2 The impact of informatics on the teaching of mathematics

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Abstract

Informatics and informatics based technologies directly impact on mathematics and its teaching. But there is also an indirect impact through developments in society which are supported by informatics. In the last thirty years developments in informatics and associated technologies have been many and fast; this has forced the understanding of their impact to change over time. For a more fundamental understanding of these impacts the role of informatics must be clear, both in real-life and in education. But also the role of mathematics needs clarification before the informatics impacts can be understood.

Keywords

Implications, information technology, modelling, visions.

INTRODUCTION

Technological developments in informatics based Information and Communication Technology (ICT) are so fast and diverse as to be bewildering. Too often in education these developments are either the main driving force behind what is happening or are ignored. Reflection is needed on the impact of informatics and related technologies on teaching and in the context of this paper specifically on mathematics teaching.

What is essential in the direct impact of informatics on mathematics and its teaching? Is it, as was stated by Sendov (1978) the concept of algorithm? Or is the impact of informatics on mathematics based on the fact that informatics is an applied logical meta-mathematics as put forward by Bauer (1980)? Or is it, as van Weert (1987) mentioned, the interaction of mathematicians with (virtual)

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mathematical environments allowing linguistic modelling in mathematics? Or is it the impact of informatics tools, especially computer algebra tools, which affect mathematics "in a very deep sense", because "it is different, a much greater emphasis being placed on numerical and algorithmic processes and on an experimental approach involving exploratory investigations" as stated by Hodgson (1995, p.32)?

Informatics not only directly impacts on mathematics, but also indirectly through its impact on society. This impact affects society in all its facets and is best illustrated by the new ways in which work is organised. Work is more and more done by teams of professionals which are partly self-managing and partly are managed through participatory management. Instead of by a hierarchical bureaucracy, the organisation is managed through horizontal team co-ordination. This horizontal co-ordination is supported by intelligent personal computers integrated in networks. To be able to function in this new type of organisation of work people need new competencies (van Weert, 1994). These are:

- abstraction;
- inductive thinking, that is: the ability to first recognise a powerful solution and then seek the problems it might solve;
- doing the work of experts enabled by the support of ICT;
- decision-making as part of the job;
- handling of dynamic situations.

These are complemented by competencies for working in professional teams and with the tools of ICT.

There is another indirect impact in that the economical and societal importance of informatics is reflected in the applications of mathematics as science which support informatics.

To find an answer to the question of what the essential elements are in the impact of informatics and related technologies, first the essence of informatics itself is defined; from that the essential elements of its impact are derived.

WHAT IS THE ESSENCE OF INFORMATICS?

Change: from number processor to interactive process

Since the first electronic computer was introduced for performing numerical calculations, a lot has changed. Turing and Church have given informatics a theoretical fundament based on the concept of algorithm. Hofstadter (1980) gives an excellent overview of the theoretical background of informatics). Informatics technology has developed from calculating device (number processor), via automating device (data processor), information system (information processor) to personal tool (interactive information processor) and intelligent agent (interacting knowledge based process).

At the basis for this development is the stored program computer (Von Neumann's brilliant idea) which can change its own program and therefore can adapt its behaviour. It can for example adapt itself to users through software embedded in microwave ovens or television sets. It can also learn to act as a personal intelligent agent in a communication network.

Application of informatics turns out to be possible and feasible in areas which are not immediately and directly associated with just data or information processing. We are moving from a world of information processing in which the concept of algorithm was central, to an even richer world of interaction in which the concept of interacting processes is central. The fundaments of informatics are changing in that: "Interaction is a more powerful paradigm than rule-based algorithms for computer problem solving, overturning the prevailing view that all computing is expressible as algorithms" (Wegner, 1997, p.80). From this viewpoint the world is a system of interacting processes waiting to be modelled in informatics processes and implemented in informatics technology.

INFORMATICS: BUILDING SYSTEMS OF INTERACTING PROCESSES

Informatics offers the concepts, the theory, the methods, the techniques and the tools to implement systems of real-world processes in computer systems. Figure 1 depicts the development cycle of computer based processes.

Informatics is an application oriented discipline

These computer system realisations are dynamic models of real-world systems of interacting processes and can be used as tools (implementing these processes) or for studying the real-world processes modelled (see the box in Figure 1). Such a process may be anything from a simple calculation or information query to a complicated process of mathematical problem solving.

This makes informatics an application oriented discipline. The informatical toolbox (see Figure 2) can be used for an abundance of applications.

