}_\mathrm{c} \tag{3.53}$$

to

$$\begin{bmatrix} \dot{e} \\ \ddot{A}_z \\ \ddot{q} \\ \ddot{\delta}_e \\ \dddot{\delta}_e \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & \dfrac{Z_\alpha}{V} & Z_\alpha & 0 & Z_\delta \\ 0 & M_\alpha/Z_\alpha & M_q & \left(M_\delta - \frac{M_\alpha Z_\delta}{Z_\alpha}\right) & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -{\omega_a}^2 & -2\zeta_a\omega_a \end{bmatrix} \begin{bmatrix} e \\ \dot{A}_z \\ \dot{q} \\ \dot{\delta}_e \\ \ddot{\delta}_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ {\omega_a}^2 \end{bmatrix} \dot{\delta}_c$$

11) Page 65, change

$$z^T Q z = z^T \begin{bmatrix} q_{11} & & & & \\ & 0 & & 0 & \\ & & 0 & & \\ & 0 & & 0 & \\ & & & & 0 \end{bmatrix} \begin{bmatrix} e \\ \dot{q} \\ \dot{A}_z \\ \dot{\delta}_e \\ \ddot{\delta}_e \end{bmatrix}, \tag{3.57}$$

to

$$z^T Q z = z^T \begin{bmatrix} q_{11} & & & & \\ & 0 & & \underset{\sim}{0} & \\ & & 0 & & \\ & \underset{\sim}{0} & & 0 & \\ & & & & 0 \end{bmatrix} \begin{bmatrix} e \\ \dot{A}_z \\ \dot{q} \\ \dot{\delta}_e \\ \ddot{\delta}_e \end{bmatrix}$$

12) Page 67, change

The time domain performance metrics of interest here are 63% rise time, 95% settling time, percent overshoot, percent undershoot (because the system is nonminimum phase), max actuator deflection, and max actuator rate in response to a constant step command. The frequency domain performance metrics are loop gain crossover frequency ω_c in Hz, the minimum of the minimum singular value of the return difference dynamics, denoted $^{-}\sigma(I+L)$, and the minimum of the minimum singular value of the stability robustness matrix $I+L^{-1}$, denoted $\underline{\sigma}\left(I+L^{-1}\right)$. The metric $\underline{\sigma}(I+L)=1/\|S\|_\infty$ and $\underline{\sigma}\left(I+L^{-1}\right)=1/\|T\|_\infty$ (see Chap. 5, Sect. 5.2 for definitions). These metrics, plotted versus ω_c, are used to determine how the increasing bandwidth of the system affects the system characteristics, indicating a desired value for q_{11}.

to

$$\underline{\sigma}(I+L)$$

13) Page 67, change

Figure 3.7 shows two frequency response metrics: the minimum of the minimum singular value of the return difference dynamics $\underline{\sigma}(I+L)$ and the minimum of the minimum singular value of the stability robustness matrix $\underline{\sigma}\left(I+L^{-1}\right)$.

As is characteristic of LQR state feedback designs (discussed in Chap. 2), the $^{-}\sigma$ $(I+L)$ is equal to unity for all q_{11} design values. This metric is not particularly useful for developing state feedback designs but is critical when output feedback is used. The $\underline{\sigma}\left(I+L^{-1}\right)$, which is the inverse of the infinity norm of the complementary sensitivity function, is a measure of the damping in the dominant poles of the closed-loop system. We would like to maximize $\underline{\sigma}\left(I+L^{-1}\right)$. The figure shows that this metric tends to favor larger gains.

to (keep together on same line)

$$\underline{\sigma}(I+L)$$

Chapter 4

1) Page 75, change

$$\|u\|_1 = \int_{-\infty}^{\infty} |u(t)|dt$$
$$\|u\|_2 = \left(\int_{-\infty}^{\infty} |u(t)|dt \right)^{\frac{1}{2}}$$
$$\|u\|_\infty = \sup_t |u(t)| \tag{4.2}$$

to

$$\|u\|_1 = \int_{-\infty}^{\infty} |u(t)|dt$$
$$\|u\|_2 = \left(\int_{-\infty}^{\infty} |u(t)|^2 dt \right)^{\frac{1}{2}}$$
$$\|u\|_\infty = \sup_t |u(t)|$$

2) Page 77, change

$$\|G\|_2^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Big| G(j\omega)|^2 d\omega = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} G(-s)G(s)ds = \frac{1}{2\pi j} \oint G(-s)G(s)ds \tag{4.9}$$

to

$$\|G\|_2^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(j\omega)|^2 d\omega = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} G(-s)G(s)ds = \frac{1}{2\pi j} \oint G(-s)G(s)ds \tag{4.9}$$

3) Page 88, change

In this section, we build a control design model that embeds the sensitivity, complementary sensitivity, and control activity weighting filters from Sect. 4.3 into a state space model and then solves for the state feedback gain matrix (4.45) using a method called γ-iteration. The design model needs to be of the form of (4.24). Define the regulated variables in vector z to comprise sensitivity, complementary sensitivity, and control activity variables.

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

- z_1 ← Sensitivity– to track commands.
- z_2 ← Complementary Sensitivity – to roll-off plant, limit bandwidth.
- z_3 ← Control Activity – minimize control usage.

From Sect. 4.3, the weighting filter W_s should be designed to be the inverse of the desired loop shape for $S(s)$, the weighting filter W_T should be designed to be the inverse of the desired loop shape for $T(s)$, and the control activity penalty to penalize control activity in the desired frequency range. To build the H_∞-controller

to

section 4.4

4) Page 94, change

$$K_\infty = [-0.92788164.7534 \ - 13.1623 \ - 0.11795 \ - 3.210460.151197] \quad (4.68)$$

to

$$K_\infty = [-0.927881 \ 64.7534 \ \ -13.1623 \ \ -0.11795 \ \ -3.21046 \ 0.151197]$$

Chapter 5

1) Page 101 change

$$K(s) = [\, K_{A_z}(s) K_q(s) \quad K_q(s) \,] \quad (5.12)$$

which is a 2 × 1 matrix. A state-space model for this controller is

to

1 × 2

2) Page 102, change

$$
\begin{aligned}
u &= Ke \\
y &= GKe + w \\
z &= GKe + w + v \\
e &= r + z = r + GKe + w + v \\
E(s) &= S(s)(R(s) + W(s) + V(s)) \qquad (5.18)
\end{aligned}
$$

to

$$
\begin{aligned}
&u = Ke \\
&y = GKe + w \\
&z = GKe + w + v \\
&e = r - z = r - GKe - w - v \\
&E(s) = S(s)(R(s) - W(s) - V(s))
\end{aligned}
$$

3) Page 114, change

114 5 Frequency Domain Analysis

2. $\det[I + L(s,\varepsilon)] = 0$ *for all* (s,ε) *in* $D_R \times [0,1]$ *and for all* R *sufficiently large*. ■

to

$$\det[I + L(s,\varepsilon)] \neq 0$$

4) Page 115, add space between x and is

5.3.5 A + B Argument

The minimum singular value $\underline{\sigma}(A)$ measures the near singularity of the matrix A. Assume that the matrix $A+B$ is singular. If $A+B$ is singular then $A+B$ is rank deficient. Since $A+B$ is rank deficient, then there exists a vector $x \neq 0$ with unit magnitude $\left(\|x\|_2 = 1\right)$ such that $(A+B)x = 0$ ($x$is in the null space of $A+B$). This leads to $Ax = -Bx$ with $\|Ax\|_2 = \|Bx\|_2$. Using the above singular value definitions in (5.20) and $\|x\|_2 = 1$, we obtain the following inequality.

