

# Erdős Magic

## Invited Talk

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The Probabilistic Method is a lasting legacy of the late Paul Erdős. We give two examples - both problems first formulated by Erdős in the 1960s with new results in the last few years and both with substantial open questions. Further in both examples we take a Computer Science vantagepoint, creating a probabilistic algorithm to create the object (coloring, packing respectively) and showing that with positive probability the created object has the desired properties.

- Given  $m$  sets each of size  $n$  (with an arbitrary intersection pattern) we want to color the underlying vertices Red and Blue so that no set is monochromatic. Erdős showed this may always be done if  $m < 2^{n-1}$ , we give a recent argument of Srinivasan and Radhukrishnan that extends this to  $m < c2^n \sqrt{n/\ln n}$ . One first colors randomly and then recolors the blemishes with a clever random sequential algorithm.

- In a universe of size  $N$  we have a family of sets, each of size  $k$ , such that each vertex is in  $D$  sets and any two vertices have only  $o(D)$  common sets. Asymptotics are for fixed  $k$  with  $N, D \rightarrow \infty$ . We want an asymptotic packing, a subfamily of  $\sim N/k$  disjoint sets. Erdős and Hanani conjectured such a packing exists (in an important special case of asymptotic designs) and this conjecture was shown by Rödl. We give a simple proof of the speaker that analyzes the random greedy algorithm.