IMPACT OF INFORMATICS ON MATHEMATICS

As application oriented discipline informatics also has an impact through its applications in mathematics. As explained before, change over time is an essential element in this impact. This change is from number processing, data processing, information processing, interactive information processing to knowledge based interacting.

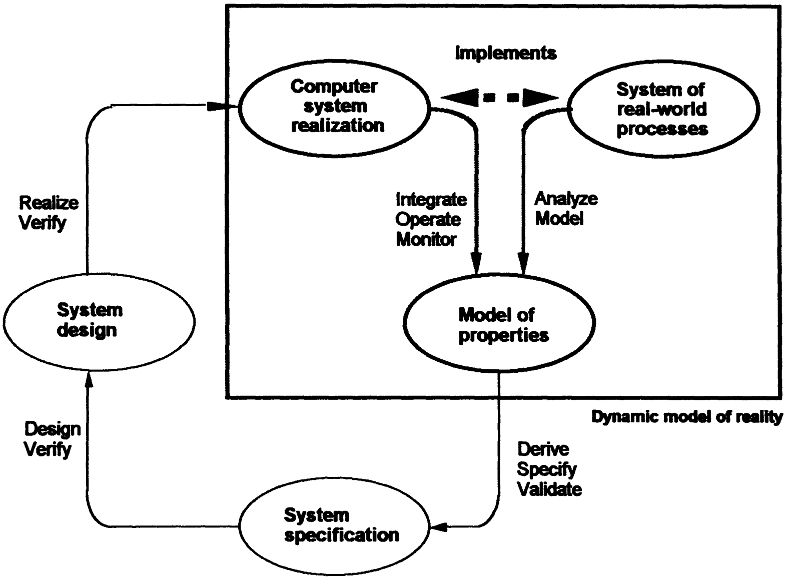

Figure 1 Development cycle within informatics.

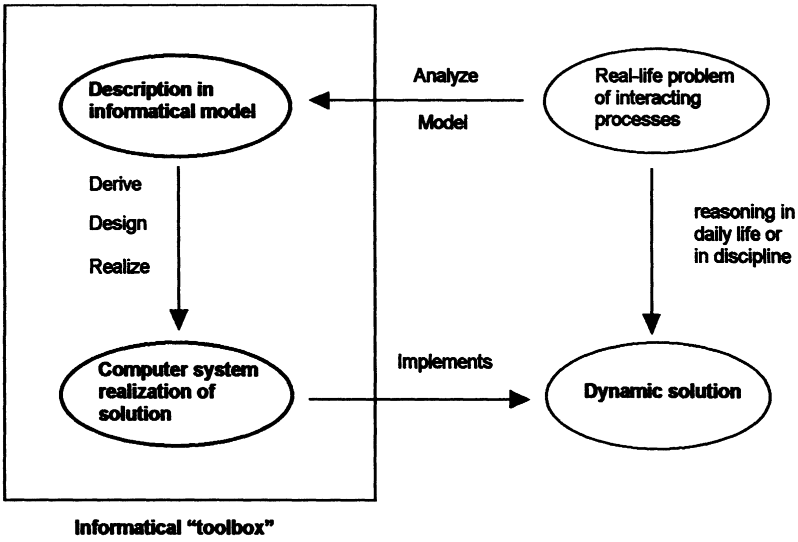

Figure 2 Dynamic modelling with an informatical toolbox.

Number and data processing

First the electronic computer supported numerical calculations, the numerical algorithm was in the centre of the attention. In the 1950's it was thought that just a few computers would suffice for the needed numerical computations. However, the miniature hand-held calculator made the slide-ruler obsolete and brought an overall change in the way in which numerical calculations were done.

Interactive information processing tool

Information processing tools may help in solving mathematical problems. Sometimes these tools are generic (e.g. spread sheets), sometimes application specific (e.g. computer algebra systems). Sometimes the technology is 'off the shelf', sometimes it is tailor-made.

Starting in the 1960's, software for symbolic computations was developed. Now powerful computer algebra tools like Mathematica, Maple and Reduce can perform both symbolic and numerical calculations: when it can be calculated, computer algebra tools can do it for you.

Knowledge based interaction

We are used to capture the world in static models built out of static (mathematical) symbols. Informatics allows us, however, to construct executable symbols which form dynamic models of reality; informatics allows us to build virtual realities of interacting processes. Modelling of reality then is 'programming' in conceptual (in this case mathematical) executable modelling languages to create 'micro-worlds' of mathematical reality. In other words: mathematical processes are analysed, developed as dynamic models and brought to life on a computer

From the end of the 1960's research has been done on the modelling of mathematical processes in formal, executable languages. Mathematical statements and concepts can be modelled in these languages, even concepts which cannot be calculated. These type of concepts can be dealt with through proofs. When there is sufficient level of detail a proof-object can be handled by a computer. The finding of a proof-object, however, is a difficult task which at the moment still has to be performed by humans (in interaction with computer tools).