5) Page 116, change

From Theorem 5.5, stability is guaranteed if $\bar{\sigma}(I+L^{-1}(s)) > \bar{\sigma}(\Delta(s))$. For $\Delta(s) = E(s) - I$, $E(s) \in R^{n_u \times n_u}$, the singular values of $\Delta(s)$ are

to

$$\underline{\sigma}\left(I+L^{-1}(s)\right) > \overline{\sigma}(\Delta(s))$$

6) Page 119, change

For this single-input single-output system, we will compute the Nyquist, Bode, $\underline{\sigma}(I+L)$, and $^{-}\sigma(I+L^{-1})$ at the plant input, and $\bar{\sigma}(S)$ and $\bar{\sigma}(T)$ at the plant output. Figures 5.14, 5.15, 5.16, and 5.17 show the plant input frequency response curves (Nyquist, Bode, $\underline{\sigma}(I+L)$, and $\underline{\sigma}(I+L^{-1})$). On the Nyquist plot in Fig. 5.14, we have drawn a circle centered at $(-1, j0)$ that has radius equal to the minimum of $\underline{\sigma}(I+L)$ (from Fig. 5.16). The classical gain and phase margins from Fig. 5.14 are 8.8 dB (2.7536) and 50°. These are also easily extracted from the Bode plot in Fig. 5.15. From Figs. 5.16 and 5.17, we have

to

$$\underline{\sigma}\left(I+L^{-1}\right)$$

7) Page 119,

with the matrices defined in (5.14). The gains are $K_a = -0.0015$, $K_q = -0.32$, $a_q = 2.0$ and $a_z = 6.0$. Substituting these values into (5.14) yields

to

$a_z = 2.0$ and $a_q = 2.0$

8) Page 122, change

$$\begin{aligned} \mathrm{GM}_{I+L^{-1}} &= [0.4324 \quad 1.5676] \\ &= [-7.28 \quad 3.90]\ \mathrm{dB} \end{aligned};\quad \mathrm{PM}_{I+L^{-1}} = \pm 42.84^\circ \qquad (5.65)$$

to

$$\begin{aligned} GM_{I+L^{-1}} &= [0.2695 \quad 1.7305]; \qquad PM_{I+L^{-1}} = \pm 42.84\ \mathrm{deg} \\ &= [-11.4 \quad 4.7]\ \mathrm{dB} \end{aligned}$$

9) Page 122, change

$$\mathrm{GM} = [-7.28 \quad 7.28]\mathrm{dB};\ \mathrm{PM} = \pm 42.84^\circ \tag{5.66}$$

to

$$GM = [-11.4\ 7.28]\ \mathrm{dB};\ \ PM = \pm\ 42.84\ \mathrm{deg}$$

10) Page 124, change

Figure 5.21 shows the frequency response of the controller $\bar{\sigma}(K)$. This figure indicates the amplification, or attenuation, of sensor noise through the controller. Although not directly related to stability margins, this frequency response should e examined to make sure the bandwidth of the controller is not too high and that high-frequency noise is not adversely amplified. The shape of the frequency response clearly shows the proportional-plus-integral control action that the controller is providing.

to

should be examined

11) Page 127, change

For a control system under no uncertainty, the controller stabilizes the plant and the return difference matrix in nonsingular at all frequencies. Stability of the nominal system implies

$$\det[I + L(s)] \neq 0\, \forall s \in D_R. \tag{5.68}$$

to

is

12) Page 130, change

This triple describes the$M$matrix in the ΔM analysis model (Fig. 5.8).

to

add space in front and after M, and change 5.8 to 5.7

13) Page 133, change

$$B_M = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1.1422 & 0.4628 & 0 & 0 \\ 0 & 0 & -6.9073 & 10.2389 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad C_M$$

$$= \begin{bmatrix} 0 & -1.1422 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.4628 & 0 \\ 0 & -6.9073 & 0 & 0 & 0 \\ 0 & 0 & 0 & -10.2389 & 0 \end{bmatrix}$$

to

$$B_M = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1.1422 & 0.4628 & 0 & 0 \\ 0 & 0 & -6.9073 & 10.2389 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$C_M = \begin{bmatrix} 0 & -1.1422 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.4628 & 0 \\ 0 & -6.9073 & 0 & 0 & 0 \\ 0 & 0 & 0 & -10.2389 & 0 \end{bmatrix}$$

14) Page 136, change

$$A = \begin{bmatrix} -0.0251 & 0.10453 & -0.99452 \\ 574.70 & 0 & 0 \\ 16.2 & 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0.1228 & -0.27630 \\ -53.610 & 33.25 \\ 195.5 & -529.40 \end{bmatrix} \quad (0.1)$$

to

$$A = \begin{bmatrix} -0.0251 & 0.10453 & -0.99452 \\ 574.7 & 0 & 0 \\ 16.2 & 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0.1228 & -0.2763 \\ 195.5 & -529.4 \\ -53.61 & 33.25 \end{bmatrix} \quad (0.2)$$

15) Page 138, change

$$\begin{bmatrix} A_p & B_p \\ C_p & D_p \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -0.0251 & 0.10453 & -0.99452 \\ 574.70 & 0 & 0 \\ 16.2 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0.1228 & -0.27630 \\ -53.610 & 33.25 \\ 195.5 & -529.40 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix} \quad (5.109)$$

to

$$\begin{bmatrix} A_p & B_p \\ C_p & D_p \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -0.0251 & 0.10453 & -0.99452 \\ 574.70 & 0 & 0 \\ 16.2 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0.1228 & -0.27630 \\ -53.610 & 33.25 \\ 195.5 & -529.40 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$$

16) Page 158, change

the $\det[I + \mathrm{KH}]$, and indicate the number of encirclements.

(b) Plot the singular values of the return difference matrix and stability robustness matrix versus frequency. Compute the singular-value gain and phase margins for this system. This is a plot of $\underline{\sigma}[I + L]$ and $^{-}\sigma[I + L^{-1}]$ versus frequency. Plot these using a log scale for frequency and magnitude in dB.

Exercise 5.2. Consider the block diagrams shown below. Each block in the diagrams is a scalar.

to

$\underline{\sigma}$

Chapter 6

1) Page 164, change

$$\begin{aligned} N_0 &= X_{n_{y1}} \quad X_{n_{y2}}^{-1} \\ B_0 &= X_{p_2} - N_0 X_{p_1} \\ A_r &= A_{22} - N_0 A_{12} \end{aligned} \tag{6.15}$$

to

$$N_0 = X_{n_{y2}} X_{n_{y1}}^{-1}$$

2) Page 166, change

$$\begin{bmatrix} \dot{e} \\ \ddot{A}_z \\ \ddot{q} \\ \dot{\delta}_e \\ \ddot{\delta}_e \end{bmatrix}] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & Z_\alpha/V & Z_\alpha & 0 & Z_\delta \\ 0 & M_\alpha/Z_\alpha & M_q & \left(M_\delta - \frac{M_\alpha Z_\delta}{Z_\alpha}\right) & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -{\omega_a}^2 & -2\zeta_a\omega_a \end{bmatrix} \begin{bmatrix} e \\ \dot{A}_z \\ \dot{q} \\ \delta_e \\ \dot{\delta}_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ {\omega_a}^2 \end{bmatrix} \dot{\delta}_c \tag{6.24}$$

to: make bracket same size as other brackets.

3) Page 169, change

$$X_n = \left[\begin{bmatrix} 2.2295\text{e}-001 \\ -9.4676\text{e}-001 \\ -1.6075\text{e}-002 \\ -5.3104\text{e}-002 \\ 2.2551\text{e}-001 \end{bmatrix} \begin{bmatrix} -1.0379\text{e}-001 & -9.1764\text{e}-002j \\ 9.1946\text{e}-001 & \\ -1.4240\text{e}-003 & -1.0897\text{e}-002j \\ -4.3324\text{e}-002 & -3.3555\text{e}-002j \\ 3.6293\text{e}-001 & -2.3622\text{e}-002j \end{bmatrix} \right.$$
$$\left. \times \begin{bmatrix} -1.0379\text{e}-001 & -9.1764\text{e}-002j \\ 9.1946\text{e}-001 & \\ -1.4240\text{e}-003 & +1.0897\text{e}-002j \\ -4.3324\text{e}-002 & +3.3555\text{e}-002j \\ 3.6293\text{e}-001 & +2.3622\text{e}-002j \end{bmatrix} \right] \tag{6.34}$$

to

$$X_{n_y} = \left[\begin{bmatrix} 2.2295e-001 \\ -9.4676e-001 \\ -1.6075e-002 \\ -5.3104e-002 \\ 2.2551e-001 \end{bmatrix} \begin{bmatrix} -1.0379e-001-9.1764e-002j \\ 9.1946e-001 \\ -1.4240e-003-1.0897e-002j \\ -4.3324e-002-3.3555e-002j \\ 3.6293e-001-2.3622e-002j \end{bmatrix} \right.$$

$$\left. \times \begin{bmatrix} -1.0379e-001+9.1764e-002j \\ 9.1946e-001 \\ -1.4240e-003+1.0897e-002j \\ -4.3324e-002+3.3555e-002j \\ 3.6293e-001+2.3622e-002j \end{bmatrix} \right]$$

4) Page 173, change

The compensator design (6.12) requires selecting a gain matrix P_0 such that the residual dynamics A_r in (6.16) are stable. The matrices needed to form A_r are N_0 and B_0.

to

$$A_{re}$$

5) Page 173, change

$$N_0 = X_{n_{y1}} X_{n_{y2}}^{-1} = \begin{bmatrix} -1.6565e-001 & -5.8888e-002 & 4.4744e+000 \\ 2.4157e+000 & 6.3925e-001 & -1.8175e+001 \end{bmatrix} \quad (6.43)$$

to

$$N_0 = X_{n_{y2}} X_{n_{y1}}^{-1} = \begin{bmatrix} -1.6565e-001 & -5.8888e-002 & 4.4744e+000 \\ 2.4157e+000 & 6.3925e-001 & -1.8175e+001 \end{bmatrix}$$

6) Page 173, change

Using the dynamic compensator in (6.12) with matrices defined in (6.17), the compensator is designed by choosing the free parameter matrix P_0 such that the residual dynamics in (6.16) are stable. For this example (6.46),

to

delete (6.46)

7) Page 173, change

$$
\begin{aligned}
A_{re} &= A_r + B_0 P_0 A_{12} \\
&= \begin{bmatrix} 4.8883e+000 & 3.3521e-001 \\ -6.6917+003 & -9.0801e+001 \end{bmatrix} \\
&\quad + \begin{bmatrix} -2.5706e-003 & -2.6871e-003 \\ 9.3759e-001 & -7.0061e-002 \end{bmatrix} P_0 \begin{bmatrix} 0 & 0 \\ 0 & -11.29 \\ -1.093 & 0 \end{bmatrix}
\end{aligned} \tag{6.46}
$$

to

$-6.6917e + 003$

8) Page 174, change

By multiplying out the matrices in (6.47), one can determine which elements of P_0 need to be chosen. This matrix is designed using a tuning process in which the elements are increased in magnitude until a suitable design is obtained (trial and error). After some tuning, the following matrix was obtained:

$$
P_0 = \begin{bmatrix} 0 & 2 & -500 \\ 0 & 2 & -2000000 \end{bmatrix} \tag{6.47}
$$

The zero elements in the first column were found not to matter. They were made zero to reduce the control usage. Substituting this P_0 into (6.47) yields

to

(6.46)

8) Page 177, change

covariance, which results from solving the algebraic filter Riccati equation (covariance equation), and Q_0 and R_0 are the process and measurement noise covariances from (6.54), respectively. The optimal control is formed using the LQR state feedback control gain matrix K_c and the estimated state feedback $\hat{x}$, given as

$$
u = -K_c \hat{x} \tag{6.55}
$$

to

(6.53)

9) Page 177, change

$$u = -K_c\hat{x} \tag{6.55}$$

Figure 6.10 combines the LQR controller (Chap. 3) with the Kalman filter state estimator (6.55) into a block diagram. This is the LQG control architecture.

to

(6.54)

10) Page 178, change

$$Q_f = Q_0 + \frac{1}{\rho}BB^T \tag{6.58}$$

where Q_0 is the nominal plant process disturbance covariance from (6.54), B is the control input distribution matrix, and $\rho$is the LTR filter compensation parameter. This parameter is adjusted to recover the LQR frequency domain characteristics

to

(6.53)

11) Page 179, change

Considering the loop broken at the plant input, LTR modifies K_f to create a system that has stability properties that asymptotically approach those of the LQR. The method uses a trial and error procedure in which the filter design is parameterized by a scalar $\rho > 0$ such that when $\rho \to 0$ we have $L_{\text{LQG}} \to L_{\text{LQR}}$ asymptotically but not necessarily uniformly. It is evident that the location of the Kalman filter eigenvalues, (6.58), alters the closed-loop frequency characteristics of the system.

The LQG/LTR approach requires that the controlled system (plant) be minimum phase (i.e., no RHP transmission zeros). The minimum phase requirement occurs

to

(6.57)

12) Page 179, change

The LQG/LTR loop transfer function matrix at the plant input, L_{LQG}, will asymptotically recover the LQR frequency domain characteristics as $\rho \to 0$. This can be shown as follows. As $\rho \to 0$, the process covariance Q_f in (6.59) becomes largely dominated by the second term $\frac{1}{\rho}BB^T$. As these elements of Q_f get large, the covariance matrix P_f has elements that get large, resulting in the Kalman gain matrix K_f getting large with the following result:

to

(6.58)

13) Page 179, change

$$u = -K_c\left(sI - \tilde{A} + \tilde{B}K_c + K_f C\right)^{-1} K_f y \qquad (6.60)$$

Substituting for the measurement $y = Cx + v$ and letting $\rho \to 0$ as in (6.60) yields

to

(6.59)

14) Page 179, change

$$\begin{aligned} L_{\mathrm{LQG}}(s) &= K_c(sI - \tilde{A} + \tilde{B}K_c + K\longrightarrow_f C)^{-1} K\longleftarrow_f C(sI - \tilde{A})^{-1}\tilde{B} \\ L_{\mathrm{LQG}}(s) &\approx K_c(sI - \tilde{A})^{-1}\tilde{B} \end{aligned} \qquad (6.59)$$

to

$$\begin{aligned} L_{LQG}(s) &= K_c\left(sI - A + BK_c + K_f C\right)^{-1} K_f C(sI - A)^{-1}B \\ L_{LQG}(s) &\approx K_c(sI - A)^{-1}B \end{aligned}$$

15) Page 179, change

It is this process that inverts the plant (within the Kalman filter) resulting in recovering the LQR L_{LQR}. It is important to note that as $\rho \to 0$, $\bar{\sigma}(P_f) \to \infty$ and $^{-}\sigma(P_f) \to 0$, creating a singular covariance matrix. In the next section, we will present the LTR method of Lavretsky [6] which prevents this condition from occurring during the recovery process.

to

$$\underline{\sigma}(P_f) \to 0$$

16) Page 179, change

$$\begin{aligned} u &= -K_c(sI - \tilde{A} + \tilde{B}K_c + K_f C)^{-1} K_f (Cx + v) \\ &= -K_c(sI - \tilde{A} + \tilde{B}K_c + K^f C)^{-1} K_f Cx - K_c(sI - \tilde{A} + \tilde{B}K_c + K_f C)^{-1} K_f v \\ &= -K_c x - K_c(sI - \tilde{A} + \tilde{B}K_c + K_c F)^{-1} K_f v \end{aligned} \tag{6.61}$$

to

$$\begin{aligned} u &= -K_c\left(sI - \widetilde{A} + \widetilde{B}K_c + K_f C\right)^{-1} K_f (Cx + v) \\ &= -K_c\left(sI - \widetilde{A} + \widetilde{B}K_c + K_f C\right)^{-1} K_f Cx - K_c\left(sI - \widetilde{A} + \widetilde{B}K_c + K_f C\right)^{-1} K_f v \\ &\approx -K_c x - K_c\left(sI - \widetilde{A} + \widetilde{B}K_c + K_f C\right)^{-1} K_f v \end{aligned}$$

17) Page 180, change

in which the first term is inverted and canceled $(K_f C)^{-1} K_f C = I$ resulting in $-K_c x$. However, the second term is not exactly canceled; $(K_f C)^{-1} K_f \neq I$, and the sensor noise v can be amplified. This feature limits the amount of recovery possible. In the use of this design method for making the LQG system robust, the sensor noise amplification in (6.62) must be examined.

The LQG/LTR controller design, examining the loop properties at the plant input, may be realized through the following synthesis technique:

Step 1: LQR controller design: K_c
Follow the robust servomechanism design approach outlined in Chap. 3. Design LQR weighting matrices Q and R such that the resulting LTFM $L_{\text{LQR}}(s) = K_c (sI - \tilde{A})^{-1} \tilde{B}$ meets performance and stability robustness requirements and exhibits the desired bandwidth. The frequency domain properties of the LQG system will not exceed those of the LQR system.

Step 2: Kalman filter design: K_f Design the Kalman filter state estimator using (6.55), with (6.59) defining the plant disturbance covariance. The LTR filter recovery parameter ρ is used to recover the LQR frequency domain characteristics over the frequency range of interest. Examine plant input and output frequency domain criteria and the sensor noise amplification in (6.62) and limit the LTR recovery so that the sensor noise is not amplified.

to

(6.62) change to (6.51)
(6.55) change to (6.54)
(6.59) change to (6.58)
(6.62) change to (6.61)

18) Page 180, change the following text:

in which the first term is inverted and canceled $(K_f C)^{-1} K_f C = I$ resulting in $-K_c x$. However, the second term is not exactly canceled; $(K_f C)^{-1} K_f \neq I$, and the sensor noise v can be amplified. This feature limits the amount of recovery possible. In the use of this design method for making the LQG system robust, the sensor noise amplification in (6.62) must be examined.

to

in which the first term approximately equals the state feedback control law, but the second term amplifies the sensor noise. This feature limits the amount of recovery possible. In the use of this design method for making the LQG system robust, the sensor noise amplification in (6.62) must be examined.

19) Page 181, change

where the first gain in K_c multiplies the integral error, and the remaining gains multiply estimates of A_z, q, δ_e, and $\dot{\delta}_e$, respectively.