Verification of a given proof is in principle quite simple: one follows the steps in the proof and verifies each step. This process can be automated as shown by de Bruijn (1994). He developed in the late 1960's a formal system, Automath, in which proofs can be formalised in such a way that these can efficiently be verified by a machine. As mentioned the formalisation can be quite difficult; therefore proof development systems support humans in this task.

Construction of a proof can be very difficult, even when we know that the proof exists. Turing has proven in 1936 that it is not possible to verify automatically that a given statement does, or does not follow, from a given set of axioms. This implies human involvement in proof construction. Proof systems are used to help a mathematician to construct the proof. A theorem for which a proof has to be found, is the 'goal' and the system asks the user along which lines the proof has to be developed (the proof tactics), generating new goals on the way. When the proof seems complete, it can be automatically verified by the system.

Computer mathematics or informatical mathematics

For the construction of computer algebra and proving systems, knowledge from both mathematics and informatics is necessary. A hybrid discipline develops, a so-called informatical discipline (van Weert, 1997), in this case informatical mathematics (see figure 3).

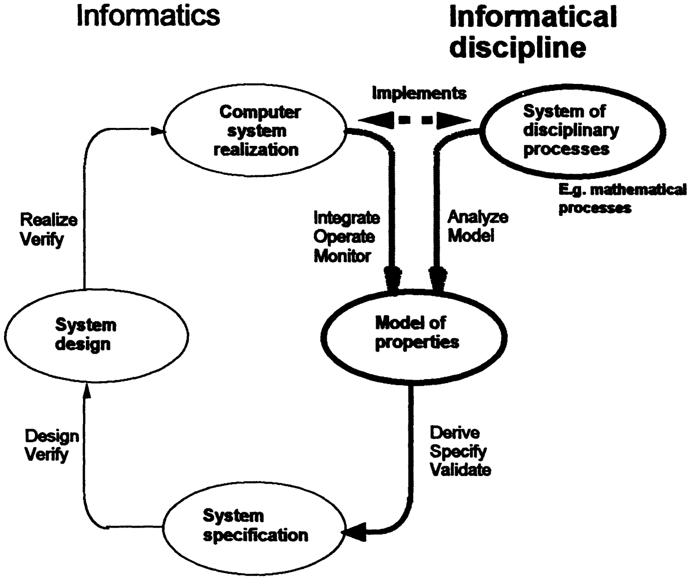

Figure 3 Relationship between informatics and informatical mathematics.

The activities, which are represented in bold in Figure 3, are the professional activities of a 'computer mathematician' or 'informatical mathematician'. The other activities are those of an application oriented informatician.

Summary: direct impact of informatics on mathematics

The direct impact of informatics on mathematics is felt in the following areas:

- numerical calculations and number crunching (calculation tools);
- computer algebra systems (mathematical information processing tools);
- dynamic mathematical models (interaction);
- modelling of mathematical processes in executable languages, e.g. proving systems (interaction).

There has been a resulting change in research topics, a move to more constructive and experimental mathematics and development of a hybrid discipline 'Informatical Mathematics'.

Most of these impacts can also be found in the list presented by Churchhouse (1992):

- new and revived areas of mathematical research;
- support for proof construction and performing of proving itself;
- experimentation in mathematics, exploratory data analysis;
- simulation, mathematical modelling;
- iterative methods (fractals);
- algorithms;
- symbolic mathematical systems;
- mathematical communication.

WHAT IS THE ESSENCE OF MATHEMATICS?

A structural description of mathematics will stress that mathematics is a system for abstraction and representation in symbols and formula through axioms, theorems and definitions, based on a system for reasoning. From this structural viewpoint one would observe that mathematics develops mathematical theories and methods.

However, from an operational viewpoint one will stress the fact that mathematics makes it possible to model the world in static symbols. Such an operational definition of mathematics will focus on the fact that mathematics reveals hidden patterns and helps to understand reality, that it offers powerful and multi-functional thinking methods:

- logical analysis through induction, deduction, discovery of patterns, i.e. structures of entities with boundary conditions;
- abstraction and modelling in symbols;
- manipulation of symbols within mathematical models;
- interpretation of results in these mathematical models.