The measurements provided by an inertial measurement unit, A_{z_m} and q_m, are available for feedback. To design the Kalman filter state estimator, we need models of the process and measurement noise covariance matrices from (6.54). At this flight condition, the process noise modeled in the state equations is

to

(6.53)

20) Page 183, change

6.2 Linear Quadratic Gaussian with Loop Transfer Recovery

where u is formed using (6.65) and was implemented using steady-state matrices obtained from the filter covariance equation

to

(6.64)

21) Page 183, change

$$K_f = P_f C^T R_0{}^{-1}$$

$$P_f = \begin{bmatrix} 2.0442e-003 & -6.0760e-006 & 2.4929e-010 & -5.6592e-008 \\ -6.0760-006 & 7.8188e-008 & -1.2264e-012 & 1.3375e-010 \\ 2.4929e-010 & -1.2264e-012 & 1.2523e-010 & -5.0000e-009 \\ -5.6592e-008 & 1.3375e-010 & -5.0000e-009 & 3.4544e-007 \end{bmatrix}$$

$$K_f = \begin{bmatrix} 3.2707-002 & -6.0760e+000 \\ -9.7217e-005 & 7.8188e-002 \\ 3.9887e-009 & -1.2264e-006 \\ -9.0547e-007 & 1.3375e-004 \end{bmatrix} \quad (6.68)$$

The controller implementing the robust servomechanism integral control with the

to: need to add the "e" e-006, e-002 to these two numbers

22) Page 185, change

Note that (6.75) is valid for plant models with no D matrix, that is, $D_p = 0$. The first state x_1 is the robust servo integrator, the vector $\hat{x}$ is the estimated state, z_{meas} contains the acceleration and pitch rate measurements, and r is the acceleration command. Writing the controller in a generic form, we have

to

(6.74)

23) Page 185, change

$$A_c = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -1.0854e+000 & -3.4041e+002 & 0 & -1.1289e+001 \\ 0 & 6.8202e-003 & -1.1116e+000 & -1.0925e+000 & 0 \\ 0 & -3.9887e-009 & 1.2264e-006 & 0 & 1.0 \\ -3.3010e+003 & -1.1944e+003 & 9.3804e+004 & -2.1408e+004 & -1.1005e+002 \end{bmatrix}$$

$$B_{c_1} = \begin{bmatrix} 1.0 & 0 \\ 3.2707e-002 & -6.0760e+000 \\ -9.7217e-005 & 7.8188e-002 \\ 3.9887e-009 & -1.2264e-006 \\ -9.0547e-007 & 1.3375e-004 \end{bmatrix}; B_{c_2} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$D_{c_1} = [0 \quad 0];\; D_{c_2} = [0] \tag{6.76}$$

to

$$\begin{bmatrix} A_c \\ C_c \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -1.0854e+000 & 3.4041e+002 & 0 & -1.1289e+001 \\ 0 & 6.8202e-003 & -1.1116e+000 & -1.0925e+000 & 0 \\ 0 & -3.9887e-009 & 1.2264e-006 & 0 & 1.0 \\ -3.3010e+003 & -1.1944e+003 & 9.3804e+004 & -2.1408e+004 & -1.1005e+002 \end{bmatrix} \\ [-4.9477e-001 \quad -1.7903e-001 \quad 1.4060e+001 \quad -2.2087e+000 \quad -1.8035e-003] \end{bmatrix}$$

$$\begin{bmatrix} B_{c_1} & B_{c_2} \\ D_{c_1} & D_{c_2} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1.0 & 0 \\ 3.2707e-002 & -6.0760e+000 \\ -9.7217e-005 & 7.8188e-002 \\ 3.9887e-009 & -1.2264e-006 \\ -9.0547e-007 & 1.3375e-004 \end{bmatrix} & \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ [0 \quad 0] & [0] \end{bmatrix}$$

24) Page 186, change

Next, we will analyze the LQG/LTR design in the frequency domain and determine the desired amount of LTR to be applied at this flight condition. Figure 6.13 shows a Nyquist plot of the LQR, LQG, and LQG/LTR designs using values of ρ from (6.74). The red circle is a unit circle centered at (−1,j0) for reference. The LQR locus (blue) demonstrates infinite gain margin (at the plant

to

(6.73)

25) Page 189

To finalize a choice of ρ, the decision should be made on maximizing $\bar{\sigma}(I+L)$ and $\bar{\sigma}(I+L^{-1})$ at the plant input, minimizing $\bar{\sigma}(S)$ and $\bar{\sigma}(T)$ at the plant output, and preventing noise amplification over a frequency range of interest. The following table summarizes these peak values:

Design	$\underline{\sigma}(I+L)$	$\underline{\sigma}(I+L^{-1})$	$\bar{\sigma}(S)$	$\bar{\sigma}(T)$
LQR	1.0000	0.7963	1.4936	1.0480
LQR	0.5506	0.9808	1.0791	1.0000
$\rho = 10^5$	0.5233	0.7136	1.0923	1.0000
$\rho = 10^4$	0.5853	0.6567	1.0599	1.0000
$\rho = 10^3$	0.7920	0.7301	1.4581	1.0000
$\rho = 10^2$	0.9160	0.7715	2.9361	2.1570

From the $\bar{\sigma}(I+L)$ values, we need $\rho \leqslant 10^4$ to meet plant input stability margin requirements. We would like $\bar{\sigma}(I+L^{-1})$ to be as large as possible, which is also satisfied by $\rho \leqslant 10^4$. We would like $\bar{\sigma}(S)$ to be minimized, which points to $\rho = 10^4$ as the desired recovery level. If $\rho = 10^3$, the peak in $\bar{\sigma}(S)$ would be too large. Thus, $\rho = 10^4$ is selected as the design. For comparison, the following table lists the Kalman filter gains:

to

$\underline{\sigma}$

26) Page 190, change

Definition 6.2. *The transfer function $G(s)$ is called strictly positive real if $G(s-\varepsilon)$ is positive real, for some $\varepsilon > 0$.*

For scalar systems $(p = 1)$, PR and SPR dynamics have their Nyquist frequency response locus located entirely in the right half complex plane. This condition for $G(s)$ can be satisfied only if the system's relative degree is zero or one. Thus, encirclements of $(-1, j0)$ cannot occur. In other words, such a system will remain stable under a large set of uncertainties, which is a highly desirable property for any system to possess.

to: This text should not be italic

27) Page 191, change

Clearly, if D is the zero matrix, then the SPR conditions (6.81) reduce to

$$\begin{aligned} PA + A^T P &= -L^T L - \varepsilon P \\ PB &= C^T \end{aligned} \tag{6.81}$$

and in this case, setting $\varepsilon = 0$, gives the PR conditions in the form

$$\begin{aligned} PA + A^T P &= -L^T L \\ PB &= C^T \end{aligned} \tag{6.82}$$

to: (6.80), and the text in the boxes should not be italic.

28) Page 192

The first relation in (6.83) is the algebraic Lyapunov equation, and $V(x) = x^T P x$ is the Lyapunov function [4]. The second relation in (6.83) enables output feedback control design, whereby the system output $y = Cx$ can be fed back into the input to control the system, while preserving closed-loop stability. Also, note that the matrices B and C define the transmission zeros of the system transfer function matrix $G(s) = C(sI - A)^{-1}B$.

We are going to modify the LQG/LTR design such that, for a class of restricted systems, the PR property is obtained asymptotically, $P_v B_v \to C^T$, with the positive tuning parameter $v \to 0$. In addition, we shall ensure that P_v remains symmetric and strictly positive definite, uniformly in v. These are the distinguishing features of LTRLM design. Similar to the previous section, in this design, the Kalman filter is no longer treated as a filter. It will continue to estimate the system state and serve as a dynamic compensator, tuned to improve the frequency domain properties of the system. The Gaussian covariance matrices for w and v are altered significantly to improve the controller robustness and to limit sensor noise amplification. So, these matrices no longer "model" the stochastic processes of the system.

We formulate the LTRLM design approach using the linear-time-invariant Gaussian design model,

$$\begin{aligned} \dot{x} &= Ax + Bu + w \\ y &= Cx + v \end{aligned} \tag{6.83}$$

where w and v are zero mean, white, uncorrelated Gaussian random processes with covariances given by

$$\begin{aligned} E\{w(t)w^T(\tau)\} &= Q_0\delta(t-\tau) \\ E\{v(t)v^T(\tau)\} &= R_0\delta(t-\tau) \end{aligned} \tag{6.84}$$

The state estimate $\hat{x}$ is formed as before, using the state estimator,

$$\dot{\hat{x}} = A\hat{x} + Bu + K_f(y_{\text{meas}} - \hat{y}) \tag{6.85}$$

and the control input is calculated using the LQR state feedback gain matrix K_c, with the estimated state feedback $\hat{x}$.

$$u = -K_c\hat{x} \tag{6.86}$$

In LTRLM, we parameterize the process and measurement noise covariance matrices using a positive scalar v,

to: Change (6.83) to (6.82). The text on this page should not be italic.

29) Page 193, change

6.3 Loop Transfer Recovery Using the Lavretsky Method 193

$$Q_v = Q_0 + \left(\frac{v+1}{v}\right)\bar{B}\bar{B}^T, \quad R_v = \frac{v}{v+1}R_0 \tag{6.87}$$

where $\bar{B}$ *is a matrix formed by adding "fictitious" columns to* B*, to make* $\bar{B} = [B \quad X]$ *have its column rank equal to the row rank of* C*, such that* $C\bar{B}$ *becomes invertible and the corresponding extended system* $C(sI-A)^{-1}\bar{B}$ *is minimum phase, that is, all its transmission zeros are located in the left half complex plane. This is the "squaring-up" step of the method. Substituting the weights from (6.88) into the filter Riccati equation, we get*

$$P_v A^T + AP_v - \left(1+\frac{1}{v}\right)P_v C^T R_0^{-1} C P_v + Q_0 + \left(1+\frac{1}{v}\right)\bar{B}\bar{B}^T = 0 \tag{6.88}$$

or, equivalently

$$P_v A^T + AP_v - P_v C^T R_0^{-1} C P_v + Q_0 + \bar{B}\bar{B}^T + \frac{1}{v}\left[\bar{B}\bar{B}^T - P_v C^T R_0^{-1} C P_v\right] = 0 \tag{6.89}$$

The gains in (6.86) are computed as

$$K_f = P_v C^T R_v^{-1} \tag{6.90}$$

Now as $v \to 0$*, one can show that the filter covariance matrix* P_v *asymptotically approaches a constant symmetric positive definite matrix* P_0*, that is,*

$$P_0 = \lim_{v\to 0} P_v = \lim_{v\to 0} P_v^T = P_0^T > 0 \tag{6.91}$$

This behavior is in contrast to the previous section, whereas the LTR parameter $\rho \to 0$, $\bar{\sigma}(P_f) \to \infty$, $\underline{\sigma}(P_f) \to 0$*, and the* P_f *matrix became singular.*

The important properties of P_0 *in (6.92) are listed below without proof (see Chap. 13, Theorem 13.1 for formal derivations):*

- P_0 *is the unique symmetric* strictly *positive definite solution of the following algebraic Lyapunov equation*

$$P_0\left(A - C^T R_0^{-1} C P_1\right)^T + \left(A - C^T R_0^{-1} C P_1\right)P_0 + Q_0 = 0 \tag{6.92}$$

to: (6.88) to (6.87), (6.86) to (6.85), (6.92) to (6.91). This page should not be italic.

30) Page 194, change

194 6 Output Feedback Control

- *There exists a unitary matrix* $W \in R^{m \times m}$ *such that*

$$P_0 C^T = \bar{B} W^T R_0^{\frac{1}{2}} \tag{6.93}$$

- *The unitary matrix W in (6.94) can be chosen as*

$$W = (UV)^T \tag{6.94}$$

where U and V are two unitary matrices defined by the singular value decomposition,

$$\bar{B}^T C^T R_0^{-\frac{1}{2}} = U \Sigma V \tag{6.95}$$

and Σ *represents the diagonal matrix of the corresponding singular values.*

For minimum phase systems, the SPR property is implied by (6.94). What the LTRLM design is trying to do is to shape the transmission zeros of the state estimator, such that the original system with the extended input becomes SPR asymptotically, as $v \to 0$. To do this, we "square-up the system" by adding extra columns to B (to form $\bar{B}$) and then apply the LTR tuning process, whereby we decrease the tuning parameter v in (6.88), until the system becomes almost SPR.

It was discussed earlier in Chap. 2 that in the LQR design problem, with the penalty matrix Q factored as $Q = Q^{\frac{T}{2}} Q^{\frac{1}{2}}$, the poles of the closed-loop system, $\lambda(A - BK_c)$, would approach the transmission zeros defined by $Q^{\frac{1}{2}}(sI - A)^{-1}B$ asymptotically as the gains grew large. If no finite transmission zeros existed, the roots would form a Butterworth pattern (or combinations of Butterworth patterns) in the left half complex plane. Thus, by the proper selection of Q, the designer places these zeros to achieve the desired response of the system. So, the selection of the LQR penalty matrix is a key tuning mechanism in the LQR controller design.

This same basic idea is in work under LTRLM. For the state estimator (aka Kalman filter), the process covariance Q_f is the equivalent to the LQR penalty matrix. Factoring the process covariance Q_f as $Q_f = L^T L$, the eigenvalues of the Kalman filter, $\lambda(A - K_f C)$, will approach the finite transmission zeros defined by $C(sI - A)^{-1}L$. Thus, the selection of the process covariance Q_f is an ideal tuning mechanism in the design of the LTRLM controller. Placing the zeros of the system in a desirable location is the key to achieving a robust design. This is achieved through the modified process covariance and measurement noise matrices in (6.88).

to: (6.94) to (6.93), (6.94) to (6.93), (6.88) to (6.87), (6.88) to (6.87). The text in the box should not be italic.

31) Page 196, change

Step 1: LQR controller design: K_c

Follow the robust servomechanism design approach outlined in Chap. 3. Design LQR weighting matrices Q and R, such that the resulting loop gain $L_{\mathrm{LQR}}(s) = K_c\left(sI - \tilde{A}\right)^{-1}\tilde{B}$ meets performance and stability robustness requirements and exhibits the desired bandwidth.

Step 2: State estimator/Kalman filter design: K_f

Select columns X to make $\bar{B} = [B \quad X]$ have column rank equal to the row rank of C and to make the extended system minimum phase. Design the Kalman filter/state estimator using (6.89), with (6.88) defining the plant process and measurement noise covariance matrices. The LTR parameter v is used to recover the LQR frequency domain characteristics over the frequency range of interest. Ad hoc adjustment of the sensor noise covariance magnitude may be needed to scale the Kalman gains to prevent large gains from occurring. Examine plant input and output frequency domain criteria and the sensor noise amplification in and limit the LTR recovery so that the sensor noise is not amplified.

to:), (6.89) to (6.88), (6.88) to (6.87).

32) Page 197, change

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_{c1} y + B_{c2} r \\ u &= C_c x_c + D_{c1} y + D_{c2} r \end{aligned} \tag{6.102}$$

Using the gains from (6.101), the state feedback controller is

$$\begin{aligned} \dot{x}_c &= [0]x_c + [1 \quad 0]y + [-1]r \\ u &= [-0.31623]x_c + [33.261 \quad 6.7127]y + [0]r \end{aligned} \tag{6.103}$$

to: (6.101) to (6.100)

33) Page 198, change

$$\dot{\hat{x}} = A\hat{x} + Bu + K_f(y - \hat{y}) \tag{6.106}$$

where u is formed using (6.102) and was implemented using steady-state matrices obtained from the filter covariance equation

$$\begin{aligned} 0 &= AP_f + P_f A^T + Q_v - P_f C^T {R_v}^{-1} C P_f \\ K_f &= P_f C^T {R_v}^{-1} \end{aligned} \tag{6.107}$$

The steady-state covariance and Kalman filter gains design (using Q_0 and R_0) are

$$\begin{aligned} P_f &= \begin{bmatrix} 9.5843\text{e}-006 & 3.8344\text{e}-007 \\ 3.8344\text{e}-007 & 4.8957\text{e}-005 \end{bmatrix} \\ K_f &= \begin{bmatrix} -0.053132 & 0.38344 \\ -0.0021257 & 48.957 \end{bmatrix} \end{aligned} \tag{6.108}$$

To analyze this observer-based design (Kalman filter), we will implement the controller in our standard model (6.103). For the LQG controller, the RSLQR control law is given by

$$\begin{aligned} u &= -K_1 \int e - K_x \hat{x} \\ e &= y_c - r = A_z - A_{z_{cmd}} \end{aligned} \tag{6.109}$$

where $\hat{x}$ is the estimated state, and the RSLQR gain matrix is partitioned as $K_c = [K_1 \quad K_x]$. To form the estimated state, we need to substitute the control (6.110) into the state estimator (6.107). Doing so gives

to: (6.102) to (6.101), (6.103) to (6.102), (6.110) to (6.109), (6.107) to (6.106)

34) Page 199, change

The LQG controller states are $x_c = \left[\int e \quad \hat{x}\right]^T$. The controller state space model using (6.110) and (6.111) is

$$\begin{aligned} \begin{bmatrix} e \\ \dot{\hat{x}} \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \int e \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ & K_f \end{bmatrix} y_{meas} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} r \\ u &= -\begin{bmatrix} K_1 & K_x \end{bmatrix} \begin{bmatrix} \int e \\ \hat{x} \end{bmatrix} + [0] y_{meas} + [0] r \end{aligned} \tag{6.111}$$

where A_{21} and A_{22} are defined as in (6.111). Substituting the gains into (6.112), we have to: (6.110) to (6.109), (6.111) to (6.110), (6.111) to (6.110), (6.112) to (6.111)

35) Page 200, change

Note the magnitude increase in the gain $K_f(2,2)$.