Mathematics as application oriented discipline

Mathematics is an application oriented discipline. Applications of mathematics are found in all facets of society and scientific disciplines. On the one hand mathematics offers tools with which one can tackle application processes, on the other hand application processes can be modelled in static models and studied. The mathematical toolbox can be used in an abundance of applications. Its use can be pictured in the same way as that of the informatical toolbox (Figure 2), but in this case for static in stead of dynamic models.

Mathematics supporting informatics

As an application oriented discipline mathematics supports informatics:

- *Reasoning and logic:* The informaticians Gries and Schneider (1993) see logic as the glue that binds together methods of reasoning, in all domains. For example traditional proof methods (proof by assumption, contradiction, mutual implication and induction) all have their basis in formal logic.
- *Supporting mathematics, a mathematical 'language':* Mathematical induction, numbers, powers, logarithms, sums and products, integer functions and elementary number theory, permutations and factorials, binomial coefficients, harmonic numbers, Fibonacci numbers, generating functions, asymptotic representations.
- *Reasoning in a formal system:* Gries and Schneider (1993) emphasise syntactic manipulation of formulas as a powerful tool for discovering and certifying truths. For example in proposition and predicate logic, temporal logic and fuzzy logic.
- *Thinking in a mathematical model:* Problem solving and symbol manipulation inside such a formal model in order to find a mathematical result which helps in solving an informatics problem. Examples of such problems and associated mathematical models are crypto systems (group theory), statical analysis (typed lambda-calculus), congestion problems in networks (stochastic processes, Markov chains) and program correctness (Floyd-Hoare calculus).
- *Thinking in an informatics model:* Problem solving and symbol manipulation in a formal, i.e. mathematically expressed, informatics model in order to find a result with a direct meaning in informatics; examples of such formal systems are linear lists, trees, multi-linked structures.
- *Using a mathematical application model:* Using of mathematical results for problem solving in application areas outside informatics; examples: differential equations, Fourier transforms.

Summary of mathematics supporting informatics

Mathematics supports informatics with:

- basic reasoning and logic;
- mathematical 'languages';
- reasoning and thinking in a formal mathematical model;
- reasoning and thinking in a formal informatics model;
- mathematics for application areas.

THE IMPACT OF INFORMATICS ON THE TEACHING OF MATHEMATICS

The indirect impact

As indicated previously, indirect impacts result from developments in society and from the role of mathematics as supporting science.

As argued in van Weert (1994), developments in society will force education, including mathematics education, to change in its focus, in its organisation, in its use of ICT and its content. The focus will change from teaching to learning, its organisation will change from rigid class based learning to flexible team based learning, ICT will be integrated into the learning process and will support both this new organisation of learning and the learning tasks of the individual student.

As to the content mathematics education the following changes may be expected:

- The content will aim at the development of high-tech, academic abilities (a shift from the basic intellectual abilities of reading, writing and arithmetic to the higher order intellectual abilities of analysing, abstracting and modelling).
- Informatics will play an important role as discipline integrated informatics: the construction of executable symbols (programming in a generalised sense) reflecting complex, dynamic, conceptual models of reality. Students in mathematics will both build dynamic models of reality—a virtual reality, and traditional models in static symbols. Students will not learn to program in traditional programming languages, but instead will learn to program in conceptual subject-oriented mathematical languages which model micro-worlds of mathematical reality.
- The use of ICT in support of learning tasks of the individual student will affect the content in that real-life problems will be tackled in a constructive and experimental way.
- Discrete mathematics and logic, mathematics as ‘language’ and reasoning and thinking in formal mathematical models will take a more prominent place in the mathematics curriculum because of their supporting role.

The direct impact

The direct impact of informatics on mathematics teaching will be felt in the following areas:

- numerical calculations and number crunching will be done by calculation tools;
- computer algebra systems will be used as mathematical information processing tools;
- dynamic mathematical modelling (simulation) will develop, including use of proving systems for the modelling of mathematical processes;

- more constructive and experimental mathematics will develop aimed at solving real-life problems in other disciplines, implying a focus on mathematics as a 'language' and reasoning in and applying of formal mathematical models.

Mathematics should be an inherently social (and cognitive) activity in which construction (action) is central, and not knowledge transfer. Mathematics is a culture in which the world is seen through mathematical glasses. Students therefore need adequate competencies for problem analysis, modelling and meaningful interpretation. Students also need an adequate tool box of mathematical knowledge and skills which they can use with insight. Mathematics learning is: following professional models, finding solutions to realistic problems (enabling transfer), identifying patterns, formulating meaningful questions (Schoenfeld, 1992).

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