The last controller in this example uses the LTRLM. The first step in the design process is to design the LQR control law. We will use the RSLQR controller from (6.102). The second step is to select columns X to make $\bar{B} = [B_p \quad X]$ have column rank equal to the row rank of C_p. To complete this design, we must look at the numbers within these matrices:

to: (6.102) to (6.101),

36) Page 201, change

Solving for the steady-state covariance and gain matrix from (6.108) yields

$$P_f = \begin{bmatrix} 0.011312 & -3.5561\text{e}-005 \\ -3.5561\text{e}-005 & 0.018303 \end{bmatrix}$$
$$K_f = \begin{bmatrix} -0.35116 & -0.19914 \\ 0.001104 & 102.5 \end{bmatrix} \quad (6.121)$$

to: (6.108) to (6.107)

37) Page 201, change

The controller is formed by substituting the gains K_f into (6.112) and results in

$$\begin{bmatrix} e \\ \hat{\dot{x}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -19.48 & 0.16584 \\ 0.36948 & -42.794 & -246.8 \end{bmatrix} \begin{bmatrix} \int e \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -0.053183 & 0.83416 \\ -0.0043243 & 237.93 \end{bmatrix} y_{meas} + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} r$$
$$u = [-0.31623 \; 33.261 \; 6.7127] \begin{bmatrix} \int e \\ \hat{x} \end{bmatrix} + [0 \; 0] y_{\text{meas}} + [0] r \quad (6.122)$$

to: (6.112) to (6.111)

38) Page 201, change

Solving for the steady-state covariance and gain matrices from (6.108) yields

$$P_f = \begin{bmatrix} 0.00017018 & 0.0017429 \\ 0.0017429 & 0.15898 \end{bmatrix}$$
$$K_f = \begin{bmatrix} -0.0052832 & 9.7605 \\ -0.054109 & 890.31 \end{bmatrix} \quad (6.124)$$

The controller is formed by substituting the gains K_f into (6.112) and results in

to: (6.108) to (6.107), (6.112) to (6.111)

39) Page 205, change

Exercise 6.1. Consider the unstable longitudinal dynamics model, as defined in Example 6.1, where $x = \begin{bmatrix} \alpha & q & \delta_e & \dot{\delta}_e \end{bmatrix}^T$. The matrices for the control design model $\dot{x} = A_p x + B_p u$ are

$$[A_p \quad B_p] = \left[\begin{bmatrix} -1.3046\text{e} & 1.0 & -0.2.1420 & 0 \\ 47.711 & 0 & -104.83 & 0 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & -12769. & -135.6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 12769 \end{bmatrix} \right]$$

to: remove the "e"

40) Page 206, change

(c) Compute the eigenstructure for (a) and (b) to show that the dominant eigenvalues are retained. Analyze this design in the frequency domain. Compute Nyquist, Bode, $^{-}\sigma[I + L]$, $^{-}\sigma[I + L^{-1}]$ frequency responses for a) and b) at the plant input. Compute $\bar{\sigma}[S]$ and $\bar{\sigma}[T]$ frequency responses for a) and b) at the plant output for the α loop. Compute the loop gain crossover frequency and singular value stability margins for the design.

to

$\underline{\sigma}$

41) Page 206 change

singular value stability margins for both designs. Determine the impact of using the Kalman filter estimator on the stability robustness of the system.

(d) Use the LTR method of Sect. 6.2 (6.59) to recover the frequency domain properties of the state feedback design in the LQG design. Evaluate the design in the frequency domain as in (c). Compute the maximum singular value of the noise-to-control transfer function matrix frequency response to examine the noise amplification in the resulting LQG/LTR design.

to: (6.59) to (6.58)

42) Page 207

(b) Simulate the LQG design and compare it to the state feedback design.

(c) Analyze this LQG design in the frequency domain. Compute Nyquist, Bode, $^{-}\sigma[I + L]$, $^{-}\sigma[I + L^{-1}]$ frequency responses for the LQG and state feedback at the plant input. Compute $\bar{\sigma}[S]$ and $\bar{\sigma}[T]$ frequency responses for (a) and (b) at the plant output for the α loop. Compute the loop gain crossover frequency and singular value stability margins for both designs. Determine the impact of using the Kalman filter estimator on the stability robustness of the system.

(d) Use the Loop Transfer Recovery method of Lavretsky, Sect. 6.2 (6.88), to recover the frequency domain properties of the state feedback design in the LQG design. Evaluate the design in the frequency domain as in (c). Compute the maximum singular value of the noise-to-control transfer function matrix frequency response to examine the noise amplification in the resulting LQG/LTR design.

to

$\underline{\sigma}$, (6.88) to (6.87)

ERRATA LIST

Robust and Adaptive Control with Aerospace Applications –
1st Edition, Springer, 2013
Eugene Lavretsky

Revised on 27 Jul 2013

Part II

Chapter 7

1. Page 222: Exercise 7.3: Change “(7.13), (7.14), (7.15), (7.16), (7.17), (7.18), (7.19), (7.20), (7.21), (7.22), and (7.23).” to “(7.13) through (7.23).”
2. Page 239: Example 8.9: Change $6\sin t - 6t\cos t - t^2 \le 6 + \underbrace{\left(t - t^2\right)}_{\le \frac{1}{4}} \le 6.25$ to

   $$6\sin t - 6t\cos t - t^2 \le 6 + \underbrace{\left(6t - t^2\right)}_{\le 9} \le 15$$

3. Page 239: Example 8.9: Change

   $$|x(t)| \le |x(t_0)|\underbrace{\exp\left(6.25 - 6\sin t_0 + 6t_0\cos t_0 + t_0^2\right)}_{c(t_0)} = |x(t_0)|\,c(t_0) \text{ to}$$

   $$|x(t)| \le |x(t_0)|\underbrace{\exp\left(15 - 6\sin t_0 + 6t_0\cos t_0 + t_0^2\right)}_{c(t_0)} = |x(t_0)|\,c(t_0)$$

Chapter 8

1. Page 261: Example 8.10: Change $\dot{x}_{ref} = Ax_{ref} + br$ to $\dot{x}_{ref} = Ax_{ref} + bK_r r$
2. Page 258: Change sentence

   “regressor vector”, which is assumed to be uniformly bounded.

   to
   ”regressor vector”, which is assumed to be uniformly bounded and Lipschitz-continuous in t.

3. Page 258: Change $V(e,\Delta K) = e^T P e + \Delta K^T \Delta K$ to $V(e,\Delta K) = e^T P e + \frac{1}{\gamma}\Delta K^T \Delta K$.
4. Page 258: Change the next equation for $\dot{V}e, \Delta K = ...$ to

$$\dot{V}(e,\Delta K) = \dot{e}^T P e + e^T P \dot{e} + 2\frac{1}{\gamma}\Delta K^T \dot{\hat{K}} = \left(A e + b\,\Delta K^T \Phi\right)^T$$
$$P e + e^T P\left(A e + b\,\Delta K^T \Phi\right) - 2\Delta K^T \Phi e^T P b = -e^T Q e \leq 0$$

5. Page 261: Change equation $\dot{x}_{ref} = A x_{ref} + b\,r$ to $\dot{x}_{ref} = A x_{ref} + b\,K_r r$.

Chapter 9

1. Page 282: Eq. (9.47): Right-justify the equation number (i.e., move it to the right).

Chapter 10

1. Page 311: Table 2.10: Change the third equation from the bottom to: $u = \left(I_{m\times m} - \hat{K}_u^T\right) u_{bl} - \hat{\Theta}^T \Phi\left(x_p\right)$.
   This equation in the current book version has an incomplete sub-index in I.
2. Page 314: Exercise 10.1: In the problem statement, replace "have" with "has".
3. Page 314: Exercise 10.2: Add the minus sign to the last equation in the problem statement: $\hat{K}_x(0) = -K_x$.

Chapter 11

1. Page 325: Replace text and equations starting from Eq. 11.26 through Eq. 11.29 with:

Using completion of squares, the sum of the first and the second terms in (11.25) can be transformed into

$$
\begin{aligned}
&-\lambda_{\min}(Q)\|e\|^2 + 2\|e\|\lambda_{\max}(P)\xi_{\max} \\
&= -\lambda_{\min}(Q)\left(\|e\| - \frac{\lambda_{\max}(P)\xi_{\max}}{\lambda_{\min}(Q)}\right)^2 + \frac{\lambda_{\max}^2(P)\xi_{\max}^2}{\lambda_{\min}(Q)}
\end{aligned}
$$

Similarly, the sum of the third and the fourth terms in (11.25) can be written as

$$
\begin{aligned}
&-2\sigma\|\Delta\Theta\|_F^2\Lambda_{\min} + 2\sigma\|\Delta\Theta\|_F\|\Theta\|_F\|\Lambda\|_F \\
&= -2\sigma\Lambda_{\min}\left(\|\Delta\Theta\|_F - \frac{1}{2}\|\Theta\|_F\frac{\|\Lambda\|_F}{\Lambda_{\min}}\right)^2 + \sigma\frac{\|\Theta\|_F^2\|\Lambda\|_F^2}{2\Lambda_{\min}}
\end{aligned}
$$

Substituting these two expressions back into (11.25), gives

$$
\begin{aligned}
\dot{V}(e,\Delta\Theta) \le & -\lambda_{\min}(Q)\left(\|e\| - \frac{\lambda_{\max}(P)\xi_{\max}}{\lambda_{\min}(Q)}\right)^2 + \frac{\lambda_{\max}^2(P)\xi_{\max}^2}{\lambda_{\min}(Q)} \\
& -2\sigma\Lambda_{\min}\left(\|\Delta\Theta\|_F - \frac{1}{2}\|\Theta\|_F\frac{\|\Lambda\|_F}{\Lambda_{\min}}\right)^2 + \sigma\frac{\|\Theta\|_F^2\|\Lambda\|_F^2}{2\Lambda_{\min}}
\end{aligned}
\tag{11.26}
$$

Hence, $\dot{V}(e,\Delta\Theta) < 0$ if at least one of the following two relations take place:

$$
\begin{gathered}
\lambda_{\min}(Q)\left(\|e\| - \frac{\lambda_{\max}(P)\xi_{\max}}{\lambda_{\min}(Q)}\right)^2 - \frac{\lambda_{\max}^2(P)\xi_{\max}^2}{\lambda_{\min}(Q)} - \sigma\frac{\|\Theta\|_F^2\|\Lambda\|_F^2}{2\Lambda_{\min}} > 0 \\
\text{OR} \\
2\sigma\Lambda_{\min}\left(\|\Delta\Theta\|_F - \frac{1}{2}\|\Theta\|_F\frac{\|\Lambda\|_F}{\Lambda_{\min}}\right)^2 - \frac{\lambda_{\max}^2(P)\xi_{\max}^2}{\lambda_{\min}(Q)} - \sigma\frac{\|\Theta\|_F^2\|\Lambda\|_F^2}{2\Lambda_{\min}} > 0
\end{gathered}
\tag{11.27}
$$

or equivalently, when

$$\|e\| > \sqrt{\frac{1}{\lambda_{\min}(Q)}\left(\frac{\lambda_{\max}^2(P)\xi_{\max}^2}{\lambda_{\min}(Q)} + \sigma\frac{\|\Theta\|_F^2\|\Lambda\|_F^2}{2\Lambda_{\min}}\right)} + \frac{\lambda_{\max}(P)\xi_{\max}}{\lambda_{\min}(Q)} = c_1$$

$$\text{OR} \qquad (11.28)$$

$$\|\Delta\Theta\|_F > \sqrt{\frac{1}{2\sigma\Lambda_{\min}}\left(\frac{\lambda_{\max}^2(P)\xi_{\max}^2}{\lambda_{\min}(Q)} + \sigma\frac{\|\Theta\|_F^2\|\Lambda\|_F^2}{2\Lambda_{\min}}\right)} + \frac{1}{2}\|\Theta\|_F\frac{\|\Lambda\|_F}{\Lambda_{\min}} = c_2$$

In other words, $\dot{V}(e, \Delta\Theta) < 0$ outside of the compact (closed and bounded) set $\Omega \subset \left(R^n \times R^{N\times m}\right)$ defined below.

$$\Omega = \left\{(e, \Delta\Theta) : \left(\|e\| \le c_1\right) \wedge \left(\|\Delta\Theta\|_F \le c_2\right)\right\} \qquad (11.29)$$

2. Page 327: Change section number from “11.3” to subsection # “11.2.3”
3. Page 328: Replace the first sentence and the second sentences with the sentence shown below in yellow:

$\sigma\left\|e^T PB\right\|$. This fact allows to arrive at a compact set [4], outside of which $\dot{V}(e, \Delta\Theta) < 0$. Once again, we can claim UUB of all trajectories. This completes the stability analysis for the $e-$modification with a guaranteed UUB-type output tracking performance.

4. Page 329: Change Eq. (11.59) to:

$$\nabla f_j = \frac{(1+\varepsilon_j^{\Theta})}{\varepsilon_j^{\Theta}(\Theta_j^{\max})^2}\nabla\left[\|\widehat{\Theta}_j \wedge\|^2\right] = \frac{2(1+\varepsilon_j^{\Theta})}{\varepsilon_j^{\Theta}(\Theta_j^{\max})^2}\widehat{\Theta}_j.$$

Chapter 12

1. Page 364: Replace δ with δR in Fig 12.5, as shown below:

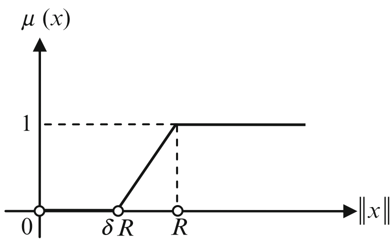


Fig. 12.5 State modulation function

2. Page 372: Change "Example 12.3" to "Example 12.1"
3. Page 373: In the sentence "In addition, we impose a restriction on the runway distance …" : Change τ_{fl} to $4\tau_h$.
4. Page 374: Change the second equation as shown below (change B(2,2) to 0.00044):

$$\underbrace{\begin{pmatrix} \dot{V} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{h} \end{pmatrix}}_{\dot{x}} = \underbrace{\begin{pmatrix} -0.038 & 18.984 & 0 & -32.174 & 0 \\ -0.001 & -0.632 & 1 & 0 & 0 \\ 0 & -0.759 & -0.518 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -250 & 0 & 250 & 0 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} V \\ \alpha \\ q \\ \theta \\ h \end{pmatrix}}_{x}$$

$$+ \underbrace{\begin{pmatrix} 10.1 & 0 \\ 0 & 0.00044 \\ 0.025 & -0.011 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} \delta_{th} \\ \delta_e \end{pmatrix}}_{u} \Leftrightarrow \boxed{\dot{x} = Ax + Bu}$$

5. Page 375: Change equation at the top of the page as shown below (change B(2,2) to 0.00044):

$$\underbrace{\begin{pmatrix} \dot{V} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{h} \end{pmatrix}}_{\dot{x}} = \underbrace{\begin{pmatrix} -0.038 & 18.984 & 0 & -32.174 & 0 \\ -0.001 & -0.632 & 1 & 0 & 0 \\ 0 & -0.759 & -0.518 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -250 & 0 & 250 & 0 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} V \\ \alpha \\ q \\ \theta \\ h \end{pmatrix}}_{x}$$

$$+ \underbrace{\begin{pmatrix} 10.1 & 0 \\ 0 & 0.00044 \\ 0.025 & -0.011 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} \delta_{th} \\ \delta_e \end{pmatrix}}_{u} + \underbrace{\begin{pmatrix} -18.984 \\ 0.632 \\ 0.759 \\ 0 \\ 0 \end{pmatrix}}_{B_g} \alpha_g(h)$$

6. Page 376: Change 4^{th} from the bottom equation as shown below:

$$Q_{lqr} = \text{diag}(\,0.2 \quad 0 \quad 0 \quad 0 \quad 1\,), \quad R_{lqr} = \text{diag}(\,10 \quad 10\,)$$

Chapter 13

1. Page 406: Insert space between "y_{cmd}" and "y" in **Eq. 13.87**. There are two equations there, not one, as shown below.

$$\underbrace{\begin{pmatrix} \dot{e}_{yI} \\ \dot{x}_p \end{pmatrix}}_{\dot{x}} = \underbrace{\begin{pmatrix} 0_{m\times m} & C_p \\ 0_{n_p\times m} & A_p \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} e_{yI} \\ x_p \end{pmatrix}}_{x} + \underbrace{\begin{pmatrix} 0_{m\times m} \\ B_p \end{pmatrix}}_{B} \Lambda \left(u + \overbrace{\Theta_d^T \Phi_d(x_p)}^{d(x_p)} \right)$$

$$+ \underbrace{\begin{pmatrix} -I_{m\times m} \\ 0_{n_p\times m} \end{pmatrix}}_{B_{ref}} y_{cmd} \; y = \underbrace{\begin{pmatrix} 0_{m\times m} & C_p \end{pmatrix}}_{C} x \tag{13.87}$$

2. Page 409: Move superscript "T" in Eq. 13.99 to the right and outside of parenthesis, as in:

$$\dot{x} = A_{ref}\, x - B\, \Lambda \underbrace{\left(\hat{\Theta} - \Theta\right)}_{\Delta\Theta}^{T} \Phi(x) + B_{ref}\, y_{cmd} \tag{13.99}$$

3. Page 416: Exercise 13.4: Change "Example 13.3" to "Example 10.2".

Chapter 14

1. Page 429: **Eq. 14.46**: Change to:

$$\dot{V}\left(e_x, \Delta\overline{\Theta}\right) = -\left(1 + \frac{1}{v}\right) e_y^T R_0^{-1} e_y - e_x^T \widetilde{P}_v Q_0 \widetilde{P}_v e_x$$

$$-\left(1 + \frac{1}{v}\right) \left\| B^T \widetilde{P}_v e_x \right\|^2 - 2\eta\, e_x^T \widetilde{P}_v e_x + 2\, e_x^T \widetilde{P}_v B \Lambda g$$

$$+2\,\mathrm{trace}\left(\Lambda \Delta\overline{\Theta}^T \left\{ \Gamma_\Theta^{-1} \dot{\hat{\overline{\Theta}}} + \overline{\Phi}(\hat{x}, u_{bl}) \underbrace{\left(e_x^T C^T\right)}_{e_y^T} R_0^{-\frac{1}{2}} W S^T \right\} \right)$$

$$+2\, e_x^T\, \mathrm{O}(v)\, \Lambda \Delta\overline{\Theta}^T\, \overline{\Phi}(\hat{x}, u_{bl})$$

2. Page 430: **Eq. 14.48**: Change to:

$$\dot{V}\left(e_x, \Delta\overline{\Theta}\right) \le -2\eta\lambda_{\min}\left(\widetilde{P}_v\right)\|e_x\|^2 - \lambda_{\min}(Q_0)\lambda_{\min}^2\left(\widetilde{P}_v\right)\|e_x\|^2$$
$$-\left(1+\frac{1}{v}\right)\lambda_{\min}\left(R_0^{-1}\right)\left\|e_y\right\|^2 - \left(1+\frac{1}{v}\right)\left\|B^T\widetilde{P}_v e_x\right\|^2 + 2\Lambda_{\max}k_g\left\|B^T\widetilde{P}_v e_x\right\|\|e_x\|$$
$$+2v\|e_x\|k\Lambda_{\max}\Delta\overline{\Theta}_{\max}\left\|\overline{\Phi}(\hat{x}, u_{bl})\right\|$$

3. Page 430: **Eq. 14.49**: Change to:

$$\dot{V}\left(e_x, \Delta\overline{\Theta}\right) \le -\left(2\eta + \lambda_{\min}(Q_0)\lambda_{\min}\left(\widetilde{P}_v\right)\right)\lambda_{\min}\left(\widetilde{P}_v\right)\|e_x\|^2$$
$$-\left(1+\frac{1}{v}\right)\lambda_{\min}\left(R_0^{-1}\right)\left\|e_y\right\|^2 - \left(1+\frac{1}{v}\right)w^2 + 2\Lambda_{\max}k_g w\|e_x\|$$
$$+2v\|e_x\|k\Lambda_{\max}\Delta\overline{\Theta}_{\max}\left\|\overline{\Phi}(\hat{x}, u_{bl})\right\|$$

4. Page 430: **Eq. 14.50**: Change to:

$$\dot{V}\left(e_x, \Delta\overline{\Theta}\right) \le -\left(2\eta + \lambda_{\min}(Q_0)\lambda_{\min}\left(\widetilde{P}_0\right)\right)\lambda_{\min}\left(\widetilde{P}_0\right)\|e_x\|^2$$
$$-\left(1+\frac{1}{v}\right)\lambda_{\min}\left(R_0^{-1}\right)\left\|e_y\right\|^2 - \left(1+\frac{1}{v}\right)w^2 + 2\Lambda_{\max}k_g w\|e_x\|$$
$$+2v\|e_x\|k\Lambda_{\max}\Delta\overline{\Theta}_{\max}\left\|\overline{\Phi}(\hat{x}, u_{bl})\right\|$$

5. Page 431: **Eq. 14.52**: Change to:

$$\dot{V}\left(e_x, \Delta\overline{\Theta}\right) \le -\left(1+\frac{1}{v}\right)\lambda_{\min}\left(R_0^{-1}\right)\left\|e_y\right\|^2$$
$$-\left[\left(2\eta + \lambda_{\min}(Q_0)\lambda_{\min}\left(\widetilde{P}_0\right)\right)\lambda_{\min}\left(\widetilde{P}_0\right) - 2vk\Lambda_{\max}\Delta\overline{\Theta}_{\max}b_2\right]\|e_x\|^2$$
$$-\left(1+\frac{1}{v}\right)w^2 + 2\Lambda_{\max}k_g w\|e_x\| + 2v\Lambda_{\max}k\Delta\overline{\Theta}_{\max}b_1\|e_x\|$$

6. Page 431: **Eq. 14.53**: Change to:

$$c_1 = \lambda_{\min}(Q_0)\,\lambda^2_{\min}\left(\widetilde{P}_0\right) - 2\,v\,k\,\Lambda_{\max}\,\Delta\Theta_{\max}\,b_2, \quad c_2 = \Lambda_{\max}\,k_g$$
$$c_3 = 1 + \frac{1}{v}, \quad c_4 = v\,\Lambda_{\max}\,k\,\Delta\Theta_{\max}\,b_1$$

7. Page 431: **Eq. 14.54**: Change to:

$$\begin{aligned}\dot{V}\left(e_x, \Delta\overline{\Theta}\right) &\le -c_3\,\lambda_{\min}\left(R_0^{-1}\right)\left\|e_y\right\|^2 - 2\eta\,\lambda^2_{\min}\left(\widetilde{P}_0\right)\|e_x\|^2 \\ &- \underbrace{\left[c_1\,\|e_x\|^2 - 2\,c_2\,\|e_x\|\,w + c_3\,w^2 - 2\,c_4\,\|e_x\|\right]}_{\varphi(\|e_x\|,w)} \\ &= -c_3\,\lambda_{\min}\left(R_0^{-1}\right)\left\|e_y\right\|^2 - 2\eta\,\lambda^2_{\min}\left(\widetilde{P}_0\right)\|e_x\|^2 - \varphi(\|e_x\|, w)\end{aligned}$$

8. Page 432: **Eq. 14.61**: Change to:

$$v_1 = \frac{\lambda_{\min}(Q_0)\,\lambda^2_{\min}\left(\widetilde{P}_0\right)}{2\,k\,\Lambda_{\max}\,\Delta\overline{\Theta}_{\max}\,b_2}$$

9. Page 432: **Eq. 14.62**: Change to:

$$\begin{aligned}&\det C = c_1\,c_3 - c_2^2 \\ &= \left(1 + \frac{1}{v}\right)\left[\lambda_{\min}(Q_0)\,\lambda^2_{\min}\left(\widetilde{P}_0\right) - 2\,v\,k\,\Lambda_{\max}\,\Delta\Theta_{\max}\,b_2\right] - \Lambda^2_{\max}\,k_g^2 = \mathrm{O}\left(\frac{1}{v}\right)\end{aligned}$$

10. Page 433: **Eq. 14.66**: Change to:

$$\dot{V}\left(e_x, \Delta\overline{\Theta}\right) \le -\left(1 + \frac{1}{v}\right)\lambda_{\min}\left(R_0^{-1}\right)\left\|e_y\right\|^2 - 2\eta\,\lambda^2_{\min}\left(\widetilde{P}_0\right)\|e_x\|^2 + |\varphi_{\min}(v)|$$

11. Page 433: **Eq. 14.67**: Change to:

$$\Omega_{e_x} = \left\{ e_x : \quad \|e_x\|^2 \le \frac{|\varphi_{\min}(v)|}{2\eta\lambda_{\min}^2\left(\widetilde{P}_0\right)} = r_{v,\eta}^2 = \mathrm{O}\left(\frac{v^2}{\eta}\right) \right\}$$

12. Page 435: **Table 14.1**: Change 11th and 12th equations in the table as shown below:

Output selection matrix for adaptive laws	$S = \begin{pmatrix} I_{m\times m} & 0_{m\times(p-m)} \end{pmatrix}$
Singular value decomposition	$\bar{B}^T C^T R_0^{-\frac{1}{2}} = U \Sigma V$

Back cover

Second paragraph: Change “The text is a **three**-part treatment” to “The text is a **two**-part treatment”."Errata List",6,1